Demographics and Real Interest Rates Across Countries and Over Time

(Preliminary version)*

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Abstract

We explore the implications of demographic trends for the evolution of real interest rates across countries and over time. To that end, we first develop a tractable three-country general equilibrium model with imperfect capital mobility and country-specific demographic trends. We calibrate the model to study how low-frequency movements in a country's real interest rate depends on its own demographics and on global factors, given a certain degree of financial integration. The more financially integrated a country is, the higher the sensitivity of its real interest rate to global developments, and the less its own real rate determinants matter. We then estimate panel error-correction models relating real interest rates to possible determinants—demographics included—imposing some restrictions motivated by lessons from the structural model. Our empirical evidence supports a meaningful role for life expectancy in determining real interest rates.

JEL codes: E52, E58, J11

Keywords: Life expectancy, population growth, demographic transition, real interest rate, imperfect capital mobility, capital flows, Secular Stagnation

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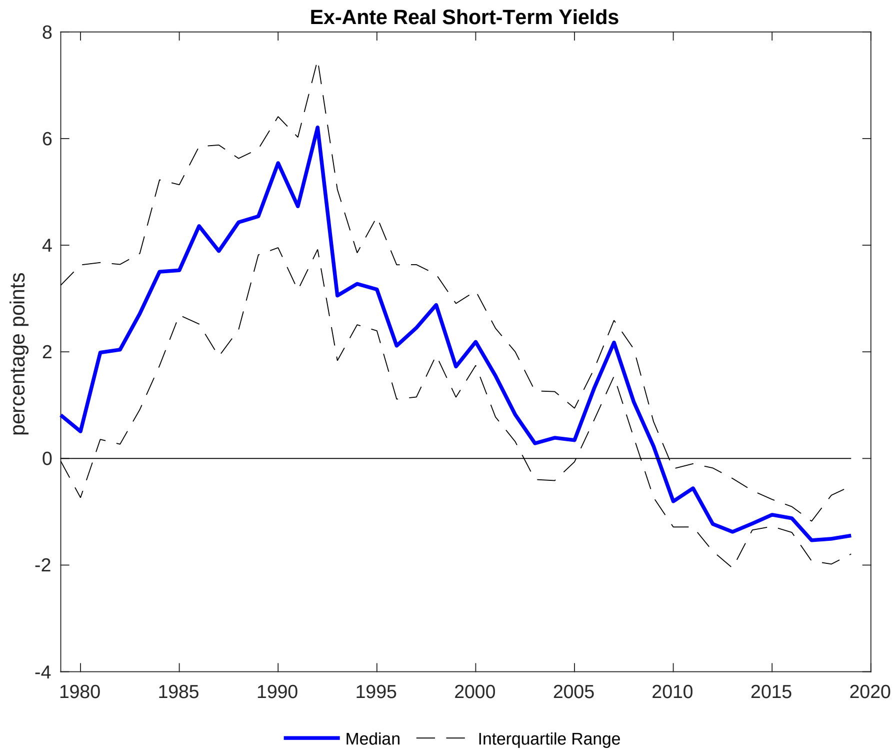
Figure 1: Real interest rates.



Note: Median and interquartile range of ex-ante real short-term interest rates for 19 OECD countries between 1979 and 2019. See section 4 for details about the calculations.

1 Introduction

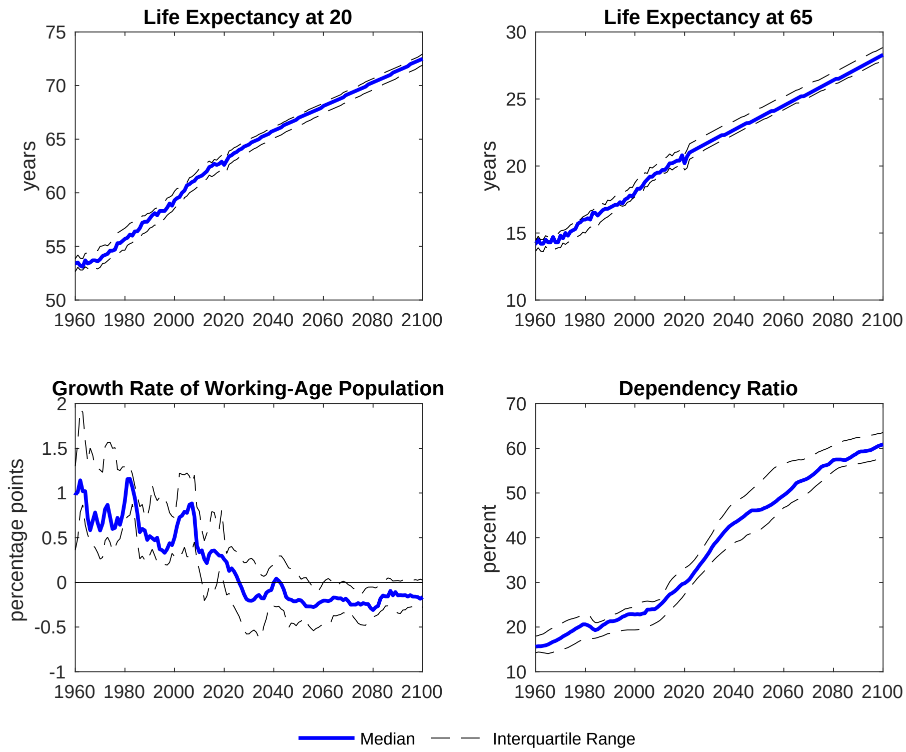
Between 1990 and 2020, real interest rates in advanced economies exhibited a persistent decline (Figure 1). In a recent commentary, Blanchard (2023) argues that once the current bout of inflation dissipates, advanced economies will revert back to a chronic condition of low real interest rates. Understanding the the foundations of this conjecture is a key macroeconomic question, which has profound ramifications for both fiscal and monetary policy, means uncovering the drivers of the secular fall in real rates.

The low-frequency nature of real interest rate movements observed before the Covid-19 pandemic suggests that long-term forces are likely playing a crucial role (Rachel and Smith, 2017).[1] In this paper, we focus on the connection between one of these long-term forces—demographic factors—and the secular decline of real interest rates.

The data clearly show that advanced economies are aging at a fast pace (Figure 2). Between 1960 and 2020, median life expectancy at 20 has increased by about 9 years, from 53.4 to 62.6 (top-left panel). Over the same period, older generations have also experienced significant longevity gains, with median

[1]The debate in the literature, however, is far from being one-sided. At the other end of the spectrum, Hamilton et al. (2016) find little evidence of permanent factors in the decline in global real interest rates. Their empirical findings attribute the decline to temporary factors, including deleveraging, tighter financial regulation, and inflation trends.

Figure 2: Demographic variables.



Note: Top-left panel: Years of life expectancy at 20. Top-right panel: Years of life expectancy at 65. Bottom-left panel: Growth rate of working-age population. Bottom-right panel: Number of people 65 and older relative to the number of people 15 to 64 years old. Sample: 19 OECD countries between 1960 and 2100 (projections after 2020). Source: United Nations World Population Prospects (2021 Revision).

life expectancy at 65 increasing from 14.2 years to 20.2 years (top-right panel). Meanwhile, fertility rates have fallen sharply, implying a correspondent decline in the growth rate of the working-age population from approximately 1% in 1960 to 0.26% in 2020 at the median (bottom-left panel). The combination of lower fertility and higher longevity has has roughly doubled the old-age dependency ratio (the ratio between people 65 years old and above to people 15 to 64 years old) from 15.6 in 1960 to 29.8 in 2020 (bottom-right panel). Going forward, the available demographic projections suggest that in advanced economies working-age population will contract at a rate of approximately 0.2% per year, while life expectancy will continue to increase, so that, by the end of this century, the dependency ratio will be over 60%.

Building on the existing literature, including our earlier work (Carvalho et al., 2016), we show that past demographic developments and available projections can explain a significant portion of the real interest rate drop observed between 1990 and 2020. The key contribution of this paper is to confront the demographics-based explanation of low real interest rates with the open economy dimension. If demographics are indeed an important driver of low-frequency movements in real rates, we should expect to see patterns in real interest rates across countries and over time that accord with the relevant demographic developments. That assessment is, however, complicated by at least two issues. First, in a world with international capital mobility, a country's real rate should depend not only on its own demographic developments, but also on global factors. Second, other variables may affect real rates, so that uncovering the role of any given driver requires carefully controlling for many other potential explanations.

To handle these two issues, we resort to both quantitative theory and econometric analysis. We begin by developing a multi-country life-cycle model in which households can invest in both domestic and foreign assets. Investment in foreign assets is subject to portfolio holding costs which proxy for various factors that can hinder capital flows in practice. As a result, international capital mobility is imperfect in the model.[2] In our framework, demographic trends affect the equilibrium real interest rate through changes to the growth rate of the labor force and to life expectancy. Population composition effects are an endogenous results of these two fundamental forces. Crucially, because households can trade assets internationally, demographic developments in one country affect the others as well.

A calibrated three-country version of the model captures the salient features of the demographic transition in developed economies. We use this framework to study the low-frequency relationship between demographics and real interest rates, and how the degree of financial integration shapes this relationship across countries and over time. In particular, we focus on how a country's real interest rate depends on its own and on global demographic developments as financial integration changes over time as in the data. Increased financial integration shifts the sensitivity of a country's real interest rate away from own demographic developments and towards global determinants.

Drawing on the lessons from the model, we then turn to an empirical analysis of the long-run relationship between demographics and real interest rates. To address the second challenge that other factors may drive real interest rates, we estimate panel error-correction models with a set of controls motivated by the literature on top of the demographic variables that the model suggests. Consistently with our theoretical analysis, we control for global factors through a measure of the relevant global real interest rate faced by each country, and interact countries' possible real rate determinants with a measure of their degree of financial integration. The global rate is always statistically significant. Among the domestic factors, demographic variables are significant in most specifications.

Our paper belongs to a recent wave of research that investigates, both theoretically and empirically, the determinants of real interest rates. A number of existing contributions focus on demographics, calibrating overlapping generation models to individual economies, such as the U.S. (Gagnon et al., 2021), the euro area (Kara and von Thadden, 2016), and Japan (Ikeda and Saito, 2014).[3] Our focus on the open economy dimension is closer in spirit to Lisack et al. (2021) and, especially, Krueger and Ludwig (2007), who also discuss the interaction between demographics and financial integration. A key difference

[2]The limiting cases of the two-country version of our model with zero and infinite portfolio costs correspond to Ferrero (2010) and Carvalho et al. (2016), respectively.

[3]Demographic variables feature prominently also among the factors that can explain the secular stagnation hypothesis (Eggertsson et al., 2019). Goodhart and Pradhan (2017) express a contrarian view, arguing that demographic trends will contribute to revert recent observed macroeconomic trends, including for real interest rates. As noted in Carvalho et al. (2016) and Blanchard (2023), this view neglects the role of increased life expectancy on workers' savings decisions during their employment spell to finance a longer retirement period.

relative to our paper is that those contributions only consider the two extreme cases of closed economies or fully-integrated capital markets. Nevertheless, we reach similar conclusions as far as real interest rate predictions are concerned.

Empirically, Lunsford and West (2019) conclude that demographic variables can explain some of the variability of US real interest rates over more than one hundred years, while Fiorentini et al. (2018) highlight the importance of the share of young workers to account for the rise and fall of real rates between 1960 and 2016. Our empirical analysis expands on this second paper by pursuing an econometric specification that is informed by our structural model and by considering a number of additional candidate explanations, such as productivity growth (Holston et al., 2017), fiscal variables (Rachel and Summers, 2019), the convenience yield (Del Negro et al., 2017; Del Negro et al., 2019), and inequality (Eggertsson et al., 2019; Mian et al., 2021). Despite this additional set of potential drivers, demographic variables remain a key determinant of real interest rates in most specifications of our panel analysis.

In our model, the real interest is the return on both government bonds, physical capital, and private claims. In practice, these returns differ. As Gomme et al. (2015) have documented for the US, while the return on safe assets (primarily government bonds in advanced economies) has declined, the return on risky assets (in particular equity) has remained roughly constant. Reis (2022) finds that this result is robust across countries, and also to different measures of capital and income. By abstracting from aggregate uncertainty and imperfect competition, our model fails to capture the rise of macroeconomic risk and markups that Farhi and Gourio (2018) and Eggertsson et al. (2021) argue are key drivers of the wedge between the return on equity and the return on government bonds.[4] Therefore, we focus on the comparison between the real interest rate in the model with the return on government bonds in the data.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 discusses the calibration and the quantitative experiments that illustrate how the relationship between demographics and real rates varies with the degrees of financial integration across countries. Section 4 reports our empirical analysis. Finally, Section 5 concludes.

2 The Model

This section presents an open economy life-cycle model with imperfect capital mobility. Building on Gertler (1999), we allow for time-varying differential demographic trends across countries. Portfolio-holding costs, as in Chang et al. (2015), hamper the free flow of capital across countries. The resulting framework nests the closed economy model of Carvalho et al. (2016) and the open-economy model of Ferrero (2010) as special cases.

[4]In addition, Farhi and Gourio (2018) also find a role for the rising importance of intangibles in production.

