Population Aging in Advanced and Emerging Economies: Capital Flows and Fiscal Spillovers*

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Abstract

Advanced economies have started aging much earlier than emerging economies and did so at a much slower pace. This study investigates the fiscal implications to emerging countries of advanced countries' earlier aging and seeks to understand the fiscal impact of emerging countries' fast catch-up. I estimated the long-run fiscal effects using a Panel FMOLS approach on a panel of advanced and emerging economies, it finds that the latter benefited from the earlier aging in the former starting in the 1990s. It also finds that aging driven more by an increase in longevity will have a lower fiscal impact. Going forward, as the emerging countries age and catch up with the advanced, they will be hit by the double-whammy of rising deficits and the abatement of capital flows, which will critically tighten their fiscal space.

Keywords: Life expectancy, Population growth, Demographic transition, Interest-growth differential, Fiscal space, Debt dynamics

JEL Classifications: E62, F41, H63, J11

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1 Introduction

It took 71 years for the old-age population (65 years or older) in advanced economies to increase from 15% of the working-age population to 35% (1950-2021). On the other hand, it will take emerging economies only 30 years to experience the same process (2019-2049) (United Nations, 2022). Advanced economies started to age much earlier than emerging economies and did so at a much slower pace. Since population aging is particularly relevant for its fiscal implications, what does the desynchronized character of this demographic change mean for emerging economies' fiscal dynamics? Moreover, what are the fiscal impacts of the fast demographic change emerging economies are going through?

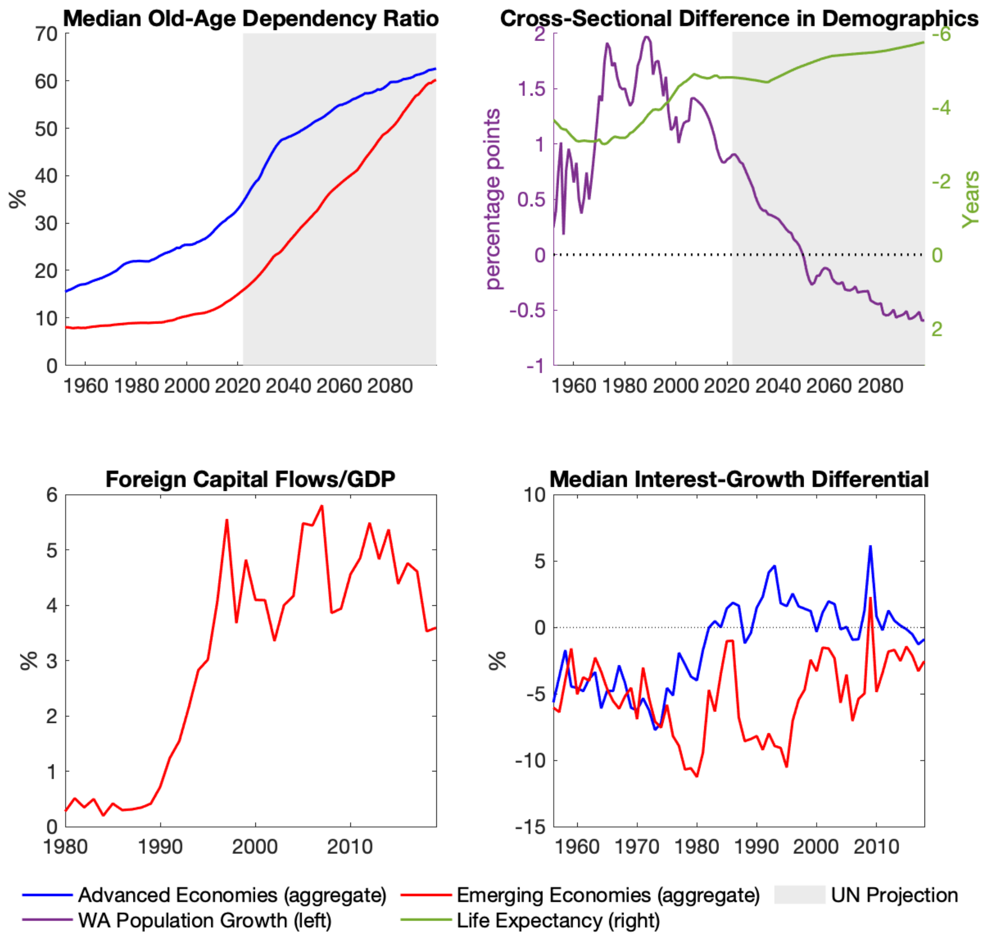
Similar to the experience of advanced economies, pension systems in emerging economies will be subject to considerable strain in the future. Fewer contributions to the system and a longer life in retirement both lead to increasing deficits. Different to the experience of advanced economies, however, the higher speed of aging makes the window for reform much shorter and countries have on average less fiscal credibility.

In addition, changes in factor prices and general equilibrium effects are important to evaluate the total fiscal cost of these changes. As Attanasio et al. (2007) has showed, projections should take into account the difference in demographic trends between regions in order to gauge the full fiscal implications of population aging. However, projections for emerging economies' pension systems do not take this into account and are potentially underestimating the negative fiscal impact of this demographic change.

A look at the data allows for a clearer picture of this backdrop. The top panels in Figure 1 show the evolution of demographic variables in advanced and emerging economies across time. The graph on the left shows the old-age dependency ratio, defined as the ratio between the population above 65 years of age and the working-age population, defined as those between 20 and 65 years old. The blue line presents the cross-section average across advanced economies weighted by population size. Note that it has been on an upward trend in advanced countries and that the trend is even expected to intensify in the near future. The red line, representing the cross-section weighted average of emerging economies, also shows that they are in the early stages of an aging process that is set to accelerate in the coming years.

The graph on the top right displays the cross-section difference between emerging and advanced economies of the average of two main drivers of the demographic transition: the fall in working-age population growth and the increase in longevity at 65 years old. Note that the lack of synchrony between each group's transition is almost entirely a function of differences in working-age population growth. This growth rate fell first in the rich world, but is now expected to plummet in emerging economies and lead them to an even slower population

Figure 1: Demographics, Capital Flows and Debt Dynamics



Note: Top left panel: Cross-sectional median of old-age dependency ratio for 23 advanced and 23 emerging economies between 1956 and 2100. Top right panel: Cross-sectional median of working-age population growth in advanced minus cross-sectional median in emerging economies (green); cross-sectional median of life expectancy at 65 in advanced minus cross-sectional median in emerging economies (purple) between 1956 and 2100. Bottom left panel: foreign capital flows to emerging economies are defined as the net change in liabilities to nonresidents in terms of foreign direct investment and portfolio investment. Bottom right panel: Cross-sectional median old-age dependency ratio for 23 advanced and 23 emerging economies between 1956 and 2100. Source: United Nations World Population Prospects, 2022 Revision (top panels); IMF (bottom left); Mauro and Zhou (2021).

growth as compared to advanced economies. The cross-sectional difference in life expectancy at 65 increased by about 2 years over time and is expected to be even higher in the future. This means that life expectancy contributed to the demographic divergence early on and will still contribute to it in the future. The demographic gap will close entirely due to the abrupt fall in fertility in emerging economies.

