Lista 1 - Introdução ao Caos

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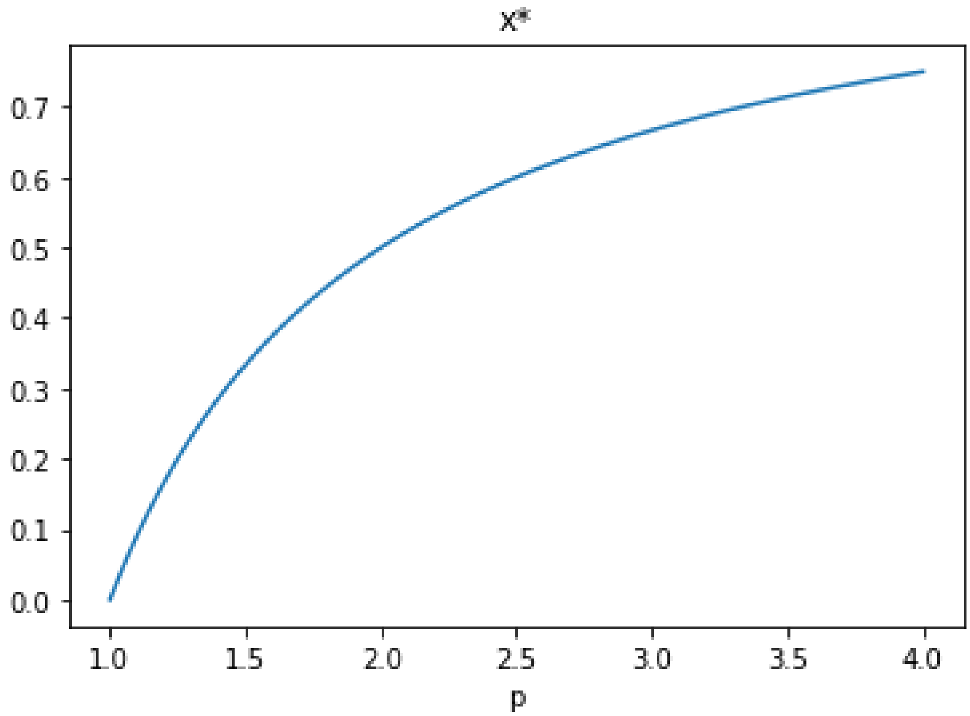
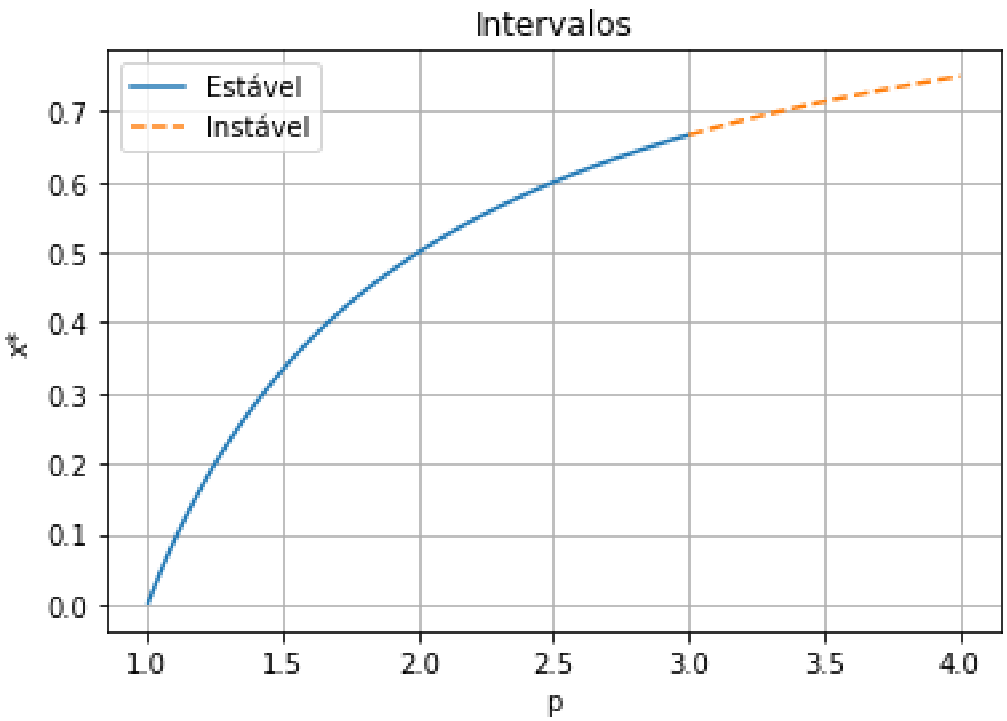
[1]:
```
from matplotlib.lines import Line2D
import matplotlib.pyplot as plt
import numpy as np

import warnings
warnings.filterwarnings("ignore")
```

1 Atividade 63

[2]:
```
p = np.linspace(1, 4, num=1000)
fx = (p-1)/p

plt.plot(p,fx)
plt.title('x*')
plt.xlabel('p')
plt.show()
```



[3]:
```
df_dx_x = p-2*p*fx

# 1) mask points where |df/dx|_x* < 1
fx_unstable = np.ma.masked_where(np.abs(df_dx_x) < 1, fx)

# 2) mask points where |df/dx|_x* >= 1
fx_stable = np.ma.masked_where(np.abs(df_dx_x) >= 1, fx)

plt.plot(p, fx_stable, '-', label='Estável')
plt.plot(p, fx_unstable, '--', label='Instável')
plt.legend()
plt.title('Intervalos')
plt.xlabel('p')
plt.ylabel('x*')
plt.grid()
plt.show()
```



