NERS 590, Homework 3

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1 Problem 1

1.1 Problem 1a

Description of problem.[1] The rate of particles leak out of one side of a purely absorbing slab is investigated. The slab material is consisted of two isotopes. The density of isotope 1 (N_1) is $0.25\,b^{-1}cm^{-1}$. The total cross section of isotope 1 (σ_1) is $2.0\,b^{-1}$. The density of isotope 2 (N_2) is $0.75\,b^{-1}cm^{-1}$. The total cross section of isotope 2 (σ_2) is $0.5\,cm^{-1}$. The length of the slab is 5 cm. The particles start at (0,0,0) and move in $+x$ direction.

Analytical solution. Let T be the transmission probability and x be the length of the slab such that T can be calculated as follows.

$$T = e^{-(N_1\sigma_1+N_2\sigma_2)x} = e^{-(0.5/cm+0.375/cm)5cm} = 0.01259 \tag{1}$$

Monte-carlo solution. The code is attached. The simulation is run in 3D. It is written in c++ and compiled with gcc v4.2.1. Ten million histories are simulated. The transmission probability is found to be 0.0125833 and its relative uncertainty is found to be 0.00280126. The analytical solution is within the uncertainty of the simulation result.

1.2 Problem 1b

Description of problem.[1] The problem set up and tally are similar to problem 1a. However, the slab is divided into two regions. The first region ($0 \leq x < 1.0\,cm$) is filled with material having a density (N_1) of $1.0\,b^{-1}cm^{-1}$ and a total cross section (σ_1) of $2.0\,b$. The second region ($1.0 < x \leq 5.0\,cm$) is filled with material having a density (N_2) of $1.0\,b^{-1}cm^{-1}$ and a total cross section of (σ_2) of $0.5\,b$.

Analytical solution. Let T be the transmission probability and x_i be the 1-D length of the region filled with the i^{th} material such that T can be calculated as follows.

$$T = e^{-N_1\sigma_1x_1} \times e^{-N_2\sigma_2x_2} = e^{-2.0/cm\times1cm} \times e^{-0.5/cm\times4cm} = 0.01832 \tag{2}$$

Monte-carlo solution. Ten million histories are simulated. The transmission probability is found to be 0.0183025 and its relative uncertainty is found to be 0.00231597. The analytical solution is within the uncertainty of the simulation result.

2 Problem 2

2.1 Problem 2a

Description of problem.[1] The rate of particles leak out of one side of a slab is investigated. The length of the slab is 10 cm. The material has an atomic density of $1.0\,b^{-1}cm^{-1}$, a total cross section of $1.00\,b$, and a scattering cross section of $0.98\,b$. The source is modeled as a point source at $x = 0$ with cosine distribution of $f(\mu) = 2\mu$, $(0 < \mu < 1)$. The scattering is isotropic.

Monte-carlo solution. Ten million histories are simulated. The source direction ($\bar{\mu}$) is sampled as follows.

$$\begin{aligned}
&\int_0^{\hat{\mu}} f(\mu)\,\mathrm{d}\mu = \xi \\
&\int_0^{\hat{\mu}} 2\mu\,\mathrm{d}\mu = \xi \\
&\hat{\mu} = \sqrt{\xi}
\end{aligned} \tag{3}$$

where ξ is a random number $(0, 1)$. The scattering direction is sampled uniformly as follows.

$$\begin{aligned}
&\int_{-1}^{\hat{\mu}} \frac{1}{2}\,\mathrm{d}\mu = \xi \\
&\frac{1}{2}(\hat{\mu}+1) = \xi \\
&\hat{\mu} = 2\xi - 1
\end{aligned} \tag{4}$$

The transmission probability is found to be 0.0417199 and its relative uncertainty is found to be 0.00151557.

2.2 Problem 2b

Description of problem.[1] The problem set up and tally are similar to problem 2a. However, the cosine distribution for scattering events is sampled following Henyey-Greenstein distribution.