The bottom panels show the evolution of two economic variables: capital flows to emerging economies as a percentage of GDP on the left and the cross-sectional median interest-growth differential on the right for advanced and emerging economies. They increase considerably in the 1990s and are sustained thereafter, which puts a downward pressure on the interest rate and stimulates output growth, shrinking the interest-growth differential and loosening fiscal conditions. In the bottom right panel we see the evolution of the differential and how it has been considerably below advanced economies'. The end of this demographic divergence can imply an increase in the differential and a tightening of fiscal conditions in emerging markets.

This paper seeks to investigate whether the disparity in demographic trends between advanced and emerging economies has led to a loosening of fiscal conditions in emerging economies and what the eventual convergence in these trends means for these conditions. I resort first to a simple 2-country Overlapping Generations model a la Diamond (1965) stripped down to the most fundamental macroeconomic relationships to derive fundamental causal relationships between the demographic processes and fiscal variables.

The analytical results show that earlier aging in advanced countries led to increased private savings and reduced investment returns, which are supportive of capital flows to young emerging countries. The influx of capital depresses interest rates and stimulates economic growth, making public debt rollover less costly. These are called fiscal spillovers and they exist as long as the demographic gap exists. As the young emerging country ages the gap closes, capital inflows decline and fiscal conditions tighten.

Another result analyzes the fiscal impact of domestic demographics. I show that changes in the old age dependency ratio are not a sufficient statistic to evaluate the fiscal impact of population aging. This means that the source of aging, whether increase in longevity or fall in fertility, matters to evaluate the fiscal impact as they will imply different effects on interest rate and GDP growth.

As a natural next step, a panel of advanced and emerging economies is used to estimate empirically the analytical results derived from the model. The estimates are consistent with the analytical results: a fall in fertility and a rise in longevity imply opposing effects on the interest-growth differential with similar quantitative importance each. The result still holds if the estimation is executed with either advanced economies only or emerging economies only. On the other hand, the spillover effects of demographic change abroad does not match with the analytical results.

Literature Review: This paper is related to a few strands of the literature. The first is the one that applies large scale OLG models to study the effects of the demographic transition on a set of economic variables in the tradition of the seminal paper or Auerbach et al. (1987). This study is closest to Attanasio et al. (2007), who study social security reform in an open economy context where there is difference in demographic trends across two groups of countries but with a focus on the side of advanced economies. Krueger and Ludwig (2007) use an open economy model to quantify the impact of the demographic transition in factor prices, capital flows and welfare; Vogel et al. (2017) also use a multi-country OLG model to address policy responses to the aging process, and Gagnon et al. (2021) use the same methodology to account for the decline in output growth and real interest rates in the United States. These papers feed the demographic transition as an exogenous process to the model to explain and project the path of macroeconomic aggregates under different policy scenarios. The results of simulations, however, are particular to the set of parameters and demographic change fed into the model. This paper, in contrast, looks for a general result and also seeks to identify the role of each demographic driver.

A second branch of the literature is the one that studies the interest-growth differential. Blanchard and Weil (2001), studies theoretical implications of dynamic (in)efficiency to debt Ponzi games under uncertainty, while more recently Blanchard (2019) studies the fiscal and welfare costs of public debt in an environment of negative interest-growth differentials. Reis (2021) in turn seek to establish a limit to the primary deficit in a context of negative interest-growth differential on public debt. However, these papers only analyze the consequences of the sign of the interest-growth differential for fiscal policy. They do not explain what drives the changes in the differential across time.

The present paper thus seeks to fill the aforementioned gaps by investigating the role demographic changes have in impacting debt dynamics through the interest-growth differential.

The rest of the paper is structured as follows: Section 2 presents a simple two-country OLG model and derives the theoretical results that will guide the empirical analysis in Section 3. Section 3 presents the panel data of advanced and emerging economies and analyzes the effects of demographics on the interest-growth differential and on the fiscal space. Section 4 concludes the paper.

2 Simple Model

The following model is a two-country version of the standard Diamond (1965) OLG model augmented with a Pay-As-You-Go (PAYGO) social security system and a survival probability.

2.1 Households

Households potentially live for two periods. They live as workers during their first period and a share $1 - \delta_{t+1}$ of them dies at the end of that period and do not reach the second period. The exogenous variable δ_{t+1} can be thus considered to be the share of the time period when agents are alive when old. The number of workers in period t is given by N_t and the number of retirees is thus $\delta_t N_{t-1}$. The number of young workers grows at the gross rate g_{Nt}: $N_t = g_{Nt} N_{t-1}$.

Households have log-utlity over consumption when young, c_{1t}, and old, c_{2t+1}, and therefore their utility over lifetime consumption is given by the function below:

$$U(c_{1t}, c_{2t+1}) = \ln c_{1t} + \beta\delta_{t+1} \ln c_{2t+1} \tag{1}$$

where β represents the discount factor.

Labor supply is inelastic. Young households receive a wage w_t in exchange for the labor supplied and also pay a lump-sum tax, τ_t. They then consume, c_{1t}, and save, s_t, out of their after-tax labor income. When old, households receive interest over their savings given by the gross interest rate, R_{t+1}, and retirement benefits, e_t.

$$c_{1t} + s_t = w_t - \tilde{\tau}_t \tag{2}$$

$$c_{2t+1} = \frac{R_{t+1}}{\delta_{t+1}} s_t + e_t \tag{3}$$

The first order conditions of this problem imply the following savings function:

$$s_t = \frac{\beta\delta_{t+1}}{1 + \beta\delta_{t+1}} (w_t - \tilde{\tau}_t) - \frac{\delta_{t+1}}{(1 + \beta\delta_{t+1}) R_{t+1}} e_{t+1} \tag{4}$$

2.2 Firms

Firms are assumed to combine labor, L_t, and capital, K_t, to produce the single good in the economy, Y_t, according to the following Cobb-Douglas utility function:

$$Y_t = (A_t L_t)^{1-\alpha} K_t^{\alpha} \tag{5}$$

with $\alpha \in (0, 1)$ and total factor productivity, A_t, that grows at rate g_{Ait}: $A_t = g_{Ait} A_{t-1}$. As a result, wage and the gross interest rate are paid their marginal products:

$$R_t = \alpha k_t^{\alpha-1} \tag{6}$$

$$w_t = (1 - \alpha) A_t k_t^{\alpha} \tag{7}$$

where $K_t \equiv k_t/(A_t N_t)$ is the capital stock measured in efficient labor units.

2.3 Government

Assume government's only tax revenue comes from the lump-sum tax levied on workers, while its only transfers comprise the social security benefits paid to retirees. For tractability, retirement benefits, E_t, are a fraction ν of the current period's wage:

$$e_t = \nu w_t \tag{8}$$

The social security budget is not forced to balance every period; the government can issue debt, b_t, if necessary. Therefore the law of motion of public debt is given by:

$$B_{t+1} = E_t + G_t - T_t + R_t B_t \tag{9}$$

where B_{t+1} is the government debt measured at the end of period t and start of period $t+1$, E_t is the total social security benefits paid in period t, T_t is the total tax revenue collected in period t and R_t is the gross interest rate paid by public debt (which in this model is equal to the marginal return on capital). Define B_{t+1} as the debt-to-output ratio measured at the end of period t (and start of period $t+1$): $B_{t+1} \equiv B_{t+1}/Y_t$ and rewrite equation (9) as:

$$b_{t+1} = g_t - \tau_t + \frac{\delta_t \nu (1-\alpha)}{g_{Nit}} + \frac{\alpha}{g_{Ait} g_{Nit}} \frac{k_{t-1}^{\alpha}}{k_t} b_t \tag{10}$$

where τ_t is the tax burden $\tau_t \equiv N_t \tilde{\tau}_t / Y_t$.