2 Atividade 64

$$
\begin{aligned}
f^2(x) &= p\left(px - px^2\right) - p\left(px - px^2\right)^2 && (1)\\
&= x(1-x)\left(p^3x^2 - p^3x + p^2\right) && (2)
\end{aligned}
$$

no ponto fixo

$$x^* = x^*(1-x^*)\left(p^3x^{*2} - p^3x^* + p^2\right)$$

cujas raízes são

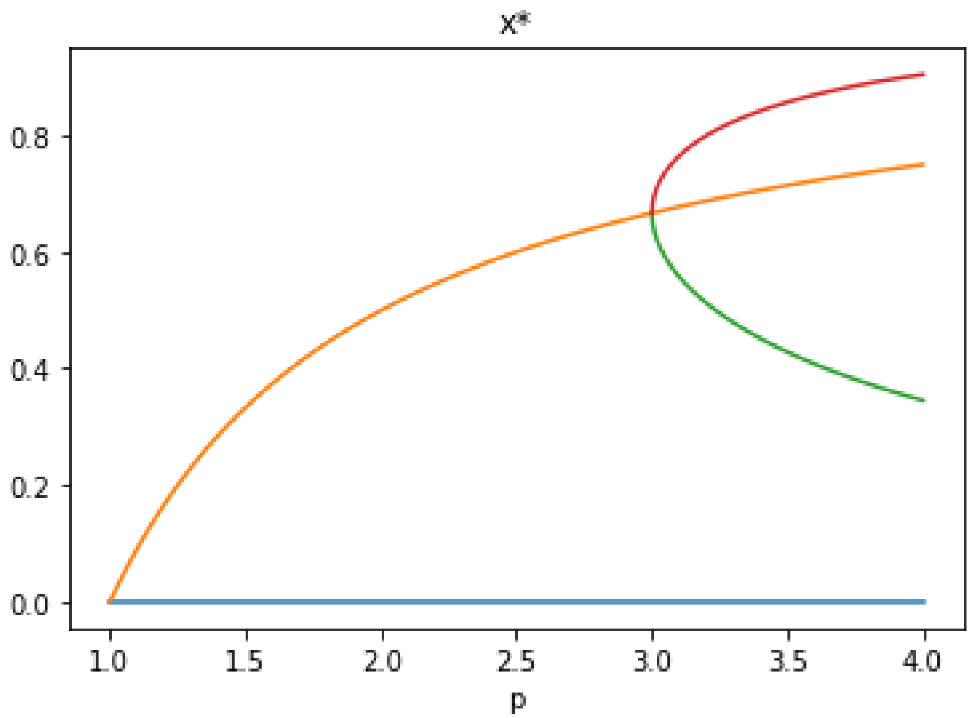
0, $\frac{p-1}{p}$, $\frac{-\sqrt{p^2-2p-3}+p+1}{2p}$ e $\frac{\sqrt{p^2-2p-3}+p+1}{2p}$

para a derivada

$$
\begin{aligned}
f^2(x) &= x(1-x)\left(p^3x^2 - p^3x + p^2\right) && (3)\\
\implies \frac{d}{dx}\left(f^2(x)\right) &= -p^2(2x-1)\left(2p(x-1)x+1\right) && (4)
\end{aligned}
$$

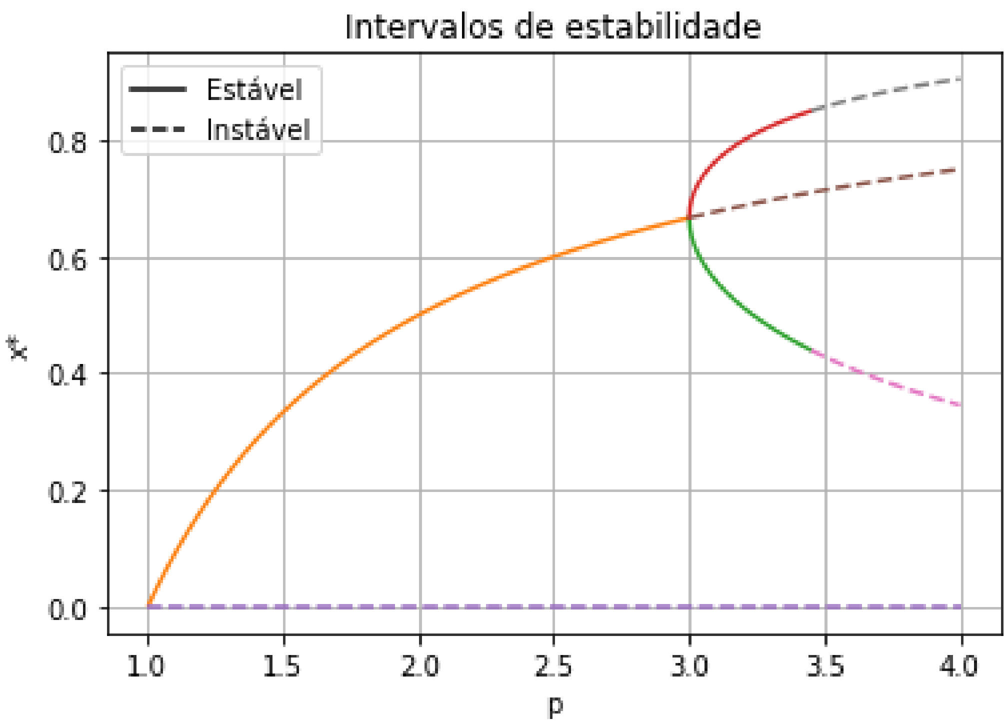
3 Atividade 65

[4]:
```
root_delta = np.sqrt(p**2-2*p-3)

x0 = np.zeros(p.size)
x1 = fx.copy()
x2 = (-root_delta+p+1)/(2*p)
x3 = (root_delta+p+1)/(2*p)

p = np.array([p]*4).T
f2x = np.array([x0,x1,x2,x3]).T

plt.plot(p,f2x)
plt.title('x*')
plt.xlabel('p')
plt.show()
```



[5]:
```
df2_dx_x = -p**2 *(2*f2x -1)*(2*p*(f2x -1)*f2x +1)

# 1) mask points where |df/dx|_x* < 1
fx_unstable = np.ma.masked_where(np.abs(df2_dx_x) < 1, f2x)

# 2) mask points where |df/dx|_x* >= 1
fx_stable = np.ma.masked_where(np.abs(df2_dx_x) >= 1, f2x)
```

