Anistropy Assisted Quasi-Phase Matching (AA-QPM)

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Introduction

The goal of this code project is to model the implementation of Anitropy-Assisted Quasi-Phase Matching (AA-QPM) for on-chip Second Harmonic Generation (SHG). This is, to the best of my knowledge, an original approach towards realizing nonlinear optical processes between phase-mismatched optical modes in integrated photonics. In traditional Quasi-Phase-Matching (QPM), the ferroelectric domains of a crystal are periodically inverted with the correct periodicity to compensate the modal mismatch. Alternatively, in AA-QPM, the effective nonlinear susceptibility is itself modulated by the path shape of the guiding structures.

The contents of the project start with this brief theoretical introduction, and is followed by the actual code in the attached Wolfram Mathematica notebook. The contents are as follows:

- Overview of the Nonlinear Susceptibility Tensor
- Coupled Mode Theory for Second Hamonic Generation
- Modelling QPM and calculating its efficiency
- Extracting effective nonlinearity from a path shape
- Modelling AA-QPM and calculating its efficiency

Importing packages and setting graphic appearances

1. Overview of the $\chi^{(2)}$ Nonlinear Susceptibility Tensor

Susceptibility Tensor

We start from the coupled mode equation for Second Harmonic Generation in a waveguide or nonlinear optical fiber (assuming no pump depletion)

$$\frac{d}{dz}A^{(2\omega)} = \frac{i\omega}{2}\epsilon_0 d_{eff}\left[A^{(\omega)}\right]^2 e^{-i(2\beta^{(\omega)}-\beta^{(2\omega)})z}\iint_{-\infty}^{\infty}\left[E_i^{(2\omega)}\right]^*\cdot\left[E_j^{(\omega)}E_k^{(\omega)}\right]\,dxdy$$

In this Equation, $A(z)$ is the normalized field amplitudes, whose z-dependence is a consequence of mode coupling induced by the optical nonlinearity. The equations tells us three main parameters that contribute to this dependence: (1) the effective nonlinear susceptibility, that is, the nonlinearity "experienced" by the optical mode in a specific polarization and propagation direction, (2) the phase-matching term, represented in the complex exponential, which implies that exclusively "forward" (rather than "reverse") conversion of power between modes can only happen when $\Delta = 2\beta^{(\omega)} - \beta^{(2\omega)}$, and (3) a spatial overlap integral, which also dictates the strength of the interaction by measuring the similarity in the spatial profiles of the optical modes.

The technique introduced in this project requires us to bring all this components together to compensate each other towards an efficient nonlinear conversion. To discuss the tensorial nature of the nonlinear susceptibility, we shall rewrite the equation back to a more general form, where χ^2 has not yet been simplified as an "effective term":

$$\frac{d}{dz}A^{(2\omega)} = \frac{i\omega}{2}\epsilon_0 d_{eff}\left[A^{(\omega)}\right]^2 e^{-i(2\beta^{(\omega)}-\beta^{(2\omega)})z}\iint_{-\infty}^{\infty}\left[E_i^{(2\omega)}\right]^* \cdot \left[\chi_{ijk}^{(2)}E_j^{(\omega)}E_k^{(\omega)}\right]\, dxdy$$

And then we are especially interested in the term that appears in the right-hand side under the spatial integral, that is

$$\kappa = \eta\iint\left[E_i^{(2\omega)}\right]^* \cdot \left[\chi_{ijk}^{(2)}E_j^{(\omega)}E_k^{(\omega)}\right]\, dxdy$$

Expanding the factor on the second brackets ($E^{(\omega)} \equiv E$), we get

$$\chi_{ijk}^{(2)}E_jE_k = \chi_{i11}^{(2)}E_1E_1 + \chi_{i12}^{(2)}E_1E_2 + \chi_{i13}^{(2)}E_1E_3 + \chi_{i21}^{(2)}E_2E_1 + \chi_{i22}^{(2)}E_2E_2 + \chi_{i32}^{(2)}E_2E_3 + \chi_{i31}^{(2)}E_3E_1 + \chi_{i32}^{(2)}E_3E_2 + \chi_{i33}^{(2)}E_3E_3$$

Now we contract the indexes by making

- $\chi_{i11} \equiv \chi_{i1}$
- $\chi_{i22} \equiv \chi_{i2}$
- $\chi_{i33} \equiv \chi_{i3}$
- $\chi_{i23} = \chi_{i32} \equiv \chi_{i4}$
- $\chi_{i31} = \chi_{i13} \equiv \chi_{i5}$
- $\chi_{i12} = \chi_{i21} \equiv \chi_{i6}$

Thus

$$\chi_{ijk}^{(2)}E_jE_k = \chi_{i1}^{(2)}E_1^2 + \chi_{i2}^{(2)}E_2^2 + \chi_{i3}^{(2)}E_3^2 + 2\chi_{i4}^{(2)}E_2E_3 + 2\chi_{i5}^{(2)}E_1E_3 + 2\chi_{i6}^{(2)}E_1E_2$$