2.4 Market clearing

The capital market clearing condition is that total savings equal the total capital and debt stocks at the beginning of the following period:

$$N_t s_t + N_t^* s_t^* = K_{t+1} + K_{t+1}^* + B_{t+1} + B_{t+1}^* \tag{11}$$

where variables with the superscript represent the foreign economy. Dividing (11) through by $A_t N_t$ and rearranging yields:

$$\frac{s_t}{A_t} + \varphi_t^* \frac{s_t^*}{A_t^*} = g_{At+1} g_{Nt+1} k_{t+1} + g_{At+1}^* g_{Nt+1}^* \varphi_t^* k_{t+1}^* + b_{t+1} k_t^{\alpha} + \varphi_t^* b_{t+1}^* (k_t^*)^{\alpha} \tag{12}$$

where $\varphi^*_t \equiv A_t N_t/(A^*_t N^*_t)$ is the relative size of the foreign economy's effective labor.

Substitute for w_t and e_{t+1} in the savings function (4) and get as a result:

$$s_t = \frac{\beta\delta_{t+1}}{1+\beta\delta_{t+1}}\left((1-\alpha)A_t(k_t)^\alpha - \tau_t A_t y_t\right) - \frac{1-\alpha}{\alpha}\frac{\delta_{t+1}}{1+\beta\delta_{t+1}}\nu A_t k_{t+1} \tag{13}$$

Plugging equation (13) into (12) yields:

$$\begin{aligned}\frac{\beta\delta_{t+1}}{1+\beta\delta_{t+1}}\left((1-\alpha)-\tau_t\right)(k_t)^\alpha - \frac{1-\alpha}{\alpha}\frac{\delta_{t+1}}{1+\beta\delta_{t+1}}\nu k_{t+1} + \\ \varphi^*_t\left(\frac{\beta^*\delta^*_{t+1}}{1+\beta^*\delta^*_{t+1}}\left((1-\alpha)-\tau^*_t\right)(k^*_t)^\alpha - \frac{1-\alpha}{\alpha}\frac{\delta^*_{t+1}}{1+\beta^*\delta^*_{t+1}}\nu^* k^*_{t+1}\right) \\ = g_{At+1}g_{Nt+1}k_{t+1} + g^*_{At+1}g^*_{Nt+1}\varphi^*_t k^*_{t+1} + b_{t+1}k^\alpha_t + \varphi^*_t b^*_{t+1}(k^*_t)^\alpha\end{aligned} \tag{14}$$

Given perfect capital mobility, the no-arbitrage condition holds and $k_t = k^*_t = k_t$. Equation (14) becomes:

$$\begin{aligned}\left(\frac{\beta\delta_{t+1}}{1+\beta\delta_{t+1}}\left((1-\alpha)-\tau_t\right) + \varphi^*_t\frac{\beta^*\delta^*_{t+1}}{1+\beta^*\delta^*_{t+1}}\left((1-\alpha)-\tau^*_t\right)\right)k^\alpha_t \\ - \frac{1-\alpha}{\alpha}\left(\frac{\delta_{t+1}}{1+\beta\delta_{t+1}}\nu + \varphi^*_t\frac{\delta^*_{t+1}}{1+\beta^*\delta^*_{t+1}}\nu^*\right)k_{t+1} \\ = (g_{At+1}g_{Nt+1} + g^*_{At+1}g^*_{Nt+1}\varphi^*_t)k_{t+1} + (b_{t+1} + \varphi^*_t b^*_{t+1})k^\alpha_t\end{aligned} \tag{15}$$

Note that given initial values for the capital stock in efficiency units k_{-1} and k_0 and given the path for the exogenous variables, the path for efficient capital is given by equations (15) and (10).

2.5 Analysis

In this section I carry out the analysis of the effects of demographic shocks on economic variables. In particular, the interest lies on the effect of such shocks on fiscal variables.

I will first analyze the effect of demographic changes on the interest-growth differential and then show that a fall in population growth has the opposite effect of an increase in longevity. The final result will hinge on this distinction.

2.5.1 Interest-Growth Differential

Proposition 1. *A fall in the growth rate of the labor force unambiguously increases the adjusted interest-growth differential next period:*

$$\frac{\partial \frac{R_{t+1}}{g_{Yt+1}}}{\partial g_{Nt+1}} < 0$$

An increase in the survival probability affects the interest-growth differential as follows:

$$\frac{\partial \frac{R_{t+1}}{g_{Yt+1}}}{\partial \delta_{it+1}} \begin{cases} \leq 0 & \textit{if } s_t \geq 0 \\ > 0 & \textit{if } s_t < 0 \end{cases}$$

Proof. Appendix. □

The results in Proposition 1 hinge on the comparison between the elasticities of the real interest rate and of the output growth rate with respect to the demographic variables. Both elasticities will in turn be functions of the elasticity of capital in efficiency units to demographics.

The elasticity of the real interest rate to the population growth rate will be lower than the elasticity of the output growth rate with respect to that same variable if the elasticity of capital is lower than one (in absolute value). That will be the case if the foreign country is sizeable and/or if there is a PAYGO social security system in place in any country. The existence of a sizeable foreign country will diminish the effect of the domestic country on the total stock of capital, whereas a PAYGO social security system will lead agents to save more in case there is a decrease in the stock of capital in efficiency units, counterbalancing changes in that stock. Both dynamics guarantee an inelastic capital stock in efficiency units with respect to the population growth rate, which leads to the first result: a fall in the population growth rate widens the interest-growth differential.

On the other hand, the increased longevity will work through the savings channel only. Savings will go up if they are positive, as the increased longevity will make agents value more the second period. Now, if agents are borrowing, an increase in longevity will diminish the interest rate adjusted by the survival probability, making consuming more in the first period more appealing. Focusing on the positive savings case, an increase in longevity will increase the stock of capital and lower the real interest rate. By the same token, output growth goes up due to the larger capital stock. The result is a smaller interest-growth differential. If savings are negative, we have the exact opposite scenario as consequence.

Proposition 2. *A fall in the growth rate of the labor force unambiguously increases the adjusted interest-growth differential beyond the following period:*

$$\frac{\partial \frac{R_{t+k}}{g_{Y\,t+k}}}{\partial g_{Nt+1}} < 0, \text{ for } k > 1$$

An increase in the survival probability affects the interest-growth differential beyond the following period as follows:

$$\frac{\partial \frac{R_{t+k}}{g_{Y\,t+k}}}{\partial \delta_{it+1}} \begin{cases} \leq 0 & \text{if } s_t \geq 0 \\ > 0 & \text{if } s_t < 0 \end{cases}$$

Proof. Appendix. □

Proposition 3. *The aging of population abroad either as a result of a fall in the growth rate of the labor force or an increase in life expectancy unambiguously decreases the interest-growth differential in the domestic economy in the current period:*

$$\frac{\partial \frac{R_{t+1}}{g_{Y\,t+1}}}{\partial g^{j}_{Nt+1}} < 0, \text{ for } k > 1$$

