Population Aging, Fiscal Space and the Interest-Growth Differential

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Abstract

Population aging strains fiscal accounts by increasing retirement benefits and decreasing income tax revenue. However, as a recent strand of the literature has argued, population aging also reduces real interest rates, implying an ambiguous total fiscal effect. This paper seeks to disentangle these two channels first analytically and then empirically using a panel of 23 advanced and 23 emerging economies. It finds that the composition of the aging process matters: a population that ages more due to a fall in population growth will experience a more negative fiscal impact than one that ages more due to an increase in life expectancy for the same change in the old-age dependency ratio. A fall of 1 pp in the working-age population growth leads to an increase of 0.8 pp in the primary balance that stabilizes that debt-to-GDP ratio, whereas an increase of 5 years in life expectancy leads to a fall of 0.4 pp in that same balance. The combination of both effects suggest that the population growth channel is stronger. The rapid population growth decline that emerging economies are going through bodes ill for their fiscal space.

Keywords: Life expectancy, Population growth, Demographic transition, Interest-growth differential, Fiscal space, Debt dynamics
JEL Classifications: E62, F41, H63, J11

1 Introduction

Public debt grows through two main channels: the accumulation of primary deficits and the accumulation of interest expenditures on existing debt. The demographic transition has poised a source of concern for debt dynamics: as the tax base shrinks with a smaller working-age population, the number of recipients of retirement benefits grows, implying a negative impact on social security budgets, higher primary deficits and higher public debt. On the other hand, a strand of the literature has presented population aging as one of the causes of the secular decline in real interest rates. As a consequence, the demographic transition might lead to two opposing effects on government's debt dynamics. This paper seeks to disentangle both effects and assess what the main determinants of their magnitudes are.

Population has been aging in advanced economies due to two dynamics: the end of the baby boom experienced in the post-war period and also an increase in longevity at old age. The top left panel of Figure 1 shows the old-age dependency ratio, defined as the ratio between the population above 65 years of age and the working-age population, defined as those between 20 and 65 years old. The blue line presents the cross-section median across advanced economies. We see that it has been on an upward trend in advanced countries that is even expected to intensify in the near future. The red line, representing the cross-section median of emerging economies, also shows that they are in the early stages of an aging process that is set to accelerate in the coming years.

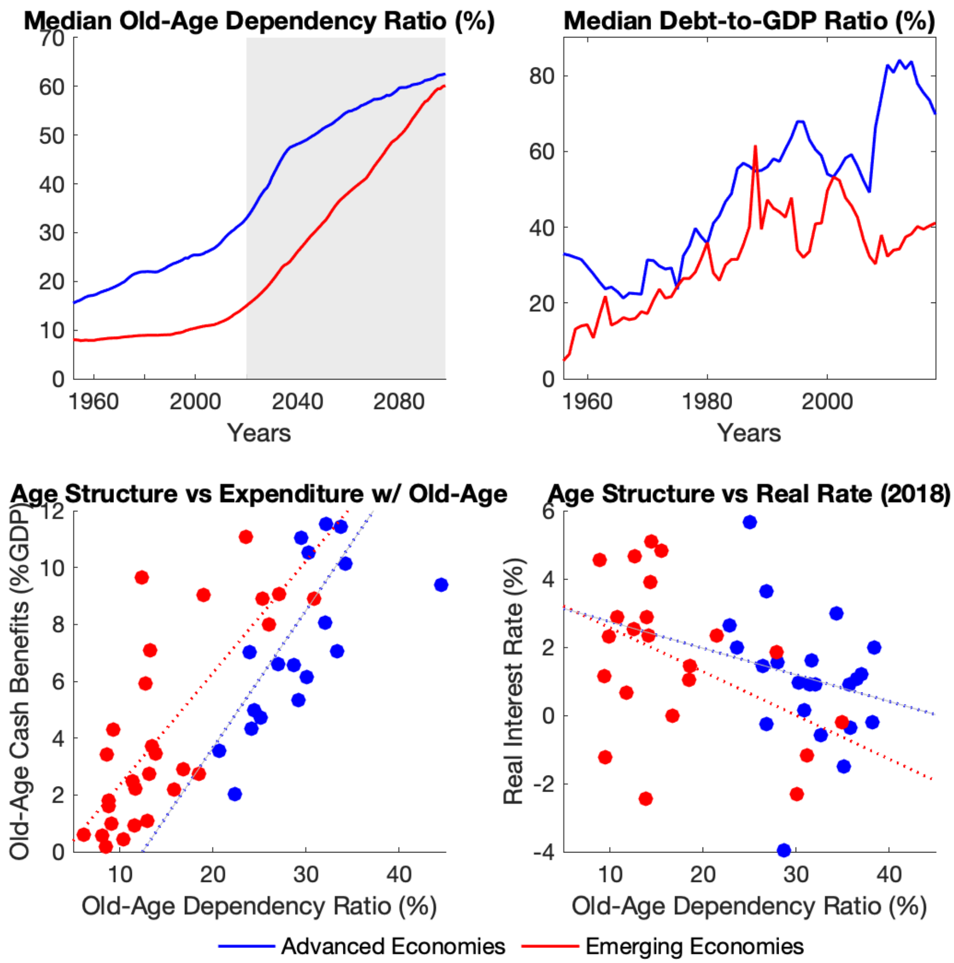
The main implications for debt dynamics alluded before can be seen first in the bottom left panel of Figure 1, where the old-age dependency ratio is plotted against old-age cash benefits for a cross-section of advanced and emerging economies. The positive correlation is clear: as countries age, the amount disbursed by governments as benefits for retirees increases. Benefits tend to be more generous in emerging economies, indicating a lack (and necessity) of reform.

The bottom right panel of Figure 1 hints at the second channel mentioned above. As countries age, there is a downward pressure on interest rates. The phenomenon is the consequence of an increase in the supply of savings: increased life expectancy lead to higher savings to finance the increased expected period in retirement, and a fall in demand for savings: lower population growth lowers the expected return on investments.