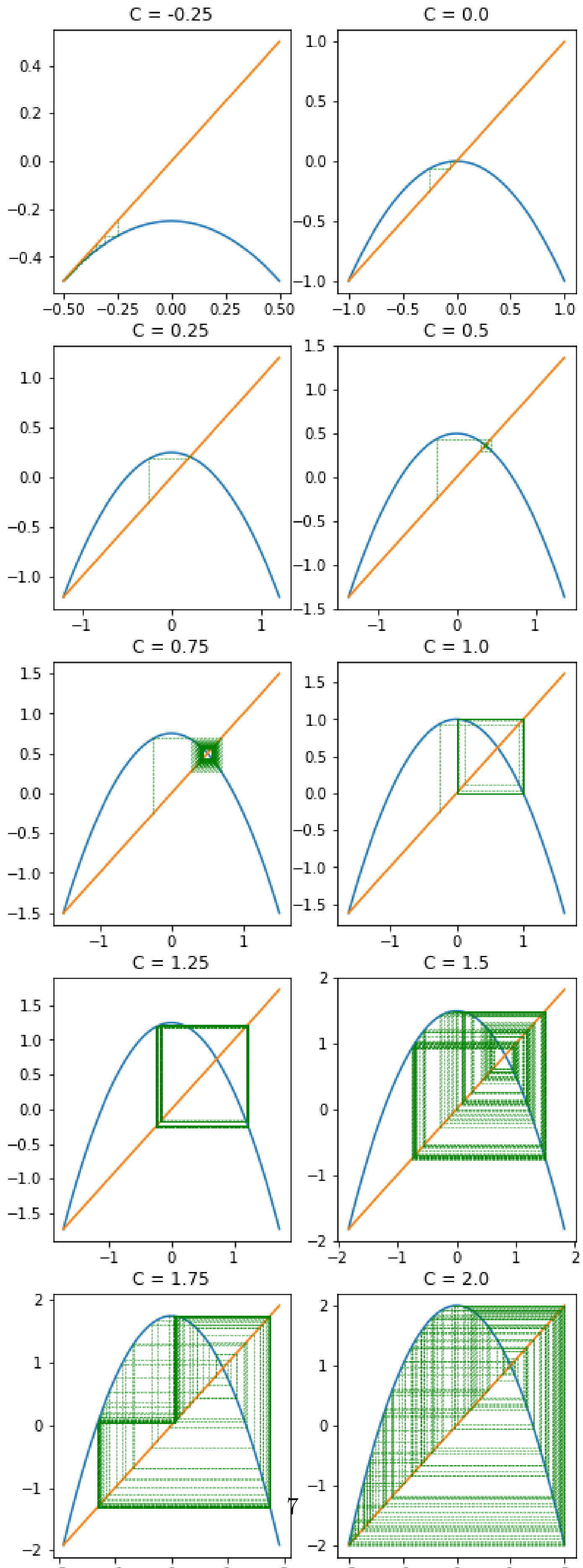
```
custom_lines = [Line2D([0], [0], color='k'),
                Line2D([0], [0], color='k', linestyle='--')]

plt.plot(p, fx_stable, '-')
plt.plot(p, fx_unstable, '--')
plt.title('Intervalos de estabilidade')
plt.xlabel('p')
plt.ylabel('x*')
plt.grid()
plt.legend(custom_lines, ['Estável', 'Instável'])
plt.show()
```



4 Atividade 66

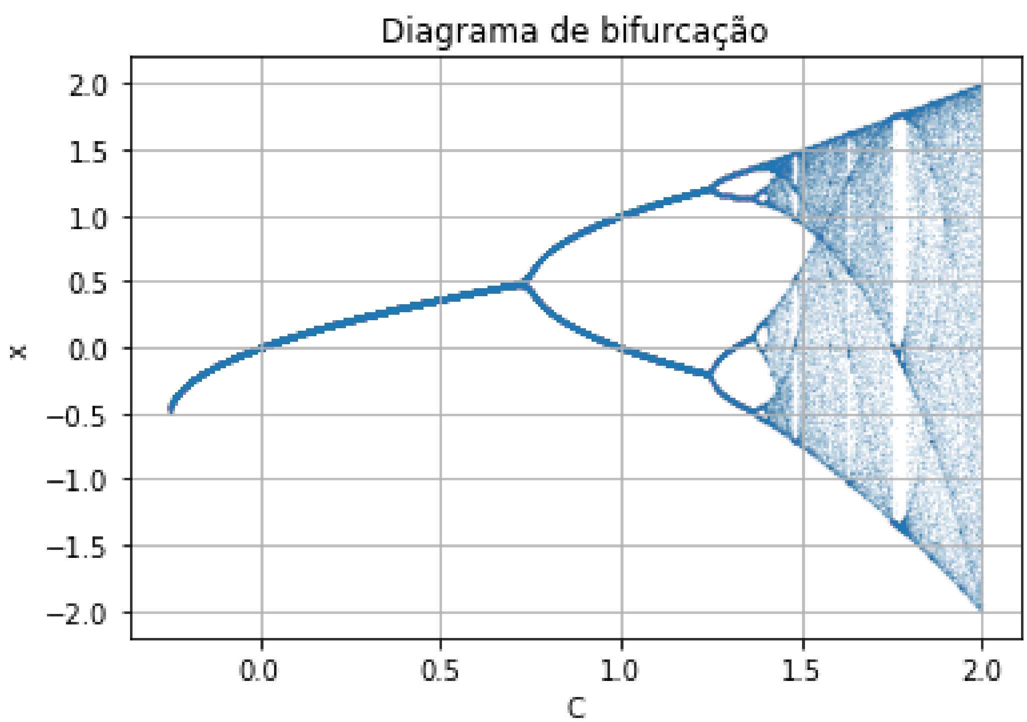
[6]:
```
fig, ax = plt.subplots(5, 2, figsize=(6.4, 19.2))

def x_n1(xn, C): return C - xn**2

C = np.linspace(-.25,2,num=10)

for i in range(10):
    x_min = -((1+4*C[i])**(1/2) + 1)/2
```

```
    x = np.linspace(x_min, -x_min)

    j = i//2
    k = i%2

    ax[j,k].plot(x, x_n1(x, C[i]))
    ax[j,k].plot(x, x)

    ax[j,k].set_title(f'C = {C[i]}')

    x1 = -.25
    for _ in range(128):
        x0 = x1
        x1 = x_n1(x0, C[i])
        ax[j,k].plot((x0,x0),(x0,x1), 'g--', linewidth=.5)
        ax[j,k].plot((x0,x1),(x1,x1), 'g--', linewidth=.5)
```