Proof. Appendix. □

2.5.2 Fiscal Space

In order to evaluate the fiscal impact of the demographic transition, it is necessary to introduce the concept of fiscal space. To that end it is useful to divide the components of the primary balance into *demographic* components and *remainder* components. The demographic components represent the parts of the primary balance that are a direct function of demographics, whereas the remainder components are simply the primary balance minus its demographic components. The remainder then becomes a function of demographics only *through* the demographic components. The breakdown is as follows:

$$\tau_t = \tau^d_t(g_{Nt}, \delta_t, \mathbf{x}_t) + \tau^r_t \tag{16}$$

$$g_t = g^d_t(g_{Nt}, \delta_t, \mathbf{x}_t) + g^r_t \tag{17}$$

$$e_t = e^d_t(g_{Nt}, \delta_t, \mathbf{x}_t) + e^r_t \tag{18}$$

Now solve equation (10) forward to get:

$$b_t = \sum_{i=0}^{\infty}\left[\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)[\tau_{t+i} - g_{t+i} - e_{t+i}]\right] + \lim_{s\to\infty}\left(\prod_{j=0}^{s} \frac{g_{Y,t+j}}{R_{t+j}}\right)[\tau_{t+s} - g_{t+s} - e_{t+s}] \quad (19)$$

Assume that the No-Ponzi condition holds and break each component of the primary surplus into demographic and remainder components.

$$b_t = \sum_{i=0}^{\infty}\left[\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\underbrace{[\tau^d_{t+i} - g^d_{t+i} - e^d_{t+i}]}_{\equiv \sigma^d_t}\right] + \sum_{i=0}^{\infty}\left[\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\underbrace{[\tau^r_{t+i} - g^r_{t+i} - e^r_{t+i}]}_{\equiv \sigma^r_t}\right] \quad (20)$$

Consider now a marginal change in an exogenous demographic variable $\varkappa_t = \{g_{Yt}, \delta_t\}$. This change will have a direct impact on the demographic primary surplus, σ^d_t, and will also have an impact on the interest-growth differential, R_t/g_{Yt}. Note that these impacts are also a function of the fiscal policy in place. The decision of how to finance the changes in the demographic primary surpluses will also affect the response of endogenous variables to the exogenous demographic variables. The financing decision will be reflected in the changes in the remainder primary surpluses. These changes will underlie the definition of fiscal space.

Definition 1. *A change in fiscal space due to a change in a demographic variable $\varkappa_t$ is defined by the present value of changes in the remainder primary surpluses, s^r_t. If this change is positive, that implies an effort to raise the primary surplus after the demographic change, i.e. there is a loss of fiscal space. Likewise, if the change is negative, then there is creation of fiscal space.*

$$\sum_{i=0}^{\infty}\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^r_{t+i}}{\partial \varkappa_t}\begin{cases} > 0, & \textit{loss of fiscal space} \\ < 0, & \textit{creation of fiscal space}\end{cases} \quad (21)$$

Now take the first derivative of the current debt-to-GDP ratio (pre-determined) with respect to the demographic variable $\varkappa_t$:

$$\begin{aligned}\frac{\partial b_t}{\partial \varkappa_t} = &\sum_{i=0}^{\infty}\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^d_{t+i}}{\partial \varkappa_t} + \sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}\frac{g_{Y,t+j}}{R_{t+j}}\right)}{\partial \varkappa_t}\sigma^d_{t+i} \\ &+ \sum_{i=0}^{\infty}\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^r_{t+i}}{\partial \varkappa_t} + \sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}\frac{g_{Y,t+j}}{R_{t+j}}\right)}{\partial \varkappa_t}\sigma^r_{t+i} = 0 \quad (22)\end{aligned}$$

The equation above implies that the change in fiscal space can be decomposed in the following two parts. Note that the impact on the interest-growth differential affects the entire primary surplus, not just the demographic part.

$$-\sum_{i=0}^{\infty}\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^r_{t+i}}{\partial \varkappa_t} = \underbrace{\sum_{i=0}^{\infty}\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^d_{t+i}}{\partial \varkappa_t}}_{\text{effect on demographic primary surplus}} + \underbrace{\sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}\frac{g_{Y,t+j}}{R_{t+j}}\right)}{\partial \varkappa_t}\sigma_{t+i}}_{\text{effect on interest-growth differential}} \tag{23}$$

The decomposition will be key in differentiating the fiscal impact of both components of the demographic transition. The first component is negative given the aging of the population, regardless its source. The second component, however, will be negative when there is a fall in population growth, but positive when there is an increase in longevity, an immediate result of Propositions 1 and 2.

In order to rigorously show the difference, we need to translate the definitions above to their correspondent variables in the model.

$$\tau^d_t = 0 \qquad \tau^r_t = \tau_t \tag{24}$$

$$g^d_t = 0 \qquad g^r_t = g_t \tag{25}$$

$$e^d_t(g_{Nt},\delta_t,\mathbf{x}_t) = e_t = \frac{(1-\alpha)\delta_t\nu}{g_{Nt}} \qquad e^r_t = 0 \tag{26}$$

Proposition 4. *A fall in the growth rate of the labor force causes a loss of fiscal space:*

$$\sum_{i=0}^{\infty}\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^r_{t+i}}{\partial g_{Nt}} > 0$$

If the domestic country is "sizeable enough", there exists a survival probability $\bar{\delta}_{t+1}$ such that:

$$\sum_{i=0}^{\infty}\left(\prod_{j=0} \frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^r_{t+i}}{\partial \delta_t}\begin{cases} \leq 0 & \textit{if } \delta_t \leq \bar{\delta}_t \\ > 0 & \textit{if } \delta_t > \bar{\delta}_t \end{cases}$$

Proof. Appendix. □

Proposition 5. *The aging of population abroad either as a result of a fall in the growth rate of the labor force or an increase in life expectancy unambiguously creates fiscal space in the domestic economy in the current period:*

$$\sum_{i=0}^{\infty}\left(\prod_{j=0}\frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^r_{t+i}}{\partial g^*_{Nt}} < 0 \qquad and \qquad \sum_{i=0}^{\infty}\left(\prod_{j=0}\frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial \sigma^r_{t+i}}{\partial \delta^*_{t}} < 0$$

Proof. Appendix. □

3 Empirical Analysis

3.1 Data

In order to estimate the relationship between the demographic variables, the fiscal balance and the interest-growth differential derived in Section 2, I will use data for 23 advanced economies[1] and 23 emerging economies [2] spanning the period 1956-2018.

The data set is comprised of different data sources. The World Population Prospects (United Nations, 2022) provides the demographic data on working-age and old-age population and life expectancy. To map the data to the model, working age population is defined as 20 to 64 years old, and old age as 65 years of age or older. Life expectancy is considered as the remaining life expectancy at 65 years old. Fiscal variables such as net and gross government debt, primary balance and interest spending are taken from the Mauro and Zhou (2021) database. Data on old-age cash benefits comes from the OECD, and data on TFP growth, the labor share and GDP in dollars is sourced from the Penn World Tables.

Since the interest here lies on the long-run effects of demographics on the differential and the fiscal space, the data was arbitrarily filtered in order to remove periods of extreme values or high volatility of the interest-growth differential. One should expect demographics not to play a part in determining such extreme values. The cutoffs were established at -15% or 15% for the level and -10 and 10 percentage points for the annual change.

In order to carry out the empirical analysis, it is necessary to evaluate the nature of each variable at hand with respect to stationarity. Table 3 shows the p-value of the ADF tests for each one of the variables and for all countries in the database. The ADF tests were carried out including a constant and choosing the number of included lags according to the Akaike criterion.