Finally, the top right panel of Figure 1 shows the upward trajectory of public debt as a percentage of GDP in the past six decades for both advanced and emerging economies, a reminder of the importance of understanding the impact of aging on debt dynamics and that there might be little fiscal space left [CITE PAPERS ON DEBT AND GROWTH].

To make sense of the forces at play, I resort first to a simple 2-country Overlapping Gen-

Figure 1: Demographics and Debt Dynamics



erations model a la Diamond (1965) stripped down to the most fundamental macroeconomic relationships to derive fundamental causal relationships between the demographic processes.

The analytical results show that the two drivers of the demographic transition have opposing effects on a country's interest-growth differential: while a fall in the growth rate of the working-age population expands the differential and thus increases the growth rate of the debt-to-GDP ratio, an increase in life expectancy shrinks the differential and thus slows down the growth of the debt-to-GDP ratio. This contrast leads to the main analytical result: the old-age dependency ratio is **not** a sufficient statistic to evaluate the fiscal impact of the demographic transition. In other words, it is not sufficient to compute the changes in resource flows into and out of the social security system, it is also necessary to know what the composition of the underlying demographic change is.

As a natural next step, a panel of advanced and emerging economies is used to estimate empirically the analytical results derived from the model. The estimates are consistent with the analytical results: a fall in fertility and a rise in longevity imply opposing effects on the interest-growth differential with similar quantitative importance each. The result still holds if the estimation is executed with either advanced economies only or emerging economies only. On the other hand, the spillover effects of demographic change abroad does not match with the analytical results.

This paper is related to a few strands of the literature. The first one is the one that applies large scale OLG models to study the effects of the demographic transition on a set of economic variables in the tradition of the seminal paper or Auerbach and Kotlikoff (1987). Among a vast number of papers one can mention Attanasio et al. (2007), who study social security reform in an open economy context; Krueger and Ludwig (2007), that also use an open economy model to quantify the impact of the demographic transition in factor prices, capital flows and welfare; Vogel et al. (2017), who also use a multi-country OLG model to address policy responses to the aging process, and Gagnon et al. (2021), that uses the same methodology to account for the decline in output growth and real interest rates in the United States. These papers feed the demographic transition as an exogenous process to the model to explain and project the path of macroeconomic aggregates under different policy scenarios. The results of simulations, however, are particular to the set of parameters and demographic change fed into the model. A broader lesson on the fiscal consequences is missing.

In particular, a spin-off of this is the literature that ascribes the recent decline in real interest rates to population aging. Some papers make use of large-scale OLG models such as the ones mentioned above or small-scale models, such as Carvalho et al. (2016). However, these papers stop short of studying the implications of the fall in interest rates to fiscal policy in the context of demographic change.

A third branch of the literature that this paper is related to is the one that studies the interest-growth differential. Blanchard and Weil (2001), studies theoretical implications of dynamic (in)efficiency to debt Ponzi games under uncertainty, while more recently Blanchard (2019) studies the fiscal and welfare costs of public debt in an environment of negative interest-growth differentials. Reis (2021) in turn seek to establish a limit to the primary deficit in a context of negative interest-growth differential on public debt. However, these papers only analyze the consequences of the sign of the interest-growth differential for fiscal policy. They do not explain the determinants differential changes across time.

The present paper thus seeks to fill the aforementioned gaps in the literature by investigating the role demographic changes have in impacting debt dynamics through the interest-growth differential.

The rest of the paper is structured as follows: Section 2 presents a simple two-country OLG model, Section 3 derives the theoretical results that will underlie the empirical analysis in Section 4. Section 4 presents the panel data of advanced and emerging economies and analyzes the effects of demographics on the interest-growth differential and on the fiscal space. Section 5 concludes the paper.

2 Simple Model

The following model is a two-country version of the standard Diamond (1965) OLG model augmented with a Pay-As-You-Go (PAYGO) social security system and a survival probability.

2.1 Households

Households in country $i \in d, f$ (domestic and foreign) potentially live for two periods. All of them live during their first period and a share $1 - \delta^i_{t+1}$ of these workers dies at the end of that period and do not reach the second period. The exogenous variable δ^i_{t+1} can be thus considered to be the share of the time period when agents are alive when old. The number of workers in period t is given by N^i_t and the number of retirees is thus $\delta^i_t N^i_{t-1}$. The number of young workers grows at the gross rate γ_{Nit}: $N^i_t = \gamma_{Nit} N^i_{t-1}$

Households have log-utlity over consumption when young, c^i_{1t}, and old, c^i_{2t+1}, and therefore their utility over lifetime consumption is given by the function below:

$$U(c^i_{1t}, c^i_{2t+1}) = \ln c^i_{1t} + \beta^i \delta^i_{t+1} \ln c^i_{2t+1} \tag{1}$$

where β^i represents the discount factor.

Labor supply is inelastic. Young households receive a wage w^i_t in exchange for the labor supplied and also pay a lump-sum tax, τ^i_t. They then consume, c^i_{1t}, and save, s^i_t, out of their after-tax labor income. When old, households receive interest over their savings given by the gross interest rate, R^i_{t+1}, and a social security payment, e^i_t each.

$$c^i_{1t} + s^i_t = w^i_t - \tilde{\tau}^i_t \tag{2}$$

$$c^i_{2t+1} = \frac{R^i_{t+1}}{\delta^i_{t+1}} s^i_t + e^i_t \tag{3}$$

The first order conditions of this problem imply the following savings function:

$$s_t^i = \frac{\beta^i \delta_{t+1}^i}{1+\beta^i \delta_{t+1}^i}(w_t^i - \tilde{\tau}_t^i) - \frac{\delta_{t+1}^i}{\left(1+\beta^i \delta_{t+1}^i\right) R_{t+1}^i} e_{t+1}^i \tag{4}$$

2.2 Firms

Firms are assumed to combine labor, L_t^i, and capital, K_t^i, to produce the single good in the economy, Y_t^i, according to the following Cobb-Douglas utility function:

$$Y_t^i = (A_t^i L_t^i)^{1-\alpha}(K_t^i)^{\alpha} \tag{5}$$

with $\alpha \in (0,1)$ and total factor productivity, A_t^i, that grows at rate γ_{Ait}: $A_t^i = \gamma_{Ait} A_{t-1}$. As a result, wage and the gross interest rate are paid their marginal products:

$$R_t^i = \alpha(k_t^i)^{\alpha-1} \tag{6}$$

$$w_t^i = (1-\alpha)A_t^i(k_t^i)^{\alpha} \tag{7}$$

where $K_t^i \equiv k_t^i/(A_t^i N_t^i)$ is the capital stock measured in efficient labor units.