2.1 Demographics

The economy consists of $\mathcal{M}$ regions. In each region $m = 1, \ldots, \mathcal{M}$, $(1-\omega_{mt}+n_{mt})N^w_{mt-1}$ new workers (w) are born in every period t, where N^w_{mt-1} is the number of workers at time $t-1$ and ω_{mt} is the probability a worker remains in the labor force between periods $t-1$ and t. Therefore, the number of workers evolves according to

$$N^w_{mt} = (1-\omega_{mt}+n_{mt})N^w_{mt-1} + \omega_{mt}N^w_{mt-1} = (1+n_{mt})N^w_{mt-1},$$

so that n_{mt} is the net growth rate of the labor force.

A worker who exits the labor force becomes a retiree (r). The probability of a retiree surviving between periods $t-1$ and t is γ_{mt}. Therefore, the number of retirees evolves according to

$$N^r_{mt} = (1-\omega_{mt})N^w_{mt-1} + \gamma_{mt}N^r_{mt-1}.$$

The (old) dependency ratio, $\psi_{mt} \equiv N^r_{mt}/N^w_{mt}$, measures the number of retirees per worker, and evolves according to

$$(1+n_{mt})\psi_{mt} = (1-\omega_{mt}) + \gamma_{mt}\psi_{mt-1}. \tag{1}$$

The growth rate of the labor force and the probability of surviving as a retiree are the fundamental demographic variables in the model. In our quantitative exercises, we will measure the growth rate of the labor force directly from the data. Conditional on a given retirement age, we will back out the probability of surviving from the evolution of the (old) dependency ratio, which is an observable variable, using equation (1).

2.2 Preferences

Retirees and workers value consumption streams according to recursive non-expected utility preferences (Epstein and Zin, 1989)

$$V^z_{mt} = \max\left[(C^z_{mt})^{\frac{\sigma-1}{\sigma}} + \beta^z_{mt+1}\mathbb{E}_t\left(V^{1-\theta}_{mt+1}|z\right)^{\frac{\sigma-1}{\sigma(1-\theta)}}\right]^{\frac{\sigma}{\sigma-1}},$$

where $z = \{w, r\}$ represents the agent's type, V^z_{mt} represents the value of utility for an agent of type z at time t, C^z_{mt} denotes consumption of the single good, $\sigma > 0$ is the elasticity of intertemporal substitution, and θ is the coefficient of risk aversion. The discount factor $\beta^z_{mt+1} > 0$ differs between retirees and workers:

$$\beta^z_{mt+1} = \begin{cases} \beta_m & \text{if } z = w \\ \beta_m\gamma_{mt+1} & \text{if } z = r. \end{cases}$$

With standard preferences (e.g. constant relative risk aversion), the assumption that the transition probability into retirement is independent of age would give rise to excessive precautionary savings early in life. To mitigate this effect while retaining the sensitivity of consumption decisions to changes in the interest rate, we assume risk neutrality with respect to income fluctuations ($\theta = 0$).[5]

2.3 Retirees

The problem of a retiree from country m born in period j and retired in period k is

$$V_{mt}^{rjk} = \max_{C_{mt}^{rjk}, \left\{A_{m\ell t}^{rjk}\right\}_{\ell=1}^{\mathcal{M}}} \left[\left(C_{mt}^{rjk}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \gamma_{mt+1} \left(V_{mt+1}^{rjk}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

subject to

$$C_{mt}^{rjk} + \left[1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{rjk} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{rjk} = \frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{rjk} + E_{mt}^{rjk}, \tag{3}$$

where $\eta_{m\ell t}^{rjk} \equiv A_{m\ell t}^{rjk} / (\sum_{p=1}^{\mathcal{M}} A_{mpt}^{rjk})$ are portfolio shares, $A_{m\ell t}^{rjk}$ are assets that a retiree of country m holds against country $\ell = 1, \ldots, \mathcal{M}$ and pay a gross return is $R_{\ell t}$. At the beginning of each period, retirees turn their wealth to a perfectly competitive mutual fund that pools the risk of death and pays an extra return equal to the inverse of the survival probability, as in Yaari (1965) and Blanchard (1985). In addition, a retiree receives a lump-sum pension benefit E_{mt}^{rjk} from the government. In forming their portfolios, retirees incur a cost that depends on the difference between the actual share invested in foreign assets $\eta_{m\ell t}^{rjk}$ and an exogenous target level $\bar{\eta}_{m\ell}$ that we assume to be independent of type and that pins down steady state gross foreign asset positions.[6] Following Chang et al. (2015), we assume that portfolio-holding costs are quadratic, and that their level depends on a time-varying parameter $\Lambda_{m\ell t} \geq 0$. By limiting capital flows across countries, portfolio costs capture, in reduced form, all the factors that prevent equalization of real interest rates across countries, even after controlling for risk premia.[7] One interpretation of our adjustment cost formulation is that retirees delegate their investment to portfolio managers who charge a fee to invest in foreign assets.[8]

Appendix A.1 shows that the share a retiree invests in country-p assets (with $p \neq m$) is independent of age and time since retirement ($\eta_{mpt}^{rjk} = \eta_{mpt}^{r}$, $\forall j$ and k), and satisfies

$$\left[1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell}\right)^2\right] (R_{pt} - R_{mt}) = \Lambda_{mpt} \left(\eta_{mpt}^{r} - \bar{\eta}_{mp}\right) R_{mt}. \tag{4}$$

[5] The resulting preferences imply linear consumption decisions (Farmer, 1990), thus facilitating aggregation among workers and retirees of different ages.

[6] The portfolio cost "rate" is positive as long as the share of foreign assets deviates from $\bar{\eta}_{mn}$. The assumption that this cost rate applies to total assets keeps the model tractable, allowing us to make substantial analytical progress based on a guess-and-verify approach.

[7] When $\Lambda_{m\ell t}$ tends to infinity, the countries in the model become closed, as in Carvalho et al. (2016). Conversely, when these parameters are equal to zero, the model corresponds to a multi-country version of Ferrero (2010).

[8] In our formulation, the government collects the costs so we can think of government regulation limiting foreign financial investment. Alternatively, we could write these costs in terms of real resources, possibly devoted to research about foreign financial markets.

In addition, Appendix A.2 shows that the same condition holds for workers. Therefore, retirees and workers invest the same share of their total financial wealth in country-p assets ($\eta^r_{mpt} = \eta^w_{mpt} = \eta_{mpt}$).

The solution of the optimization problem for a retiree yields a consumption function linear in the sum of total financial wealth and the present discounted value of pension benefits (S^{rjk}_{mt}):

$$C^{rjk}_{mt} = \xi^r_{mt} \left(\frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^{rjk}_{m\ell t-1} + S^{rjk}_{mt} \right), \tag{5}$$

where

$$S^{rjk}_{mt} = E^{rjk}_{mt} + \frac{\gamma_{mt+1} S^{rjk}_{mt+1}}{\tilde{R}_{mt}},$$

and where we have defined the adjusted gross return

$$\tilde{R}_{mt} \equiv \frac{\sum_{\ell=1}^{\mathcal{M}} \eta_{m\ell t} R_{\ell t}}{1 + \sum\limits_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} (\eta_{m\ell t} - \bar{\eta}_{m\ell})^2}. \tag{6}$$

The marginal propensity to consume is also independent of individual characteristics (birth and retirement age), and evolves according to the Euler equation

$$\frac{1}{\xi^r_{mt}} = 1 + \gamma_{mt+1} \beta^\sigma_m \tilde{R}^{\sigma-1}_{mt} \frac{1}{\xi^r_{mt+1}}. \tag{7}$$

2.4 Workers

In every period, workers need to take into account the possibility of retirement. Thus, with probability ω_{mt+1}, the continuation value for an individual born in period j and currently employed is V^{wj}_{mt+1}, and is V^{rjt+1}_{mt+1} with the complementary probability. The full optimization problem is

$$V^{wj}_{mt} = \max_{C^{wj}_{mt}, \{A^{wj}_{m\ell t}\}_{\ell=1}^{\mathcal{M}}} \left\{ \left(C^{wj}_{mt} \right)^{\frac{\sigma-1}{\sigma}} + \beta_m \left[\omega_{mt+1} V^{wj}_{mt+1} + (1-\omega_{mt+1}) V^{rjt+1}_{mt+1} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

subject to

$$C^{wj}_{mt} + \left[1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta^{wj}_{m\ell t} - \bar{\eta}_{m\ell} \right)^2 \right] \sum_{\ell=1}^{\mathcal{M}} A^{wj}_{m\ell t} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^{wj}_{m\ell t-1} + W^w_{mt} - T^w_{mt}, \tag{9}$$

where W^w_t is the real wage and T^w_{mt} are lump-sum taxes, which only workers pay.[9]

As already mentioned, all workers optimally choose the same portfolio shares, which also equal the choice of retirees. Workers' consumption is linear in the sum of total financial wealth, human wealth, and

[9] As in Ferrero (2010) and Carvalho et al. (2016), we assume that workers inelastically supply one unit of labor and that retirees do not work. Gertler (1999) shows how to relax both these assumptions. With endogenous labor, the optimal response to a declining growth rate of the labor force and an increase in life expectancy would be to supply more hours. Such a behavior of hours worked would be inconsistent with the data for most OECD economies (OECD, 2018).

the present discounted value of pension benefits:

$$C_{mt}^{wj} = \xi_{mt}^{w} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{wj} + H_{mt}^{wj} + Z_{mt}^{wj} \right), \tag{10}$$

where human wealth is

$$H_{mt}^{wj} = W_{mt}^{w} - T_{mt}^{w} + \frac{\omega_{mt+1} H_{mt+1}^{wj}}{\Omega_{mt+1} \tilde{R}_{mt}},$$

and the present discounted value of pension benefits for workers is

$$Z_{mt}^{wj} = \frac{1}{\Omega_{mt+1} \tilde{R}_{mt}} \left[(1 - \omega_{mt+1}) \left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}} \right)^{\frac{1}{1-\sigma}} S_{mt+1}^{rjt+1} + \omega_{mt+1} Z_{mt+1}^{wj} \right].$$

Workers discount both variables by the adjusted gross return (6) times an additional factor that takes into account the probability of retiring and the heterogeneity in the marginal propensity to consume between the two groups:

$$\Omega_{mt} \equiv \omega_{mt} + (1 - \omega_{mt}) \left(\frac{\xi_{mt}^{r}}{\xi_{mt}^{w}} \right)^{\frac{1}{1-\sigma}}. \tag{11}$$

Because retirees and workers discount the future at different rates, Ricardian equivalence does not hold in this model, even though taxes are lump-sum.

Finally, as for retirees, the marginal propensity to consume for workers is independent of individual characteristics and evolves according to

$$\frac{1}{\xi_{mt}^{w}} = 1 + \beta_m^{\sigma} \left(\Omega_{mt+1} \tilde{R}_{mt} \right)^{\sigma-1} \frac{1}{\xi_{mt+1}^{w}}. \tag{12}$$

2.5 Aggregation

Since marginal propensities to consume are independent of individual characteristics and consumption functions are linear, we can aggregate among workers and among retirees by simply adding over individuals in each group.[10]

Aggregate retirees' consumption is given by

$$C_{mt}^{r} = \xi_{mt}^{r} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{r} + S_{mt}^{r} \right).$$

Note that, in the aggregate, the extra-return the mutual fund offers corresponds to the fraction of retirees who survive between two periods because of the law of large numbers. Similarly, aggregate workers'

[10] In dropping reference to the birth and retirement period, we use the notation $\sum_r C_{mt}^{rjk} = N_{mt}^{r} C_{mt}^{rjk} \equiv C_{mt}^{r}$ and $\sum_w C_{mt}^{wj} = N_{mt}^{w} C_{mt}^{wj} \equiv C_{mt}^{w}$. The same notation applies to asset holdings, human wealth, and the present discounted value of pensions for retirees and workers.

consumption is given by

$$C_{mt}^{w}=\xi_{mt}^{w}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A_{m\ell t-1}^{w}+H_{mt}^{w}+Z_{mt}^{w}\right).$$

Aggregate consumption in country m is simply the sum of retirees' and workers' consumption:

$$C_{mt}\equiv C_{mt}^{w}+C_{mt}^{r}.$$

Finally, because of the heterogeneity between workers and retirees over the life cycle, we need to keep track of the distribution of wealth between these two groups. The result that retirees and workers from a given country choose the same portfolio shares is useful in this respect. First, given the total amount of country-n assets held by country-m agents, we define the share held by retirees as

$$\lambda_{mpt}\equiv\frac{A_{mpt}^{r}}{A_{mpt}^{r}+A_{mpt}^{w}}. \tag{13}$$

Second, from the definition of portfolio shares, $A_{mpt}^{z}=\eta_{mpt}A_{mt}^{z}$ for $z=\{r,w\}$, where $A_{mt}^{z}=\sum_{\ell=1}^{\mathcal{M}}A_{m\ell t}^{z}$. Using this definition in equation (13), we obtain

$$\lambda_{mpt}=\frac{A_{mt}^{r}}{A_{mt}^{r}+A_{mt}^{w}}=\lambda_{mt},$$

that is, the retirees' share of country-p assets held by country-m agents corresponds to the share of wealth accruing to retirees in country m. In appendix A.3, we show that, combining the budget constraints of retirees and workers, we can derive the evolution of the distribution of wealth, which obeys

$$\begin{aligned}&\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell}}{2}\left(\eta_{m\ell t}-\bar{\eta}_{m\ell}\right)^{2}\right]\left[\lambda_{mt}-(1-\omega_{mt+1})\right]A_{mt}\\&\qquad=\omega_{mt+1}\left[(1-\xi_{mt}^{r})\lambda_{mt-1}A_{mt-1}\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}\eta_{m\ell t-1}+E_{mt}-\xi_{mt}^{r}S_{mt}\right],\end{aligned} \tag{14}$$

where $A_{mt}\equiv A_{mt}^{r}+A_{mt}^{w}$ represents the total amount of assets held by country-m residents.