[1] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States.

[2] Argentina, Bolivia, Brazil, Bulgaria, Chile, China, Colombia, Costa Rica, Korea, Dominican Republic, Honduras, Hungary, India, Indonesia, Mexico, Nicaragua, Panama, Paraguay, Peru, Philippines, Poland, Romania, Thailand.

The tests show us that when treated independently, the interest-growth differential, the primary balance that stabilizes the debt-to-GDP ratio, and the TFP growth rate are the variables for which we reject the null hypothesis of the ADF test more often than not with 28, 29, and 43 of such cases out of a total of 46, respectively. The Im, Pesaran and Shin's W-statistic also point out to these variables as being stationary.

On the other hand, the growth rate of the working-age population, life expectancy at 65, the debt-to-GDP ratio and the labor share are clearly non-stationary with only 4, 0, 8, and 4 rejections of the null, respectively, and also a non-rejection by the W-statistic. This result suggests the presence of a unit root in each one of these series.

Carrying out the ADF test assuming independence across countries allows us to capture unit roots in variables of each cross-section unit, which in turn may even be cointegrated among each other, but it makes us miss the cointegration *across* cross-section units that can be potentially found in a panel data context. Therefore, it is useful to look for common factors across units and to assess their stationarity.

When there is cross-sectional dependence, it is useful to decompose the variable into two components: a vector of common factors to all cross-section units, F_{zt}, and a idiosyncratic element, ε_{zit}. In such case, a variable can present a unit root stemming either from the common factors or from its idiosyncratic component:

$$z_{it} = d_{it} + \beta'_{zi} F_{zt} + \varepsilon_{zit} \tag{27}$$

where

$$\begin{aligned} (1-L)F_{zt} &= C(L)u_{zt} \\ \varepsilon_{zit} &= \rho\varepsilon_{zit-1} + \nu_{zit} \end{aligned}$$

In order to perform the decomposition and test both components for nonstationarity, we can resort to Bai and Ng (2004)'s PANIC test. The test follows a series of steps to decompose each panel variable and to test for stationarity. The results can be found in Table 4.

3.2 Methods

Given that the series are nonstationary, regular OLS estimation provides superconsistent estimates. The only issue with performing a simple OLS estimation is that the associated standard errors are not estimated consistently and thus not suitable for statistical inference on the cointegrating vector. A straightforward approach that correct for this issue is the fully-modified OLS estimator (FMOLS, Phillips and Hansen (1990)).

Fully-modified OLS uses a nonparametric procedure to adjust the cointegrating variables and address the long-run correlation between the innovations in the regressors and the errors of the cointegrating equation. By doing so, it corrects the standard errors and allows for the use of standard t and F-statistics for hypothesis testing. It first carries out a preliminary estimation of the cointegration relationship by OLS to obtain the residuals and use them to adjust the independent variable such that the estimator is not asymptotically biased. It then estimates the long-term regression:

3.3 Aging and Fiscal Spillovers in Emerging Economies

Finding a sufficient statistic that captures the essence of fiscal space implied by Definition 1 is not straightforward.

One correlated measure is the primary balance that stabilizes the current debt-to-GDP ratio assuming the interest-growth differential to be constant at its current level. The definition of the stabilizing primary balance, $\bar{\sigma}_t$ is:

$$\sigma^*_{it} \equiv b_{it}\left(\frac{R_{it}}{g_{Yit}} - 1\right) \tag{28}$$

where b_{it} is the current debt-to-GDP ratio and R_t/g_{Yit} is the interest-growth differential.

The interest-growth differential of interest here is the one that is relevant to debt dynamics. As such, the interest rate is the implicit interest rate calculated as the ratio between interest spending and the debt stock. That provides the implicit one-year interest rate on government debt. The output growth rate is the growth rate of nominal GDP measured in local currency.

The long-run relationship between the stabilizing primary balance and demographics will be estimated by the following equation:

$$\sigma^*_{it} = \alpha_i + d'_{it}\beta + \bar{d}'_{-it}\gamma + X'_{it}\theta_0 + \bar{X}'_{-it}\theta_1 + u_{it} \tag{29}$$

where $d_{it} = [g_{Nit}, \ell_{it}]$ is the vector of country i's demographic variables: working-age population growth, g_{Nit}, and life expectancy at 65, ℓ_{it}; $\bar{d}_{-it}$ is the cross-section average of demographic variables of the other countries weighted by their GDP; X_{it} represents the domestic controls: labor share and TFP growth; and $\bar{X}_{-it}$ represents the cross-section average of foreign controls weighted by countries' GDP: labor share, TFP growth, and the debt-to-GDP ratio. FMOLS time-demeans all regressors.

Table 1 shows the estimation of equation (29) for the full sample of emerging economies, from 1956 to 2019. Equations (1) through (6) estimate equation (29) using Panel FMOLS assuming that the marginal effects are homogeneous across countries. They come in two

Table 1: Emerging Economies (1956-2018)

Dep. Variable: Stabilizing Balance	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
AR(1)	0.7496		0.7198		0.7011		0.6573	
	(0.0000)***		(0.0000)***		(0.0000)***		(0.0000)***	
Working-Age Pop. Growth (%)	-0.2588	-0.9441	-0.2722	-0.7402	-0.2309	-0.5094	-0.1717	-0.4291
	(0.0247)**	(0.0000)***	(0.0258)**	(0.0001)***	(0.0491)**	(0.0009)***	(0.1416)	(0.2537)
Longevity at 65	-0.0136	-0.1006	-0.018	-0.0276	-0.1971	-0.2416	-0.1628	-0.1828
	(0.7575)	(0.1426)	(0.7247)	(0.7288)	(0.0071)***	(0.0109)**	(0.0411)**	(0.4826)
Advanced WA Pop. Growth (%)					0.1815	0.2093	-0.1815	-0.3266
					(0.6687)	(0.7061)	(0.7281)	(0.7078)
Advanced Longevity at 65					-0.0793	0.2293	-0.1877	0.1056
					(0.4961)	(0.1323)	(0.0775)*	(0.7374)
Domestic Controls	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Foreign Controls	No	No	No	No	Yes	Yes	Yes	Yes
Countries	23	23	23	23	23	23	23	23
Observations:	755	769	729	742	729	742	744	744
Starting Year	1956	1956	1956	1956	1956	1956	1956	1956
R-squared:	0.6389	0.2616	0.6641	0.2831	0.6761	0.3016	0.6856	0.3011

p-values in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

different flavors: with and without the autoregressive term. The addition of the AR(1) term seeks to capture the persistence of shocks to the dependent variable.

Equations (1) and (2) only include both demographic drivers as regressors. The growth rate of the working-age population is already statistically significant in this simple specification. The long-run effect is one-to-one, as in equation (1) an increase in the working-age population growth leads to a fall of $0.2588/(1 - 0.7496) = 1.03$ percentage points in the primary balance needed to stabilize the debt-to-GDP ratio. Equation (2) is similar, with a fall of 0.94 percentage points for the same change in the working-age population growth.