C = -0.25
C = 0.0
C = 0.25
C = 0.5
C = 0.75
C = 1.0
C = 1.25
C = 1.5
C = 1.75
C = 2.0

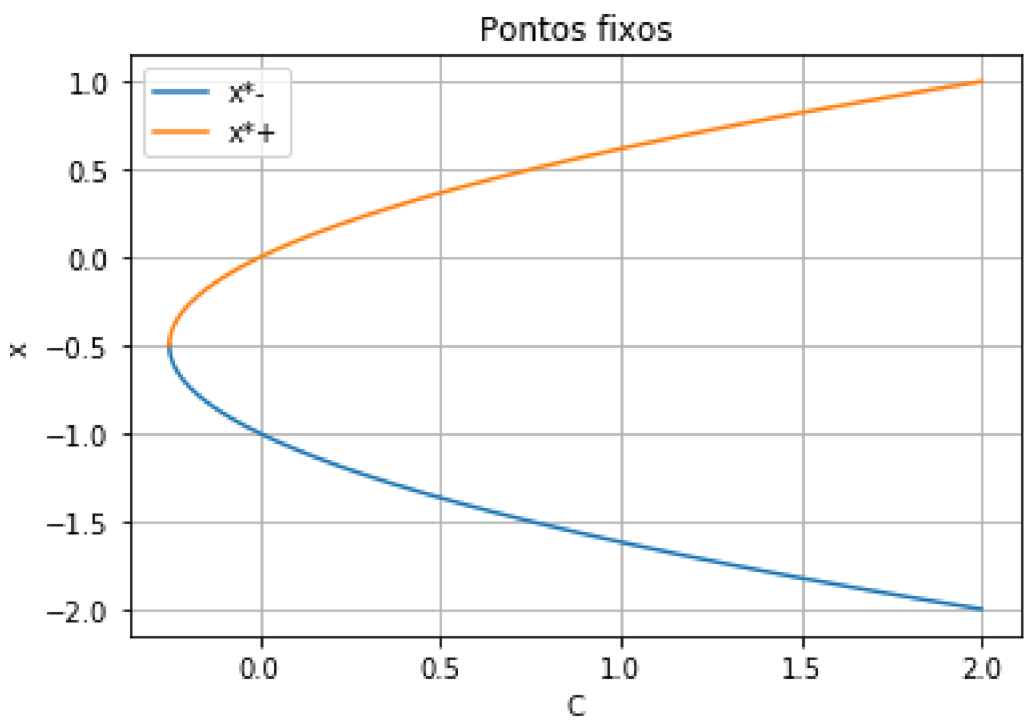
5 Atividade 67

[7]:
```
NT = N = 64

C = np.linspace(-.25,2,num=2250)

x = np.empty((NT+N+1, C.size))
x[0] = .25

for i in range(NT+N):
    x[i+1] = x_n1(x[i], C)

C_plot = np.full(x[-N:].shape, C)

plt.scatter(C_plot, x[-N:], s=.001)
plt.xlabel('C')
plt.ylabel('x')
plt.title('Diagrama de bifurcação')
plt.grid()
plt.show()
```



6 Atividade 68

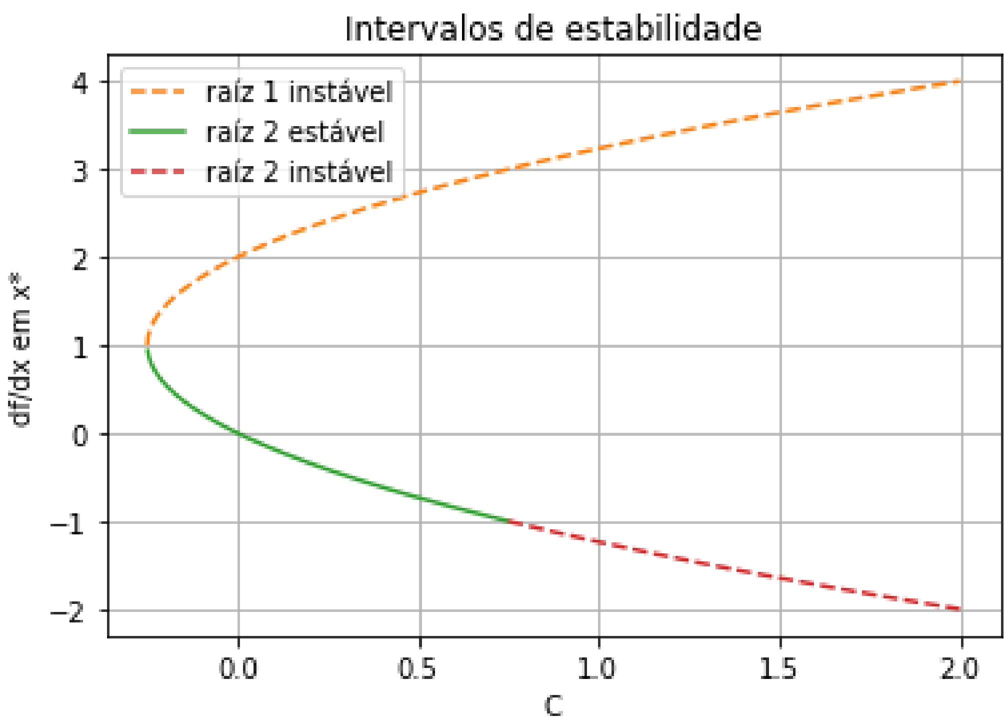
[8]:
```
root_delta = (1+4*C)**(1/2)

root1 = (-1-root_delta)/2
root2 = (-1+root_delta)/2

plt.plot(C,root1,label='x*-')
plt.plot(C,root2,label='x*+')
plt.xlabel('C')
plt.ylabel('x')
plt.title('Pontos fixos')
plt.legend()
plt.grid()
plt.show()
```