2.6 Firms

A continuum of measure one of perfectly competitive firms operate in each country. Firms hire workers and accumulate capital K_{mt} to produce the single good according to a labor-augmenting Cobb-Douglas technology, identical across countries:

$$Y_{mt}=\left(X_{mt}N_{mt}^{w}\right)^{\alpha}K_{mt-1}^{1-\alpha},$$

where $\alpha \in (0,1)$ and Y_{mt} represents output. The productivity factor grows exogenously at a rate x_{mt} between periods $t-1$ and t:

$$X_{mt} = (1 + x_{mt})X_{mt-1}.$$

The law of motion of capital is standard:

$$K_{mt} = (1 - \delta)K_{mt-1} + I_{mt},$$

where I_{mt} stands for investment and $\delta \in (0,1)$ is the depreciation rate.

The first order conditions for firms are standard. The wage is equal to the marginal product of labor

$$W^w_{mt} = \alpha \frac{Y_{mt}}{N^w_{mt}},$$

while the real interest rate is equal to the marginal product of capital

$$R_{mt} = (1 - \alpha)\frac{Y_{mt+1}}{K_{mt}} + (1 - \delta).$$

2.7 Government

In each period, the government issues one-period bonds, B_{mt}, and levies lump-sum taxes on workers $T_{mt} \equiv N^w_{mt}T^w_{mt}$ to fund pension benefits $E_{mt} \equiv N^r_{mt}E^r_{mt}$ and wasteful spending G_{mt}, and to repay maturing debt inclusive of interests to bondholders $R_{mt-1}B_{mt-1}$. The government also collects the amount of resources foreign investors forego to hold positions in country-m assets (the portfolio-holding costs). Its budget constraint is:

$$B_{mt} = R_{mt-1}B_{mt-1} + G_{mt} + E_{mt} - \left[T_{mt} + \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2}(\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell t} \right].$$

We assume that debt, spending, and pensions are exogenous fractions of output:

$$G_{mt} = g_{mt}Y_{mt}, \qquad B_{mt} = b_{mt}Y_{mt}, \qquad E_{mt} = e_{mt}Y_{mt},$$

so that the government budget constraint determines taxes residually.

2.8 Balance of Payments and Equilibrium

Country-m assets held by its residents correspond to the amount of capital and bonds not owned by foreigners:

$$A_{mmt} = K_{mt} + B_{mt} - \sum_{\ell \neq m} A_{\ell mt}.$$

Net foreign assets for country m equal the gross amount of foreign assets owned by its residents net of country-m assets held by foreigners:

$$F_{mt} \equiv \sum_{\ell \neq m} (A_{m\ell t} - A_{\ell mt}).$$

Net foreign assets evolve according to

$$F_{mt} = F_{mt-1} + \sum_{\ell \neq m} (R_{\ell t-1} - 1) A_{m\ell t-1} - (R_{mt-1} - 1) \sum_{\ell \neq m} A_{\ell mt-1} \\ - \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 A_{mt} + \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2} (\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell t} + NX_{mt},$$

where the trade balance is the difference between production and domestic absorption:

$$NX_{mt} \equiv Y_{mt} - C_{mt} - G_{mt} - I_{mt}.$$

Finally, the global asset market-clearing condition is:

$$\sum_{\ell=1}^{\mathcal{M}} F_{\ell t} = 0.$$

Because the labor force and technology grow over time, we focus on a solution for de-trended variables.[11] Given exogenous processes for the growth rate of the labor force, life expectancy, the growth rate of technology, and fiscal variables, a competitive equilibrium for the world economy requires that, in each country: retirees and workers maximize utility subject to their budget constraints; firms maximize profits given their technological constraints; the government satisfies its budget constraint; and labor markets clear. In addition, the asset market clears at the global level.

We solve the model with an "extended-path" approach, according to which agents form expectations in each period assuming that the exogenous processes will remain constant at their current values into the indefinite future. The extended-path solution concatenates the pointwise equilibrium values obtained in each period by solving for the perfect-foresight path from each period onward under those "constant beliefs." Like perfect foresight, the extended-path approach allows us to obtain a fully non-linear solution for the transition between steady states focusing on low-frequency dynamics (i.e., abstracting from fluctuations that are the focus of business cycle models with aggregate shocks). However, because of the constant-beliefs assumption, the extended-path approach avoids the excessive "front-loading" of responses that characterizes the standard perfect-foresight solution.

[11] For a generic variable D_t, the stationary counterpart is $d_t \equiv D_t/(X_t N_t^w)$. The model admits a well-defined steady state in terms of de-trended variables.

Table 1: Demographic variables in the initial steady state (in %).

Parameter	Young Economy	Old Economy	Rest of the World
Relative size ($N^w_{m0}/N^w_{\mathcal{W}0}$)	0.38	0.38	100
Growth rate of the labor force (n_{m0})	1.13	0.59	0.75
Dependency ratio (ψ_{m0})	20.98	24.46	22.29

3 Quantitative Analysis

Our main quantitative experiment characterizes the macroeconomic transition of the world economy between two steady states driven by differential demographic developments across countries and time-varying degrees of financial integration. For simplicity, we assume that technology and fiscal variables remain constant and equal across countries.

3.1 Calibration

Each period corresponds to one year. We specialize our multi-country model to a world with three regions: a small young economy ($\mathcal{Y}$), a small old economy ($\mathcal{O}$), and the rest of the world ($\mathcal{W}$). The "young" economy has a relatively high growth rate of the labor force and a relatively low dependency ratio, while the opposite is true for the "old" economy. The rest of the world captures the median demographic profile.

The source for the demographic data is the United Nation World Population Database (2019). We assume that the initial size of the rest of the world is equal to the sum of the working-age population across the twenty OECD economies in our sample in 1990 (3.78 billion), and we set the initial relative size of both the young and the old economy to 0.4% to match the first quartile of the cross-sectional distribution of sizes in the same year.

In the initial steady state, the young economy has a relatively high growth rate of the labor force and a relatively low dependency ratio. We target the initial growth rate of the working-age population and the dependency ratio in this region to match the third and first quartile, respectively, of their empirical counterparts, which gives $n_{\mathcal{Y}0} = 1.13\%$ and $\psi_{\mathcal{Y}0} = 20.98\%$. Vice versa, for the old economy, we target the first quartile for the growth rate of the working-age population and the third quartile for the dependency ratio ($n_{\mathcal{O}0} = 0.59\%$ and $\psi_{\mathcal{O}0} = 24.46\%$, respectively). Finally, for the rest of the world, we simply target the weighted average of the countries in our sample for both growth rate of the working-age population and dependency ratio ($n_{\mathcal{W}0} = 0.75\%$ and $\psi_{\mathcal{W}0} = 22.29\%$).[12] Table 1 reports the calibrated values for the demographic variables in the initial steady state.

We follow Gertler (1999) and Carvalho et al. (2016) in setting most of the remaining parameters that

[12] Conditional on the growth rate of the working-age population and on the probability of retiring (which we discuss in the text), the steady state version of equation (1), $\psi_m = (1-\omega)/(1+n_m-\gamma_m)$, provides a unique mapping between the dependency ratio of country m and a value for its probability of surviving.

Table 2: Common parameters across countries.

Parameter value			Description
ω	$=$	0.978	Average employment duration
σ	$=$	0.500	Elasticity of intertemporal substitution
α	$=$	0.667	Labor share
δ	$=$	0.100	Depreciation rate
x	$=$	0.005	Growth rate of productivity
$\bar{\eta}$	$=$	0	Steady state net foreign asset position
b	$=$	0.600	Debt/GDP
g	$=$	0.250	Government spending/GDP
e	$=$	0.075	Pensions/GDP

are common to all countries (Table 2). Agents are born workers at the age of 20. We fix the probability of remaining employed ω at 0.978 to match an average employment duration of 45 years so that on average individuals retire at 65. We set the elasticity of intertemporal substitution σ to 0.5, consistent with the evidence in Hall (1988) and Yogo (2004), who report values significantly lower than one. We further set the labor share α equal to 0.667 and the depreciation rate δ equal to 0.1, which are standard values in the literature. We assume that the growth rate of technology is $x = 0.5\%$, roughly in line with the average growth rate for the countries in our dataset since 1990. We calibrate fiscal variables (debt, government spending and pensions) as a fraction of GDP to match the average values for OECD countries since 1990, which implies $b = 60\%$, $g = 25\%$ and $e = 7.5\%$, respectively.

The remaining parameters to calibrate are the target shares for foreign asset holdings, the initial value of the portfolio holding costs and the individual discount factors. For simplicity, we assume the target shares for foreign asset holdings $\bar{\eta}$ to be zero in all countries.[13] Finally, for each country, we jointly choose the initial value of the portfolio-holding cost parameter $\Lambda_{mn0}(= \Lambda_{nm0})$ and of the individual discount factor β_m to target the real interest rate and the external position in the initial steady state. The real interest rate measure that we use is a three-year moving average centered in 1990 of the ex-ante short yield (the same data plotted in Figure 1). For the external position, we target the cross-country distribution of gross foreign debt relative to GDP from Lane and Milesi-Ferretti (2017). The focus on debt aligns well our real interest rate measure with the appropriate asset class in the data. While the model only keeps track of net foreign assets, we use the gross position as the empirical asset measure to limit the heterogeneity in discount factors necessary to match the initial dispersion of real interest rates. For the young economy, we match the third quartile of the empirical distribution of real interest rates (7.01%) and the first quartile of the gross foreign debt liabilities relative to GDP (39.36%), which gives $\beta_{\mathcal{Y}} = 0.987$, $\Lambda_{\mathcal{Y}\mathcal{O}0} = 300$ and $\Lambda_{\mathcal{Y}\mathcal{W}0} = 32.8595$.[14] Similarly, for the old economy, we match the

[13]In spite of the target, however, the foreign asset position in the initial steady state actually differs from zero because of the cross-country demographic differences.

[14]The high value for $\Lambda_{\mathcal{Y}\mathcal{O}0} = \Lambda_{\mathcal{O}\mathcal{Y}0}$ is a by-product of the calibration strategy. A high value of the portfolio-holding cost parameter corresponds to a country being in autarky. Starting from autarky, we then progressively lower the friction

Table 3: Discount factors and portfolio-holding costs.

Parameter	**Young Economy**	**Old Economy**	**Rest of the World**
Discount factor (β_m)	0.987	1.013	1.003
Portfolio holding costs ($\Lambda_{\mathcal{Y}n0}$)	0	300	32.8595
Portfolio holding costs ($\Lambda_{\mathcal{O}n0}$)	300	0	68.5684
Portfolio holding costs ($\Lambda_{\mathcal{W}n0}$)	32.8595	68.5684	0

first quartile of the distribution of real interest rates (3.56%) and the first quartile of gross foreign debt assets relative to GDP (16.79%), which gives $\beta_{\mathcal{O}} = 1.013$ and $\Lambda_{\mathcal{OW}0} = 68.5684$. Finally, for the global economy, we match the median real interest rate (5.28%) and a balanced external position, which gives $\beta_{\mathcal{W}} = 1.003$.[15] Table 3 summarizes this part of the calibration.

3.2 Experiment

In our baseline experiment, the size of the three economies and the degree of financial integration evolve endogenously over time in response to the evolution of the demographic variables and to changes of the portfolio-holding cost parameters. The targets for these variables are the same as for the steady state (growth rate of working-age population, dependency ratio, and gross foreign debt positions relative to GDP). In order to obtain projections over an arbitrary long simulation horizon, we feed the model with the trend component of a HP filter (Hodrick and Prescott, 1997) applied to the data (see Figure 3).[16] Due to the limited availability of the data for foreign debt, we consider the period 1990-2020 as our "sample." The simulation, however, uses the projected trends for demographic variables and foreign debt until 2070. We refer to the results for the period 2021-2070 as "projections."