For Lithium niobate (3m group crystal), the only terms remaining will be χ_{15}, χ_{16}, χ_{24}, χ_{21}, χ_{22}, χ_{31}, χ_{32}, χ_{33}. Notice that this is a significant reduction, since each of the terms should span a set of three terms corresponding to $i = 1, 2, 3$. We are left with

$$\sum_i \chi_{ijk}^{(2)}E_jE_k = \chi_{21}^{(2)}E_1^2 + \chi_{31}^{(2)}E_1^2 + \chi_{22}^{(2)}E_2^2 + \chi_{32}^{(2)}E_2^2 + \chi_{33}^{(2)}E_3^2 + 2\chi_{24}^{(2)}E_2E_3 + 2\chi_{15}^{(2)}E_1E_3 + 2\chi_{16}^{(2)}E_1E_2$$

The tensor is

$$\chi^{(2)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \chi_{15} & \chi_{16} \\ \chi_{21} & \chi_{22} & 0 & \chi_{24} & 0 & 0 \\ \chi_{31} & \chi_{32} & \chi_{33} & 0 & 0 & 0 \end{pmatrix}$$

Now, exploring Kleinman symmetry (exchange of i index) [1], many of those terms are set equal

- $\chi_{15} = \chi_{31}$
- $\chi_{16} = \chi_{21}$
- $\chi_{24} = \chi_{32}$

$$\chi^{(2)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \chi_{31} & \chi_{21} \\ \chi_{21} & \chi_{22} & 0 & \chi_{32} & 0 & 0 \\ \chi_{31} & \chi_{32} & \chi_{33} & 0 & 0 & 0 \end{pmatrix}$$

And again for properties specific to 3m group crystals [1]

- $\chi_{21} = -\chi_{22}$

- $\chi_{32} = \chi_{31}$

Such that

$$\chi^{(2)} = \begin{pmatrix} 0 & 0 & 0 & 0 & \chi_{31} & -\chi_{22} \\ -\chi_{22} & \chi_{22} & 0 & \chi_{31} & 0 & 0 \\ \chi_{31} & \chi_{31} & \chi_{33} & 0 & 0 & 0 \end{pmatrix}$$

And now the expression for the product appearing in the overlap integral goes to

$$\sum_i \chi^{(2)}_{ijk} E_j E_k = -\chi^{(2)}_{22} E_1^2 + \chi^{(2)}_{31} E_1^2 + \chi^{(2)}_{22} E_2^2 + \chi^{(2)}_{31} E_2^2 + \chi^{(2)}_{33} E_3^2 + 2\chi^{(2)}_{31} E_2 E_3 + 2\chi^{(2)}_{31} E_1 E_3 - 2\chi^{(2)}_{22} E_1 E_2$$

And simply grouping some terms

$$\sum_i \chi^{(2)}_{ijk} E_j E_k = -\chi^{(2)}_{22} \left(E_1^2 - E_2^2 + 2E_1 E_2\right) + \chi^{(2)}_{31} \left(E_1^2 + E_2^2 + 2E_2 E_3 + 2E_1 E_3\right) + \chi^{(2)}_{33} \left(E_3^2\right)$$

Finally, the overlap integral becomes

$$\kappa = \eta \iint \begin{pmatrix} E_1^{*(2\omega)} & E_2^{*(2\omega)} & E_3^{*(2\omega)} \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 & \chi_{31} & -\chi_{22} \\ -\chi_{22} & \chi_{22} & 0 & \chi_{31} & 0 & 0 \\ \chi_{31} & \chi_{31} & \chi_{33} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_1^2 \\ E_2^2 \\ E_3^2 \\ 2E_2 E_3 \\ 2E_3 E_1 \\ 2E_1 E_2 \end{pmatrix} dxdy$$

Conveniently, we can write

$$\frac{1}{\eta}\kappa = (d_{31} I_A + d_{22} I_B + d_{33} I_C)$$
$$I_A = \iint \left(2E_1^{*(2\omega)} E_3 E_1 + 2E_2^{*(2\omega)} E_2 E_3 + E_3^{*(2\omega)} [E_1^2 + E_2^2]\right) dxdy$$
$$I_B = -\iint \left(2E_1^{*(2\omega)} E_1 E_2 + E_2^{*(2\omega)} [-E_1^2 + E_2^2]\right) dxdy$$
$$I_C = \iint \left(E_3^{*(2\omega)} E_3^2\right) dxdy$$

In almost every case of practical importance, I_A and I_B can be ignored in comparison to I_C. However, we will se that in our treatment of anisotropy-assisted quasi-phase matching, we must consider how the field amplitude is distributed among the crystalline orientation directions.

This project now continues in the Mathematica Notebook.

References

[1] V. G. Dmitriev, G. G. Gurzadyan, D. N. Nikogosyan, Handbook of Nonlinear Optical Crystals, sections 2.10, 3.1.8.

[2] R. W. Boyd, Nonlinear Optics, sections 1.5.