# Vector Boson decays of the Higgs Boson

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#### **Abstract**

We derive the width of the Higgs boson into vector bosons. General formulas are derived both for the on–shell decay  $H \to VV$  as well for the off–shell decays,  $H \to V^*V$  and  $H \to V^*V^*$ , where  $V = \gamma, W^{\pm}, Z^0$ . For the off-shell decays the width of the decaying vector boson is properly included. The formulas are valid both for the Standard Model as well as for arbitrary extensions. As an example we study in detail the gauge-invariant effective Lagrangian models where we can have sizable enhancements over the Standard Model that could be observed at LEP.

#### 1 Introduction

In recent years it has been established [1] with great precision (in some cases better than 0.1%) that the interactions of the gauge bosons with the fermions are described by the Standard Model (SM) [2]. However other sectors of the SM have been tested to a much lesser degree. In fact only now we are beginning to probe the self–interactions of the gauge bosons through their pair production at the Tevatron [3] and LEP [4] and the Higgs sector, responsible for the symmetry breaking has not yet been tested.

A more complicated symmetry breaking sector can introduce modifications in the couplings of the Higgs boson with the vector bosons. It is therefore important to have expressions for the decay widths of the Higgs boson into vector bosons that are valid for an arbitrary extension of the SM. For the region of the Higgs boson mass relevant for searches at LEP II and LHC it is necessary that the vector bosons in the decays can be off-shell.

In this paper we derive the complete set of formulas for the decay widths of the Higgs boson in vector bosons. The formulas are valid both for the Standard Model (SM) and for any arbitrary extension. For the case of the decay into the  $W^{\pm}$  and  $Z^{0}$  the formulas are also valid for off-shell decays. This is important for Higgs boson masses close to the threshold of the production of one or two real vector bosons. Many of these results have appeared before in the literature [5, 6, 7, 8, 9, 10], sometimes for particular cases, but we think that it will be very useful for the Higgs boson search at LEP and at LHC to have the general results in a consistent notation.

The paper is organized as follows. In Section 2 the decays  $H \to VV$  where  $V = W^{\pm}, Z^0$  are calculated. The decays  $H \to \gamma\gamma$  and  $H \to \gamma Z^0$ , that in the SM proceed at one-loop level, are reviewed in Sections 3 and 4, respectively. In Section 5 the off-shell 3-point functions  $Z^* \to H\gamma$  and  $\gamma^* \to H\gamma$  are given in a consistent notation both for the SM as well as for any of its extensions. In Section 6 we give an example of physics Beyond de Standard Model (BSM) and in Section 7 a brief discussion of our results and a comparison with previous ones is presented.

#### 2 The Decays $H \rightarrow VV$

#### 2.1 The HVV Couplings

We consider the most general couplings of the Higgs H with the  $W^{\pm}$  and  $Z^{0}$ . These are

$$H - \frac{P}{V_{\nu}} = \frac{i g M_V \left(g_{\mu\nu} + T_{\mu\nu}^V\right)}{i g M_V \left(g_{\mu\nu} + T_{\mu\nu}^V\right)}$$
 (1)
and  $T^W$  and  $T^Z$  are the extra contributions from new physics Beyond the

where V = W, Z and  $T_{\mu\nu}^W$  and  $T_{\mu\nu}^Z$  are the extra contributions from new physics Beyond the Standard Model (BSM). In general they will depend on the momenta  $P, k_1$  and  $k_2$ , but as we will see, we will not need their exact expressions to get the final formulas.

#### 2.2 The On–Shell Decay $H \rightarrow VV$

We now consider the on–shell decay  $H \to VV$ . To be precise we derive the expression for  $H \to W^+W^-$  and then present a final result valid also for  $H \to Z^0Z^0$ . We consider the kinematics given in Fig. 1.

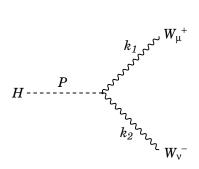


Fig. 1

The differential width is given by [11]

$$d\Gamma = \frac{1}{32\pi^2} \sum_{pol} |\mathcal{M}|^2 \frac{|\vec{k}_1|}{M_H^2} d\Omega_{\vec{k}_1}$$
 (2)

where

$$\mathcal{M} = i g M_W \epsilon^{\mu}(k_1) \epsilon^{\nu}(k_2) \left( g_{\mu\nu} + T_{\mu\nu}^W \right) \tag{3}$$

