CENTRALESUPÉLEC AND BNP PARIBAS

Portfolio Construction and Risk Minimization

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TODAY'S HIGHLIGHTS

1. Brief Introduction
2. Methodology
3. Returns and Variance Forecast
4. Portfolio Construction
5. Portfolio Comparison
6. Conclusion

Discussion Outline

What is Modern Portfolio Theory?

HARRY MARKOWITZ - 1952

Theory of how **risk averse** investors can construct portfolios that **maximize returns** under a given **risk exposure**

Key Concepts of Modern Portfolio Theory

Risk Aversion Utility

Rational Investors seek to minimize risk

Efficient Frontier

Certain combinations of assets in a portfolio have higher returns for the same level of risk

Risk Exposure

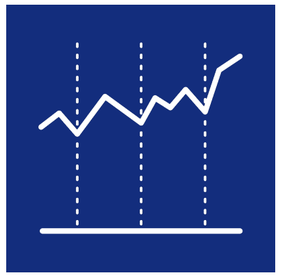
How much can I afford to lose in my portfolio

Purpose of this project

IMPLEMENTING MODERN PORTFOLIO THEORY

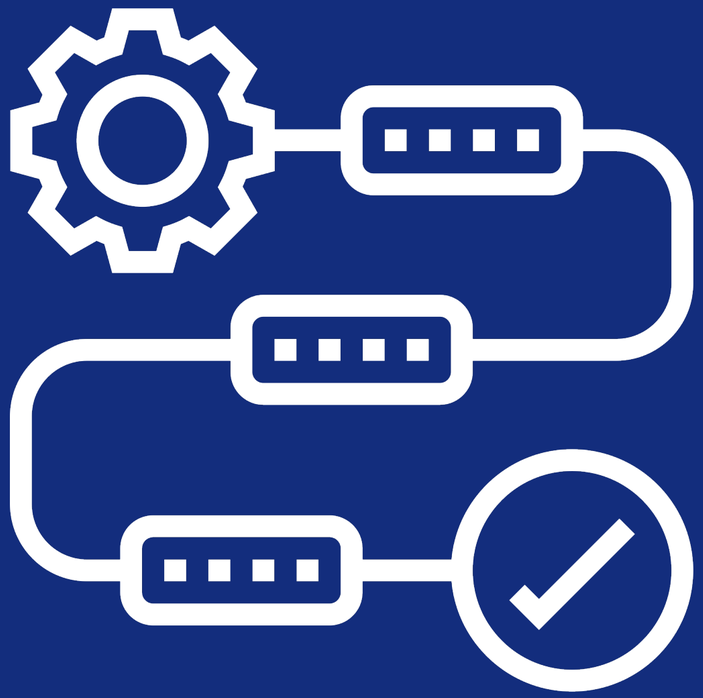
Build several **US equity** market portfolios under multiple constraints

Compare different portfolio models in their ability to allocate capital



Evaluate their **Risk Exposures and Returns** for different horizons

Our Methodology



- Forecasting Expected Returns and Variance
- Creating and Training Portfolio Models
- Back testing and Model Comparison

RETURNS AND VARIANCE FORECAST

Price Time Series

- Prices are not a stationary series
- High Noise/Signal Ratio
- Difficulty to predict

ARIMA

- Statistical Model for Time Series
- Integration parameter
- Prediction quickly turns constant

Recommendations

- Scoring system according to banks' recommendations
- Combined with Expected returns to generate stronger signal

GARCH

- Estimate future variance of assets
- Use Correlation to determine Covariance matrix