7 Atividade 69

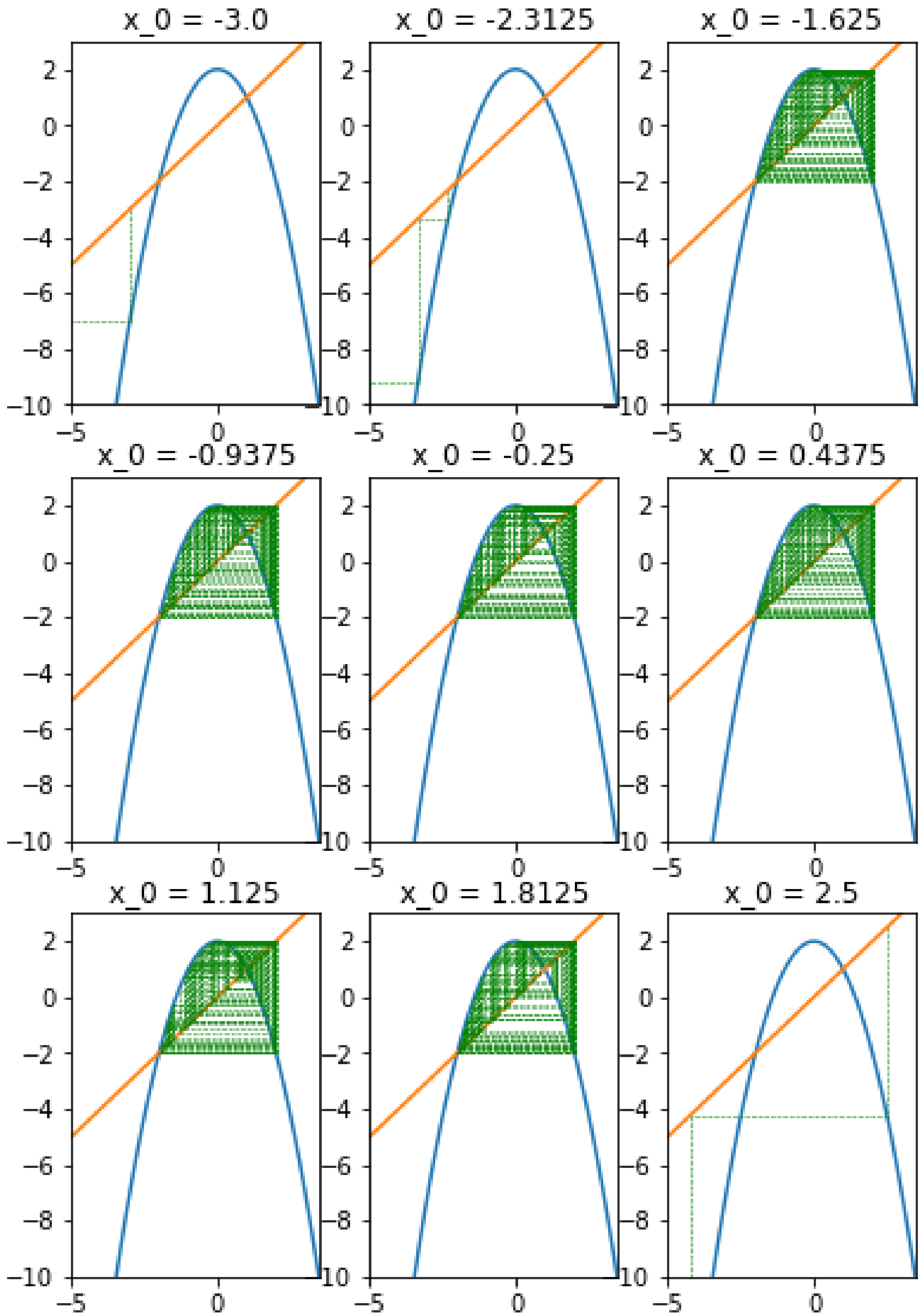
[9]:
```
df_dx_root1 = -2*root1
df_dx_root2 = -2*root2

# 1) mask points where |df/dx|_x* < 1
df_dx_root1_unstable = np.ma.masked_where(np.abs(df_dx_root1) < 1,
```

```
                                        df_dx_root1)
df_dx_root2_unstable = np.ma.masked_where(np.abs(df_dx_root2) < 1,
                                          df_dx_root2)

# 2) mask points where |df/dx|_x* >= 1
df_dx_root1_stable = np.ma.masked_where(np.abs(df_dx_root1) >= 1,
                                        df_dx_root1)
df_dx_root2_stable = np.ma.masked_where(np.abs(df_dx_root2) >= 1,
                                        df_dx_root2)

plt.plot(C, df_dx_root1_stable, '-')
plt.plot(C, df_dx_root1_unstable, '--', label='raíz 1 instável')
plt.plot(C, df_dx_root2_stable, '-', label='raíz 2 estável')
plt.plot(C, df_dx_root2_unstable, '--', label='raíz 2 instável')
plt.title('Intervalos de estabilidade')
plt.xlabel('C')
plt.ylabel('df/dx em x*')
plt.grid()
plt.legend()
plt.show()
```