We get therefore

$$\sum_{pol} |\mathcal{M}|^{2} = (gM_{W})^{2} \left(-g^{\mu\alpha} + \frac{k_{1}^{\mu}k_{1}^{\alpha}}{M_{W}^{2}}\right) \left(-g^{\nu\beta} + \frac{k_{2}^{\nu}k_{2}^{\beta}}{M_{W}^{2}}\right) \left(g_{\mu\nu} + T_{\mu\nu}^{W}\right) \left(g_{\alpha\beta} + T_{\alpha\beta}^{W}\right) 
= \left[2 + \frac{(k_{1} \cdot k_{2})^{2}}{M_{W}^{4}} + 2T_{\alpha}^{W\alpha} - 2\frac{k_{1}^{\alpha}k_{1}^{\beta}}{M_{W}^{2}}T_{\alpha\beta}^{W} - 2\frac{k_{2}^{\alpha}k_{2}^{\beta}}{M_{W}^{2}}T_{\alpha\beta}^{W} + 2\frac{k_{1} \cdot k_{2}k_{1}^{\alpha}k_{2}^{\beta}}{M_{W}^{4}}T_{\alpha\beta}^{W} \right] 
+ T_{\mu\nu}^{W}T^{W\mu\nu} - \frac{k_{1\mu}k_{1}^{\alpha}}{M_{W}^{2}}T^{\mu\nu}T_{\alpha\nu}^{W} - \frac{k_{2\nu}k_{2}^{\beta}}{M_{W}^{2}}T^{\alpha\nu}T_{\alpha\beta}^{W} + \frac{k_{1}^{\mu}k_{2}^{\nu}}{M_{W}^{2}}T_{\mu\nu}^{W}\frac{k_{1}^{\alpha}k_{2}^{\beta}}{M_{W}^{2}}T_{\alpha\beta}^{W} \right] (4)$$

Now, using

$$k_1 \cdot k_2 = \frac{1}{2} (M_H^2 - 2M_W^2)$$

$$= \frac{1}{2} \sqrt{M_H^4 \lambda(M_W^2, M_W^2; M_H^2) + 4M_W^4}$$
(5)

where

$$\lambda(x,y;z) = \left(1 - \frac{x}{z} - \frac{y}{z}\right)^2 - 4\frac{xy}{z^2} \tag{6}$$

and defining

$$X(p_{1}, p_{2}, M_{H}, T^{V}) \equiv 4 \left[ 2 \frac{p_{1}^{2} p_{2}^{2}}{M_{H}^{4}} T_{\alpha}^{V\alpha} - 2 \frac{p_{2}^{2}}{M_{H}^{2}} \frac{p_{1}^{\alpha} p_{1}^{\beta}}{M_{H}^{2}} T_{\alpha\beta}^{V} - 2 \frac{p_{1}^{2}}{M_{H}^{2}} \frac{p_{2}^{\alpha} p_{2}^{\beta}}{M_{H}^{2}} T_{\alpha\beta}^{V} + 2 \frac{p_{1}^{2} p_{2}^{2}}{M_{H}^{4}} T_{\alpha\beta}^{V} + \frac{p_{1}^{2} p_{2}^{2}}{M_{H}^{4}} T_{\mu\nu}^{V} T^{V\mu\nu} - \frac{p_{2}^{2}}{M_{H}^{2}} \frac{p_{1\mu} p_{1}^{\alpha}}{M_{H}^{2}} T^{V\mu\nu} T_{\alpha\nu}^{V} - \frac{p_{1}^{2} p_{2}^{2}}{M_{H}^{2}} T^{V\mu\nu} T_{\alpha\beta}^{V} + \frac{p_{1}^{2} p_{2}^{2}}{M_{H}^{2}} T^{V\mu\nu} \frac{p_{1}^{\alpha} p_{2}^{\beta}}{M_{H}^{2}} T^{V\mu\nu} T_{\alpha\beta}^{V} \right]$$

$$- \frac{p_{1}^{2}}{M_{H}^{2}} \frac{p_{2\nu} p_{2}^{\beta}}{M_{H}^{2}} T^{V\alpha\nu} T_{\alpha\beta}^{V} + \frac{p_{1}^{\mu} p_{2}^{\nu}}{M_{H}^{2}} T_{\mu\nu}^{V} \frac{p_{1}^{\alpha} p_{2}^{\beta}}{M_{H}^{2}} T^{V}_{\alpha\beta}$$

$$(7)$$

we can write

$$\sum_{pol} |\mathcal{M}|^2 = (gM_W)^2 \frac{M_H^4}{4M_W^4} \left[ \lambda(M_W^2, M_W^2; M_H^2) + 12 \frac{M_W^4}{M_H^4} + X(k_1, k_2, M_H, T^W) \right]$$
(8)

It is easy to see that the 4-momenta  $k_1$  and  $k_2$  will only appear in the square bracket of Eq. (8) as the scalar products like  $k_1 \cdot k_2$ ,  $P \cdot k_1$  and  $P \cdot k_2$ . These can all be written in terms of the masses and therefore there is no angular dependence in  $d\Gamma$ . Noticing also that

$$|\vec{k}_1| = \frac{1}{2} M_H \sqrt{\lambda(M_W^2, M_W^2; M_H^2)}$$
(9)

we can finally write

$$\Gamma = \frac{g^2 M_H^3}{64\pi M_W^2} \sqrt{\lambda(M_W^2, M_W^2; M_H^2)} \left[ \lambda(M_W^2, M_W^2; M_H^2) + \frac{M_W^4}{M_H^4} + X(k_1, k_2, M_H, T^W) \right]$$
(10)

which can be written in terms of  $G_F$  as

$$\Gamma = \frac{G_F M_H^3}{8\pi\sqrt{2}} \sqrt{\lambda(M_W^2, M_W^2; M_H^2)} \left[ \lambda(M_W^2, M_W^2; M_H^2) + 12 \frac{M_W^4}{M_H^4} + X(k_1, k_2, M_H, T^W) \right]$$
(11)