2.3 Government

Assume government's only tax revenue comes from the lump-sum tax levied on workers, while its only transfers comprise the social security benefits paid to retirees. For simplicity social security payments, E_t^i, are a fraction ν^i of the current period's wage:

$$e_t^i = \nu^i w_t^i \tag{8}$$

The social security budget is not forced to balance every period; the government can issue debt, b_t^i, if necessary. Therefore the law of motion of public debt is given by:

$$B_{t+1}^i = E_t^i + G_t^i - T_t^i + R_t^i B_t^i \tag{9}$$

where B_{t+1}^i is the government debt measured at the end of period t and start of period $t+1$, E_t^i is the total social security benefits paid in period t, T_t^i is the total tax revenue collected in period t and R_t^i is the gross interest rate paid by public debt (which in this model is equal to the marginal return on capital). Define B^i_{t+1} as the debt-to-output ratio measured at the end of period t (and start of period $t+1$): $B^i_{t+1} \equiv B^i_{t+1}/Y^i_t$ and rewrite equation (9) as:

$$b^i_{t+1} = g^i_t - \tau^i_t + \frac{\delta^i_t \nu^i (1-\alpha)}{\gamma_{Nit}} + \frac{\alpha}{\gamma_{Ait}\gamma_{Nit}} \frac{(k^i_{t-1})^\alpha}{k^i_t} b^i_t \tag{10}$$

where τ^i_t is the tax burden $\tau^i_t \equiv N^i_t \tilde{\tau}^i_t / Y^i_t$.

2.4 Market clearing

The capital market clearing condition is that total savings equal the total capital and debt stocks at the beginning of the following period:

$$\sum_{i=d,f} N^i_t s^i_t = \sum_{i=d,f} K^i_{t+1} + \sum_{i=d,f} B^i_{t+1} \tag{11}$$

Dividing (11) through by $A^d_t N^d_t$ and rearranging yields:

$$\frac{s^d_t}{A^d_t} + \varphi^*_t \frac{s^*_t}{A^*_t} = g^d_{At+1} g^d_{Nt+1} k^d_{t+1} + \gamma^*_{At+1}\gamma^*_{Nt+1}\varphi^*_t k^*_{t+1} + b^d_{t+1}(k^d_t)^\alpha + \varphi^*_t b^*_{t+1}(k^*_t)^\alpha \tag{12}$$

where $\varphi^*_t \equiv A^d_t N^d_t / (A^*_t N^*_t)$ is the relative size of the foreign economy's effective labor.

Substitute for w^i_t and e^i_{t+1} in the savings function (4) and get as a result:

$$s^i_t = \frac{\beta^i \delta^i_{t+1}}{1+\beta^i\delta^i_{t+1}} \left((1-\alpha) A^i_t (k^i_t)^\alpha - \tau^i_t A^i_t y^i_t \right) - \frac{1-\alpha}{\alpha} \frac{\delta^i_{t+1}}{1+\beta^i\delta^i_{t+1}} \nu^i A^i_t (k^i_t)^\alpha (k^i_{t+1})^{1-\alpha} \tag{13}$$

Plugging equation (13) into (12) yields:

$$\begin{aligned} &\frac{\beta^d \delta^d_{t+1}}{1+\beta^d\delta^d_{t+1}} \left((1-\alpha) - \tau^d_t \right) (k^d_t)^\alpha - \frac{1-\alpha}{\alpha} \frac{\delta^d_{t+1}}{1+\beta^d\delta^d_{t+1}} \nu^d (k^d_t)^\alpha (k^d_{t+1})^{1-\alpha} + \\ &\qquad \varphi^*_t \left(\frac{\beta^* \delta^*_{t+1}}{1+\beta^*\delta^*_{t+1}} \left((1-\alpha) - \tau^*_t \right) (k^*_t)^\alpha - \frac{1-\alpha}{\alpha} \frac{\delta^*_{t+1}}{1+\beta^*\delta^*_{t+1}} \nu^* (k^*_t)^\alpha (k^*_{t+1})^{1-\alpha} \right) \\ &\qquad\qquad = g^d_{At+1} g^d_{Nt+1} k^d_{t+1} + \gamma^*_{At+1}\gamma^*_{Nt+1}\varphi^*_t k^*_{t+1} + b^d_{t+1}(k^d_t)^\alpha + \varphi^*_t b^*_{t+1}(k^*_t)^\alpha \end{aligned} \tag{14}$$

Given perfect capital mobility, the no-arbitrage condition holds and $k^d_t = k^*_t = k_t$. Equation (14) becomes:

$$\left(\frac{\beta\delta_{t+1}}{1+\beta\delta_{t+1}}\left((1-\alpha)-\tau_t\right)+\varphi_t^*\frac{\beta^*\delta_{t+1}^*}{1+\beta^*\delta_{t+1}^*}\left((1-\alpha)-\tau_t^*\right)\right)k_t^\alpha$$
$$-\frac{1-\alpha}{\alpha}\left(\frac{\delta_{t+1}}{1+\beta\delta_{t+1}}\nu+\varphi_t^*\frac{\delta_{t+1}^*}{1+\beta^*\delta_{t+1}^*}\nu^*\right)k_{t+1}$$
$$=(\gamma_{At+1}\gamma_{Nt+1}+\gamma_{At+1}^*\gamma_{Nt+1}^*\varphi_t^*)k_{t+1}+(b_{t+1}+\varphi_t^*b_{t+1}^*)k_t^\alpha \quad (15)$$

Note that given initial values for the capital stock in efficiency units k_{-1} and k_0 and given the path for the exogenous variables, the path for efficient capital is given by equations (15) and (10).