3.3 Results

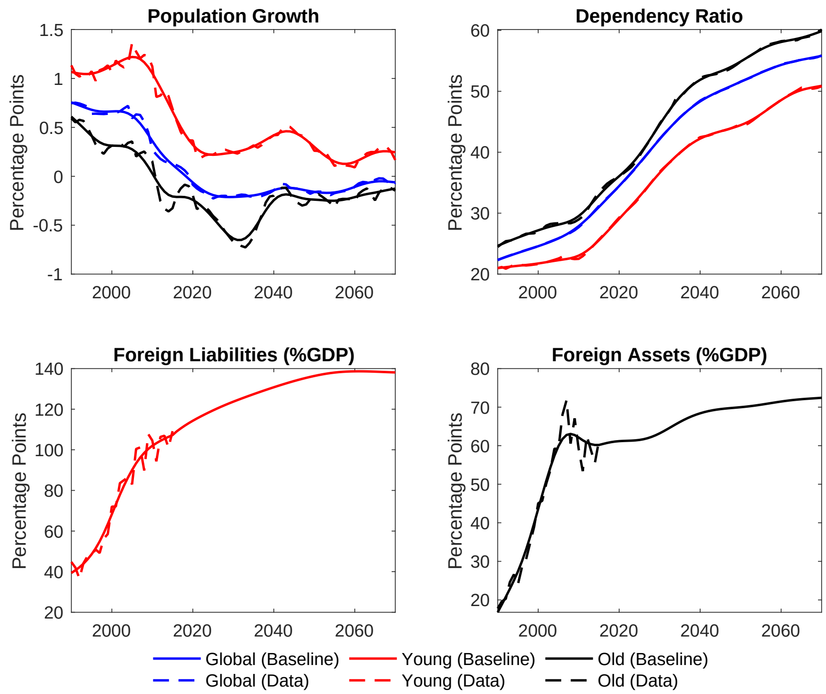
Figure 4 shows the results of the basic experiment for the period 1990-2070. In sample (1990-2020), the real interest rate of the global economy (solid blue line) falls by more than two percentage points, from its initial value of 5.28% to 3.10%. In the young economy, the decline is almost three percentage points (from 7% to 4.15%). Over the entire sample, the interest rate falls also in the old economy, by about one and a half percentage point (from 3.56% to 2.19%). However, differently from the other two countries, the dynamics are not monotone in this case. Before starting to decline, the real interest rate actually increases for the first decade and a half of the sample to reach almost 4%.

to match the target for the gross foreign debt position. Given the relative size of the three countries, the bulk of financial flows occurs between each small country and the global economy so that we can simply adjust the portfolio holding cost parameter $\Lambda_{m\mathcal{W}0}$.

[15] The household problem is well defined even when the individual discount factor is bigger than one as long as $\beta_m \gamma_{mt} < 1$, which is always the case in our experiments.

[16] We set the HP filter smoothing parameter for both the demographic variables and the financial integration to 40 so that the smoothed series approximate well the variables of interest.

Figure 3: Calibration of demographic processes and financial integration.

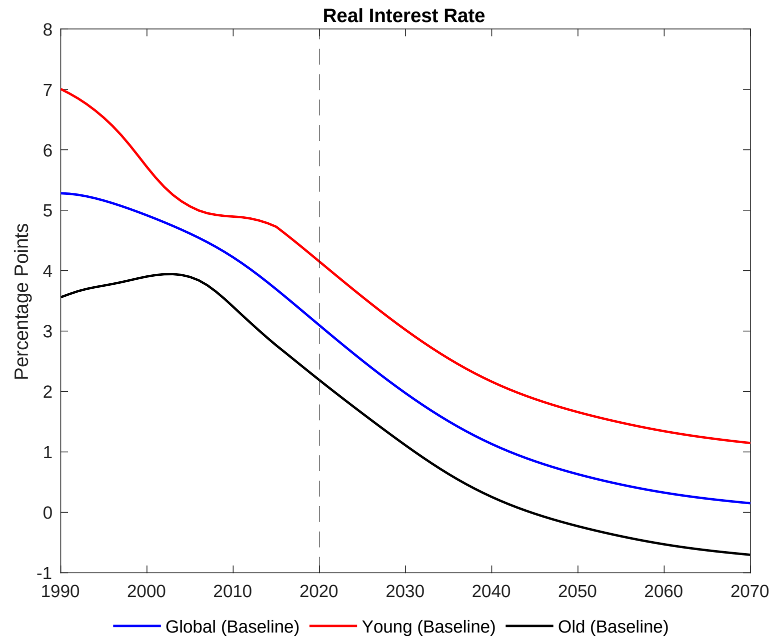


Note: Growth rate of working-age population (top-left panel), dependency ratio (top-right panel), foreign debt liabilities (bottom-left), and foreign debt assets (bottom-right). Red lines correspond to the young economy, black lines to the old economy, blue lines to the rest of the world. The dashed lines are data, the solid lines are the fitted processes (trend from a HP filter).

These rich dynamics reflect the interaction between demographic variables and financial integration. In general, the evolution of demographic variables exerts downward pressure on the real interest rate. As countries move towards full financial integration, real interest rates across countries converge. Given the respective starting points, the real interest rate falls in the young economy but increases in the old economy. The dynamics of foreign debt liabilities as a percentage of GDP (red line in Figure 3) suggest that the process of financial integration has slowed down following the financial crisis of 2008. Even more strikingly, foreign debt assets as a percentage of GDP fell slightly (black line in Figure 3), thus determining the divergence of real interest rates for both small regions relative to the global economy in the last few years of the sample .

Going forward, the projections for demographic variables by the United Nations suggest that the aging process will continue everywhere in the world, with some further decline in the growth rate of the labor force and a progressive increase in the dependency ratio. The implication is that real interest rates will continue to decline, reaching a level near zero globally in 2070. As for financial integration, we can only assess the evolution of foreign debt assets and liabilities based on the projected trends from our filtering procedure, which implies a further increase in liabilities and a stabilization of assets. The model predicts that the dispersion of about one percentage point on each side of the global real interest rate prevailing

Figure 4: Real interest rates in the baseline simulation.

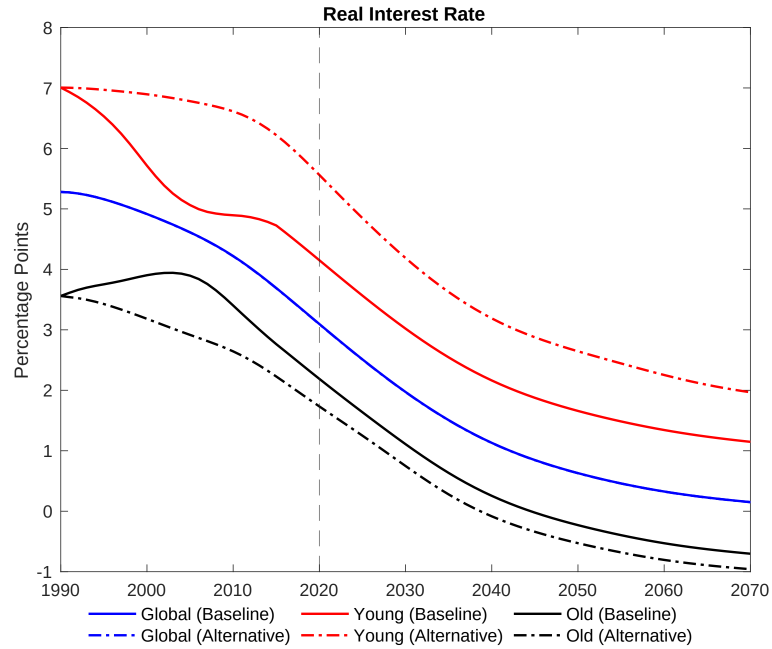


Note: The figure plots the simulated real interest rate from the model (baseline experiment) for the global economy (blue line), the young economy (red line), and the old economy (black line). The vertical line denotes the last period for which data on foreign debt are available.

in 2020 will persist throughout the simulation horizon. Should financial integration experience a further boost in the future, real interest rates will converge once again, as we observed especially between 1990 and 2005.

Figure 5 isolates the role of financial integration in determining the dynamics of real interest rates across regions. The solid lines are again the real interest rate in the three regions from the baseline experiment. The dashed-dotted lines correspond to the simulation that keeps the degree of financial integration at its initial value. Because the size of the three regions changes over time, in the counterfactual we adjust the friction so that in each period we match the same target for foreign debt assets and liabilities as in the initial steady state. Not surprisingly, financial integration does not matter for the global economy. The real interest rate is essentially unchanged in the two cases for country $\mathcal{W}$, to the extent that the results from the two simulations are virtually indistinguishable. Conversely, financial integration makes a big difference for the two small economies. As discussed, the evolution of demographic variables still exerts downward pressure on the real interest rate in both regions. In line with the closed economy results in Carvalho et al. (2016), the decline is less pronounced in country $\mathcal{Y}$ and more so in country $\mathcal{O}$. The main difference with the baseline simulation is that, with constant financial integration, the real interest rate of the old economy now falls monotonically, and also by more. As a result, the real interest rates in the two small economies do not converge (in fact diverge slightly). With constant financial integration,

Figure 5: Real interest rates with constant financial integration.



Note: The figure plots the simulated real interest rate from the model for the global economy (blue lines), the young economy (red lines), and the old economy (black lines). The solid lines correspond to the baseline simulation. The dashed-dotted lines are the counterfactual simulation in which the degree of financial integration remains at its initial value.

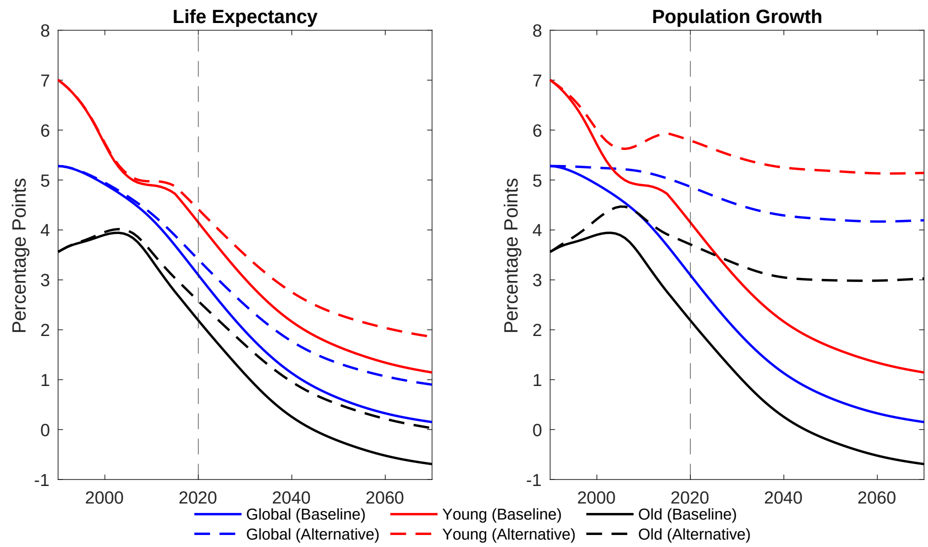
domestic demographic developments dominate. Conversely, with increasing financial integration, global demographic trends become progressively more important over time.

The dashed-dotted lines in Figure 6 show the path of the real interest rate in the three regions in two counterfactual simulations. The left panel presents the result of the simulation in which we fix the growth rate of the working-age population at its initial value. The right panel displays the experiment in which we hold the probability of surviving constant at its initial value. The increase in life expectancy associated with the higher probability of surviving explains about 80% of the overall effect, depending on the region. Conversely, the lower growth rate of the labor force explains less than 20% of the overall effect.

The intuition for this decomposition is the same as in Carvalho et al. (2016). The increase in life expectancy induces households to save more in anticipation of a longer retirement period. This saving-for-retirement motive is stronger for workers, who face a longer expected lifespan, but also affects retirees since their life expectancy continues to increase even after leaving work.[17] The fall in the growth rate of the working-age population has only a modest effect on the real interest rate because two effects

[17] An increase in the retirement age would mitigate this effect. In many OECD countries, pension reforms are moving in this direction. In addition, people work for more years, even though the official retirement age has not changed (Scott, 2021). Yet, Carvalho et al. (2016) show that the increase in retirement age necessary to fully offset the consequences of higher life expectancy on the real interest rate is substantial—well above the changes currently being discussed and implemented in most countries.

Figure 6: Real interest rates in two demographics counterfactuals.



Note: The figure plots the simulated real interest rate from the model for the global economy (blue lines), the young economy (red lines), and the old economy (black lines). The solid lines correspond to the baseline simulation. The dashed-dotted lines are the counterfactual simulations in which the growth rate of the working age population (left) and the probability of surviving (right) remain at their initial values in each country.

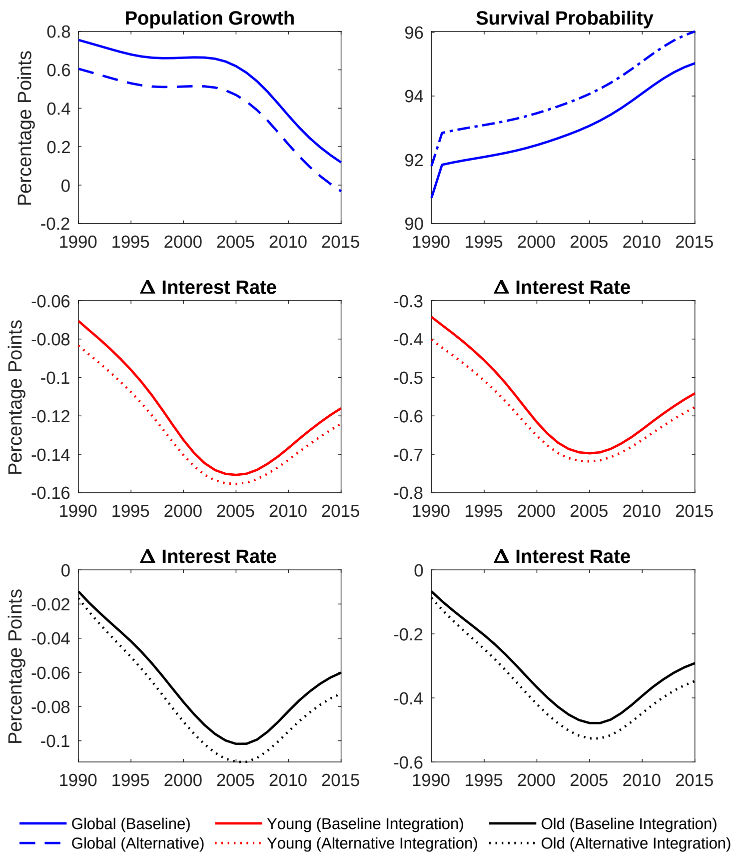
tend to compensate each other. On the one hand, a lower growth rate of the working-age population increases capital per-worker and thus tends to depress returns on financial assets. On the other hand, the reduction in the number of workers implies a change in the composition of the population. Since, retirees have a higher marginal propensity to consume, aggregate savings fall and the real interest rate rises. On balance, the first effect associated with the lower growth rate of the working-age population dominates in the simulation but the quantitative implications are nonetheless small in our model. Overall, the notable consequence of the actual and predicted changes in demographic variables on the real interest rate is due to the increase in life expectancy.