Now for the decay  $H \to Z^0 Z^0$  everything is similar except that we have to divide by a factor of 2 because we have two identical particles in the final state. Introducing  $\delta_V = 2(1)$  for V = W(Z), respectively, we can write both decays in a single formula

$$\Gamma = \delta_V \frac{G_F M_H^3}{16\pi\sqrt{2}} \sqrt{\lambda(M_V^2, M_V^2; M_H^2)} \left[ \lambda(M_V^2, M_V^2; M_H^2) + 12 \frac{M_V^4}{M_H^4} + X(k_1, k_2, M_H, T^V) \right]$$
(12)

where  $\lambda$  and X are given in Eq. (6) and Eq. (7). The SM part of Eq. (12) agrees with Eq. (5) of ref. [10]. The term proportional to X represents the extra contributions from physics beyond the SM and is in agreement with the results of ref. [8] as we will explain in Section 6.

#### 2.3 The Off-Shell Decay $H \to VV^*$

We now consider the off-shell decay  $H \to VV^*$ . To be precise we derive the expression for  $H \to W^+W^{-*} \to W^+f_i\overline{f'}_i$  and then present a final result valid for all cases. We consider the kinematics given in Fig. 2.

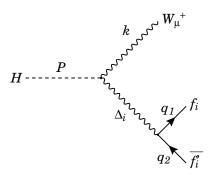


Fig. 2

where  $(f_i, \overline{f'}_i)$  represents one of the decay channels of the  $W^-$ , for instance,  $(e^-, \overline{\nu}_e)$ . Using the conventions of ref. [11], we can write the differential width as

$$d\Gamma = \frac{(2\pi)^4}{2M_H} \sum_{pol} |\mathcal{M}|^2 d\Phi_3 \tag{13}$$

where  $d\Phi_3$  is the phase space for 3 particles that we write as [11]

$$d\Phi_3(P; k, q_1, q_2) = d\Phi_2(P; k, \Delta_i) d\Phi_2(\Delta_i; q_1, q_2) (2\pi)^3 d\Delta_i^2$$
(14)

with

$$\Delta_i = q_1 + q_2 \quad ; \quad \Delta_i^2 = (q_1 + q_2)^2$$
 (15)

But the 2-body phase space in the rest frame of the decaying W can be written as

$$d\Phi_2(\Delta_i; q_1, q_2) = (2\pi)^{-6} \frac{|\vec{q^*}_1|}{4\sqrt{\Delta_i^2}} d\Omega_1^* = \frac{(2\pi)^{-6}}{8} d\Omega_1^*$$
(16)

where the last equality holds for massless decaying products of the W that we will assume and  $\Omega_1^*$  is the solid angle of the particle with momentum  $q_1$  in the rest frame of the decaying W. Also the 2-body phase space of the decaying H can be written as

$$d\Phi_2(P; k, \Delta_i) = \frac{(2\pi)^{-6}}{8} \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)} d\Omega_{\vec{k}}$$
 (17)

Putting everything together we get

$$d\Gamma = \frac{(2\pi)^{-5}}{128M_H} \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)} \sum_{pol} |\mathcal{M}|^2 d\Omega_{\vec{k}} d\Delta_i^2 d\Omega_1^*$$
 (18)

Neglecting the fermion masses the matrix element  $\mathcal{M}$  is

$$\mathcal{M} = (gM_W)\epsilon^{\alpha}(k) \left(g_{\mu\alpha} + T_{\mu\alpha}^W\right) \frac{1}{D(\Delta_i^2)} \frac{g}{2\sqrt{2}} \overline{u}(q_1)\gamma^{\mu} \left(1 - \gamma_5\right) v(q_2) \tag{19}$$

where

$$D(\Delta_i^2) = \Delta_i^2 - M_W^2 + iM_W\Gamma_W \tag{20}$$

We obtain for the matrix element squared

$$\sum_{pol} |\mathcal{M}|^{2} = (gM_{W})^{2} \frac{1}{|D(\Delta_{i}^{2})|^{2}} \left(-g^{\alpha\beta} + \frac{k^{\alpha}k^{\beta}}{M_{W}^{2}}\right) \left(g_{\mu\alpha} + T_{\mu\alpha}^{W}\right) \left(g_{\nu\beta} + T_{\nu\beta}^{W}\right)$$

$$= \frac{g^{2}}{8} \operatorname{Tr}\left[\not q_{1}\gamma^{\mu}(1-\gamma_{5})\not q_{2}\gamma^{\nu}(1-\gamma_{5})\right]$$

$$= (gM_{W})^{2} \frac{1}{|D(\Delta_{i}^{2})|^{2}} \left(-g^{\alpha\beta} + \frac{k^{\alpha}k^{\beta}}{M_{W}^{2}}\right) \left(g_{\mu\alpha} + T_{\mu\alpha}^{W}\right) \left(g_{\nu\beta} + T_{\nu\beta}^{W}\right)$$