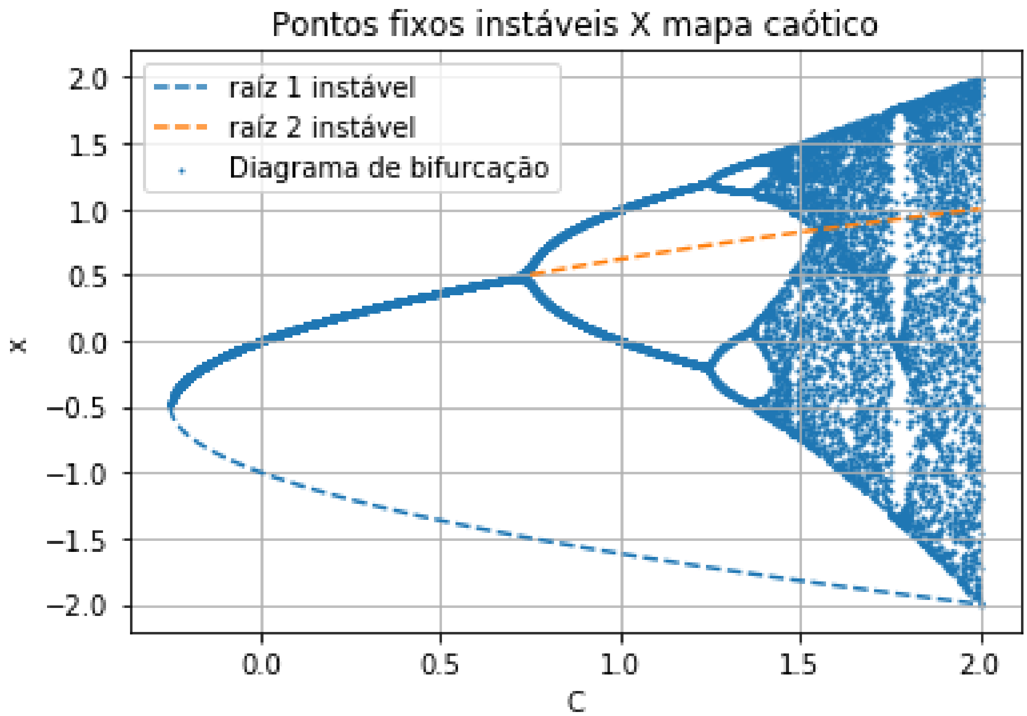
8 Atividade 70

[10]:
```
fig, ax = plt.subplots(3, 3, figsize=(6.4, 9.6))

C = 2
x_min = -((1+4*C)**(1/2) + 1)/2

xi = np.linspace(x_min-1, .5-x_min, num=9)

for i in range(9):
    x = np.linspace(x_min-3, 1.5-x_min)

    j = i//3
    k = i%3

    ax[j,k].plot(x, x_n1(x, C))
    ax[j,k].plot(x, x)
    ax[j,k].set_xlim(x_min-3, 1.5-x_min)
    ax[j,k].set_ylim(-10, 3)

    ax[j,k].set_title(f'x_0 = {round(xi[i],6)}')

    x1 = xi[i]
    for _ in range(128):
        x0 = x1
        x1 = x_n1(x0, C)
        ax[j,k].plot((x0,x0),(x0,x1), 'g--', linewidth=.5)
        ax[j,k].plot((x0,x1),(x1,x1), 'g--', linewidth=.5)
```


x_0 = -3.0
x_0 = -2.3125
x_0 = -1.625
x_0 = -0.9375
x_0 = -0.25
x_0 = 0.4375
x_0 = 1.125
x_0 = 1.8125
x_0 = 2.5
2
0
−2
−4
−6
−8
−10
−5
0

9 Atividade 71

[11]:
```python
NT = N = 64

C = np.linspace(-.25,2,num=2250)

x = np.empty((NT+N+1, C.size))
x[0] = .25

for i in range(NT+N):
    x[i+1] = x_n1(x[i], C)

C_plot = np.full(x[-N:].shape, C)

# 1) mask points where |df/dx|_x* < 1
x_root1_unstable = np.ma.masked_where(np.abs(df_dx_root1) < 1,
                                      root1)
x_root2_unstable = np.ma.masked_where(np.abs(df_dx_root2) < 1,
                                      root2)

plt.plot(C, x_root1_unstable, '--', label='raíz 1 instável')
plt.plot(C, x_root2_unstable, '--', label='raíz 2 instável')

plt.scatter(C_plot[::7], x[-N::7], s=1, label='Diagrama de bifurcação')
plt.xlabel('C')
plt.ylabel('x')
plt.title('Pontos fixos instáveis X mapa caótico')
plt.grid()
plt.legend()
plt.show()
```



10 Atividade 72

Partimos da expressão do mapa logístico

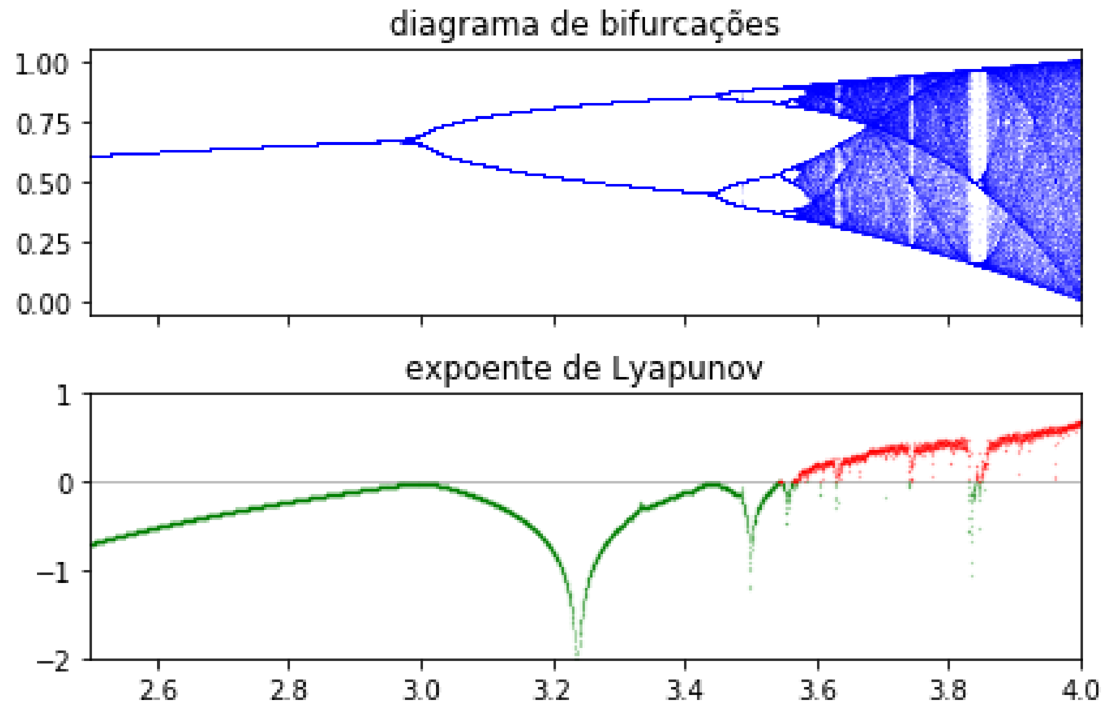
$$
\begin{aligned}
px(1-x) &= px^2 - px && (5)\\
&= px^2 - px - \frac{p}{4} + \frac{p}{4} && (6)\\
&= -\left(\sqrt{p}x - \frac{\sqrt{p}}{2}\right)^2 + \frac{p}{4} && (7)
\end{aligned}
$$