3.3.1 Demographic Comparative Statics

In this section, we perform two comparative-static exercises to understand how global demographic developments affect the real interest rate in the two small economies, and how this effect depends on the degree of financial integration with the rest of the world. We perturb the path of one demographic variable at a time (growth rate of the labor force or probability of surviving) as to generate a parallel upward shift in the dependency ratio relative to the baseline by one percentage point, and assess the impact of the change on the real interest rate of the three regions given different degrees of financial integration.

In the left column of Figure 7, we consider a downward shift of the growth rate of the working age population in the global economy by 15 basis points each year (dashed line) relative to the baseline

Figure 7: Real interest rate sensitivity to alternative demographic profiles.



Note: Change in the real interest rate of the small economy (right) for baseline (solid red line) and high (dotted red line) level of financial integration in response to (i) a more pronounced decline of the growth rate of the labor force in the large economy (dashed blue line, top-left panel); and (ii) a more pronounced increase of the probability of surviving in the large economy (dashed-dotted blue line, bottom-left panel). The solid blue lines in the left column correspond to the baseline processes for demographic variables.

scenario (solid line), displayed in the top-left panel. The middle-left panel reports the change of the real interest rate relative to the baseline scenario in the young economy. The solid line keeps the degree of financial integration at the same level as in the main simulation. The dotted line corresponds to a higher degree of financial integration (a 25% reduction of the portfolio-holding cost). The downward shift in the trajectory of global population growth leads to a lower real interest rates in the young economy, and this effect is more pronounced the more financially integrated the country is with the global economy. The bottom-left panel plots the same experiment for the old economy. In this case, the consequences of a downward shift of the growth rate of working age population in the global economy for the domestic real interest rate are smaller. The reason is that the old economy is less financially integrated with the global economy than the young economy, and thus is relatively less sensitive to demographic developments in the rest of the world.

The right column of Figure 7 shows an upward shift of the probability of surviving in the global economy by 1 percentage point each year (dashed-dotted line) relative to the baseline scenario (solid

Table 4: Real interest rate in 2020 relative to the baseline with different calibrations.

Factor	Baseline	Alternative	$\mathbf{R}^{\mathbf{Alt}}_{\mathcal{Y},\mathbf{2020}} - \mathbf{R}^{\mathbf{Base}}_{\mathcal{Y},\mathbf{2020}}$	$\mathbf{R}^{\mathbf{Alt}}_{\mathcal{O},\mathbf{2020}} - \mathbf{R}^{\mathbf{Base}}_{\mathcal{O},\mathbf{2020}}$
TFP (x)	0.5%	0.6%	8	10
Debt/GDP (b)	60%	70%	21	11
Gov't Spending/GDP (g)	25%	26%	32	17
Pensions/GDP (e)	7.5%	8.5%	66	39
Age of Retirement (ω)	65	66	66	47

Note: For each experiment, the first column (Factor) reports the parameter that changes, the second (Baseline) the value in the baseline calibration, the third (Alternative) the value in the alternative calibration, the fourth ($R^{Alt}_{\mathcal{Y},2020} - R^{Base}_{\mathcal{Y},2020}$) the difference (in basis points) between the real interest rate in 2020 under the alternative calibration relative to the baseline in the young economy, and the fifth ($R^{Alt}_{\mathcal{O},2020} - R^{Base}_{\mathcal{O},2020}$) the difference (in basis points) between the real interest rate in 2020 under the alternative calibration relative to the baseline in the old economy.

line), displayed in the top-right panel. As in the case of the growth rate of the working age population, the effect of a higher probability of surviving is stronger in the young economy than in the old economy, and is larger with higher financial integration.

Together with the main counterfactual analysis of Figure 5, both comparative static exercises in this section reinforce the message that global demographic developments influence the real rate of a small economy progressively more as its financial integration with the rest of the world increases. This point will serve as a guide to interpret the empirical results that we present below.

3.3.2 Other Factors

The existing literature has identified a number of factors that contribute to explain the observed decline of global real interest rates over time. While our analysis focuses on demographic variables, our model is suitable to analyze the effects at least a subset of these other potential drivers. We keep this part deliberately simple and only perform another set of comparative static exercises. Namely, we modify the calibrated value of one factor at a time, and compare the real interest rate so generated with the baseline simulation under the same demographic transition. The last two columns of Table 4 report the difference (in basis points) between the real interest rate in the alternative and in the baseline calibration in 2020 for the young and old economy, respectively, in each of these comparative static exercises.

The first line (TFP) shows the consequence of increasing TFP growth (x in the model) from 0.5% as in the baseline calibration to 0.6%. The real interest rate increases by about 8 basis points in country $\mathcal{Y}$ and 10 basis points in country $\mathcal{O}$. The model-implied elasticity of the real interest rate to TFP growth is broadly consistent with some of the recent literature, for example, Holston et al. (2017) and Rachel and Smith (2017), that discusses the importance of the slowdown in trend GDP for the secular decline of the equilibrium real interest rate.

The next three lines (Debt/GDP, Pensions/GDP and Gov't Spending/GDP) correspond to changes in b, e, and g, respectively. In the case of debt, we consider a ten percentage point increase relative to the baseline, while for government spending and pensions the change is one percentage point. In all cases, a more expansionary fiscal policy causes a higher interest rate. A 10 percentage point increase in debt/GDP raises the real interest rate by 21 basis points in the small young economy and 11 in the small old economy. The effect of a one percentage point increase in government spending is slightly larger, 32 and 17 basis points, respectively. Finally, a one percentage point increase in pensions as a fraction of GDP causes a 66 basis points increase in the real interest rate of the young economy and 39 basis points for the old economy. Because of the life-cycle features of the model, government bonds are net wealth for the private sector. Therefore, a higher level of debt relative to GDP supports private sector's consumption and thus contributes to increase the real interest rate. For government spending, the mechanism works through a crowding out of private consumption and investment. The increase in pensions has a large effect on the real interest rate because, effectively, the government transfers resources from agents with lower marginal propensity to consume (workers) to agents with a higher marginal propensity to consume (retirees). Overall, our results about fiscal policy are closely in line with the findings in Rachel and Summers (2019).

The last factor that we consider is the retirement age. In this experiment, we increase the parameter ω so that the average duration of employment is 46 years compared to 45 in the baseline. This exercise approximates the reforms that governments in many countries are implementing to make their public finances—and in particular their pension systems—more sustainable (OECD, 2019). The rise in interest rate in this case is 66 basis points in the young economy and 47 in the old. The intuition is that a longer employment span reduces the incentives for workers to save during their years on the job and thus diminishes the downward pressure on the real interest rate.

Overall, the message from these additional comparative static exercises is that a rebound of TFP growth, a fiscal expansion, or an increase in the retirement age are all factors that could contribute to lift the real interest rate. While the exact magnitudes may depend on the details of the model, we take the direction of the effects as the main lesson to inform our empirical analysis below.

4 Empirical Analysis

The analysis presented in the previous section illustrates how demographic variables affect real interest rates in a world of imperfect capital mobility. In particular, the model shows that both domestic and global demographics affect the real interest rate of a given country. Furthermore, the relative importance of country-specific and global determinants varies with the degree of financial integration. The more financially integrated a country is, the higher the sensitivity of its real interest rate to global demographic developments. The process of financial integration, coupled with different initial conditions, may generate non-monotonic dynamics in the process of convergence towards the world real interest rate. Finally, size matters too. For given demographics, the larger the relative size is, the more domestic variables affect the equilibrium of the global economy.

These implications of the calibrated model serve as guidelines for our empirical analysis of the relationship between demographics and real interest rates. To this end, we exploit a panel of countries, thus leveraging variation both across countries and over time.

The model suggests a particular specification of the panel regressions. First, demographic trends should imply low-frequency movements in real interest rates. We take into account the long-run nature of this relationship by employing a panel error-correction model (panel ECM), which allows for cointegrating relationships between real interest rates and their determinants—in particular demographic variables. Second, the relative importance of domestic and global factors for a country's real interest rate should vary over time with the degree of financial integration. In light of this consideration, we weight global factors by the degree of financial openness and domestic factors by its complement.

For the sake of parsimony, we summarize global factors with a measure of the foreign real interest rate faced by each country. An alternative would be to include all determinants of global interest rates as regressors, interacted with openness. Because of the relatively small sample, this second approach would lead to a loss of power. We then separately add country-specific demographic variables and other determinants of real rates, interacted with one minus the country's degree of openness. The resulting panel error-correction model (ECM) for country m is

$$\begin{aligned}\Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta \Theta_{m,t-1} r^*_{m,t-1} + \sum_j \psi_j (1-\Theta_{m,t-1}) D_{m,j,t-1} + \sum_k \Psi_k (1-\Theta_{m,t-1}) X_{m,k,t-1} \\ + \lambda \Delta(\Theta_{m,t} r^*_{m,t}) + \sum_j \phi_j \Delta[(1-\Theta_{m,t}) D_{m,j,t}] + \sum_k \chi_k \Delta[(1-\Theta_{m,t}) X_{m,k,t}] + \epsilon_{m,t}, \quad (15)\end{aligned}$$

where $r_{m,t}$ is the ex-ante real interest rate, $r^*_{m,t}$ is the foreign real interest rate faced by the country, α_m is a country fixed effect, $\Theta_{m,t}$ is an index of financial openness, $D_{m,j,t}$ includes j demographic variables (described below), $X_{m,k,t}$ collects k other potential determinants of real interest rates, and Δ is the first-difference operator. Based on this specification, the estimated long-run effects of each variable on interest rate corresponds to its estimated coefficient ($\hat{\theta}$, $\hat{\psi}_j$ or $\hat{\Psi}_k$) divided by $-\hat{\gamma}$. The next section explains how we construct the variables in the regression.

4.1 Data

We estimate the panel ECM in equation (15) using annual data for a set of 19 OECD countries. Our sample covers the period 1979-2019 and includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, United Kingdom, and the United States.[18] As Figure 1 shows, starting in 1979 allows us to include in the sample a decade during which real interest rates increased, so that spurious correlations of purely trending variables should not contaminate the empirical analysis.

We rely on different data sources to construct the variables that enter our regression. The ex ante

[18]The sample excludes countries that experienced episodes of high inflation (above 25% in a given year) between 1970 and 2019.

short-term real interest rate $r_{m,t}$ equals the difference between the time-t short-term nominal rate $i_{m,t}$ and one-period-ahead expected inflation $\mathbb{E}_t\pi_{m,t+1}$. The data for short-term nominal interest rate are either overnight or three-month official rates (see Table A1 for details). To construct expected inflation, we follow the approach in Hamilton et al. (2016) and calculate the one-year-ahead forecast from AR(1) regressions with rolling windows of 20 years, that is, we estimate the regression $\pi_{m,t} = a_m + b_m\pi_{m,t-1} + \varepsilon_{m,t}$ with OLS so that $\mathbb{E}_t\pi_{m,t+1} = \hat{a}_m + \hat{b}_m\pi_{m,t}$. For all countries, we use the headline CPI inflation rate obtained from the OECD.

Using data from Lane and Milesi-Ferretti (2017), we build a measure of financial integration of country m ($LMF_{m,t}$) as the sum of financial assets and liabilities expressed as a fraction of GDP. The index of financial openness that we use in the regression ($\Theta_{m,t}$) to weight the global interest rate is a transformation of $LMF_{m,t}$

$$\Theta_{m,t} = \frac{LMF_{m,t}}{100 + LMF_{m,t}},$$

which makes $\Theta_{m,t}$ an index between zero and one, consistent with the specification of equation (15).[19]

The global real interest rate that an individual country faces is a weighted average of all other countries' real interest rates, where the weight associated with each country is the working-age population ($POP_{m,t}$) share adjusted by the index of financial openness

$$r^*_{m,t} = \sum_{\ell \neq m} \left(\frac{\Theta_{\ell,t} POP_{\ell,t}}{\sum_{\ell \neq m} \Theta_{\ell,t} POP_{\ell,t}} \right) r_{\ell,t}.$$

The vector $D_{m,j,t}$ contains two demographic variables: life expectancy ($LE_{m,t}$) and the growth rate of working-age population ($DPOP_{m,t}$). The data source for both demographic variables is the United Nations World Population Prospects 2019. Life expectancy (measured at 20) comes straight from the database. We use the data on the number of individuals between 20 and 65 years old ($POP_{m,t}$) to construct the growth rate of working-age population

$$DPOP_{m,t} = 100 \times \left(\frac{POP_{m,t}}{POP_{m,t-1}} - 1 \right).$$

The model introduced in Section 2 isolates the effects of demographic trends on real interest rates given financial openness. In practice, as discussed in Section 3.3.2, a growing literature has identified a number of other forces at play that may contribute to explain the dynamics of real interest rates. At the end of Section 3, we have discussed a subset of these forces that constitute parameters in our model through simple comparative static exercises. The vector $X_{m,k,t}$ in the panel ECM (15) includes these and other factors.