Returns - Variance

Minimum Variance
Maximum Returns

Risk Based

Equal Weights
Inverse Variance
Minimum Variance

Uncertain Returns

Robust Optimisation
Black-Litterman Model

Portfolio Construction

MULTIPLE OPTIMISATION STRATEGIES

MINIMUM VARIANCE

Minimize Variance of the Portfolio over constraint on Minimum Returns

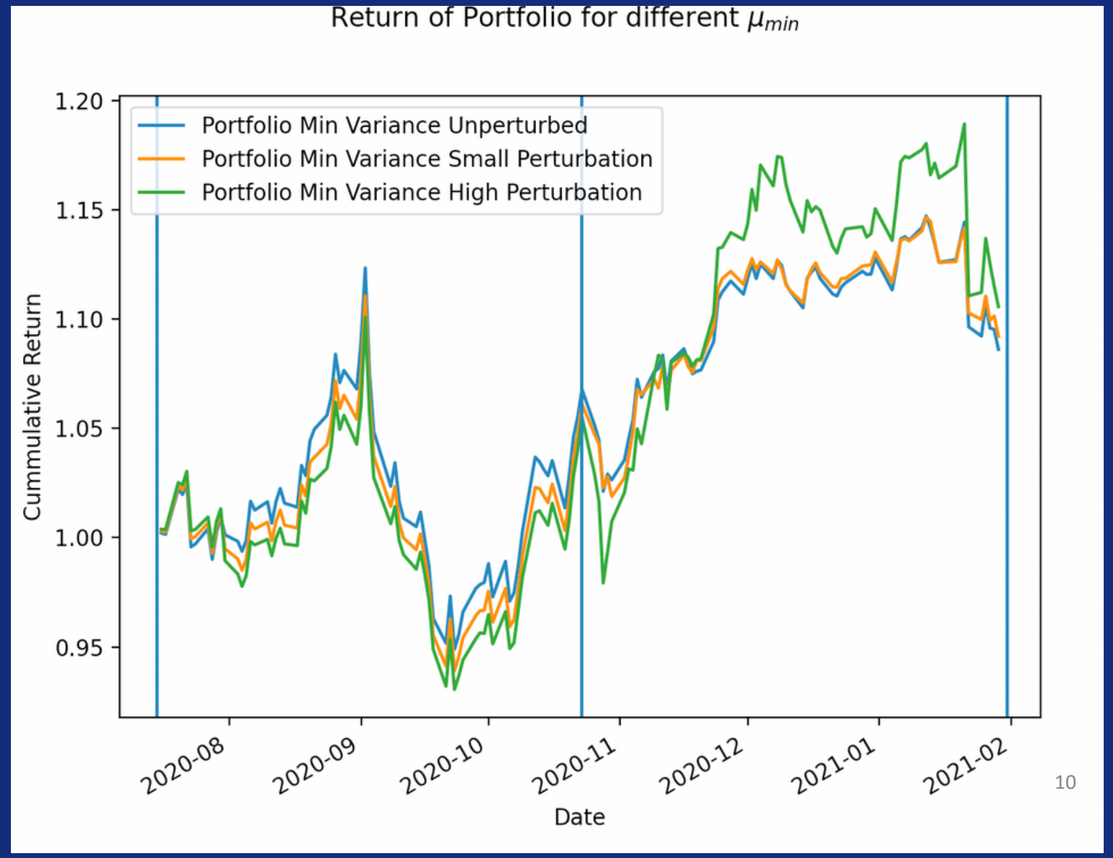
$$\text{find} \quad \omega \in argmin\, \omega^T \Sigma \omega \quad \text{such that } \mu^T \omega \geq \mu^*$$

High Sensitivity on parameter μ

Trajectories are completely different uppon change

Elevate Sensitivity to Noise

Uncertainty in the returns widely propagates in the performance



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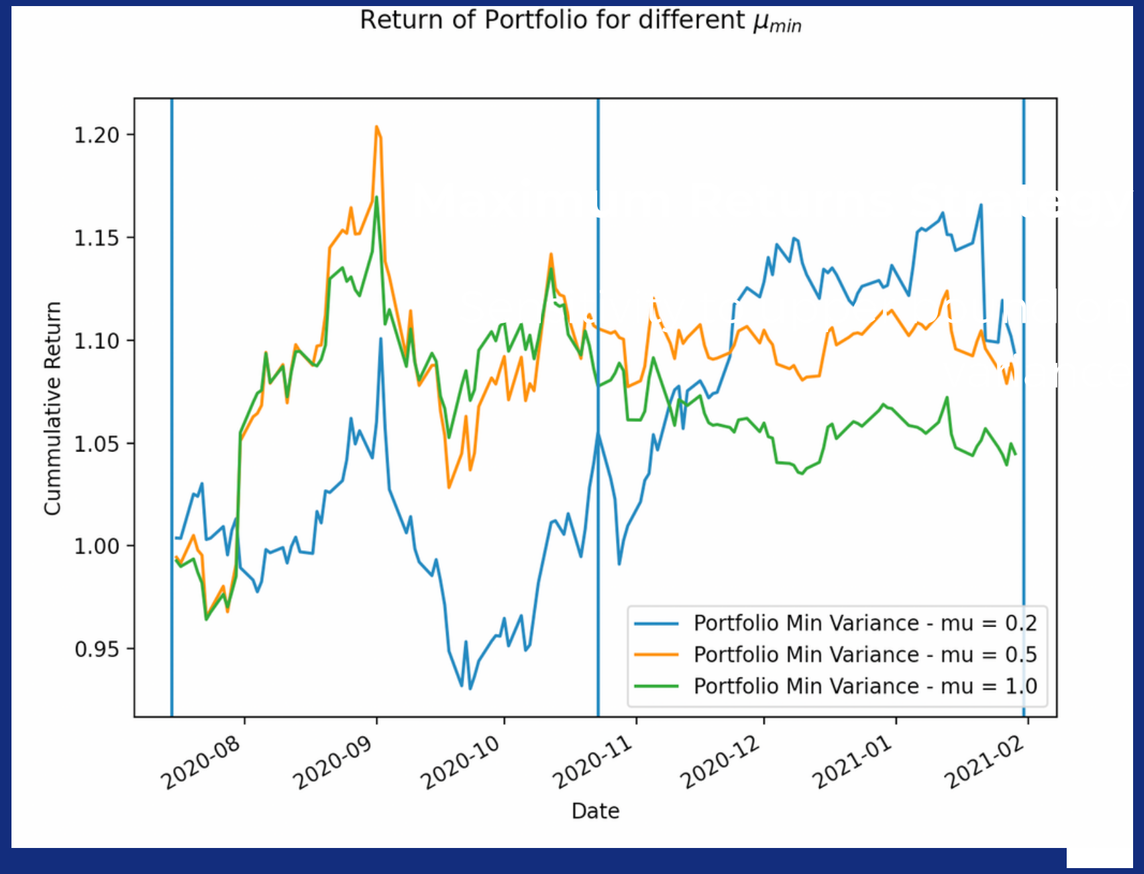
Minimum Variance Strategy