Substituindo $y = \sqrt{p}x - \frac{\sqrt{p}}{2}$ e $C = \frac{p}{4}$ resulta na expressão do mapa quadrático $C - y^2$

11 Atividade 73

[12]:
```
def logistic(r, x):
    return r * x * (1 - x)

n = 10001
r = np.linspace(1., 4., n)

iterations = 1000
```

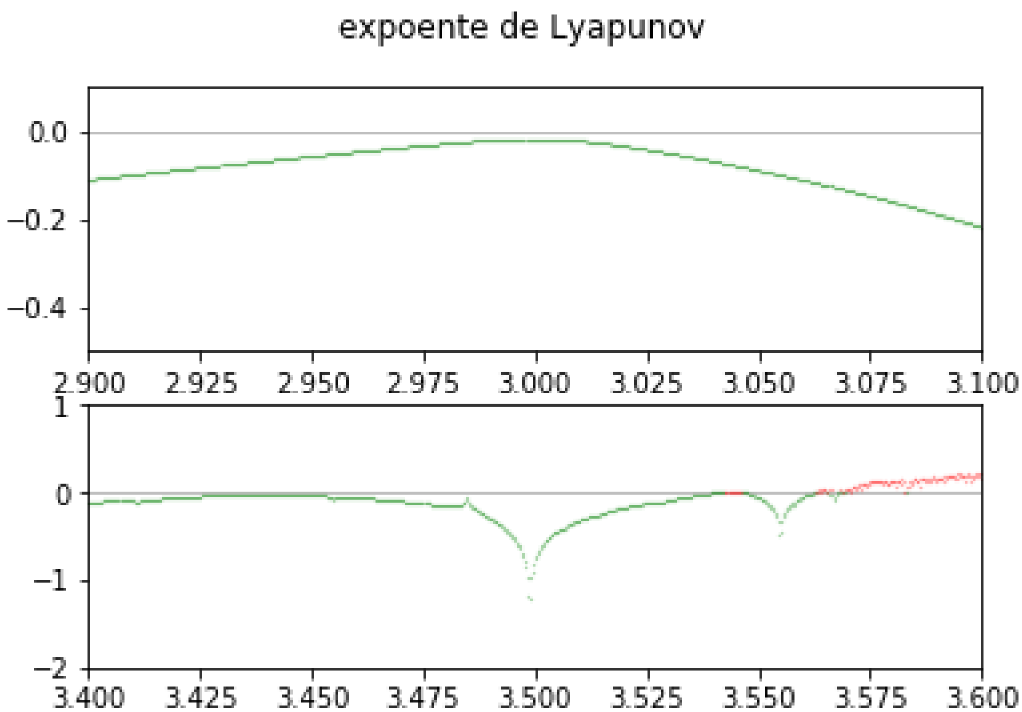
```
x = .3 * np.ones(n)

lyapunov = np.zeros(n)

fig, (ax1, ax2) = plt.subplots(2, 1,sharex=True)
for i in range(NT+N):
    x = logistic(r, x)

    lyapunov += np.log(abs(r - 2 * r * x))

    if i >= (NT):
        ax1.plot(r, x, 'b,', alpha=.1)
ax1.set_xlim(2.5, 4)
ax1.set_title("diagrama de bifurcações")

ax2.axhline(0, color='k', lw=.5, alpha=.5)

ax2.plot(r[lyapunov < 0],
         lyapunov[lyapunov < 0] / (NT+N),
         'g.', alpha=.5, ms=.5)

ax2.plot(r[lyapunov >= 0],
         lyapunov[lyapunov >= 0] / (NT+N),
         '.r', alpha=.5, ms=.5)
ax2.set_xlim(2.5, 4)
ax2.set_ylim(-2, 1)
ax2.set_title("expoente de Lyapunov")
plt.tight_layout()
```



12 Atividade 74

[13]:
```
fig, (ax1, ax2) = plt.subplots(2, 1)

ax1.axhline(0, color='k', lw=.5, alpha=.5)

ax1.plot(r[lyapunov < 0],
         lyapunov[lyapunov < 0] / (NT+N),
         '.g', alpha=.5, ms=.5)

ax1.plot(r[lyapunov >= 0],
         lyapunov[lyapunov >= 0] / (NT+N),
         '.r', alpha=.5, ms=.5)
ax1.set_xlim(2.9, 3.1)
ax1.set_ylim(-.5, .1)

ax2.axhline(0, color='k', lw=.5, alpha=.5)

ax2.plot(r[lyapunov < 0],
         lyapunov[lyapunov < 0] / (NT+N),
         '.g', alpha=.5, ms=.5)

ax2.plot(r[lyapunov >= 0],
```