2.5 Analysis

In this section I carry out the analysis of the effects of demographic shocks on economic variables. In particular, the interest lies on the effect of such shocks on fiscal variables.

I will first analyze the effect of demographic changes on the interest-growth differential and then show that a fall in population growth has the opposite effect of an increase in longevity. The final result will hinge on this distinction.

2.5.1 Interest-Growth Differential

Proposition 1. *A fall in the growth rate of the labor force unambiguously increases the adjusted interest-growth differential next period:*

$$\frac{\partial \frac{R_{t+1}}{g_{Yt+1}^i}}{\partial g_{Nt+1}^i}<0$$

An increase in the survival probability affects the interest-growth differential as follows:

$$\frac{\partial \frac{R_{t+1}}{g_{Yt+1}^i}}{\partial \delta_{it+1}}\begin{cases}\leq 0 & \text{if } s_t \geq 0\\ >0 & \text{if } s_t<0\end{cases}$$

Proof. Appendix. □

The results in Proposition 1 hinge on the comparison between the elasticities of the real interest rate and of the output growth rate with respect to the demographic variables. Both elasticities will in turn be functions of the elasticity of capital in efficiency units to demographics.

The elasticity of the real interest rate to the population growth rate will be lower than the elasticity of the output growth rate with respect to that same variable if the elasticity of capital is lower than one (in absolute value). That will be the case if the foreign country is sizeable and/or if there is a PAYGO social security system in place in any country. The existence of a sizeable foreign country will diminish the effect of the domestic country on the total stock of capital, whereas a PAYGO social security system will lead agents to save more in case there is a decrease in the stock of capital in efficiency units, counterbalancing changes in that stock. Both dynamics guarantee an inelastic capital stock in efficiency units with respect to the population growth rate, which leads to the first result: a fall in the population growth rate widens the interest-growth differential.

On the other hand, the increased longevity will work through the savings channel only. Savings will go up if they are positive, as the increased longevity will make agents value more the second period. Now, if agents are borrowing, an increase in longevity will diminish the interest rate adjusted by the survival probability, making consuming more in the first period more appealing. Focusing on the positive savings case, an increase in longevity will increase the stock of capital and lower the real interest rate. By the same token, output growth goes up due to the larger capital stock. The result is a smaller interest-growth differential. If savings are negative, we have the exact opposite scenario as consequence.

Proposition 2. *A fall in the growth rate of the labor force unambiguously increases the adjusted interest-growth differential beyond the following period if [CONDITION]:*

$$\frac{\partial \frac{R_{t+k}}{g^i_{Yt+k}}}{\partial g^i_{Nt+1}} < 0,\ \textit{for } k > \textit{1}$$

If [CONDITION], an increase in the survival probability affects the interest-growth differential beyond the following period as follows:

$$\frac{\partial \frac{R_{t+k}}{g^i_{Yt+k}}}{\partial \delta_{it+1}} \begin{cases} \leq 0 & \textit{if } s_t \geq 0 \\ > 0 & \textit{if } s_t < 0 \end{cases}$$

Proof. Appendix. □

Proposition 3. *The aging of population abroad either as a result of a fall in the growth rate of the labor force or an increase in life expectancy unambiguously decreases the interest-growth differential in the domestic economy in the current period:*

$$\frac{\partial \frac{R_{t+1}}{g^{i}_{Yt+1}}}{\partial g^{j}_{Nt+1}} < 0, \ \textit{for } k > 1$$

Proof. Appendix. □

2.5.2 Fiscal Space

In order to evaluate the fiscal impact of the demographic transition, it is necessary to introduce the concept of fiscal space. To that end it is useful to divide the components of the primary balance into *demographic* components and *remainder* components. The demographic components represent the parts of the primary balance that are a direct function of demographics, whereas the remainder components are simply the primary balance minus its demographic components. The remainder then becomes a function of demographics only *through* the demographic components. The breakdown is as follows:

$$\tau_t = \tau_t^d(\gamma_{Nt}, \delta_t, \mathbf{x}_t) + \tau_t^r \tag{16}$$
$$g_t = g_t^d(\gamma_{Nt}, \delta_t, \mathbf{x}_t) + g_t^r \tag{17}$$
$$e_t = e_t^d(\gamma_{Nt}, \delta_t, \mathbf{x}_t) + e_t^r \tag{18}$$

Now solve equation (10) forward to get:

$$b_t = \sum_{i=0}^{\infty}\left[\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)[\tau_{t+i} - g_{t+i} - e_{t+i}]\right] + \lim_{s\to\infty}\left(\prod_{j=0}^{s}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)[\tau_{t+s} - g_{t+s} - e_{t+s}] \tag{19}$$

Assume that the No-Ponzi condition holds and break each component of the primary surplus into demographic and remainder components.

$$b_t = \sum_{i=0}^{\infty}\left[\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\underbrace{[\tau^d_{t+i} - g^d_{t+i} - e^d_{t+i}]}_{\equiv\sigma^d_t}\right] + \sum_{i=0}^{\infty}\left[\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\underbrace{[\tau^r_{t+i} - g^r_{t+i} - e^r_{t+i}]}_{\equiv\sigma^r_t}\right] \tag{20}$$

Consider now a marginal change in an exogenous demographic variable $\varkappa_t = \{\gamma_{Yt}, \delta_t\}$. This change will have a direct impact on the demographic primary surplus, σ_t^d, and will also have an impact on the interest-growth differential, R_t/γ_{Yt}. Note that these impacts are also a function of the fiscal policy in place. The decision of how to finance the changes in the demographic primary surpluses will also affect the response of endogenous variables to the exogenous demographic variables. The financing decision will be reflected in the changes in the remainder primary surpluses. These changes will underlie the definition of fiscal space.