Equations (3) and (4) control for domestic TFP growth and for the domestic labor share, but do not differ much from the results in (1) and (2). The long-run effect of the working-age population growth is still nearly one-to-one. On the other hand, when foreign controls are added in equations (5) and (6) both domestic demographic drivers come out significant. The foreign controls utilized are the average of working-age population growth, life expectancy at 65, TFP growth, debt-to-GDP ratio and labor share across all advanced economies weighted by real GDP in 2017 dollars. Here the long-run effect of working-age population growth is lower: -0.77 in specification (5) and -0.51 in specification (6). However the effect of longevity is much more pronounced: one more year of life expectancy at 65 translates to a fall of 0.66 percentage points in the stabilizing primary balance in equation (5) and a fall of 0.24 percentage points in equation (6). This is particularly strong once one considers that over time life expectancy increased 5 years for every 1 percentage point fall in working-age population growth. In addition, the effect of longevity on fiscal space is ambiguous in theory, which makes this result even surprising.

The point estimates of foreign demographics in equation (5) confirm the analytical results. A fall of 1 percentage point in the foreign working-age population growth lead to a long-term fall of 0.61 percentage points in the stabilizing balance, while a one-year increase in foreign life expectancy at 65 leads to a fall of 0.27 percentage points in the stabilizing primary balance.

Equations (7) and (8) carry out the same estimation as (5) and (6), but use a panel fixed effects estimation with standard errors clustered at the country level as a benchmark for comparison. The estimates are not very significant for most demographic variables even though their signs and magnitudes are similar to equations (5) and (6).

3.4 Aging and Fiscal Spillovers in Advanced Economies

Advanced economies have been aging for longer than emerging economies and have also been more financially integrated. This suggests that the estimation of the fiscal effects of domestic and foreign demographics should result in more accurate estimates. Table 2 contains the estimations of equation (29) for the subset of advanced economies and, as expected, the results are more in line with the theory.

Equations (2) and (4) present strong fiscal effects for both demographic drivers. Both a fall in the working-age population and an increase in longevity at 65 lead to an increase in the stabilizing primary balance. Estimations (1) and (3) do not yield significant results, however. As foreign controls are added, however, the long-run effect of the working-age population growth gets more pronounced, -0.8 in equation (5) and -1.03 in (6). The sign of the effect of life expectancy, on the other hand, flips, and becomes negative: -0.48 is the long-run effect in (5), albeit insignificant, and -0.24 is the effect in (6), strikingly similar to the effect estimated for emerging economies with the same specification.

The effect of fiscal spillovers are more distinct in advanced economies. Regression (5) finds a significant beneficial fiscal spillover of an increase in life expectancy abroad. An increase of 1 year in the average longevity at 65 of other advanced economies leads to a long-run decrease of 1.75 percentage points in the stabilizing primary balance, a very strong effect. The effect of longevity estimated by regression (6) is much less pronounced: a fall of 0.03 percentage points for every one-year increase in longevity abroad, but not significant. Equation (5) shows a negative, but not significant effect of working-age population growth, whereas equation (6) shows a very strong positive effect: for a fall of one percentage point in the working age population growth abroad, the stabilizing balance falls by 3.29 percentage points. Surprisingly the results are very similar in magnitude and significance in equations (7) and (8) where the estimation is done with a panel fixed effects OLS.

Table 2: Advanced Economies (1956-2018)

Dep. Variable: Stabilizing Balance	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
AR(1)	0.8817		0.8452		0.8118		0.7295	
	(0.0000)***		(0.0000)***		(0.0000)***		(0.0000)***	
Working-Age Pop. Growth (%)	0.0441	-0.6563	-0.08	-0.7707	-0.1592	-1.0348	-0.2205	-1.0144
	(0.6259)	(0.0002)***	(0.3208)	(0.0000)***	(0.0355)**	(0.0000)***	(0.0354)**	(0.0005)***
Longevity at 65	0.0151	0.3724	-0.0507	0.3193	-0.0902	-0.2413	-0.0696	-0.1876
	(0.565)	(0.0000)***	(0.0926)*	(0.0000)***	(0.248)	(0.0484)**	(0.1522)	(0.3677)
Advanced WA Pop. Growth (%)					-0.114	3.2877	0.4121	3.5534
					(0.7261)	(0.0000)***	(0.1377)	(0.0000)***
Advanced Longevity at 65					-0.329	-0.0386	-0.2769	0.0173
					(0.0031)***	(0.8233)	(0.0062)***	(0.9435)
Domestic Controls	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Foreign Controls	No	No	No	No	Yes	Yes	Yes	Yes
Countries	23	23	23	23	23	23	23	23
Observations:	1342	1344	1341	1343	1341	1343	1344	1344
Starting Year	1956	1956	1956	1956	1956	1956	1956	1956
R2	0.6305	0.233	0.6873	0.2766	0.7022	0.3218	0.708	0.3237

p-values in parentheses, * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4 Conclusion

The demographic transition has been an ongoing slow-moving process for advanced economies that picked up especially in the post war era. Emerging economies have sustained a younger age structure of the population for a longer time and are now at the early stages of the aging process. The non-synchronized character of this transition has implications for emerging economies as it drives capital flows from the aging economies to the younger ones.

This study finds that emerging economies were affected positively by the aging of advanced economies starting the nineties, as financial integration increased between the developed and developing worlds.

It also finds that the different drivers of demographic change has opposing fiscal effects: a fall in the domestic working-age population growth is negative and calls for a fiscal adjustment, whereas an increase in domestic longevity increases savings and can thus lead to a positive fiscal effect.

Projections of future population aging by the United Nations (United Nations, 2022) show a very rapid fall in emerging countries' growth rate of working-age population, much faster than the one experienced by advanced economies. This suggests that as emerging economies age and catch up with advanced economies, they will be hit by a double-whammy of rising deficits and the abatement of capital flows which will critically tighten their fiscal space.

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Appendices

A Proofs of Propositions

A.1 Proposition 1

Proof.

Part 1: It suffices to show that the elasticity of the real interest rate with respect to the population growth rate is bigger than the elasticity of the output growth rate to the population growth rate.

$$\frac{\partial R_t}{\partial g_{Nt}}\frac{g_{Nt}}{R_t} = (\alpha - 1)\frac{\partial k_t}{\partial g_{Nt}}\frac{g_{Nt}}{k_t} \tag{30}$$

$$\frac{\partial g_{Yt}}{\partial g_{Nt}}\frac{g_{Nt}}{g_{Yt}} = 1 + \alpha\frac{\partial k_t}{\partial g_{Nt}}\frac{g_{Nt}}{k_t} \tag{31}$$

$$\frac{\partial R_t}{\partial g_{Nt}}\frac{g_{Nt}}{R_t} > \frac{\partial g_{Yt}}{\partial g_{Nt}}\frac{g_{Nt}}{g_{Yt}} \iff \frac{\partial k_t}{\partial g_{Nt}}\frac{g_{Nt}}{k_t} > -1 \tag{32}$$

which means that we just need to check whether the capital stock in efficiency units is inelastic to the population growth rate. Apply the implicit function theorem to the market clearing equation (15):

$$\frac{\partial k_t}{\partial g_{Nt}}\frac{g_{Nt}}{k_t} = -\frac{g_{At}g_{Nt}}{\frac{1-\alpha}{\alpha}\left(\frac{\delta_t}{1+\beta\delta_t}\nu g_{At} + \varphi^*_{t-1}\frac{\delta^*_t}{1+\beta^*\delta^*_t}\nu^* g^*_{At}\right) + (g_{At}g_{Nt} + \varphi^*_{t-1}g^*_{At}g^*_{Nt})} \tag{33}$$

As long as $\varphi^*_{t-1} > 0$ and/or $\nu \neq 0$, the capital stock in efficiency units will be inelastic to the population growth rate and, as a result, the interest-growth differential will expand when the population growth rate falls.