Sensitivity to lower noise

Minimum Variance Strategy

Sensitivity to lower bound on returns



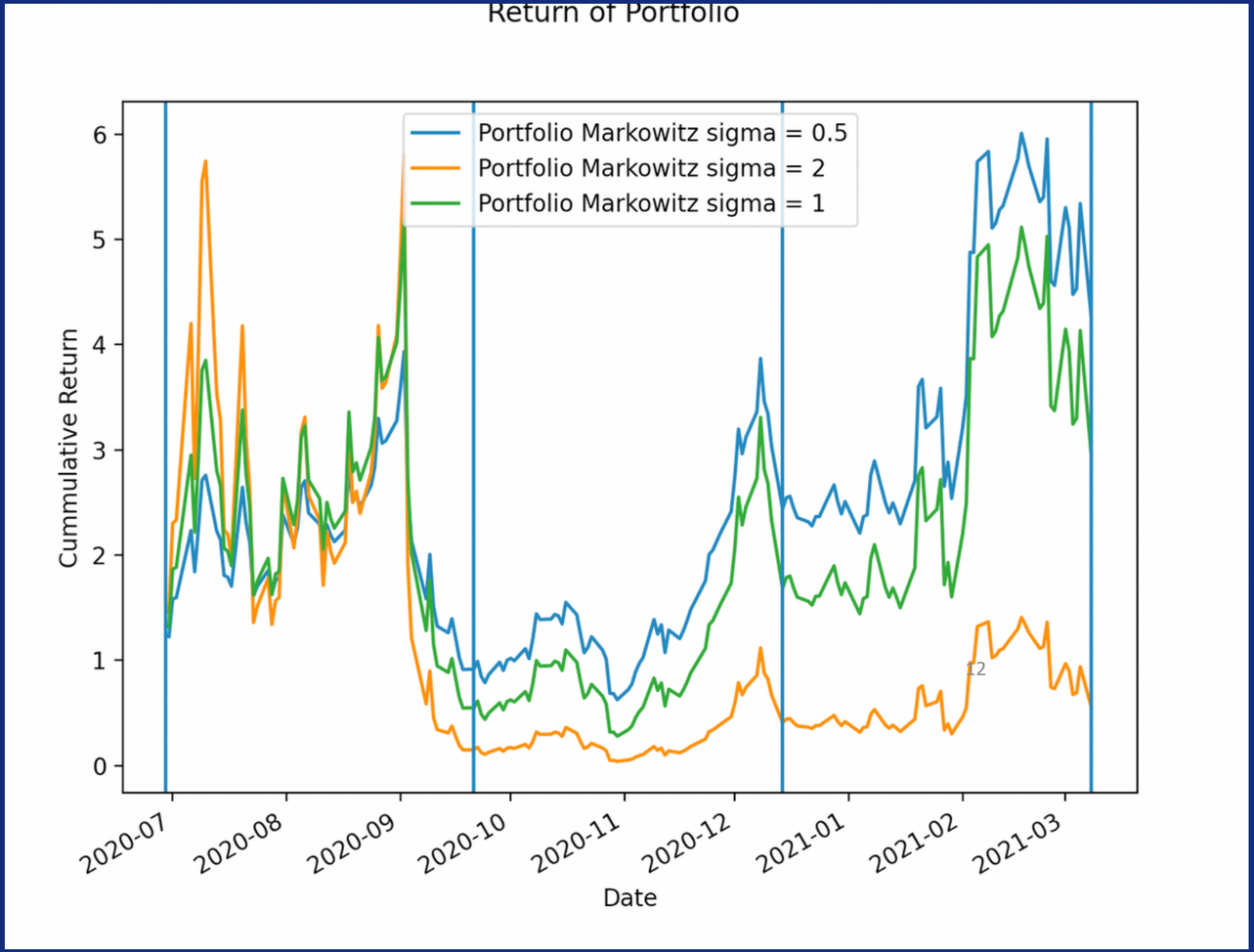
MAXIMUM RETURNS

Maximize Returns of the Portfolio over constraint on Maximum Variance

$$\text{find} \quad \omega \in argmax\, \mu^T \omega \quad \text{such that} \quad \omega^T \Sigma \omega \leq \sigma^*$$

Not Monotoneosly improved when increasing sigma

Increasing upper bound allows for a higher overall return but also imply higher risk