Definition 1. *A change in fiscal space due to a change in a demographic variable $\varkappa_t$ is defined by the present value of changes in the remainder primary surpluses, s_t^r. If this change is positive, that implies an effort to raise the primary surplus after the demographic change, i.e. there is a loss of fiscal space. Likewise, if the change is negative, then there is creation of fiscal space.*

$$\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^{r}}{\partial\varkappa_t}\begin{cases}>0, \text{ loss of fiscal space}\\ <0, \text{ creation of fiscal space}\end{cases} \tag{21}$$

Now take the first derivative of the current debt-to-GDP ratio (pre-determined) with respect to the demographic variable $\varkappa_t$:

$$\begin{aligned}\frac{\partial b_t}{\partial\varkappa_t} = &\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^{d}}{\partial\varkappa_t}+\sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)}{\partial\varkappa_t}\sigma_{t+i}^{d}\\ &+\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^{r}}{\partial\varkappa_t}+\sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)}{\partial\varkappa_t}\sigma_{t+i}^{r}=0\end{aligned} \tag{22}$$

The equation above implies that the change in fiscal space can be decomposed in the following two parts. Note that the impact on the interest-growth differential affects the entire primary surplus, not just the demographic part.

$$-\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^{r}}{\partial\varkappa_t} = \underbrace{\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^{d}}{\partial\varkappa_t}}_{\text{effect on demographic primary surplus}} + \underbrace{\sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)}{\partial\varkappa_t}\sigma_{t+i}}_{\text{effect on interest-growth differential}} \tag{23}$$

The decomposition will be key in differentiating the fiscal impact of both components of the demographic transition. The first component is negative given the aging of the population, regardless its source. The second component, however, will be negative when there is a fall in population growth, but positive when there is an increase in longevity, an immediate result of Propositions 1 and 2.

In order to rigorously show the difference, we need to translate the definitions above to their correspondent variables in the model.

$$\tau_t^d = 0 \qquad \tau_t^r = \tau_t \tag{24}$$

$$g_t^d = 0 \qquad g_t^r = g_t \tag{25}$$

$$e_t^d(\gamma_{Nt},\delta_t,\mathbf{x}_t) = e_t = \frac{(1-\alpha)\delta_t\nu}{\gamma_{Nt}} \qquad e_t^r = 0 \tag{26}$$

Proposition 4. *A fall in the growth rate of the labor force causes a loss of fiscal space:*

$$\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^r}{\partial\gamma_{Nt}} > 0$$

If the domestic country is "sizeable enough", there exists a survival probability $\bar{\delta}_{t+1}^i$ *such that:*

$$\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^r}{\partial\delta_t}\begin{cases}\leq 0 & \textit{if } \delta_t^i \leq \bar{\delta}_t^i\\ > 0 & \textit{if } \delta_t^i > \bar{\delta}_t^i\end{cases}$$

Proof. Appendix. □

Proposition 5. *The aging of population abroad either as a result of a fall in the growth rate of the labor force or an increase in life expectancy unambiguously creates fiscal space in the domestic economy in the current period:*

$$\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^r}{\partial\gamma_{Nt}} > 0$$

If the domestic country is "sizeable enough", there exists a survival probability $\bar{\delta}_{t+1}^i$ *such that:*

$$\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^r}{\partial\delta_t}\begin{cases}\leq 0 & \textit{if } \delta_t^i \leq \bar{\delta}_t^i\\ > 0 & \textit{if } \delta_t^i > \bar{\delta}_t^i\end{cases}$$

Proof. Appendix. □

3 Empirical Analysis

3.1 Data

In order to estimate the relationship between the demographic variables, the fiscal balance and the interest-growth differential derived in Section 2, I will use data for 23 advanced economies[1] and 23 emerging economies [2] spanning the period 1956-2018.

The data set is comprised of different data sources. The United Nations World Population Prospects (2022) provides the demographic data on working-age and old-age population and life expectancy. To map the data to the model, working age population is defined as 20 to 64 years old, and old age as 65 years of age or older. Life expectancy is considered as the remaining life expectancy at 65 years old. Fiscal variables such as net and gross government debt, primary balance and interest spending are taken from the Mauro and Zhou (2020) database. Data on old-age cash benefits comes from the OECD, and data on TFP growth and the labor share is sourced from the Penn World Tables. The time series on the United States 3-month Treasury Bill is taken from the St Louis Fed database and world GDP comes from the World Bank.

Since the interest here lies on the long-run effects of demographics on the differential and the fiscal space, the data was arbitrarily filtered in order to remove periods of extreme values or high volatility of the interest-growth differential. One should expect demographics not to play a part in determining such extreme values. The cutoffs were established at -15% or 15% for the level and -10 and 10 percentage points for the annual change. The results including such extreme values can be found in section XZ of the Appendix.

[Descriptive statistics of variables]

In order to carry out the empirical analysis, it is necessary to evaluate the nature of each variable at hand with respect to stationarity. Table **??** shows the p-value of the ADF tests for each one of the variables and for all countries in the database. The ADF tests were carried out including a constant and choosing the number of included lags according to the Akaike criterion.

The tests show us that when treated independently, the interest-growth differential, the primary balance that stabilizes the debt-to-GDP ratio, and the TFP growth rate are the variables for which we reject the null hypothesis of the ADF test more often than not with 29, 35, and 43 of such cases out of a total of 46, respectively. The Im, Pesaran and Shin's

[1] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States.

[2] Argentina, Bolivia, Brazil, Bulgaria, Chile, China, Colombia, Costa Rica, Korea, Dominican Republic, Honduras, Hungary, India, Indonesia, Mexico, Nicaragua, Panama, Paraguay, Peru, Philippines, Poland, Romania, Thailand.

W-statistic also point out to these variables as being stationary.

On the other hand, the growth rate of the working-age population, life expectancy at 65, the debt-to-GDP ratio and the labor share are clearly non-stationary with only 4, 0, 8, and 4 rejections of the null, respectively, and also a non-rejection by the W-statistic. This result suggests the presence of a unit root in each one of these series.

Carrying out the ADF test assuming independence across countries allows us to capture unit roots in variables of each cross-section unit, which in turn may even be cointegrated among each other, but it makes us miss the cointegration *across* cross-section units that can be potentially found in a panel data context. Therefore, it is useful to look for common factors across units and to assess their stationarity.