Part 2: Here the result will depend on current savings.

$$\frac{\partial R_t}{\partial \delta_t}\frac{\delta_t}{R_t} = (\alpha - 1)\frac{\partial k_t}{\partial \delta_t}\frac{\delta_t}{k_t} \tag{34}$$

$$\frac{\partial g_{Yt}}{\partial \delta_t}\frac{\delta_t}{g_{Yt}} = \alpha\frac{\partial k_t}{\partial \delta_t}\frac{\delta_t}{k_t} \tag{35}$$

$$\frac{\partial R_t}{\partial \delta_t}\frac{\delta_t}{R_t} \gtrless \frac{\partial g_{Yt}}{\partial \delta_t}\frac{\delta_t}{g_{Yt}} \iff \frac{\partial k_t}{\partial \delta_t}\frac{\delta_t}{k_t} \lesseqgtr 0 \tag{36}$$

$$\frac{\partial k_t}{\partial \delta_t} = \frac{\frac{\beta}{(1+\beta\delta_t)^2}((1-\alpha)-\tau_{t-1})k_{t-1}^{\alpha} - \frac{1-\alpha}{\alpha}\frac{1}{(1+\beta\delta_t)^2}\nu g_{At}k_t}{\frac{1-\alpha}{\alpha}\left(\frac{\delta_t}{1+\beta\delta_t}\nu g_{At} + \varphi_{t-1}^{*}\frac{\delta_t^{*}}{1+\beta^{*}\delta_t^{*}}\nu^{*}g_{At}^{*}\right) + (g_{At}g_{Nt} + \varphi_{t-1}^{*}g_{At}^{*}g_{Nt}^{*})}$$

$$= \frac{1}{\delta_t(1+\beta\delta_t)}\frac{s_t/A_t}{\left[\frac{1-\alpha}{\alpha}\left(\frac{\delta_t}{1+\beta\delta_t}\nu g_{At} + \varphi_{t-1}^{*}\frac{\delta_t^{*}}{1+\beta^{*}\delta_t^{*}}\nu^{*}g_{At}^{*}\right) + (g_{At}g_{Nt} + \varphi_{t-1}^{*}g_{At}^{*}g_{Nt}^{*})\right]}$$

$$s_t \gtrless 0 \iff \frac{\partial k_t}{\partial \delta_t}\frac{\delta_t}{k_t} \gtrless 0 \iff \frac{\partial \frac{R_t}{g_{Yt}}}{\partial \delta_t} \lessgtr 0 \tag{37}$$

□

$$\frac{\partial R_{t+2}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{R_{t+2}} = (\alpha-1)\frac{\partial k_{t+2}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{k_{t+2}} \tag{38}$$

$$\frac{\partial g_{Yt+2}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{g_{Yt+2}} = \alpha\left(\frac{\partial k_{t+2}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{k_{t+2}} - \frac{\partial k_{t+1}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{k_{t+1}}\right) \tag{39}$$

$$\frac{\partial R_{t+2}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{R_{t+2}} < \frac{\partial g_{Yt+2}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{g_{Yt+2}} \iff \frac{\partial k_{t+2}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{k_{t+2}} > \alpha\frac{\partial k_{t+1}}{\partial g_{Nt+1}}\frac{g_{Nt+1}}{k_{t+1}} \tag{40}$$

1.2 Proposition 2

Proof.

□

1.3 Proposition 5

Proof.

Part 1: The effect of changes in population growth on the fiscal space is straightforward given that the impacts on the primary surplus and on the interest-growth differential go in the same direction.

The impact of a *fall* in the population growth rate on the primary balance is negative.

$$\sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{g_{Y,t+j}}{R_{t+j}}\right)\frac{\partial\sigma_{t+i}^{d}}{\partial g_{Nt}}=\frac{g_{Y,t}}{R_t}\frac{\nu(1-\alpha)\delta_t}{g_{Nt}^2}>0 \tag{41}$$

$$\begin{aligned}
\sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}^{i}\frac{g_{Y,t+j}}{R_{t+j}}\right)}{\partial g_{Nt}}\sigma_{t+i}=&\frac{\partial\frac{g_{Y,t}}{R_t}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t}}{R_t}}\frac{\frac{g_{Y,t}}{R_t}}{g_{Nt}}\sigma_t+\\
&+\left(\frac{\partial\frac{g_{Y,t}}{R_t}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t}}{R_t}}+\frac{\partial\frac{g_{Y,t+1}}{R_{t+1}}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t+1}}{R_{t+1}}}\right)\frac{\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}}{g_{Nt}}\sigma_{t+1}+\\
&+\left(\frac{\partial\frac{g_{Y,t}}{R_t}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t}}{R_t}}+\frac{\partial\frac{g_{Y,t+1}}{R_{t+1}}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t+1}}{R_{t+1}}}+\frac{\partial\frac{g_{Y,t+2}}{R_{t+2}}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t+2}}{R_{t+2}}}\right)\frac{\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}\frac{g_{Y,t+2}}{R_{t+2}}}{g_{Nt}}\sigma_{t+2}+\cdots
\end{aligned}$$

Now rearrange according to the changes in the interest-growth differential in each period:

$$\begin{aligned}
\sum_{i=0}^{\infty}\frac{\partial\left(\prod_{j=0}^{i}\frac{g_{Y,t+j}}{R_{t+j}}\right)}{\partial g_{Nt}}\sigma_{t+i}=&\frac{\partial\frac{g_{Y,t}}{R_t}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t}}{R_t}}\frac{1}{g_{Nt}}\left(\frac{g_{Y,t}}{R_t}\sigma_t+\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}\sigma_{t+1}+\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}\frac{g_{Y,t+2}}{R_{t+2}}\sigma_{t+2}+\cdots\right)\\
&+\frac{\partial\frac{g_{Y,t+1}}{R_{t+1}}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t+1}}{R_{t+1}}}\frac{1}{g_{Nt}}\left(\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}\sigma_{t+1}+\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}\frac{g_{Y,t+2}}{R_{t+2}}\sigma_{t+2}+\cdots\right)\\
&+\frac{\partial\frac{g_{Y,t+2}}{R_{t+2}}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t+2}}{R_{t+2}}}\frac{1}{g_{Nt}}\left(\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}\frac{g_{Y,t+2}}{R_{t+2}}\sigma_{t+2}+\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}\frac{g_{Y,t+2}}{R_{t+2}}\frac{g_{Y,t+3}}{R_{t+3}}\sigma_{t+3}+\cdots\right)\\
&+\cdots\\
&=\frac{1}{g_{Nt}}\left(\frac{\partial\frac{g_{Y,t}}{R_t}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t}}{R_t}}b_t+\frac{\partial\frac{g_{Y,t+1}}{R_{t+1}}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t+1}}{R_{t+1}}}\frac{g_{Y,t}}{R_t}b_{t+1}+\frac{\partial\frac{g_{Y,t+2}}{R_{t+2}}}{\partial g_{Nt}}\frac{g_{Nt}}{\frac{g_{Y,t+2}}{R_{t+2}}}\frac{g_{Y,t}}{R_t}\frac{g_{Y,t+1}}{R_{t+1}}b_{t+2}+\cdots\right)\\
&>0 \text{ using Propositions 1 and 2, if } b_{t+i}\geq 0, \forall i=1,2,\ldots
\end{aligned}$$