Monte-carlo solution. Ten million histories are simulated. The cosine distribution is sampled as follows

$$\begin{aligned}
&\int_{-1}^{\hat{\mu}} \frac{1-g^2}{2(1+g^2-2g\mu)^{3/2}e}\,\mathrm{d}\mu = \xi \\
&\hat{\mu} = \frac{1}{2g}(1+g^2+(\frac{1-g^2}{1+g(2\xi-1)})^2)
\end{aligned} \tag{5}$$

where g is a parameter describing the Henyey-Greenstein distribution. In this problem, $g = 0.25$. The transmission probability is found to be 0.0633666 and its relative uncertainty is found to be 0.00121578.

3 Problem 3

3.1 Problem 3a

Description of problem.[1] The number of neutrons that leak out of the system per neutron source is tallied. The system is a fuel with reflecting boundary condition at $x = 0$. The length of the fuel is 4 cm. The material has a capture cross section of 0.3 b, a fission cross section of 0.2 b, and a scattering cross section of 1.5 b. The atomic density of the slab is $0.1\,b^{-1}cm^{-1}$. The source is uniform and isotropic. The fission neutron multiplicity is sampled as follows

$$\nu = [\bar{\nu} + \xi]. \tag{6}$$

where $\bar{\nu}$ is the average neutron multiplicity ($\bar{\nu} = 2.8$) and $[]$. is a floor operator.

Monte-carlo solution. Ten million histories are simulated. Let N_i be the number of neutrons that leak out of the system for the i^{th} neutron source. The average number of neutrons that leak out per neutron source is calculated as follows.

$$< N >= \frac{\sum_{i=1}^{1\times10^7} N_i}{1\times10^7} \tag{7}$$

The variance is calculated as follows.

$$\sigma^2 = \frac{\sum_{i=1}^{1\times10^7}(N_i - < N >)^2}{1\times10^7} \tag{8}$$

The average number of neutrons that leak out of the system per neutron source is found to be 1.08787 and its variance is found to be 1.71149. The probability of mass function (PMF) is available in Fig. 1.

3.2 Problem 3b

Description of problem.[1] The problem set up and tally is similar to problem 3a. However, the neutron multiplicity is sampled following Terrell distribution.

Monte-carlo solution. Ten million histories are simulated. The fission neutron multiplicity is sampled using discrete CDF. The CDF for ν number of neutron is the following.

$$P_{(\nu)} = \frac{1}{2}(erf(\frac{(\nu - \bar{\nu} + 1/2 + b)/\gamma}{\sqrt{2}}) + 1) \tag{9}$$

where $\bar{\nu}$ is the average neutron multiplicity ($\bar{\nu} = 2.8$), b is a parameter ($b = 1.41 \times 10^{-3}$), and γ is likewise a parameter ($\gamma = 1.1$). The average number of neutrons that leak out of the system per source neutron is found to be 1.08724 and its variance is found to be 2.0941. The probability of mass function is available in Fig. 1.

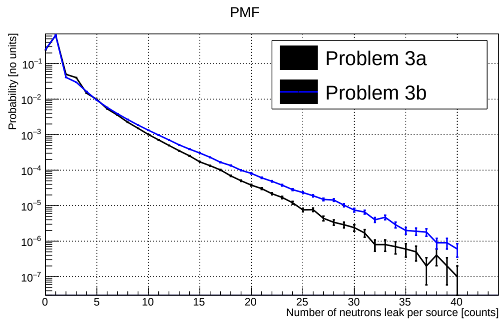

Figure 1: The probability mass function. On problem 3a, the fission neutron multiplicity model is simplified. On problem 3b, the fission neutron multiplicity is sampled following Terrell distribution.

Problem 3a and 3b yield similar mean number of neutrons that leak out of the system per neutron source. However, they yield different values of variance. Problem 3a uses a simplified method of sampling and therefore, simulation can be performed faster. However, it is necessary to use Terrell distribution if one is trying to tally any observables that are strongly affected by the fission neutron multiplicity (i.e., the PMF).