$$= \frac{48\pi\Gamma_{i}}{M_{W}} \left[q_{1}^{\mu}q_{2}^{\nu} + q_{2}^{\mu}q_{1}^{\nu} - g^{\mu\nu}q_{1} \cdot q_{2}\right]$$
(21)

where  $\Gamma_i = g^2/(48\pi) M_W$  is the decay width  $W \to f_i \overline{f'}_i$ . Looking at Eq. (18) and Eq. (21) we realize that the only dependence on the solid angle  $\Omega_1^*$  is inside the square bracket in Eq. (21). Then the integrals we have to evaluate are of the form

$$I^{\alpha\beta} = \int d\Omega_1^* q_1^{\alpha} q_2^{\beta} \tag{22}$$

These can be easily done if we realize that in the rest frame of the decaying W the only 4-vector available is  $\Delta_i$ . We should have then

$$I^{\alpha\beta} = A\Delta_i^{\alpha}\Delta_i^{\beta} + B\Delta_i^2 g^{\alpha\beta} \tag{23}$$

Multiplying the last equation respectively with  $g_{\alpha\beta}$  and with  $\Delta_{i\alpha}\Delta_{i\beta}$  and noticing that  $\Delta_i \cdot q_1 = \Delta_i \cdot q_2 = 1/2\Delta_i^2$  we get a system of equations for A and B

$$\begin{cases} A + 4B = 2\pi \\ A + B = \pi \end{cases} \tag{24}$$

which gives  $A = 2\pi/3$  and  $B = \pi/3$ . We get then

$$\int d\Omega_1^* q_1^{\alpha} q_2^{\beta} = \frac{\pi}{3} \left( 2\Delta_i^{\alpha} \Delta_i^{\beta} + \Delta_i^2 g^{\alpha\beta} \right) \tag{25}$$

and

$$\int d\Omega_1^* \left[ q_1^{\mu} q_2^{\nu} + q_2^{\mu} q_1^{\nu} - g^{\mu\nu} q_1 \cdot q_2 \right] = \frac{4\pi}{3} \left( \Delta_i^{\mu} \Delta_i^{\nu} - \Delta_i^2 g^{\mu\nu} \right) \tag{26}$$

Doing the integration in  $\Omega_i^*$  we get

$$\int d\Omega_{1}^{*} \sum_{pol} |\mathcal{M}|^{2} = (gM_{W})^{2} \frac{1}{|D(\Delta_{i}^{2})|^{2}} \left( -g^{\alpha\beta} + \frac{k^{\alpha}k^{\beta}}{M_{W}^{2}} \right) \left( g_{\mu\alpha} + T_{\mu\alpha}^{W} \right) \left( g_{\nu\beta} + T_{\nu\beta}^{W} \right)$$

$$\frac{48\pi\Gamma_{i}}{M_{W}} \frac{4\pi}{3} \left( \Delta_{i}^{\mu} \Delta_{i}^{\nu} - \Delta_{i}^{2} g^{\mu\nu} \right)$$
(27)

If we compare Eq. (8) with Eq. (27) we can write this last equation in the form

$$\int d\Omega_1^* \sum_{pol} |\mathcal{M}|^2 = (gM_W)^2 \frac{1}{|D(\Delta_i^2)|^2} \frac{M_H^4}{4M_W^2} \frac{48\pi\Gamma_i}{M_W} \frac{4\pi}{3} \left[ \lambda(M_W^2, \Delta_i^2; M_H^2) + 12 \frac{M_W^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^W) \right]$$
(28)

We get therefore

$$\frac{d\Gamma}{d\Delta_i^2 d\Omega_{\vec{k}}} = \frac{(2\pi)^{-5}}{128M_H} \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)} (gM_W)^2 \frac{1}{|D(\Delta_i^2)|^2} \frac{M_H^4}{4M_W^2} \frac{48\pi\Gamma_i}{M_W} \frac{4\pi}{3} \left[ \lambda(M_W^2, \Delta_i^2; M_H^2) + 12 \frac{M_W^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^W) \right]$$
(29)

Next we realize that in Eq. (29) there is no dependence on the solid angle of the real W. We can therefore trivially perform that integration. We get

$$\frac{d\Gamma}{d\Delta_{i}^{2}} = \frac{(2\pi)^{-5}}{128M_{H}} (4\pi) \sqrt{\lambda(M_{W}^{2}, \Delta_{i}^{2}; M_{H}^{2})} (gM_{W})^{2} \frac{1}{|D(\Delta_{i}^{2})|^{2}} \frac{M_{H}^{4}}{4M_{W}^{2}} \frac{48\pi\Gamma_{i}}{M_{W}} \frac{4\pi}{3} \left[\lambda(M_{W}^{2}, \Delta_{i}^{2}; M_{H}^{2}) + 12\frac{M_{W}^{2}\Delta_{i}^{2}}{M_{H}^{4}} + X(k, \Delta_{i}, M_{H}, T^{W})\right] \tag{30}$$