Following Holston et al. (2017), the first variable that we consider is TFP growth, which we obtain from the Penn World Tables. Secondly, in line with the analysis in Rachel and Smith (2017) on the role of fiscal policy, we add government debt as a fraction of GDP (from the AMECO database) to control

[19] Appendix B reports results using trade openness as an alternative proxy for international integration (Table A2).

Table 5: Panel Error Correction Model (ECM)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Global Rate	0.68***	0.66***	0.70***	0.96***	1.02***	1.18***	1.52***
	(0.17)	(0.17)	(0.13)	(0.15)	(0.15)	(0.21)	(0.19)
Life Expectancy	0.14***	0.14***	-0.24***	-1.27***	-1.37***	-1.40***	-1.84***
	(0.04)	(0.04)	(0.06)	(0.35)	(0.37)	(0.54)	(0.58)
Growth Rate of Labor Force	0.24	0.30	6.03***	5.20***	4.92***	7.47***	9.09***
	(1.02)	(1.01)	(0.98)	(1.02)	(1.11)	(1.69)	(1.65)
TFP Growth		0.49	0.02	0.07	-0.04	0.10	0.08
		(0.34)	(0.30)	(0.30)	(0.36)	(0.40)	(0.39)
Government Debt			0.03	0.03	0.01	0.06*	0.04
			(0.02)	(0.02)	(0.03)	(0.03)	(0.05)
Pension Spending			2.31***	3.02***	2.94***	2.85***	3.64***
			(0.41)	(0.45)	(0.60)	(0.61)	(0.83)
Retirement Age				0.79***	0.91***	0.83*	0.96**
				(0.27)	(0.26)	(0.44)	(0.38)
Gini Coefficient					-0.03		0.22
					(0.22)		(0.32)
Convenience Yield						-0.28	-0.39
						(1.36)	(1.77)
R-Squared	0.24	0.24	0.39	0.38	0.36	0.55	0.57
Adjusted R-Squared	0.21	0.22	0.35	0.34	0.31	0.49	0.50
Observations	743	743	505	505	445	206	169
Clusters	19	19	19	19	18	7	7

Note: Results from the estimation of equation (15). Robust standard errors, reported in parenthesis, are clustered at the country level.

for the fiscal stance. Still in relation to fiscal policy, albeit on a different dimension, we also include data on pension spending as a fraction of GDP and the effective retirement age (both from the OECD database) as separate potential explanatory variables. (Papetti, 2021) explores the possibility that the generosity of pensions may affect the real interest rate by reducing the incentive of workers to save while employed. While within each country the retirement age typically exhibits very few changes over time, in our panel, the cross-country variation may be informative for the dispersion of real interest rates. Third, we consider versions of our regressions augmented with the convenience yields obtained from Del Negro et al. (2019).[20] Since the convenience yield is only available for seven major advanced economies, these regressions have a smaller sample size. Finally, Mian et al. (2021) have recently highlighted the potential importance of widening inequality to explain the secular decline of real interest rates. To account for this potential explanation, we also control for the Gini coefficient of each country, which we obtain from the World Bank.

Table 5 presents the main results from the estimation of equation (15). For each variable, we report the estimated long-run coefficients from the panel ECM, that is, $\hat{\theta}$, $\hat{\psi}_j$, and $\hat{\Psi}_k$ divided by $-\hat{\gamma}$, weighted by the country's index of financial openness. Standard errors are clustered at the country level.

Column (1) considers a basic specification that only includes the demographic variables. The global

[20] We thank the authors for kindly sharing their series for Canada, France, Germany, Italy, Japan, United Kingdom, and the United States.

real interest rate is statistically significant at the 1% confidence level, with a point estimate just below 0.7. Life expectancy is statistically significant but enters with the wrong sign compared to the predictions of the model. The growth rate of working-age population has the right sign but is not statistically significant. Column (2) adds TFP growth, which is not statistically significant and does not meaningfully alter the results. The picture changes once we control for fiscal variables (government debt and pension spending) in column (3). Pension spending, in particular, is significant at the 1% confidence level. The sign is in line with the prediction of the model and the point estimate is associated with a large economic magnitude. More importantly, both life expectancy and the growth rate of the labor force now become significant at the 1% confidence level and enter with the sign predicted by the model. Adding the effective retirement age in column (4) reinforces these findings. The coefficients on the global rate and, in particular, on life expectancy increase substantially. The retirement age is significant at the 1% confidence level and the effect is in line with the comparative static analysis of our model. Column (5) and (6) introduce the Gini coefficient and the convenience yield, respectively. While the point estimate of the coefficients on these two variables is in line with the literature, neither is statistically significant. The coefficients on the other variables remain comparable with the other specifications. Finally, column (7) reports the regression with both the Gini coefficient and the convenience yield, which are again not statistically significant and do not change the results for the other variables.

Because of the interaction between each factor and financial openness, the estimated coefficients are effectively country-specific and time-varying. To get a sense of the magnitude of each factor on average, we can multiply every coefficient by the cross-country time average of $1 - \Theta_t$, except for the coefficient on the global rate, which should be multiplied by the average value of Θ_t. Across the six regression specifications considered in Table 5, Θ_t ranges from 0.66 to 0.76. As a result, a one percentage point change in the global real interest rate is associated with a change of the domestic real rate between 45 and 115 basis points. In column (3) to (7), the effect of life expectancy ranges from 6 to 63 basis points and the effect of the growth rate of working-age population falls between 144 and 310 basis points. This adjustment is also likely to account for why the point estimates become larger as we move to richer specifications, in particular, since the countries for which we have data on the convenience yield are more financially open.

On balance, the panel error correction model suggests a stable link between the global rate and domestic real interest rates. Once we control for fiscal policy, the effects of life expectancy and the growth rate of working-age population become significant, in line with the predictions of the model, and broadly consistent across specifications.[21] TFP growth is never statistically significant, while government debt only marginally so in one specification. Conversely, pension spending and the retirement age are highly significant in all specifications. Lastly, neither the Gini coefficient nor the convenience yield are

[21] We have also experimented with a number of additional specifications that include only a subset of the control variables at a time. While the results are broadly robust, we have found cases in which the statistical significance, or even the sign, of the coefficients changes. In general, controlling for both government debt and pension spending is important for the statistical significance and the sign of the coefficient on life expectancy, while introducing even just one of those two variables is sufficient to make the coefficient on the growth rate of working age population positive and statistically significant.

statistically significant, whether included separately or together.[22]

Between 1990 and 2019, the median real interest rate in our sample went from 5.5% to -1.5%. In the data, the growth rate of working-age population for the median country in our sample fell from 0.52% in 1990 to 0.26% in 2020. Therefore, the regressions suggest that the contribution of this factor to the decline of the real interest rate for the median country in the data is between 31 and 80 basis points over the sample period. Over the same time span, median life expectancy at 20 increased by five years (from 57.6 to 62.6), which implies a total effect on the real interest rate between 29 and 313 basis points. Overall, the empirical analysis is broadly consistent with the results in the model. The mid-range of the estimated effects for demographic variables (56 basis points for the growth rate of working age population and 171 basis points for life expectancy) accounts for about 230 basis points of the decline of the real interest rate between 1990 and 2020. In addition, the total effect of the increase in life expectancy is more than three times large than the effect of the decline in the growth rate of the working-age population, although the range of estimates across specifications is rather wide.

Among the other controls, pension spending and the retirement age are the two variables that are consistently significant across the various specifications. According to the panel ECM, a one percent increase in pension spending adds between 55 and 124 basis points to the real interest rate, while an additional year of retirement age is associated with an increase by 19 to 33 basis points. Both effects are consistent with the comparative static exercises in section 3.3.2. The magnitude of the coefficient on pension spending is significantly larger than in our theoretical exercise while the one for retirement age is slightly smaller. In practice, changes in retirement age take time to come in full effect and are often implemented at the same time as reforms to the generosity of pensions, which may introduce problems in the empirical estimates. The absence of a significant link between real interest rates and the amount of government debt may depend on two forces pushing in opposite directions. On the one hand, a standard crowding out argument and the fact that higher levels of debt may increase default risk would tend to push up the real interest rate. On the other hand, the quantity of government debt may also be interpreted as a proxy for the availability of safe assets (Caballero et al., 2017), thus implying a negative correlation with the level of the real interest rate.

Table 6 reports the results for the same specifications as in Table 5, but without interacting the explanatory variables with the degree of financial openness

$$\Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta r^*_{m,t-1} + \sum_j \psi_j (1 - \Theta_{m,t-1}) D_{m,j,t-1} + \sum_k \Psi_k X_{m,k,t-1} + \lambda \Delta r^*_{m,t} + \sum_j \phi_j \Delta D_{m,j,t} + \sum_k \chi_k \Delta X_{m,k,t} + \epsilon_{m,t}. \quad (16)$$

In this case, the values and statistical significance of the coefficients are much less stable across specifications. Life expectancy is the only variable that is consistently significant across specifications.

[22]The convenience yield is statistically significant if introduced without fiscal policy variables. In this case, all other variables are not statistically significant, except for life expectancy, which however enters with the wrong sign.

Table 6: Panel ECM without interaction.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Global Rate	0.32**	0.30**	-0.23	-0.20	-0.24*	-0.24	-0.21
	(0.13)	(0.13)	(0.15)	(0.16)	(0.14)	(0.21)	(0.18)
Life Expectancy	-0.76***	-0.72***	-1.49***	-1.45***	-1.42***	-0.77**	-1.01***
	(0.14)	(0.13)	(0.16)	(0.16)	(0.16)	(0.32)	(0.33)
Growth Rate of Labor Force	-0.58	-0.41	0.58	0.58	0.24	1.44*	1.15*
	(0.35)	(0.36)	(0.41)	(0.41)	(0.36)	(0.76)	(0.64)
TFP Growth		0.31**	0.24*	0.23	0.06	0.35	0.11
		(0.14)	(0.14)	(0.15)	(0.14)	(0.21)	(0.19)
Government Debt			0.01	0.01*	-0.02**	0.01	-0.02*
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Pension Spending			0.47**	0.49**	0.10	0.73**	0.11
			(0.21)	(0.22)	(0.20)	(0.30)	(0.28)
Retirement Age				0.14	0.39**	0.42	0.18
				(0.18)	(0.17)	(0.33)	(0.30)
Gini Coefficient					0.18**		0.04
					(0.09)		(0.15)
Convenience Yield						-2.39**	-0.77
						(0.95)	(0.90)
R-Squared	0.24	0.24	0.30	0.29	0.30	0.52	0.55
Adjusted R-Squared	0.21	0.21	0.25	0.24	0.24	0.46	0.47
Observations	743	743	505	505	445	206	169
Clusters	19	19	19	19	18	7	7

Note: Results from the estimation of equation (16). Robust standard errors, reported in parenthesis, are clustered at the country level.

The global rate becomes statistically not significant once we add the fiscal variables, the Gini coefficient and the convenience yield. The growth rate of the labor force is significant in four out of the seven specifications. Among the control variables, TFP growth is significant in four out of six specifications, although only at the 10% confidence level. Government debt and pension spending are significant in three out of five specifications, the retirement age in one out of four, and the convenience yield in one out of two. The good news is that the sign of the coefficient of all these variables is in line with the theoretical prediction of our model or literature. The Gini coefficient is significant only if added separately, but not together with the convenience yield, and its sign is inconsistent with the idea that higher inequality should be associated with a lower real interest rate.

Overall, the comparison between the results reported on Tables 5 and 6 further highlights the importance interacting the explanatory variables of real interest rate with the degree of financial integration, as suggested by the model.

5 Conclusions

The demographic trends that most advanced economies are undergoing are a natural explanation for the prolonged decline of global real interest rates observed between 1990 and 2020. In this paper, we have explored the interaction of these trends with the process of increased financial integration that took place globally over the same period. Our analysis has proceeded in two steps. First, we have developed a multi-country, general-equilibrium model with imperfect capital mobility and differential demographic trends. A calibrated three-country version of the model demonstrates how low-frequency movements in a country's real interest rate depend on its own as well as on global demographic developments. The weight on global demographic variables is increasing in the degree of global financial integration. Conversely, domestic demographic developments matter less for the real interest rate of a highly financially integrated country. Drawing on the lessons from the model, we have then estimated several specifications of a panel error-correction model that relates real interest rates to demographic variables and other possible drivers, interacted with a measure of financial integration. A "world" real interest rate, which summarizes global factors, is consistently significant in all specifications, with an effect close to one for one. Nevertheless, domestic demographic variables remain an important determinant of real interest rates, together with pension spending and the retirement age.

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Appendix

A Derivations

This section presents the derivations of the retirees and workers' problems.