4 Problem 4

4.1 Problem 4a

Description of problem.[1] The simulation contains a sphere of radius 4 cm (centered at the origin) and a cylindrical detector having a radius of 0.5 cm and a length of 5 cm (parallel to x axis). The edge of the detector is 10 cm from the edge of the sphere. Atomic density of the sphere is 0.1 $b^1 cm^1$. The sphere contains two isotopes (first isotope: $\sigma_s = 19$ b, $\sigma_c = 1$ b, linearly anisotropic scattering $\mu_0 = 1/3$, fraction $= 2/3$, second isotope: $\sigma_s = 4$ b, $\sigma_c = 1$ b, fraction $= 1/3$, isotropic scattering). The detector contains one isotope (atomic density $= 0.005$ $b^1 cm^1$, $\sigma_s = 1$ b, $\sigma_c = 59$ b, isotropic scattering). Isotropic sources are placed at three points, (1,1,2), (-1,0,-3), and (0,-2,1), and emission probabilities are 1/6, 1/3, and 1/2 respectively.

Monte-carlo solution. One hundred million histories are simulated. For the linearly anisotropic scattering, the μ is sampled as follows

$$\mu = \frac{\sqrt{b^2 + 4b\xi - 2b + 1} - 1}{b} \tag{10}$$

where ξ is a uniform random number (0.1) and b is $(3 \times \bar{\mu})$. The absorption rate is found to be 7.37757e-05. The relative uncertainty is found to be 0.0100999.

4.2 Problem 4b

Description of problem.[1] The problem is similar to problem 4a. However, the source is distributed evenly throughout the volume of a spherical shell with inner radius of 1 cm and outer radius of 2 cm (centered at the origin). The source is emitted isotropically.

Monte-carlo solution. One hundred million histories are simulated. The absorption rate is found to be 9.91967e-05. The relative uncertainty is found to be 0.00894413.

4.3 Problem 4c

Description of problem.[1] The problem is similar to problem 4a. However, the source position is sampled randomly from a disk centered at (1,0,5) with radius of 2 cm. The direction is pointed in the positive z direction.

Monte-carlo solution. One hundred million histories are simulated. The absorption rate is found to be 4.1102e-05. The relative uncertainty is found to be 0.0132568.

5 Problem 5

Description of problem.[1] In Figure 2, material A (lighter gray) has properties as follows: atomic density = 0.1 $b^1cm^1, \sigma_c = 10b, \sigma_s = 10b$, isotropic scattering. Material B (darker gray) has properties as follows: atomic density = 0.1 $b^1cm^1, \sigma_c = 100b$. The system is infinite in the z direction. An isotropic point source is located at (1.5,1.5,0). An infinite (in z direction) cylindrical detector (white in color) is centered at (6.5, 1.5) with radius of 0.5 cm. The detector material has properties as follows: atomic density = $0.005b^1cm^1, \sigma_s = 1b, \sigma_c = 59b$, isotropic scattering.

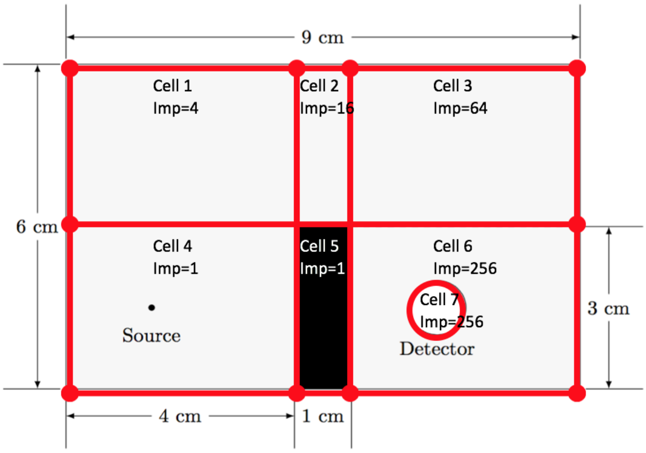

Figure 2: The simulation scheme and the assigned importances

Monte-carlo solution. One hundred million histories are simulated. Figure 2 shows the subdivisions and the assigned importances. Cell 7 is the detector cell. The absorption rate is found to be 1.00587e-08 . The relative uncertainty is found to be 0.0433733. The FOM is found to be 2.54867e-06.