Maximum Returns Strategy

Sensitivity to upper bound on variance

Risk Based Portfolios

Equally Weighted

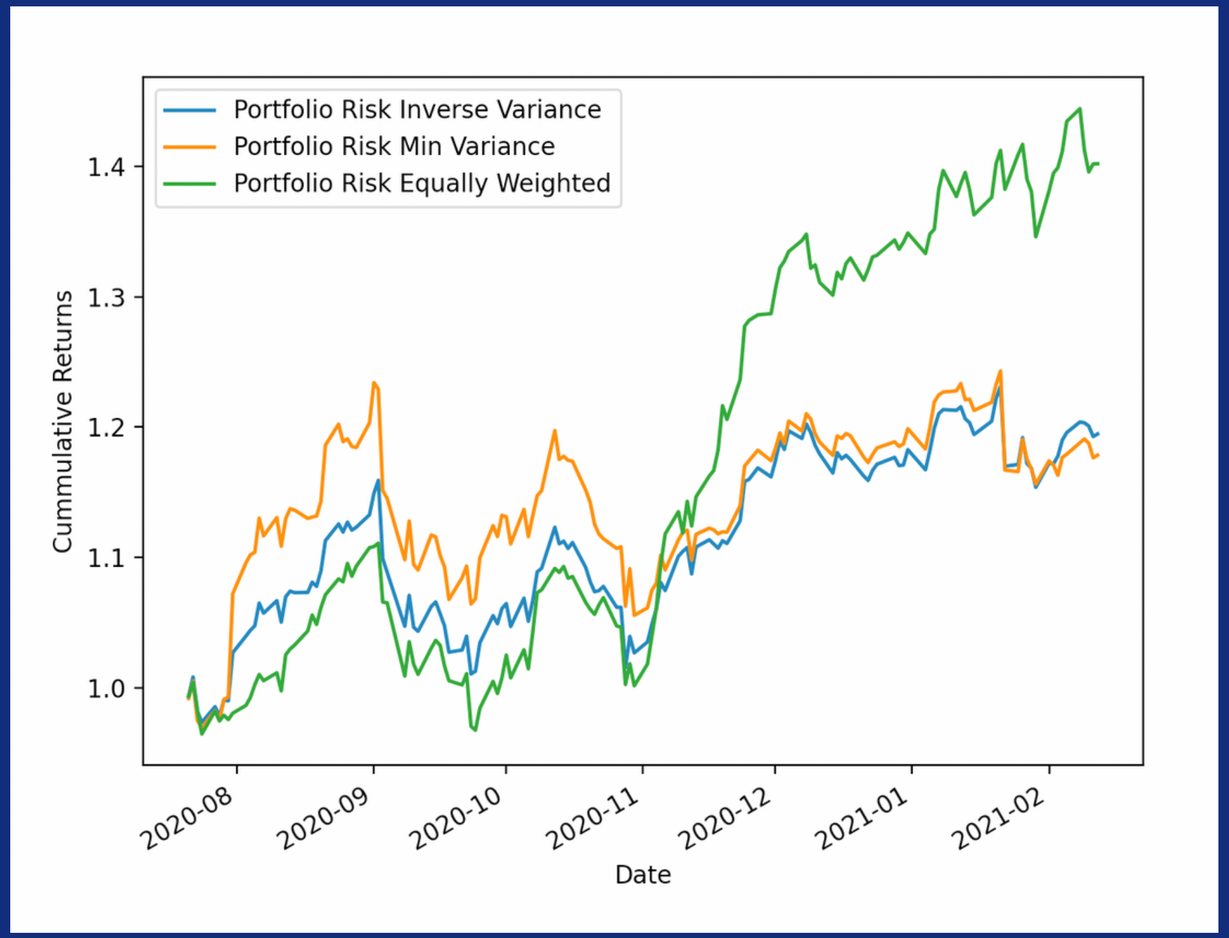
All Stocks are allocated the same amount of capital

Inverse Variance

Stocks are allocated capital proportionally to the inverse of their variance

Minimal Variance

Stocks are allocated capital in the same proportion as their variance

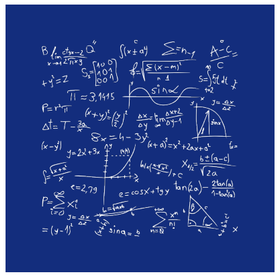


Path of different risk-based portfolios

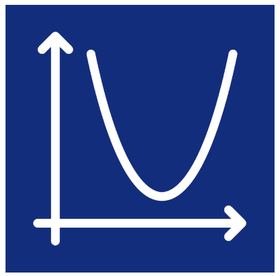
- Equally Weighted Outperforms – More exposure to Risks
- Similar Behavior for Min Variance and Inverse Variance - Assets not correlated