When there is cross-sectional dependence, it is useful to decompose the variable into two components: a vector of common factors to all cross-section units,F_{zt}, and a idiosyncratic element, ε_{zit}. In such case, a variable can present a unit root stemming either from the common factors or from its idiosyncratic component:

$$z_{it} = d_{it} + \beta'_{zi} F_{zt} + \varepsilon_{zit} \tag{27}$$

where

$$(1-L)F_{zt} = C(L)u_{zt}$$
$$\varepsilon_{zit} = \rho\varepsilon_{zit-1} + \nu_{zit}$$

In order to perform the decomposition and test both components for nonstationarity, we can resort to Bai and Ng's PANIC test. The test follows a series of steps....

The results can be found in Table ZZ.

The

3.2 Econometric Methods

FMOLS

3.3 The Interest-Growth Differential

As a first test, it is worth checking whether the effects of changes in population growth and longevity on the interest-growth differential are really opposite as the theory implies.

The interest-growth differential of interest here is the one that is relevant to debt dynamics. As such, the interest rate is the implicit interest rate calculated as the ratio between interest

Table 1: Interest-Growth Differential

Dep. Variable: Differential Sample	(1) Full	(2) Full	(3) Advanced	(4) Advanced	(5) Emerging	(6) Emerging
Differential (-1)	0.7174***		0.7786***		0.6143***	
	(0.0000)		(0.0000)		(0.0000)	
Working-Age Pop. Growth (%)	-0.5927***	-1.5246***	-0.3970***	-1.4288***	-1.0102***	-1.8298***
	(0.0000)	(0.0000)	(0.0015)	(0.0000)	(0.0000)	(0.0000)
Longevity at 65	-0.3835***	-0.2134*	-0.2522**	-0.2639	-0.5774***	-0.6531***
	(0.0000)	(0.0912)	(0.0418)	(0.1975)	(0.0002)	(0.0004)
Foreign WA Pop. Growth (%)	0.6156**	1.9824***	0.3225	3.0718***	0.3514	-0.9084
	(0.0232)	(0.0000)	(0.2871)	(0.0000)	(0.5526)	(0.1969)
Foreign Longevity at 65	0.5673***	1.1308***	0.3297***	1.3272***	0.6721***	0.8628***
	(0.0000)	(0.0000)	(0.0093)	(0.0000)	(0.0000)	(0.0000)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2070	2085	1341	1343	729	742
R2	0.7289	0.4357	0.7489	0.4187	0.6393	0.3490

p-values in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

spending and the debt stock. That provides an average one-year interest rate on government debt. The output growth rate is the growth rate of nominal GDP measured in local currency.

The long-run relationship between the differential and demographics will be estimated by the following equation:

$$y_{it} = \alpha_{i0} + \rho y_{it-1} + \alpha_{i1}\gamma_{Nit} + \alpha_{i2}\ell_{it} + X'_{it}\beta + \epsilon_{it} \tag{28}$$

where y_{it} represents the interest-growth differential, γ_{Nit} is the growth rate of the working-age population, ℓ_{it} is life expectancy at 65 and X_{it} represents the controls. The auto-regressive component is added so as to capture the protracted effects that changes in the regressors and the error have on the differential.

Table 1 shows the estimation of equation (28) for the full sample and two sub samples: advanced economies only and emerging economies only. Three methods were chosen: pooled and grouped FMOLS and fixed effects panel data estimation. In terms of the controls, the model suggests the inclusion of public debt as a percentage of GDP, the labor share, the growth rate of TFP and foreign demographics. Foreign demographics are included as advanced economies' cross-sectional average of working-age population growth and life expectancy at 65 weighted by countries' real GDP.

As the discussion in section 3.2 made clear, an adequate method to estimate the long-run relationship between the dependent variable and the regressors with the presence of unit

Table 2: Stabilizing Primary Balance

Dep. Variable: Stabilizing Balance Sample	(1) Full	(2) Full	(3) Advanced	(4) Advanced	(5) Emerging	(6) Emerging
Stabilizing Balance (-1)	0.7800***		0.8263***		0.7004***	
	(0.0000)		(0.0000)		(0.0000)	
Working-Age Pop. Growth (%)	-0.1900***	-0.7648***	-0.1171	-1.0101***	-0.2444*	-0.4655***
	(0.0024)	(0.0000)	(0.1436)	(0.0000)	(0.0561)	(0.0068)
Longevity at 65	-0.1779***	-0.0811	-0.1369*	-0.2939**	-0.2062***	-0.2250**
	(0.0008)	(0.3286)	(0.0980)	(0.0374)	(0.0099)	(0.0339)
Foreign WA Pop. Growth (%)	0.1330	0.6598***	-0.0333	0.9777***	-0.2609	-1.4240***
	(0.4090)	(0.0091)	(0.8689)	(0.0044)	(0.3911)	(0.0004)
Foreign Longevity at 65	0.1902***	0.4190***	0.0828	0.7155***	0.1544**	0.0732
	(0.0004)	(0.0000)	(0.3318)	(0.0000)	(0.0327)	(0.4469)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2070	2085	1341	1343	729	742
R2	0.6987	0.3192	0.6902	0.2948	0.6706	0.2948

p-values in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

roots would be the FMOLS. The pooled estimation assumes that the marginal effects are homogeneous across countries and is shown in the first 6 specifications. Note that a significant negative marginal effect is already present for the growth rate of the working-age population even without domestic or foreign controls. On the other hand, the sign of the marginal effect of longevity at 65 is estimated to be significantly positive when not controlling for the effects of foreign demographics. The addition of foreign demographics recover a negative effect for a country's own life expectancy. The effect of foreign population growth is positive, which implies that as fertility falls in foreign countries, that shrinks the domestic country's interest-growth differential, a positive fiscal impact. However, the marginal effect of foreign life expectancy is positive, at odds with the model, implying a negative fiscal impact of increased life expectancy abroad.