and finally we get for the width

$$\Gamma = \frac{G_F M_H^3}{8\pi\sqrt{2}} \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)} \frac{1}{\pi} \int d\Delta_i^2 \frac{\Gamma_i M_W}{|D(\Delta_i^2)|^2} \left[ \lambda(M_W^2, \Delta_i^2; M_H^2) + 12 \frac{M_W^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^W) \right]$$
(31)

or

$$\Gamma = \frac{1}{\pi} \int d\Delta_i^2 \frac{\Gamma_i M_W}{|D(\Delta_i^2)|^2} \, \Gamma_0^W(k, \Delta_i, M_H) \tag{32}$$

where

$$\Gamma_0^W(k, \Delta_i, M_H) = \frac{G_F M_H^3}{8\pi\sqrt{2}} \sqrt{\lambda(M_W^2, \Delta_i^2; M_H^2)} \left[ \lambda(M_W^2, \Delta_i^2; M_H^2) + 12 \frac{M_W^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^W) \right]$$
(33)

If we sum over all the final states of the W we can substitute  $\Gamma_i$  with  $\Gamma_W$ . Eq. (32) is in agreement with Eq. (6) of ref. [5] in the zero width limit. Similar considerations apply to the case of the decay  $H \to Z^0 + f_i \overline{f}_i$ . We can summarize the final result in the formula,

$$\Gamma = \frac{1}{\pi} \int d\Delta_i^2 \frac{\Gamma_V M_V}{|D(\Delta_i^2)|^2} \, \Gamma_0^V(k, \Delta_i, M_H) \tag{34}$$

where

$$\Gamma_0^V(k, \Delta_i, M_H) = \delta_V \frac{G_F M_H^3}{16\pi\sqrt{2}} \sqrt{\lambda(M_V^2, \Delta_i^2; M_H^2)} \left[ \lambda(M_V^2, \Delta_i^2; M_H^2) + 12 \frac{M_V^2 \Delta_i^2}{M_H^4} + X(k, \Delta_i, M_H, T^V) \right]$$
(35)

 $\delta_V$  was defined before, X is given in Eq. (7) and  $k^2 = M_V^2$ .

# 2.4 The Off–Shell Decay $H \to V^*V^*$

We now consider the off-shell decay  $H \to V^*V^*$ . To be precise we derive the expression for  $H \to W^{+*}W^{-*} \to (f_i\overline{f'}_i) + (f_j\overline{f'}_j)$  and then present a final result valid for all cases. We consider the kinematics given in Fig. 3.

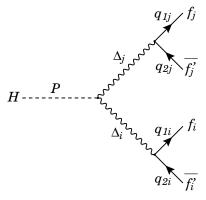


Fig. 3

where  $(f_i, \overline{f'}_i)$  represents one of the decay channels of the  $W^-$  and  $(f_j, \overline{f'}_j)$  represents one of the decay channels of the  $W^+$ . After we have done the case  $H \to VV^*$  it is very easy to do this case.

The expression for the width is [11],

$$d\Gamma = \frac{(2\pi)^4}{2M_H} \sum_{pol} |\mathcal{M}|^2 d\Phi_4 \tag{36}$$

where  $d\Phi_4$  is the phase space for 4 particles that we write as [11]

$$d\Phi_4(P; k, q_1, q_2) = d\Phi_2(P; \Delta_i, \Delta_j) d\Phi_2(\Delta_i; q_{i1}, q_{i2}) (2\pi)^3 d\Delta_i^2 d\Phi_2(\Delta_j; q_{j1}, q_{j2}) (2\pi)^3 d\Delta_j^2$$
(37)

with

$$\Delta_i = q_{i1} + q_{i2} \; ; \; \Delta_i^2 = (q_{i1} + q_{i2})^2 \; ; \; \Delta_j = q_{j1} + q_{j2} \; ; \; \Delta_i^2 = (q_{j1} + q_{j2})^2$$
 (38)

But the 2-body phase spaces can be written as

$$d\Phi_{2}(\Delta_{i}; q_{i1}, q_{i2}) = \frac{(2\pi)^{-6}}{8} d\Omega_{i1}^{*}$$

$$d\Phi_{2}(\Delta_{j}; q_{j1}, q_{j2}) = \frac{(2\pi)^{-6}}{8} d\Omega_{j1}^{*}$$

$$d\Phi_{2}(P; \Delta_{i}, \Delta_{j}) = \frac{(2\pi)^{-6}}{8} \sqrt{\lambda(\Delta_{i}^{2}, \Delta_{j}^{2}; M_{H}^{2})} d\Omega_{\vec{\Delta}_{i}}$$
(39)

where, as before, we consider that the decays products of the  $W^{\pm}$  are massless. Putting everything together we have

$$d\Gamma = \frac{(2\pi)^{-8}}{2^{10}M_H} \sqrt{\lambda(\Delta_i^2, \Delta_j^2; M_H^2)} \sum_{pol} |\mathcal{M}|^2 d\Delta_i^2 d\Delta_j^2 d\Omega_{\vec{\Delta}_i} d\Omega_{i1}^* d\Omega_{j1}^*$$
(40)