Robust Optimisation

Introduces an **Uncertainty** on the Returns Parameter

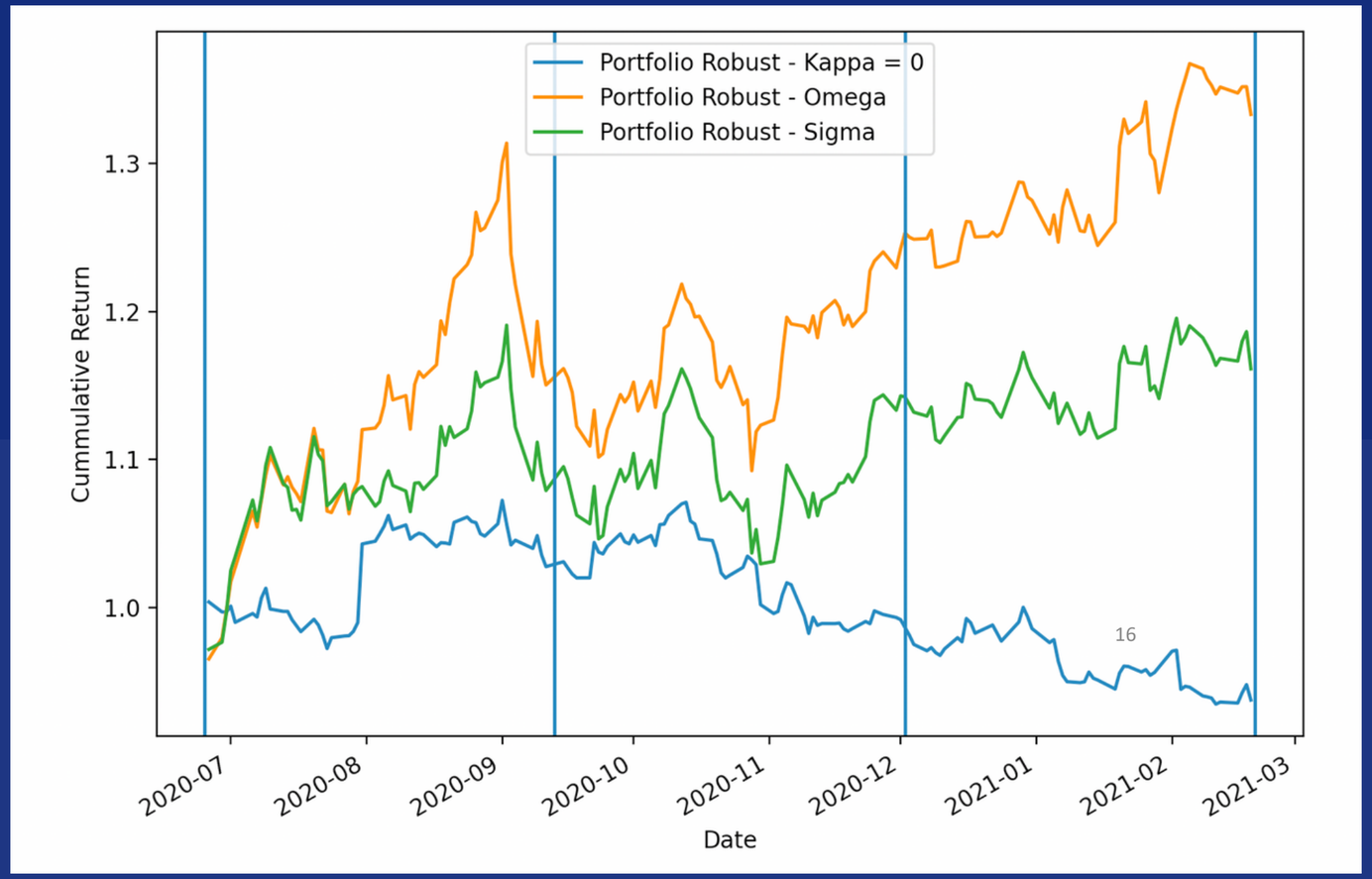


Parametric description: **Worst-case** scenario optimisation



$$\max \mu^T \omega \; - \kappa \sqrt{\omega^T \, \Omega \, \omega} \; - \lambda \, \omega^T \, \Sigma \, \omega$$