A.1 Retirees

Retirees maximize (2) subject to (3). After substituting the constraint into the objective function, we can rewrite the unconstrained maximization problem as

$$V_{mt}^{r} = \max_{\{A_{m\ell t}^{r}\}_{\ell=1}^{\mathcal{M}}} \left\{ \left[\frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{r} + E_{mt}^{r} - \left(1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{nm} \right)^{2}}{2} \right) \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{r} \right]^{\frac{\sigma-1}{\sigma}} + \gamma_{mt+1} \beta_{m} (V_{mt+1}^{r})^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}.$$

The first-order condition with respect to foreign assets ($A_{mpt}^{r},\ p \neq m$) is

$$\left[\left(1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right)^{2} \right) - \sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^{r} + \Lambda_{mpt} \left(\eta_{mpt}^{r} - \bar{\eta}_{mp} \right) \left(1 - \eta_{mpt}^{r} \right) \right] \left(C_{mt}^{r} \right)^{-\frac{1}{\sigma}} = \beta_{m} \gamma_{mt+1} \left(V_{mt+1}^{r} \right)^{-\frac{1}{\sigma}} \frac{\partial V_{mt+1}^{r}}{\partial A_{mpt}^{r}},$$

while the first-order condition with respect to domestic assets (A_{mmt}^{r}) is

$$\left[\left(1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right)^{2} \right) - \sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^{r} \right] \left(C_{mt}^{r} \right)^{-\frac{1}{\sigma}} = \beta_{m} \gamma_{mt+1} \left(V_{mt+1}^{r} \right)^{-\frac{1}{\sigma}} \frac{\partial V_{mt+1}^{r}}{\partial A_{mmt}^{r}}.$$

By the Envelope Theorem, the partial derivatives above are

$$\frac{\partial V_{mt+1}^{r}}{\partial A_{mpt}^{r}} = \left(V_{mt+1}^{r} \right)^{\frac{1}{\sigma}} \left(C_{mt+1}^{r} \right)^{-\frac{1}{\sigma}} \frac{R_{pt}}{\gamma_{mt+1}}, \quad \forall\ p = 1, ..., \mathcal{M}. \tag{A.1}$$

Substituting (A.1) into the first-order conditions above gives

$$\left[\left(1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right)^{2} \right) - \sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t} \left(\eta_{m\ell t}^{r} - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^{r} + \Lambda_{mpt} \left(\eta_{mpt}^{r} - \bar{\eta}_{mp} \right) \left(1 - \eta_{mpt}^{r} \right) \right] \left(C_{mt}^{r} \right)^{-\frac{1}{\sigma}} = \beta_{m} R_{pt} (C_{mt+1}^{r})^{-\frac{1}{\sigma}}, \tag{A.2}$$

and

$$\left[\left(1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{r}-\bar{\eta}_{m\ell}\right)^{2}\right)-\sum_{\ell=1}^{\mathcal{M}}\Lambda_{m\ell t}\left(\eta_{m\ell t}^{r}-\bar{\eta}_{m\ell}\right)\eta_{m\ell t}^{r}\right]\left(C_{mt}^{r}\right)^{-\frac{1}{\sigma}}=\beta_{m}R_{mt}(C_{mt+1}^{r})^{-\frac{1}{\sigma}}. \tag{A.3}$$

Dividing (A.2) by (A.3) and rearranging yields:

$$\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{r}-\bar{\eta}_{m\ell}\right)^{2}\right]\left(R_{pt}-R_{mt}\right)=\Lambda_{mp}\left(\eta_{mpt}^{r}-\bar{\eta}_{mp}\right)R_{mt},$$

which correspond to equation (4) in the main text.

Next, if we multiply equation (A.2) by η_{mmt}^{r} and equation (A.3) by $\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}\eta_{m\ell t}^{r}$, and we add them up, we obtain the Euler equation for the optimal path of consumption of retirees

$$C_{mt+1}^{r}=\left[\frac{\beta_{m}\sum_{\ell=1}^{n}\eta_{m\ell t}^{r}R_{\ell t}}{1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{r}-\bar{\eta}_{m\ell}\right)^{2}}\right]^{\sigma}C_{mt}^{r}. \tag{A.4}$$

In order to find the difference equation for the marginal propensity to consume out of wealth for retirees, we substitute the retirees budget constraint (3) into the policy function (5). After rearranging, we obtain

$$\frac{1-\xi_{mt}^{r}}{\xi_{mt}^{r}}C_{mt}^{r}=\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{r}-\bar{\eta}_{m\ell}\right)^{2}\right]\sum_{\ell=1}^{\mathcal{M}}A_{m\ell t}^{r}+\frac{\gamma_{mt+1}S_{mt+1}^{r}}{\tilde{R}_{mt}},$$

where the present discounted value of pension benefits to retirees S_{mt}^{r} and the adjusted return $\tilde{R}_{mt}$ are defined in the text. Replacing for current consumption from the Euler equation (A.4) , we obtain

$$\frac{1-\xi_{mt}^{r}}{\xi_{mt}^{r}}C_{mt+1}^{r}(\beta_{m}\tilde{R}_{mt})^{-\sigma}=\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{r}-\bar{\eta}_{m\ell}\right)^{2}\right]\sum_{\ell=1}^{\mathcal{M}}A_{m\ell t}^{r}+\frac{\gamma_{mt+1}S_{mt+1}^{r}}{\tilde{R}_{mt}}$$

Finally, we can substitute the guess of the consumption function at $t+1$ for C_{mt+1}^{r} to obtain

$$\begin{aligned}\frac{1-\xi_{mt}^{r}}{\xi_{mt}^{r}}\xi_{mt+1}^{r}&\left(\frac{1}{\gamma_{mt+1}}\sum_{\ell=1}^{\mathcal{M}}R_{\ell t}A_{m\ell t}^{r}+S_{mt+1}^{r}\right)(\beta_{m}\tilde{R}_{mt})^{-\sigma}\\&=\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{r}-\bar{\eta}_{m\ell}\right)^{2}\right]\sum_{\ell=1}^{\mathcal{M}}A_{m\ell t}^{r}+\frac{\gamma_{mt+1}S_{mt+1}^{r}}{\tilde{R}_{mt}}\end{aligned}$$

Dividing by $\sum_{\ell=1}^{\mathcal{M}}A_{m\ell t}^{r}$ and using the definition of η_{mpt} allows us to obtain (7) in the text.

Finally, we guess and verify that the value function is linear in the level of consumption:

$$V_{mt}^{r}=\Delta_{mt}^{r}C_{mt}^{r}.$$

Substituting the guess into the functional equation (2) together with the Euler equation (A.4) to eliminate

C^r_{mt+1}, we obtain

$$\Delta^r_{mt} C^r_{mt} = \left[(C^r_{mt})^{\frac{\sigma-1}{\sigma}} + \beta_m \gamma_{mt+1} \left(\Delta^r_{mt+1}\right)^{\frac{\sigma-1}{\sigma}} (\beta_m \tilde{R}_{mt})^{\sigma-1} (C^r_{mt})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

We can simplify the last expression by eliminating the terms in C^r_{mt}. After rearranging, we obtain

$$\frac{1}{(\Delta^r_{mt})^{\frac{\sigma-1}{\sigma}}} = 1 - \gamma_{mt+1} \beta^\sigma_m \tilde{R}^{\sigma-1}_{mt} \left(\frac{\Delta^r_{mt+1}}{\Delta^r_{mt}} \right)^{\frac{\sigma-1}{\sigma}}. \tag{A.5}$$

Comparing (A.5) with the difference equation for the marginal propensity to consume (7), we can see that

$$\Delta^r_{mt} = (\xi^r_{mt})^{-\frac{\sigma}{\sigma-1}}.$$

A.2 Workers

The workers' problem is to maximize (8) subject to (9). After substituting the constraint into the objective, the unconstrained maximization problem becomes

$$V^w_{mt} = \max_{\{A^w_{m\ell t}\}_{\ell=1}^{\mathcal{M}}} \Bigg\{ \left[\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^w_{m\ell t-1} + W^w_{mt} - T^w_{mt} - \left(1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{nm}\right)^2 \right) \sum_{\ell=1}^{\mathcal{M}} A^w_{m\ell t} \right]^{\frac{\sigma-1}{\sigma}} \\ + \beta_m \left[\omega_{mt+1} V^w_{mt+1} + (1-\omega_{mt+1}) V^{rt+1}_{mt+1} \right]^{\frac{\sigma-1}{\sigma}} \Bigg\}^{\frac{\sigma}{\sigma-1}}.$$

The first-order condition with respect to country-p assets (with $p \neq m$) is

$$\left[\left(1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 \right) - \sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right) \eta^w_{m\ell t} + \Lambda_{mpt} \left(\eta^w_{mpt} - \bar{\eta}_{mp}\right) \left(1 - \eta^w_{mpt}\right) \right] (C^w_{mt})^{-\frac{1}{\sigma}} \\ = \beta_m \left[\omega_{mt+1} V^w_{mt+1} + (1-\omega_{mt+1}) V^r_{mt+1} \right]^{-\frac{1}{\sigma}} \left[\omega_{mt+1} \frac{\partial V^w_{mt+1}}{\partial A^w_{mpt}} + (1-\omega_{mt+1}) \frac{\partial V^r_{mt+1}}{\partial A^r_{mpt}} \right], \tag{A.6}$$

while the first-order condition with respect to domestic assets is

$$\left[\left(1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 \right) - \sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right) \eta^w_{m\ell t} \right] (C^w_{mt})^{-\frac{1}{\sigma}} \\ = \beta_m \left[\omega_{mt+1} V^w_{mt+1} + (1-\omega_{mt+1}) V^r_{mt+1} \right]^{-\frac{1}{\sigma}} \left[\omega_{mt+1} \frac{\partial V^w_{mt+1}}{\partial A^w_{mmt}} + (1-\omega_{mt+1}) \frac{\partial V^r_{mt+1}}{\partial A^r_{mmt}} \right]. \tag{A.7}$$

As for retirees, we use the Envelope Theorem to calculate the partial derivatives above

$$\frac{\partial V^w_{mt+1}}{\partial A^w_{mpt}} = \left(V^w_{mt+1}\right)^{\frac{1}{\sigma}} \left(C^w_{mt+1}\right)^{-\frac{1}{\sigma}} R_{pt}. \tag{A.8}$$

To solve the workers' problem, we need to guess the functional form of the value function at this stage. Like for retirees, we conjecture that the value function is linear in consumption and the slope is the same function of the marginal propensity to consume

$$V_{mt}^{w} = \Delta_{mt}^{w} C_{mt}^{w}, \quad \text{with} \quad \Delta_{mt}^{w} = \left(\xi_{mt}^{w}\right)^{-\frac{\sigma}{\sigma-1}}. \tag{A.9}$$

By substituting equation (A.8) and the guess (A.9) into equation (A.7) we get

$$\begin{aligned}
&\left[\left(1+\sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)^{2}\right)-\sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right) \eta_{m\ell t}^{w}\right]\left(C_{mt}^{w}\right)^{-\frac{1}{\sigma}} \\
&=\beta_{m}\left[\omega_{mt+1} \Delta_{mt+1}^{w} C_{mt+1}^{w}+\left(1-\omega_{mt+1}\right) \Delta_{mt+1}^{r} C_{mt+1}^{r}\right]^{-\frac{1}{\sigma}}\left[\omega_{mt+1}\left(\Delta_{mt+1}^{w}\right)^{\frac{1}{\sigma}}+\left(1-\omega_{mt+1}\right)\left(\Delta_{mt+1}^{r}\right)^{\frac{1}{\sigma}}\right].
\end{aligned} \tag{A.10}$$

Multiplying both sides of (A.10) by $(\Delta_{mt+1}^{w})^{\frac{1}{\sigma}}$ and rearranging yields

$$\begin{aligned}
&\left[\omega_{mt+1} C_{mt+1}^{w}+\left(1-\omega_{mt+1}\right) \frac{\Delta_{mt+1}^{r}}{\Delta_{mt+1}^{w}} C_{mt+1}^{r}\right]^{\frac{1}{\sigma}} \\
&\qquad =\frac{\beta_{m}\left[\omega_{mt+1}+\left(1-\omega_{mt+1}\right)\left(\frac{\Delta_{mt+1}^{r}}{\Delta_{mt+1}^{w}}\right)^{\frac{1}{\sigma}}\right] R_{mt}\left(C_{mt}^{r}\right)^{\frac{1}{\sigma}}}{\left[1+\sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)^{2}\right]-\sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right) \eta_{m\ell t}^{w}}.
\end{aligned}$$

Using the solution for the value function of retirees and the guess for the value function of workers, we can rewirte the last expression as

$$\begin{aligned}
&\left[\omega_{mt+1} C_{mt+1}^{w}+\left(1-\omega_{mt+1}\right)\left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{\sigma}{1-\sigma}} C_{mt+1}^{r}\right]^{\frac{1}{\sigma}} \\
&\qquad =\frac{\beta_{m}\left[\omega_{mt+1}+\left(1-\omega_{mt+1}\right)\left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{1}{1-\sigma}}\right] R_{mt}\left(C_{mt}^{r}\right)^{\frac{1}{\sigma}}}{\left[1+\sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)^{2}\right]-\sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right) \eta_{m\ell t}^{w}}.
\end{aligned} \tag{A.11}$$

Following the same steps for equation (A.6), we obtain

$$\begin{aligned}
&\left[\omega_{mt+1} C_{mt+1}^{w}+\left(1-\omega_{mt+1}\right)\left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{\sigma}{1-\sigma}} C_{mt+1}^{r}\right]^{\frac{1}{\sigma}}= \\
&\frac{\beta_{m}\left[\omega_{mt+1}+\left(1-\omega_{mt+1}\right)\left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{1}{1-\sigma}}\right] R_{pt}\left(C_{mt}^{r}\right)^{\frac{1}{\sigma}}}{\left[1+\sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)^{2}\right]-\sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right) \eta_{m\ell t}^{w}+\Lambda_{mp}(\eta_{mpt}^{w}-\bar{\eta}_{mp})}.
\end{aligned} \tag{A.12}$$

Dividing equation (A.11) by equation (A.12) shows that workers choose asset shares according to the same condition as retirees

$$\left[1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] (R_{pt} - R_{mt}) = \Lambda_{mpt} \left(\eta^w_{mpt} - \bar{\eta}_{mp}\right) R_{mt},$$

which implies $\eta^w_{mpt} = \eta^r_{mpt} = \eta_{mpt}\ \forall p$.