□

2 Unit Root Tests

Table 3: Unit Root Tests - Independent Cross-Sections

	Differential	Stabilizing Balance	Population Growth	Life Expectancy	Debt-to-GDP	Labor Share	TFP Growth
Argentina	0.1969	0.4522	0.5666	0.5208	0.9382	0.9700	0.0585
Australia	0.0390	0.1453	0.1087	0.9998	0.2094	0.9230	0.0000
Austria	0.0132	0.0012	0.1976	0.9980	0.8296	0.6827	0.0000
Belgium	0.1869	0.3311	0.1683	0.9984	0.4458	0.7859	0.2398
Bolivia	0.0233	0.0120	0.0757	0.9988	0.3134	0.4853	0.0207
Brazil	0.0483	0.1609	0.9995	0.9743	0.3870	0.9680	0.0658
Bulgaria	0.3397	0.0893	0.9926	0.7257	0.0285	0.3833	0.0000
Canada	0.2829	0.3374	0.8436	0.9984	0.6270	0.8113	0.0000
Chile	0.0027	0.0000	0.0861	0.5406	0.0001	0.2648	0.0679
China	0.0076	0.1396	0.7908	0.9291	0.9999	0.4956	0.0163
Colombia	0.0280	0.0027	0.9643	0.9996	0.7207	0.2450	0.0073
Costa Rica	0.0028	0.0000	0.9995	0.8327	0.9072	-	0.0039
Korea	0.0082	0.0003	0.6474	0.9996	0.9849	0.1709	0.0000
Denmark	0.7697	0.5113	0.5659	0.9998	0.4174	0.6249	0.0000
Dominican Republic	0.0502	0.0106	0.8799	0.7963	0.6691	0.9207	0.0054
Finland	0.0089	0.0324	0.8418	0.9986	0.8678	0.8264	0.0000
France	0.5096	0.3555	0.5064	0.9866	0.9967	0.6620	0.0027
Germany	0.0216	0.1956	0.3852	0.9984	0.8387	0.4528	0.0010
Greece	0.1602	0.9950	0.6503	0.8625	0.9966	0.8945	0.0000
Honduras	0.0168	0.0060	0.5278	0.9997	0.8953	0.5658	0.0001
Hungary	0.2622	0.1583	0.9802	0.8393	0.0438	0.7251	0.0014
Iceland	0.1323	0.0040	0.4711	0.9985	0.4205	0.0228	0.0002
India	0.0001	0.0230	1.0000	0.8550	0.5489	0.9077	0.0000
Indonesia	0.0685	0.0327	0.7977	0.9886	0.0000	0.9694	0.0143
Ireland	0.0047	0.0101	0.7614	0.9990	0.0648	0.9996	0.0000
Israel	0.3320	0.3184	0.0175	0.7304	0.0411	0.7131	0.0000
Italy	0.1964	0.1826	0.1433	0.9920	0.8798	0.8367	0.3383
Japan	0.5218	0.7234	0.9043	0.9609	0.9947	0.8471	0.0002
Mexico	0.0402	0.0406	0.9794	0.5953	0.4405	0.6842	0.0000
Netherlands	0.1891	0.0736	0.9488	0.9949	0.2589	0.9117	0.0327
New Zealand	0.1077	0.0200	0.8673	1.0000	0.4928	0.8167	0.0000
Nicaragua	0.1757	0.3621	0.9996	0.5961	0.7942	0.8374	0.0145
Norway	0.0204	0.0939	0.3131	0.9999	0.0270	0.6220	0.0041
Panama	0.0006	0.0001	0.6047	0.9939	0.6747	0.6572	0.0003
Paraguay	0.1808	0.2194	1.0000	0.9916	0.1635	0.5343	0.0018
Peru	0.0469	0.0254	0.6955	0.9997	0.4627	0.9998	0.0231
Philippines	0.0432	0.0001	0.7099	0.9810	0.3053	0.0006	0.0000
Poland	0.0004	0.0001	0.9997	0.4503	0.6178	0.6392	0.0192
Portugal	0.0597	0.0253	0.5909	0.9969	0.9042	0.0000	0.0005
Romania	0.0082	0.0088	0.9999	0.4497	0.5440	0.1877	0.1035
Spain	0.5357	0.1978	0.2574	0.9789	0.5948	0.9562	0.0159
Sweden	0.0885	0.0368	0.1079	0.9998	0.3483	0.3748	0.0001
Switzerland	0.0598	0.0306	0.0511	0.9873	0.2795	0.0018	0.0000
Thailand	0.0082	0.0074	0.9194	0.9506	0.0873	1.0000	0.0000
United Kingdom	0.1963	0.0473	0.2044	0.9992	0.1943	0.8233	0.0000
United States	0.0013	0.0001	0.7999	0.9843	0.8451	0.6445	0.0000
IPS W-stat	0.0000	0.0000	1.0000	1.0000	0.5936	0.9982	0.0000

Table 4: Unit Root Tests - Dependent Cross-Sections

Idiosyncratic factors

Countries	MCF	MCF Advanced	MCF Emerging
Argentina	0.0153	-	0.1357
Australia	0.0167	0.3321	-
Austria	0.0175	0.0221	-
Belgium	0.267	0.8477	-
Bolivia	0.4891	-	0.6003
Brazil	0.2324	-	0.5119
Bulgaria	0.9808	-	0.3904
Canada	0.0098	0.0239	-
Chile	0.9631	-	0.9272
China	0.526	-	0.4909
Colombia	0.5237	-	0.1604
Costa Rica	0.9999	-	0.3764
Korea	0.8805	-	0.0032
Denmark	0.7159	0.6539	-
Dominican Republic	0.9215	-	0.4308
Finland	0.0199	0.0002	-
France	0.0178	0	-
Germany	0.3259	0.0002	-
Greece	0.2945	0.1631	-
Honduras	0.7761	-	0
Hungary	0.2644	-	0.3804
Iceland	0.0004	0.141	-
India	0.0018	-	0.7319
Indonesia	0.9650	-	0.8442
Ireland	0.0405	0.0325	-
Israel	0.0000	0.0688	-
Italy	0.0365	0.002	-
Japan	0.9999	0	-
Mexico	0.4014	-	0.6527
Netherlands	0.0103	0.5154	-
New Zealand	0.9999	0	-
Nicaragua	0.9879	-	0.7819
Norway	0.1115	0.3983	-
Panama	0.0035	-	0.3492
Paraguay	0.5425	-	0.9909
Peru	0.0000	-	0.9996
Philippines	0.5637	-	0.1931
Poland	0.7385	-	0.0332
Portugal	0.9999	0.0593	-
Romania	0.0002	-	0.0017
Spain	0.0274	0.1182	-
Sweden	0.9185	0.6688	-
Switzerland	0.0011	0.0038	-
Thailand	0.1035	-	0.0038
United Kingdom	0.0394	0.0013	-
United States	0.0088	0.0123	-

Common factors			
Total number	3	1	8
Nonstationary	3	1	8
Pooled Statistic	0.0000	0.0000	0.0000
Cross-sections	46	23	23
Balanced Observations	19	32	21