Ω: Symetric Posite Semi $-$ definite matrix

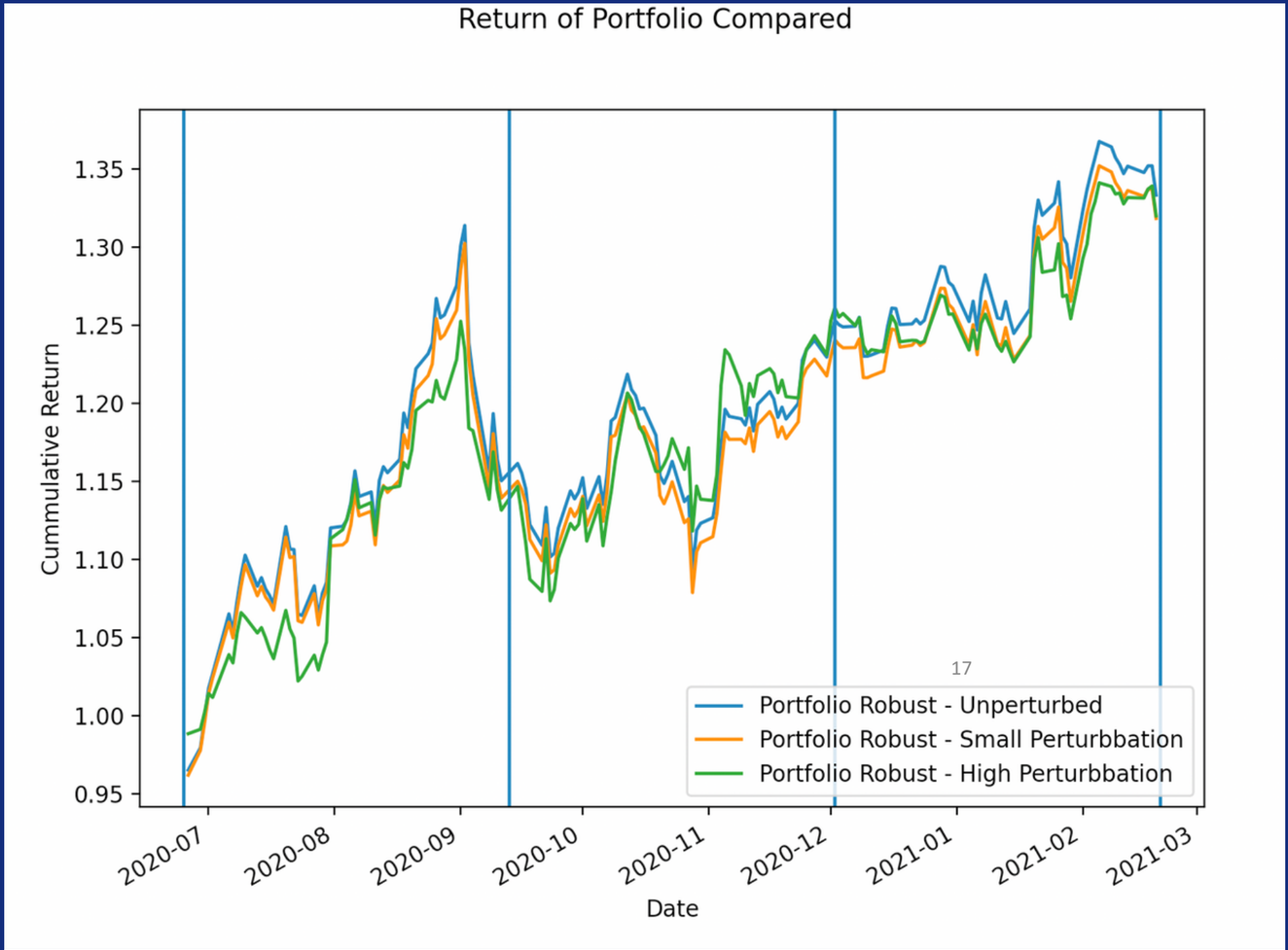


Better Overall Performance

Higher cumulative returns for both cases where error term was introduced

Uncertainty Region

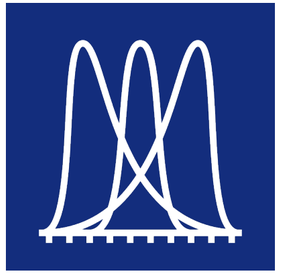
Results depend highly on parametrization: trade-off in utility function



BETTER NOISE SENSITIVITY

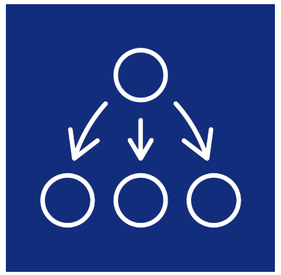
Noise in the input is much less propagated in returns