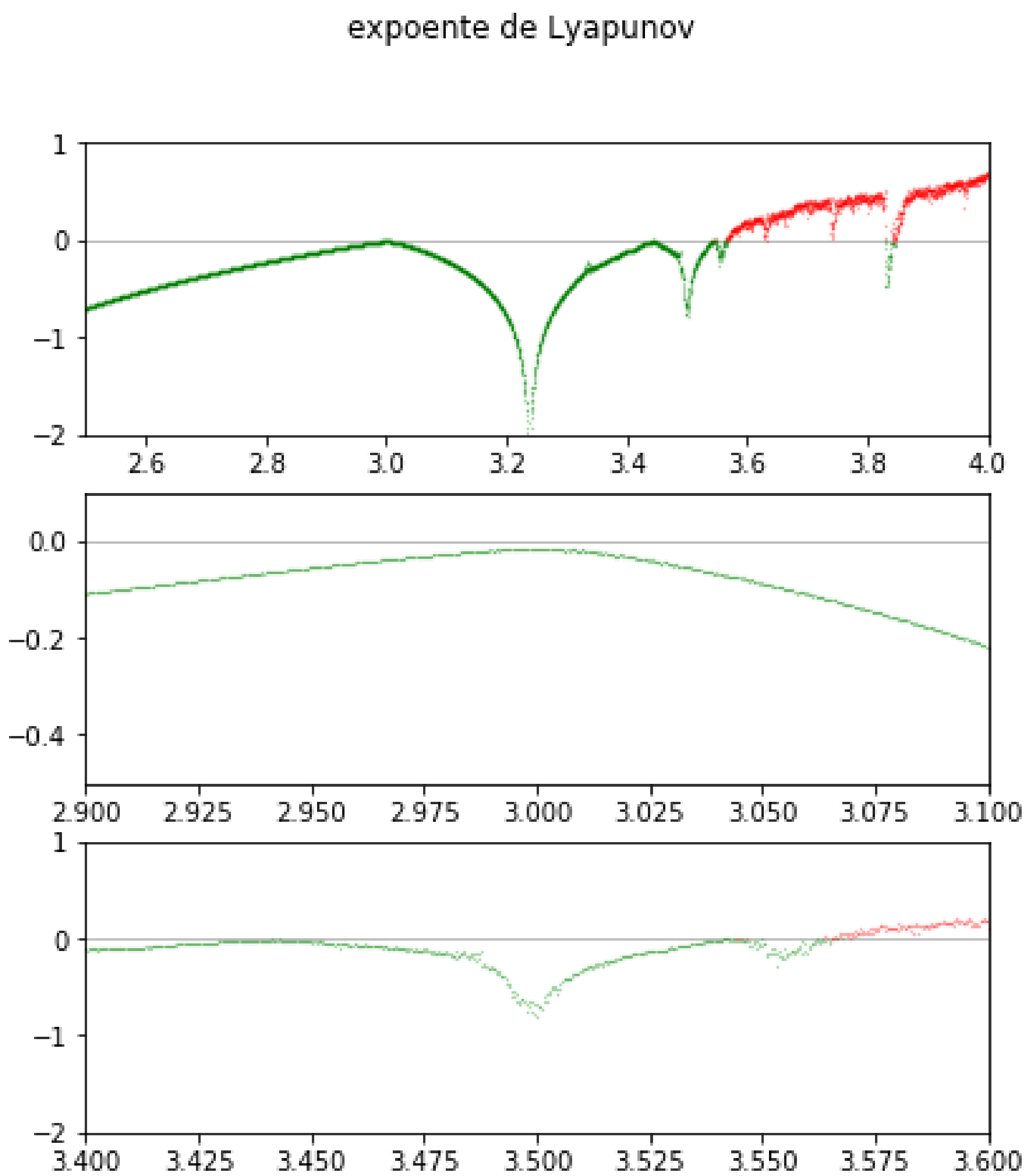
```
        lyapunov[lyapunov >= 0] / (NT+N),
        '.r', alpha=.5, ms=.5)
ax2.set_xlim(3.4, 3.6)
ax2.set_ylim(-2, 1)

fig.suptitle("expoente de Lyapunov")
plt.show()
```



13 Atividade 75

[14]:
```
rng = np.random.default_rng()

r = np.linspace(1., 4., n)
x = .3 * np.ones(n)
lyapunov = np.zeros(n)

fig, (ax1, ax2, ax3) = plt.subplots(3, 1, figsize=[6.4, 7.2])
for i in range(NT+N):
    x = logistic(r, x)

    lyapunov += np.log(abs(r - 2 * r * x))

    x += 0.001*(0.5-rng.random(x.size))
```

```
ax1.axhline(0, color='k', lw=.5, alpha=.5)

ax1.plot(r[lyapunov < 0],
         lyapunov[lyapunov < 0] / (NT+N),
         'g.', alpha=.5, ms=.5)

ax1.plot(r[lyapunov >= 0],
         lyapunov[lyapunov >= 0] / (NT+N),
         '.r', alpha=.5, ms=.5)
ax1.set_xlim(2.5, 4)
ax1.set_ylim(-2, 1)


ax2.axhline(0, color='k', lw=.5, alpha=.5)

ax2.plot(r[lyapunov < 0],
         lyapunov[lyapunov < 0] / (NT+N),
         '.g', alpha=.5, ms=.5)

ax2.plot(r[lyapunov >= 0],
         lyapunov[lyapunov >= 0] / (NT+N),
         '.r', alpha=.5, ms=.5)
ax2.set_xlim(2.9, 3.1)
ax2.set_ylim(-.5, .1)

ax3.axhline(0, color='k', lw=.5, alpha=.5)

ax3.plot(r[lyapunov < 0],
         lyapunov[lyapunov < 0] / (NT+N),
         '.g', alpha=.5, ms=.5)

ax3.plot(r[lyapunov >= 0],
         lyapunov[lyapunov >= 0] / (NT+N),
         '.r', alpha=.5, ms=.5)
ax3.set_xlim(3.4, 3.6)
ax3.set_ylim(-2, 1)

fig.suptitle("expoente de Lyapunov")
plt.show()
```



14 Atividade 76

[15]:
```
delta = 4.
 ↪66920160910299067185320382046620161725818557747576863274565134300413433021131473713868974402
alpha = 2.
 ↪50290787509589282228390287321821578638127137672714997733619205677923546317959020670329964974

print(f'A fração que dá a largura da janela de príodo 16 é {delta**-3}')
```

```python
print(f'A fração relativa ao tamanho do atrator {alpha**-3}')
```

```
A fração que dá a largura da janela de príodo 16 é 0.009823632798772396
A fração relativa ao tamanho do atrator 0.06377719370777242
```