6 Problem 6

Description of problem 6a. [1] Figure. 3 shows the geometry of the problem. The monoenergetic source (1.0 MeV) is uniformly distributed in the inner surface of the truncated cone. Its angular distribution follows a probability density function as follows.

$$P(\mu) = \frac{1}{6}(1 + (\mu + 1)^3) \tag{11}$$

All neutron sources start at time t = 0. The cross section σ depends on energy E following Eq. 12.

$$\sigma = xsa + \frac{xsb}{\sqrt{E}} \tag{12}$$

All neutrons start at time t = 0 with an energy of 1.0 MeV. The sphere has an atomic density of $0.1b^{-1}cm^{-1}$, A = 2.0, capture's xsa = 0.0, capture's xsb = 0.1, scattering's xsa = 2.4, scattering's

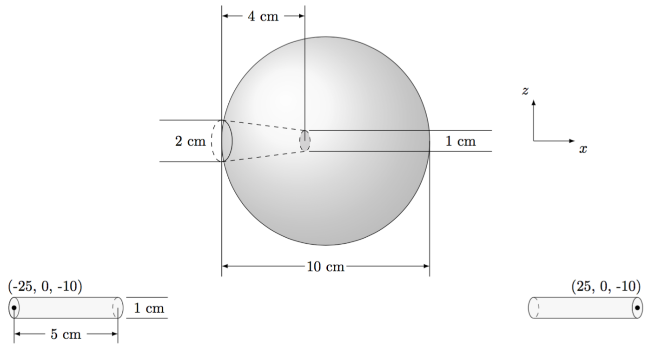

Figure 3: The geometry of problem 6 [1]

xsb = 0.0, linearly anisotropic scattering distribution in the center of mass frame ($\mu = 0.2$) . The cylinders (the detectors) has an atomic density of $0.1b^{-1}cm^{-1}$, scattering's xsa = 1.0, scattering's xsb = 0.5, isotropic scattering distribution in center of mass frame, atomic mass of 1. We tally the number of scattering with time binning (from t = 0 ns to t = 100 ns). The bin width is 5 ns.

Description of problem 6b. [1] The problem is similar to problem 6a, except that the sphere has an atomic mass of 6 (Lithium).

Description of problem 6c. [1] The problem is similar to problem 6a, except that the sphere has an atomic mass of 12 (Carbon).

Monte-carlo solution. The number of history used is 1e8. The results show the expected trend. As the atomic number (A)of the sphere increases, the number of particles that arrive at the detector at later time decreases. The neutron loses less energy when it collides with heavier nuclei and therefore, the neutron should stay fast longer. Since the total cross section is inversely proportional to the energy of the neutron, the neutron is less likely to interact with the nuclei and should arrive at the detectors earlier. The left detector shows higher results in comparison with the right detector. This is not quite expected since the source is forward peaked. However, the fact that the source is closer to the left detector may explain such trend.

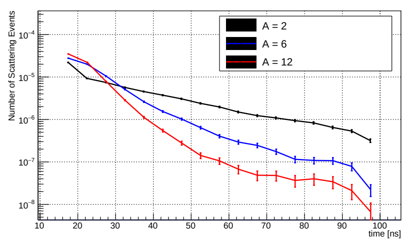

Figure 4: The time histogram of the detector response (left detector)

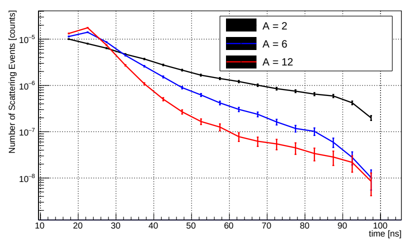

Figure 5: The time histogram of the detector response (right detector).

7 The code

The code is attached. To run the code, please use the following command lines:
make
./a.out

References

[1] Kiedrowski, B. *NERS 590, Homework 3 problem set.* University of Michigan, Nuclear Engineering and Radiological Science, Winter 2017.