The matrix element is

$$\mathcal{M} = (gM_W) \frac{1}{D(\Delta_i^2)} \frac{1}{D(\Delta_j^2)} \frac{g}{2\sqrt{2}} \overline{u}(q_{i1}) \gamma^{\mu} (1 - \gamma_5) v(q_{i2})$$
$$\frac{g}{2\sqrt{2}} \overline{u}(q_{j1}) \gamma^{\mu} (1 - \gamma_5) v(q_{j2})$$
(41)

and the same procedure that we used for the  $H \to VV^*$  case gives

$$\int d\Omega_{1i}^* d\Omega_{1j}^* \sum_{pol} |\mathcal{M}|^2 = (gM_W)^2 \frac{1}{|D(\Delta_i^2)|^2} \frac{1}{|D(\Delta_j^2)|^2} \frac{M_H^4}{4} \frac{(48\pi)^2 \Gamma_i \Gamma_j}{M_W^2} \left(\frac{4\pi}{3}\right)^2$$

$$\left[\lambda(\Delta_i^2, \Delta_j^2; M_H^2) + 12 \frac{\Delta_i^2 \Delta_j^2}{M_H^4} + X(\Delta_i, \Delta_j, M_H, T^W)\right]$$
(42)

and after doing the  $d\Omega_{\vec{\Delta}_i}$  integration we obtain

$$\Gamma = \frac{G_F M_H^3}{8\pi\sqrt{2}} \sqrt{\lambda(\Delta_i^2, \Delta_j^2; M_H^2)} \frac{1}{\pi} \int d\Delta_i^2 \frac{\Gamma_i M_W}{|D(\Delta_i^2)|^2} \frac{1}{\pi} \int d\Delta_j^2 \frac{\Gamma_j M_W}{|D(\Delta_j^2)|^2} \left[ \lambda(\Delta_i^2, \Delta_j^2; M_H^2) + 12 \frac{\Delta_i^2 \Delta_i^2}{M_H^4} + X(\Delta_i, \Delta_j, M_H, T^W) \right]$$
(43)

Summing over all final states we get

$$\Gamma = \frac{1}{\pi} \int d\Delta_i^2 \frac{\Gamma_V M_V}{|D(\Delta_i^2)|^2} \frac{1}{\pi} \int d\Delta_j^2 \frac{\Gamma_V M_V}{|D(\Delta_j^2)|^2} \Gamma_0^V(\Delta_i, \Delta_j, M_H)$$
(44)

 $where^{1}$ 

$$\Gamma_0^V(\Delta_i, \Delta_j, M_H) = \delta_V \frac{G_F M_H^3}{16\pi\sqrt{2}} \sqrt{\lambda(\Delta_i^2, \Delta_j^2; M_H^2)} \left[ \lambda(\Delta_i^2, \Delta_j^2; M_H^2) + 12\frac{\Delta_i^2 \Delta_j^2}{M_H^4} + X(\Delta_i, \Delta_j, M_H, T^V) \right]$$
(46)

$$\sum_{Final\ States} \Gamma\left[H \to (W^{+*} \to i_+) + (W^{-*} \to i_-)\right] \propto \sum_{i_+} \sum_{i_-} \Gamma(W^{+*} \to i_+) \Gamma(W^{-*} \to i_-)$$

$$= \sum_{i_+} \Gamma(W^{+*} \to i_+) \sum_{i_-} \Gamma(W^{-*} \to i_-)$$

$$= \Gamma_W \Gamma_W \tag{45}$$

and  $\delta_W = 2$  because of the constants we factored out. For the  $H \to (Z^* \to i) + (Z^* \to j)$  case one should be more careful. If  $i \neq j$  than we should divide by 2 otherwise we would be double counting in the product  $(\Gamma_1 + \Gamma_2 + \cdots)(\Gamma_1 + \Gamma_2 + \cdots)$ . For i = j there is no double counting in the above product, but now we have two pairs of identical particles in the final state but we also have 2 diagrams. Then we should square the sum of the amplitudes and divide by 4. In general this would not give a factor of 1/2 because of the interference term. However the interference will be negligible because the momenta squared in the denominators cannot be equal to  $M_Z$  in all 4 lines (of the product of the 2 diagrams) at the same time. Therefore if we neglect the interference we should divide also by 2 in this case. Therefore  $\delta_Z = 1$ .

One might worry about the factor  $\delta_V$ . For the  $W^{+*}W^{-*}$  final case there is no problem because the final states of the  $W^{+*}$  are different from the final states of the  $W^{-*}$ . Therefore

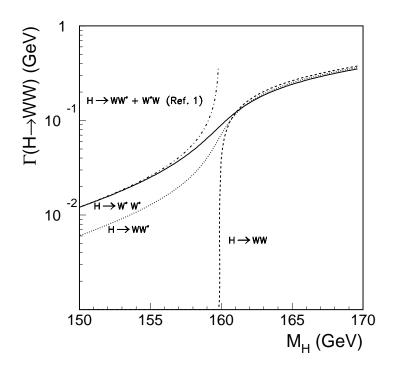


Figure 4: Comparison of the off-shell and on-shell formulas for  $H \to W^+W^-$ . The dashed line corresponds to the on-shell formula Eq. (12), the dotted line to the case that only one W is off-shell Eq. (31), and the solid line corresponds to the case where both W's are off-shell Eq. (43). For comparison is also shown Eq. (6) of ref. [5].