Black-Litterman Model

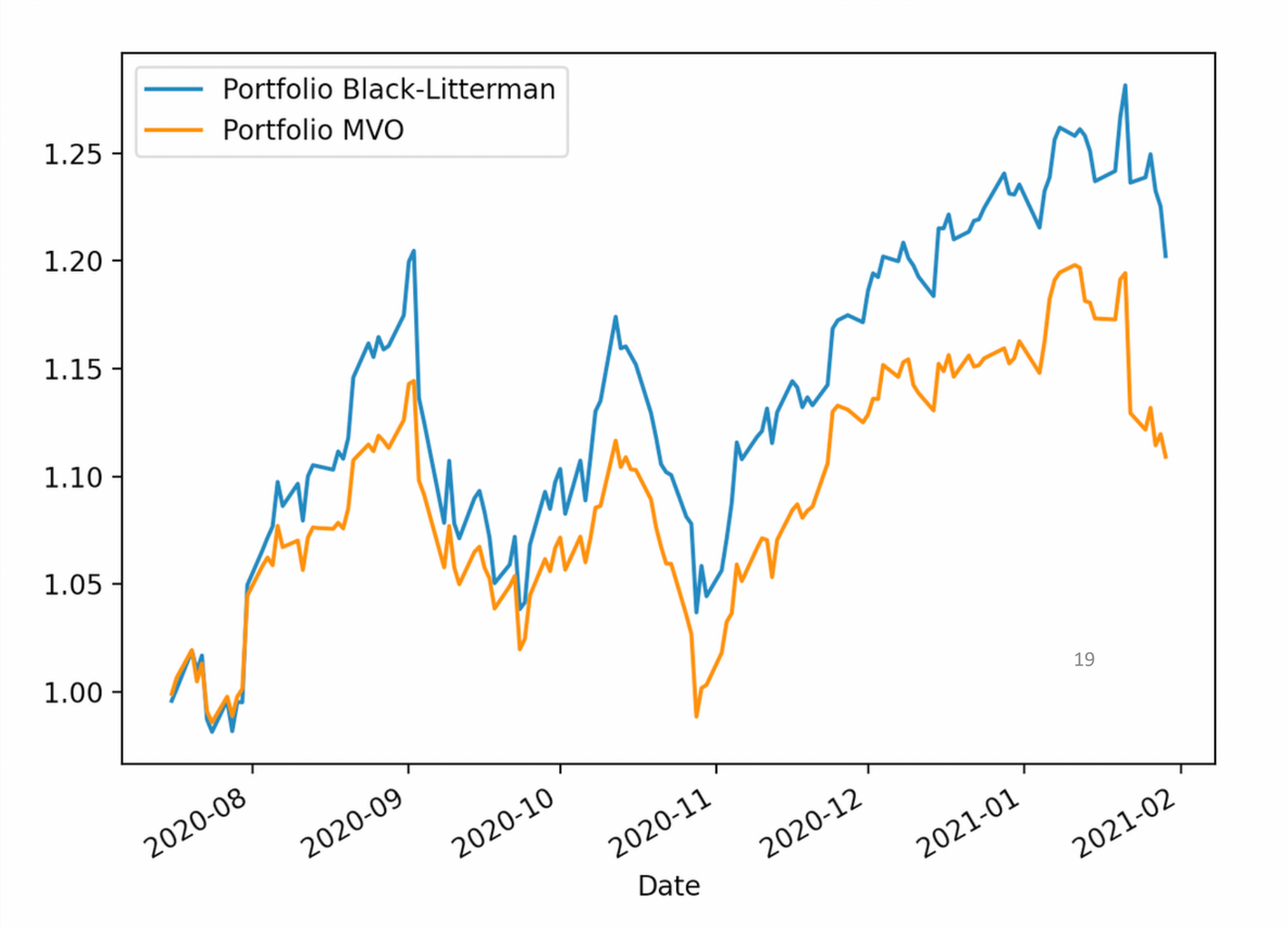


Based in **Bayesian** Statistics - improve the **estimative of returns**

Uses **Market Capitalisations** to **reverse optimise** and define a prior distribution for returns



Can use any **other base strategy** to allocate capital

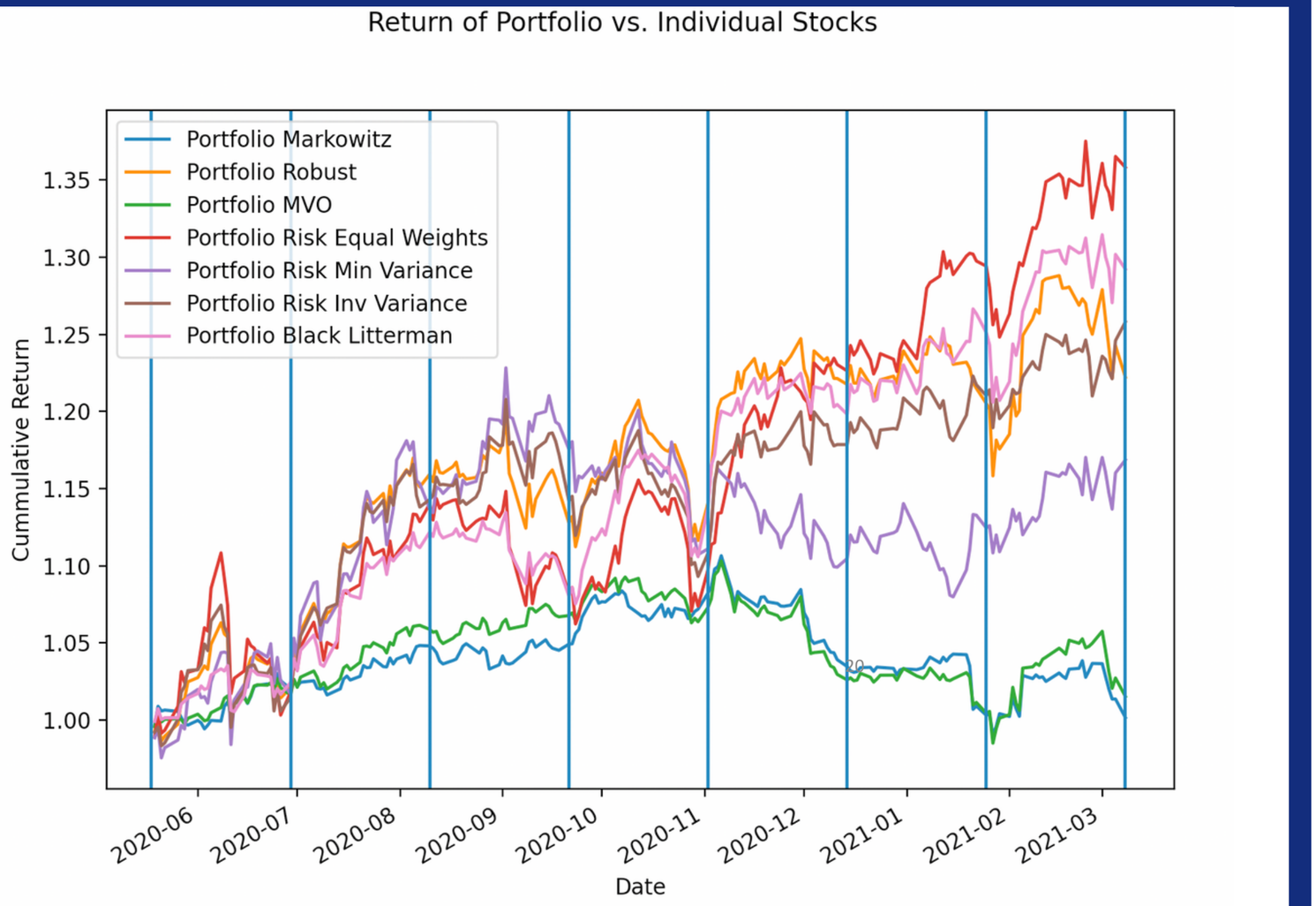


Same Base Estimator

Minimum Variance Strategy for both portfolios

Increased Performance

With the Black-Litterman Model, we have significantly higher returns



BACKTEST AND COMPARISON

Testing all models in for a universe composed of 25 different stocks, randomly chosen

Base Line: S&P 500 index had a 28% returns in 1 year whereas Dow Jones had a 47.5% increase.