Next, we check the correlation of the residuals

3.4 The Fiscal Space

The interest-growth differential is a simple observable component of the law of motion of the debt-to-GDP ratio. However, finding a sufficient statistic that captures the essence of fiscal space implied by Definition 1 is not straightforward.

One possible measure is the primary balance that stabilizes the current debt-to-GDP ratio assuming a constant interest-growth differential at its current level.

4 Conclusion

The demographic transition has presented countries many challenges as contributions to the pension system fall and disbursements rise. On the other hand, as many studies have argued, it also depress interest rates as agents save more to finance a longer period in retirement and the scarcer labor force leads to higher wages and lower returns on capital. At a first glance, the impact on debt dynamics is not clear, one has to know what the relative magnitudes of both effects are.

This study finds that the fiscal space (i.e. the present value of the primary balance that finances the social security budget when it is not balanced out) will certainly shrink when population ages due to a fall in population growth, but might shrink or expand when it ages due to an increase in life expectancy. The implication is that it is not sufficient to evaluate the net flow of resources in or out of the social security budget to infer the fiscal impact of population aging, one also has to consider the source of aging. Different compositions of demographic change will have different effects on the interest-growth differential and thus on fiscal space.

The empirical analysis confirms the analytical results to an extent. A panel of advanced and emerging economies is utilized to estimate the long-run relationship between, first, the interest-growth differential and demographics, and, second, a measure of fiscal space and demographics. The estimates suggest an increase of 1.52 percentage points (pp) in the interest-growth differential following a 1 pp fall in the growth rate of working-age population and a decrease of 1 pp in the differential following a 5-year increase in life expectancy at 65 years old, comparable changes in demographics across time.

The estimated effects on the primary balance that stabilizes the debt-to-GDP ratio are similar in sign: an increase of 0.76 percentage points (pp) in the interest-growth differential of a 1 pp fall in the growth rate of working-age population and a decrease of 1 pp of a 5-year increase in life expectancy at 65 years old.

Projections of future population aging by the United Nations (United Nations, 2022) show a very rapid fall in emerging countries' growth rate of working-age population, much faster than the one experienced by advanced economies. This suggests that those economies are in for a much more negative fiscal impact than what was experienced by advanced economies, a scenario that calls for crucial social security reform.

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Appendices

A Proofs of Propositions

A.1 Proposition 1

Proof.

Part 1: It suffices to show that the elasticity of the real interest rate with respect to the population growth rate is bigger than the elasticity of the output growth rate to the population growth rate.

$$\frac{\partial R_t}{\partial \gamma_{Nt}}\frac{\gamma_{Nt}}{R_t} = (\alpha - 1)\frac{\partial k_t}{\partial \gamma_{Nt}}\frac{\gamma_{Nt}}{k_t} \tag{29}$$

$$\frac{\partial \gamma_{Yt}}{\partial \gamma_{Nt}}\frac{\gamma_{Nt}}{\gamma_{Yt}} = 1 + \alpha\frac{\partial k_t}{\partial \gamma_{Nt}}\frac{\gamma_{Nt}}{k_t} \tag{30}$$

$$\frac{\partial R_t}{\partial \gamma_{Nt}}\frac{\gamma_{Nt}}{R_t} > \frac{\partial \gamma_{Yt}}{\partial \gamma_{Nt}}\frac{\gamma_{Nt}}{\gamma_{Yt}} \iff \frac{\partial k_t}{\partial \gamma_{Nt}}\frac{\gamma_{Nt}}{k_t} > -1 \tag{31}$$

which means that we just need to check whether the capital stock in efficiency units is inelastic to the population growth rate. Apply the implicit function theorem to the market clearing equation (15):

$$\frac{\partial k_t}{\partial \gamma_{Nt}}\frac{\gamma_{Nt}}{k_t} = -\frac{\gamma_{At}\gamma_{Nt}}{\frac{1-\alpha}{\alpha}\left(\frac{\delta_t}{1+\beta\delta_t}\nu\gamma_{At} + \varphi^*_{t-1}\frac{\delta^*_t}{1+\beta^*\delta^*_t}\nu^*\gamma^*_{At}\right) + (\gamma_{At}\gamma_{Nt} + \varphi^*_{t-1}\gamma^*_{At}\gamma^*_{Nt})} \tag{32}$$

As long as $\varphi^*_{t-1} > 0$ and/or $\nu \neq 0$, the capital stock in efficiency units will be inelastic to the population growth rate and, as a result, the interest-growth differential will expand when the population growth rate falls.

Part 2: Here the result will depend on current savings.

$$\frac{\partial R_t}{\partial \delta_t}\frac{\delta_t}{R_t} = (\alpha - 1)\frac{\partial k_t}{\partial \delta_t}\frac{\delta_t}{k_t} \tag{33}$$

$$\frac{\partial \gamma_{Yt}}{\partial \delta_t}\frac{\delta_t}{\gamma_{Yt}} = \alpha\frac{\partial k_t}{\partial \delta_t}\frac{\delta_t}{k_t} \tag{34}$$

$$\frac{\partial R_t}{\partial \delta_t}\frac{\delta_t}{R_t} \gtreqless \frac{\partial \gamma_{Yt}}{\partial \delta_t}\frac{\delta_t}{\gamma_{Yt}} \iff \frac{\partial k_t}{\partial \delta_t}\frac{\delta_t}{k_t} \lesseqgtr 0 \tag{35}$$