We can find the Euler equation for workers' consumption following the same steps we did for retirees and get

$$\omega_{mt+1} C^w_{mt+1} + (1 - \omega_{mt+1}) \left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{\sigma}{1-\sigma}} C^r_{mt+1} = \left(\beta_m \Omega_{mt+1} \tilde{R}_{mt}\right)^{\sigma} C^r_{mt}, \tag{A.13}$$

with Ω_{mt} defined in (11) in the text.

Next, we substitute the guesses for the policy functions, (5) and (10), for C^r_{mt+1} and C^w_{mt+1}, respectively, in (A.13) to obtain

$$\begin{aligned}
\omega_{mt+1}\xi^w_{mt+1} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t} A^w_{m\ell t} + H^w_{mt+1} + Z^w_{mt+1}\right) + (1-\omega_{mt+1}) \left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{1}{1-\sigma}} \xi^w_{mt+1} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t} A^r_{m\ell t} + S^r_{mt+1}\right) \\
= \left(\beta_m \Omega_{mt+1} \tilde{R}_{mt}\right)^{\sigma} \xi^w_{mt} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^w_{m\ell t-1} + H^w_{mt} + Z^w_{mt}\right).
\end{aligned}$$

Dividing this expression by ξ^w_{mt+1} and $\sum_{\ell=1}^{\mathcal{M}} \eta_{m\ell t} R_{\ell t}$ gives us

$$\begin{aligned}
\omega_{mt+1} \left(\sum_{\ell=1}^{\mathcal{M}} A^w_{m\ell t} + \frac{H^w_{mt+1} + Z^w_{mt+1}}{\sum_{\ell=1}^{\mathcal{M}} \eta_{m\ell t} R_{\ell t}}\right) + (1 - \omega_{mt+1}) \left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{1}{1-\sigma}} \left(\sum_{\ell=1}^{\mathcal{M}} A^r_{m\ell t} + \frac{S^r_{mt+1}}{\sum_{\ell=1}^{\mathcal{M}} \eta_{m\ell t} R_{\ell t}}\right) \\
= \left[\frac{\beta_m \Omega_{mt+1}}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta^w_{m\ell t} - \bar{\eta}_{m\ell}\right)^2}\right]^{\sigma} \left(\sum_{\ell=1}^{\mathcal{M}} \eta_{m\ell t} R_{\ell t}\right)^{\sigma-1} \frac{\xi^w_{mt}}{\xi^w_{mt+1}} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^w_{m\ell t-1} + H^w_{mt} + Z^w_{mt}\right).
\end{aligned}$$

Note that, for a worker who retires, $\sum_{\ell=1}^{\mathcal{M}} A^w_{m\ell t} = \sum_{\ell=1}^{\mathcal{M}} A^r_{m\ell t}$. Therefore, we can simplify the previous equation using the workers' budget constraint as to obtain

$$\begin{aligned}
\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^w_{m\ell t-1} + W^w_{mt} + T^w_{mt} - C^w_{mt} + \frac{\omega_{mt+1}(H^w_{mt+1} + Z^w_{mt+1})}{\Omega_{mt+1} \tilde{R}_{mt}} + \frac{(1 - \omega_{mt+1}) \left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{1}{1-\sigma}} S^r_{mt+1}}{\Omega_{mt+1} \tilde{R}_{mt}} \\
= \beta^{\sigma}_m \left(\Omega_{mt+1} \tilde{R}_{mt}\right)^{\sigma-1} \frac{\xi^w_{mt}}{\xi^w_{mt+1}} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^w_{m\ell t-1} + H^w_{mt} + Z^w_{mt}\right).
\end{aligned}$$

Substituting the guess for C^w_{mt} and using the recursive definitions of H^w_{mt+1} and Z^w_{mt+1}, we get the difference equation for the marginal propensity to consume of workers (12)

The last step to characterize the workers' problem is to verify the guess for the value function. After substituting the guess into equation (8) and rearranging, we get

$$\Delta_{mt}^{w} C_{mt}^{w} = \left\{ \left(C_{mt}^{w}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \left[\omega_{mt+1} C_{mt+1}^{w} + (1 - \omega_m t + 1) \frac{\Delta_{mt+1}^{r}}{\Delta_{mt+1}^{w}} C_{mt+1}^{r} \right]^{\frac{\sigma-1}{\sigma}} \left(\Delta_{mt+1}^{w}\right)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}. \quad (A.14)$$

We can then substitute the Euler equation (A.13) into (A.14) and write

$$\Delta_{mt}^{w} C_{mt}^{w} = \left\{ \left(C_{mt}^{w}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \left[(\beta_m \Omega_{mt+1} \tilde{R}_{mt})^{\sigma} C_{mt}^{w} \right]^{\frac{\sigma-1}{\sigma}} \left(\Delta_{mt+1}^{w}\right)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}.$$

Simplifying C_{mt}^{w} from the equation and rearranging leads to

$$\left(\Delta_{mt}^{w}\right)^{\frac{\sigma-1}{\sigma}} = 1 + \beta_m^{\sigma} \left(\Omega_{mt+1} \tilde{R}_{mt} \right)^{\sigma-1} \left(\Delta_{mt+1}^{w}\right)^{\frac{\sigma-1}{\sigma}}. \quad (A.15)$$

Comparing equation (A.15) to equation (12) shows that the guess for the policy function is correct provided that

$$\Delta_{mt}^{w} = \left(\xi_{mt}^{w}\right)^{-\frac{\sigma}{\sigma-1}}.$$

A.3 Assets

The heterogeneity between workers and retirees makes it necessary to keep track of the distribution of wealth between the two groups. We start by writing the law of motion of the amount of assets held by retirees

$$\sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{r} = \frac{\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{r} + E_{mt} - C_{mt}^{r}}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2} + (1 - \omega_{mt+1}) \frac{\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{w} + W_{mt} - T_{mt} - C_{mt}^{w}}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2}. \quad (A.16)$$

From the workers' aggregate budget constraint, we substitute the total amount of workers' assets into the second term of the right-hand side of equation (A.16). Next, we substitute out retirees' consumption, and rewrite retirees and workers' total value of non-human assets as shares of total assets using the definition in the text

$$\lambda_{mt} \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t} - \frac{1 - \omega_{mt+1}}{\omega_{mt+1}} (1 - \lambda_{mt}) \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t} = \frac{\lambda_{mt-1} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + E_{mt} - \xi_{mt}^{r} \left(\lambda_{mt-1} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + S_{mt} \right)}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2}.$$

After rearranging and using the definition of aggregate assets $A_{mt} \equiv \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}$, we obtain equation (14) in the text.

Table A1: Sources for nominal short-term interest rates used to construct ex-ante real interest rates.

Country	Source	Description
Australia	World Bank	Lending interest rate
Austria	OECD	1-day central bank yield
Belgium	OECD	3-month interbank yield
Canada	OECD	1-day central bank yield
Denmark	OECD	1-day central bank yield
Finland	OECD	1-day central bank yield
France	OECD	3-month interbank rate
Germany	AMECO	Short term interest rate
Ireland	OECD	3-month interbank rate
Italy	AMECO	Short term interest rate
Japan	OECD	1-day central bank yield
Netherlands	AMECO	Short term interest rate
New Zealand	OECD	3-month bankbill yield
Norway	OECD	3-month interbank yield
Spain	OECD	3-month interbank rate
Sweden	OECD	3-month interbank rate
Switzerland	OECD	3-month interbank loan rate
United Kingdom	OECD	3-month interbank loan rate
United States	IFS	Money market rate

B Empirical Analysis

This section describes the sources of the data for short-term nominal interest rates and presents the estimates of the coefficients for the panel ECM based on a measure of trade integration.

B.1 Interest Rates

For each country in our sample, Table A1 reports the source and the maturity of the nominal interest rate i_t used to construct the ex-ante real interest rate r_t, according to

$$r_t = i_t - \mathbb{E}_t \pi_{t+1}.$$

As discussed in the text, we construct expected inflation following Hamilton et al. (2016). Specifically, for each country m, we first estimate a regression of inflation on its own lag with rolling windows of 20 years

$$\pi_{m,t} = a_m + b_m \pi_{m,t-1} + \varepsilon_{m,t}. \tag{B.17}$$

We then calculate the one-year-ahead forecast

$$\mathbb{E}_t \pi_{m,t+1} = \hat{a}_m + \hat{b}_m \pi_{m,t},$$

where $\hat{a}_m$ and $\hat{b}_m$ are the OLS estimates of the coefficients in (B.17). For all countries, we use the headline CPI inflation rate obtained from the OECD.

Table A2: Panel ECM with trade integration measure.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Global Rate	2.61***	2.43***	2.48***	1.74***	1.10***	1.07	1.04**
	(0.32)	(0.33)	(0.35)	(0.30)	(0.27)	(0.65)	(0.53)
Life Expectancy	0.20*	0.17*	-0.26**	-1.29***	-1.54***	-0.84***	-1.20***
	(0.10)	(0.10)	(0.12)	(0.17)	(0.21)	(0.26)	(0.34)
Growth Rate of Labor Force	0.49	0.58	3.25***	1.85***	1.31**	3.21***	2.63***
	(0.44)	(0.44)	(0.70)	(0.57)	(0.51)	(1.04)	(0.94)
TFP Growth		0.56**	0.59**	0.41**	0.05	0.35	0.00
		(0.22)	(0.25)	(0.19)	(0.19)	(0.27)	(0.27)
Government Debt			-0.00	0.02**	-0.02	0.02	-0.01
			(0.01)	(0.01)	(0.01)	(0.02)	(0.02)
Pension Spending			1.57***	1.34***	0.78**	1.46***	0.69
			(0.37)	(0.29)	(0.31)	(0.39)	(0.42)
Retirement Age				0.96***	1.14***	0.56***	0.86***
				(0.13)	(0.12)	(0.21)	(0.21)
Gini Coefficient					0.38***		0.19
					(0.14)		(0.23)
Convenience Yield						-2.45***	-1.00
						(0.91)	(0.89)
R-Squared	0.25	0.25	0.31	0.33	0.34	0.53	0.53
Adjusted R-Squared	0.22	0.22	0.27	0.28	0.28	0.47	0.45
Observations	742	742	505	505	445	206	169
Clusters	19	19	19	19	18	7	7

Notes: Results from the estimation of equation (B.18). Robust standard errors, reported in parenthesis, are clustered at the country level.

B.2 Panel ECM with Trade Integration Measure

In this section, we consider an alternative interaction term $\widetilde{\Theta}_{m,t} \equiv TO_{m,t}/(100+TO_{m,t})$, where $TO_{m,t}$ is a measure of trade integration, defined as the sum of exports and imports of goods and services measured as a share of GDP (from the World Bank). The panel ECM regression thus becomes

$$\Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta \widetilde{\Theta}_{m,t-1} r^*_{m,t-1} + \sum_j \psi_j (1-\widetilde{\Theta}_{m,t-1}) D_{m,j,t-1} + \sum_k \Psi_k (1-\widetilde{\Theta}_{m,t-1}) X_{m,k,t-1} + \lambda \Delta(\widetilde{\Theta}_{m,t} r^*_{m,t}) + \sum_j \phi_j \Delta[(1-\widetilde{\Theta}_{m,t}) D_{m,j,t}] + \sum_k \chi_k \Delta[(1-\widetilde{\Theta}_{m,t}) X_{m,k,t}] + \epsilon_{m,t}. \quad \text{(B.18)}$$

Table A2 reports the coefficient estimates for the same six specifications as in the baseline analysis. The results are largely in line with those in the main text. The global rate is always significant, except for the specification in which we add the convenience yield (but not the Gini coefficient). The coefficient on life expectancy is always significant and its sign becomes consistent with the predictions of the model once we control for fiscal variables, which is also when the coefficient on the growth rate of the working-age population becomes significant. TFP growth is significant in three specifications, compared to one in the baseline, but loses statistical significance once we introduce either the Gini coefficient or the convenience yield (or both). Pension spending and the retirement age are almost always significant and enter with the expected sign, while government debt is only significant in one specification. The Gini coefficient and the convenience yield are significant if introduced separately but not jointly. In addition, the Gini coefficient enters with the opposite sign compared to the predictions from the literature.