Highest Returns

Minimal Variance Allocation

Highest Sharpe

Black-Litterman Model with 0.138

Lower VaR 95%

Maximum Returns with -4.1%

Lowest Returns

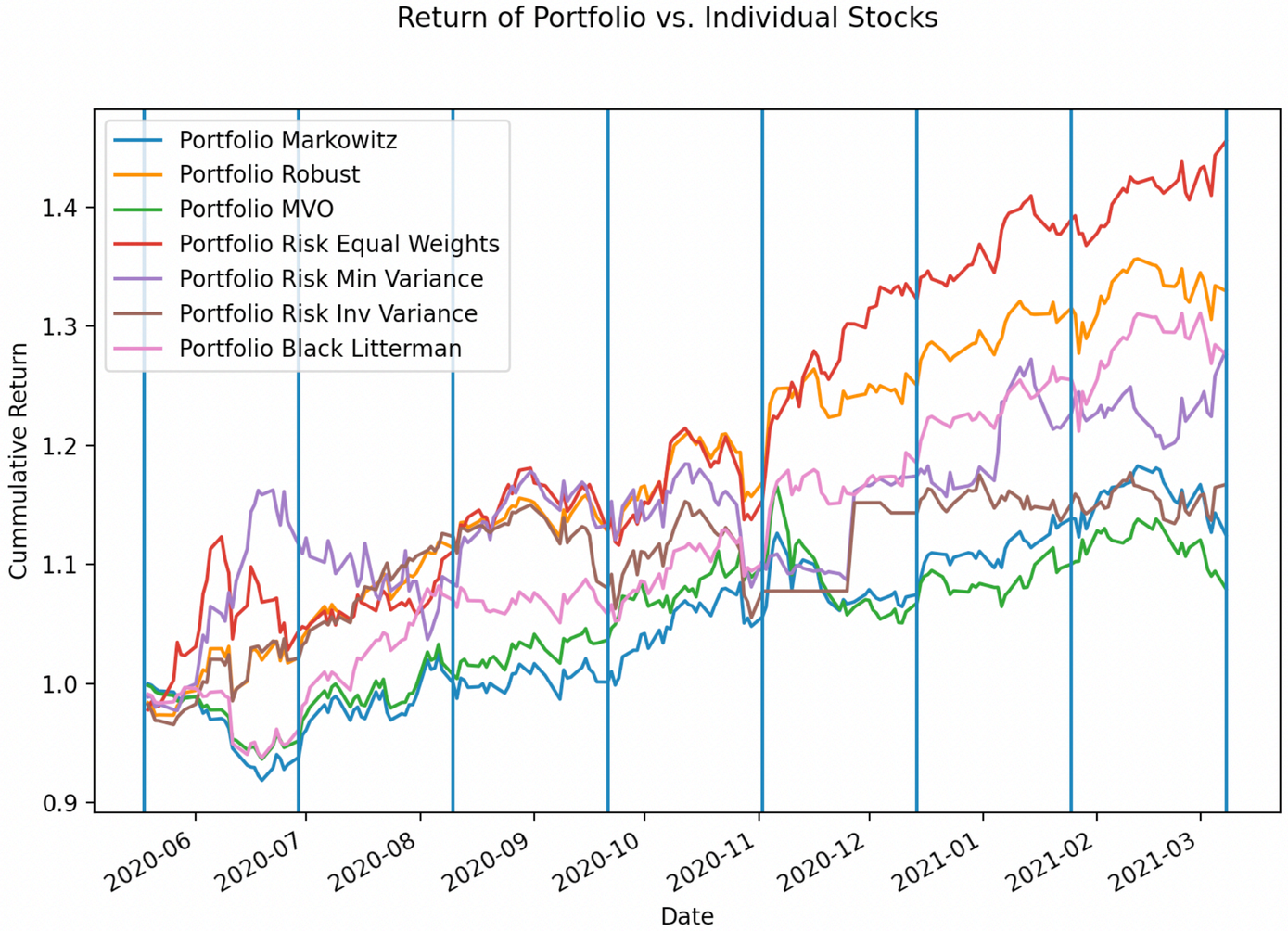
Maximum Returns Allocation

Smallest Sharpe

Maximum Returns with 0.004

Higher VaR 95%

Minimum Variance at -9.5%



BACKTEST AND COMPARISON

Further constraint in rebalancing: new weights should be close to previous ones

Base Line: S&P 500 index had a 28% return in 1 year whereas Dow Jones had a 47.5% increase.

Highest Returns

Equal Weights Allocation

Highest Sharpe

Equal Weights Allocation with 0.17

Lower VaR 95%

Minimum Variance Allocation with -7%

Lowest Returns

Minimum Variance Allocation

Smallest Sharpe

Maximum Returns with 0.065

Higher VaR 95%

Minimum Variance at -7.9%

WHERE SHOULD YOUR MONEY GO ?

- NO SINGLE UNIQUE BEST STRATEGY

  No model is better in every criteria

- INTRODUCTION OF RISK-FREE ASSET

  Possibility to invest with little to no risk

- CONSUMMER PORTFOLIO

  Reduction of risk and returns

ROOM FOR IMPROVEMENT

Returns Prediction

Our results shows us that strategies with less confidence in our signal performs better

Trasaction Costs

Introduce costs upon buying assets, proportional to their liquidity

Consider Discounted Prices

Money has a value in time, so we need to consider it to evaluate investments

Index Sampling

It is not feasible to invest in all stocks , models should be able to chose the best

ANY QUESTIONS ?