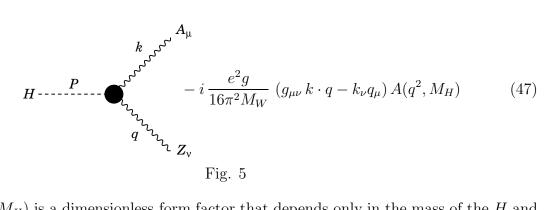
This result is in agreement with [9, 10], except for the value of  $\delta_Z$ . One should mention that formulas for off-shell decays of the type of Eqs. (34) and (44) for other decays are known in the literature [15].

#### 2.5 A comparison of the various formulas

Perhaps it is useful to indicate the domain of validity of the various formulas for the widths. This will depend on the value of the Higgs boson mass. In Figure 4 we plot the various formulas for the case of  $H \to W^+W^-$ . From this figure it is clear that the proper way to calculate the width below the two W's threshold is to use Eq. (43) with the two W's off–shell. The two integrations in Eq. (43) automatically take care of the fact that either one of the W's can be close to be on–shell. In Eq. (6) of ref. [5] this is done by adding the two possibilities, but as the width is neglected the formula is only good below the threshold.

### 3 The Decay $H \to \gamma Z$

Due to the electromagnetic gauge invariance the most general expression for the coupling  $H\gamma Z$  is



where  $A(q^2, M_H)$  is a dimensionless form factor that depends only in the mass of the H and on the square of the momentum of the Z (if the Z is on–shell then  $q^2 = M_Z^2$ ). In the SM the lowest contribution to A is at the 1–loop level. If we are considering physics Beyond the Standard Model (BSM) then we should have

$$A = A_{SM} + A_{BSM} \tag{48}$$

where the SM contribution is given [6, 12, 13] by

$$A_{SM} = A_W + A_F \tag{49}$$

 ${\rm with}^2$ 

$$A_W = -4\cot\theta_W \left[ (3 - \tan\theta_W^2) \ J_1(q^2, M_H^2, M_W^2) + \left( -5 + \tan_W^2 \theta_W - \frac{1}{2} \frac{M_H^2}{M_W^2} (1 - \tan\theta_W^2) \right) \ J_2(q^2, M_H^2, M_W^2) \right]$$
(50)

and

$$A_F = -\sum_f \frac{4g_V^f Q_f}{\sin \theta_W \cos \theta_W} \left[ -J_1(q^2, M_H^2, M_f^2) + 4J_2(q^2, M_H^2, M_f^2) \right] . \tag{51}$$

where  $Q_f$  is the charge, in units of |e|, of the fermion f in the loop, and  $g_V^f = 1/2T_3^f - Q_f \sin^2 \theta_W$ . The explicit form of the functions  $J_1$  and  $J_2$  can be found in Appendix B. In the following we will use this general coupling to evaluate both the on–shell and the off–shell decays of the Higgs boson.

<sup>&</sup>lt;sup>2</sup>Our convention here for the coupling, Eq. (47) is as in ref. [6]. It differs from our previous convention, ref. [13], by a factor of  $-1/\sin\theta_W$ . Our conventions are explained in Appendix A.

#### 3.1 The On–Shell Decay $H \rightarrow \gamma Z$

The differential width is, like before (see Eq. (2))

$$d\Gamma = \frac{1}{32\pi^2} \sum_{pol} |\mathcal{M}|^2 \frac{|\vec{k}|}{M_H^2} d\Omega_{\vec{k}}$$
 (52)

where

$$\mathcal{M} = \epsilon^{\mu}(k)\epsilon^{\nu}(q) \frac{e^2 g}{16\pi^2 M_W} (g_{\mu\nu} k \cdot q - k_{\nu} q_{\mu}) A(q^2, M_H)$$
 (53)

We get therefore

$$\sum_{pol} |\mathcal{M}|^2 = \left(\frac{e^2 g}{16\pi^2 M_W}\right)^2 2(k \cdot q)^2 |A|^2 \tag{54}$$

Now using

$$|\vec{k}| = \frac{k \cdot q}{M_H} = \frac{1}{2} M_H \sqrt{\lambda(M_Z^2, 0; M_H^2)}$$
 (55)

where

$$\lambda(M_Z^2, 0; M_H^2) = \left(1 - \frac{M_Z^2}{M_H^2}\right)^2 \tag{56}$$

we get finally

$$\Gamma = \frac{G_F M_H^3}{4\pi\sqrt{2}} \frac{\alpha^2}{16\pi^2} \lambda (M_Z^2, 0; M_H^2)^{3/2} |A|^2$$
 (57)

This result is in agreement for the SM with refs. [6, 12] but it differs by a factor of two from ref. [8] that claims to have the same definition of A as we and ref. [6] do.