$$
\begin{aligned}
\frac{\partial k_t}{\partial \delta_t} &= \frac{\frac{\beta}{(1+\beta\delta_t)^2}((1-\alpha)-\tau_{t-1})k_{t-1}^{\alpha} - \frac{1-\alpha}{\alpha}\frac{1}{(1+\beta\delta_t)^2}\nu\gamma_{At}k_t}{\frac{1-\alpha}{\alpha}\left(\frac{\delta_t}{1+\beta\delta_t}\nu\gamma_{At} + \varphi_{t-1}^{*}\frac{\delta_t^{*}}{1+\beta^{*}\delta_t^{*}}\nu^{*}\gamma_{At}^{*}\right) + (\gamma_{At}\gamma_{Nt} + \varphi_{t-1}^{*}\gamma_{At}^{*}\gamma_{Nt}^{*})} \\
&= \frac{1}{\delta_t(1+\beta\delta_t)}\frac{s_t/A_t}{\left[\frac{1-\alpha}{\alpha}\left(\frac{\delta_t}{1+\beta\delta_t}\nu\gamma_{At} + \varphi_{t-1}^{*}\frac{\delta_t^{*}}{1+\beta^{*}\delta_t^{*}}\nu^{*}\gamma_{At}^{*}\right) + (\gamma_{At}\gamma_{Nt} + \varphi_{t-1}^{*}\gamma_{At}^{*}\gamma_{Nt}^{*})\right]}
\end{aligned}
$$

$$
s_t \gtrless 0 \iff \frac{\partial k_t}{\partial \delta_t}\frac{\delta_t}{k_t} \gtrless 0 \iff \frac{\partial \frac{R_t}{\gamma_{Yt}}}{\partial \delta_t} \lessgtr 0 \tag{36}
$$

□

$$
\frac{\partial R_{t+2}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{R_{t+2}} = (\alpha-1)\frac{\partial k_{t+2}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{k_{t+2}} \tag{37}
$$

$$
\frac{\partial \gamma_{Yt+2}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{\gamma_{Yt+2}} = \alpha\left(\frac{\partial k_{t+2}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{k_{t+2}} - \frac{\partial k_{t+1}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{k_{t+1}}\right) \tag{38}
$$

$$
\frac{\partial R_{t+2}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{R_{t+2}} < \frac{\partial \gamma_{Yt+2}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{\gamma_{Yt+2}} \iff \frac{\partial k_{t+2}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{k_{t+2}} > \alpha\frac{\partial k_{t+1}}{\partial \gamma_{Nt+1}}\frac{\gamma_{Nt+1}}{k_{t+1}} \tag{39}
$$

1.2 Proposition 2

Proof.

□

1.3 Proposition 5

Proof.
Part 1: The effect of changes in population growth on the fiscal space is straightforward given that the impacts on the primary surplus and on the interest-growth differential go in the same direction.

The impact of a *fall* in the population growth rate on the primary balance is negative.

$$\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^{d}}{\partial\gamma_{Nt}}=\frac{\gamma_{Y,t}}{R_t}\frac{\nu(1-\alpha)\delta_t}{\gamma_{Nt}^2}>0 \tag{40}$$

$$\begin{aligned}
\sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)}{\partial\gamma_{Nt}}\sigma_{t+i} &= \frac{\partial\frac{\gamma_{Y,t}}{R_t}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t}}{R_t}}\frac{\frac{\gamma_{Y,t}}{R_t}}{\gamma_{Nt}}\sigma_t+\\
&+\left(\frac{\partial\frac{\gamma_{Y,t}}{R_t}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t}}{R_t}}+\frac{\partial\frac{\gamma_{Y,t+1}}{R_{t+1}}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t+1}}{R_{t+1}}}\right)\frac{\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}}{\gamma_{Nt}}\sigma_{t+1}+\\
&+\left(\frac{\partial\frac{\gamma_{Y,t}}{R_t}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t}}{R_t}}+\frac{\partial\frac{\gamma_{Y,t+1}}{R_{t+1}}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t+1}}{R_{t+1}}}+\frac{\partial\frac{\gamma_{Y,t+2}}{R_{t+2}}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t+2}}{R_{t+2}}}\right)\frac{\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}\frac{\gamma_{Y,t+2}}{R_{t+2}}}{\gamma_{Nt}}\sigma_{t+2}+\cdots
\end{aligned}$$

Now rearrange according to the changes in the interest-growth differential in each period:

$$\begin{aligned}
\sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}^{i}\frac{\gamma_{Y,t+j}}{R_{t+j}}\right)}{\partial\gamma_{Nt}}\sigma_{t+i} &= \frac{\partial\frac{\gamma_{Y,t}}{R_t}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t}}{R_t}}\frac{1}{\gamma_{Nt}}\left(\frac{\gamma_{Y,t}}{R_t}\sigma_t+\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}\sigma_{t+1}+\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}\frac{\gamma_{Y,t+2}}{R_{t+2}}\sigma_{t+2}+\cdots\right)\\
&+\frac{\partial\frac{\gamma_{Y,t+1}}{R_{t+1}}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t+1}}{R_{t+1}}}\frac{1}{\gamma_{Nt}}\left(\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}\sigma_{t+1}+\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}\frac{\gamma_{Y,t+2}}{R_{t+2}}\sigma_{t+2}+\cdots\right)\\
&+\frac{\partial\frac{\gamma_{Y,t+2}}{R_{t+2}}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t+2}}{R_{t+2}}}\frac{1}{\gamma_{Nt}}\left(\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}\frac{\gamma_{Y,t+2}}{R_{t+2}}\sigma_{t+2}+\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}\frac{\gamma_{Y,t+2}}{R_{t+2}}\frac{\gamma_{Y,t+3}}{R_{t+3}}\sigma_{t+3}+\cdots\right)\\
&+\cdots\\
&=\frac{1}{\gamma_{Nt}}\left(\frac{\partial\frac{\gamma_{Y,t}}{R_t}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t}}{R_t}}b_t+\frac{\partial\frac{\gamma_{Y,t+1}}{R_{t+1}}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t+1}}{R_{t+1}}}\frac{\gamma_{Y,t}}{R_t}b_{t+1}+\frac{\partial\frac{\gamma_{Y,t+2}}{R_{t+2}}}{\partial\gamma_{Nt}}\frac{\gamma_{Nt}}{\frac{\gamma_{Y,t+2}}{R_{t+2}}}\frac{\gamma_{Y,t}}{R_t}\frac{\gamma_{Y,t+1}}{R_{t+1}}b_{t+2}+\cdots\right)\\
&>0 \text{ using Propositions 1 and 2, if } b_{t+i}\geq 0, \forall i=1,2,\ldots
\end{aligned}$$

□