# 3.2 The Off–Shell Decay $H \to \gamma Z^*$

We consider for definiteness the the decay  $H \to \gamma f_i \overline{f}_i$  as represented in Fig. 6

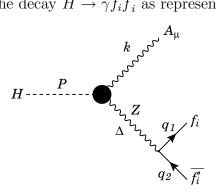


Fig. 6

The differential width can be written as in Eq. (18)

$$d\Gamma = \frac{(2\pi)^{-5}}{128M_H} \sqrt{\lambda(0, \Delta^2; M_H^2)} \sum_{pol} |\mathcal{M}|^2 d\Omega_{\vec{k}} d\Delta^2 d\Omega_1^*$$
 (58)

where the matrix element  $\mathcal{M}$  is (we again neglect the fermion masses)

$$\mathcal{M} = \epsilon^{\mu}(k) \frac{e^2 g}{16\pi^2 M_W} \left( g_{\mu\nu} \, k \cdot \Delta - k_{\nu} \Delta_{\mu} \right) A(\Delta^2, M_H) \frac{1}{D(\Delta^2)} \frac{g}{\cos \theta_W} \overline{u}(q_1) \gamma^{\nu} (g_V^f - g_A^f \gamma_5) v(q_2)$$

$$\tag{59}$$

and

$$\Delta = q_1 + q_2 \tag{60}$$

Our conventions for the couplings of the Z to the fermion f are given in Appendix A. The sum over polarizations and spins of the matrix element squared gives now

$$\sum_{pol} |\mathcal{M}|^2 = \left(\frac{e^2 g}{16\pi^2 M_W}\right)^2 \frac{1}{|D(\Delta^2)|^2} \left(\frac{g}{\cos \theta}\right)^2 8|A|^2 \left(g_V^{f\,2} + g_A^{f\,2}\right)$$

$$\left[k \cdot \Delta \ k \cdot q_1 \ \Delta \cdot q_2 + k \cdot \Delta \ k \cdot q_2 \ \Delta \cdot q_1 - k \cdot q_1 \ k \cdot q_2 \ \Delta \cdot \Delta\right]$$
(61)

Using now Eq. (25) to perform the integration over the solid angle in the center of mass frame of the decaying Z we get

$$\int d\Omega_1^* \sum_{pol} |\mathcal{M}|^2 = \left(\frac{e^2 g}{16\pi^2 M_W}\right)^2 \frac{1}{|D(\Delta^2)|^2} \left(\frac{g}{\cos \theta}\right)^2 |A|^2 \left(g_V^{f\,2} + g_A^{f\,2}\right) \frac{32\pi}{3} (k \cdot \Delta)^2 \Delta^2 \tag{62}$$

We can now perform the integration over the solid angle of the photon and obtain

$$\frac{d\Gamma}{d\Delta^2} = \frac{1}{32\pi^2 M_H} \lambda(\Delta^2, 0; M_H^2)^{3/2} M_H^3 \left(\frac{eg^2}{16\pi^2 M_W}\right)^2 \frac{\Gamma_i}{M_Z} \Delta^2 \frac{1}{|D(\Delta^2)|^2}$$
(63)

where we have used the expression for the partial width  $\Gamma_i$  of  $Z \to f_i \overline{f}_i$ 

$$\Gamma_i = \frac{1}{12\pi} \left( \frac{g}{\cos \theta_W} \right)^2 \left( g_V^{f2} + g_A^{f2} \right) \tag{64}$$

Summing over all the final states we obtain finally

$$\Gamma = \frac{1}{\pi} \int d\Delta^2 \frac{\Gamma_Z}{M_Z} \frac{\Delta^2}{|D(\Delta^2)|^2} \Gamma^{\gamma Z}(M_H, \Delta^2)$$
 (65)

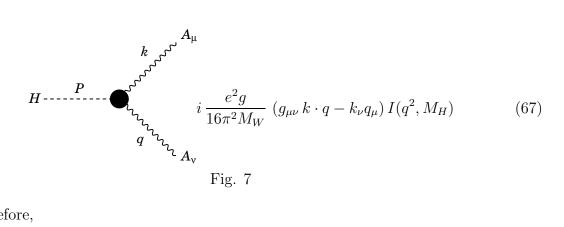
where

$$\Gamma^{\gamma Z}(M_H, \Delta^2) = \frac{G_F M_H^3}{4\pi\sqrt{2}} \frac{\alpha^2}{16\pi^2} \lambda(\Delta^2, 0; M_H^2)^{3/2} |A(\Delta^2, M_H^2)|^2$$
 (66)

is the decay into an off-shell Z and gives back Eq. (57) when  $\Delta^2 = M_Z^2$ .

### 4 The decay $H \rightarrow \gamma \gamma$

For completeness we also give the general formula for this decay. Due to the electromagnetic gauge invariance the most general expression for the coupling  $^3H\gamma^*\gamma$  is



where, as before,

$$I = I_{SM} + I_{BSM} \tag{68}$$

The Standard Model contribution is given by [8, 6, 12]

$$I_{SM} = I_W + I_F \tag{69}$$

where

$$I_W = -4 \left[ -4J_1(q^2, M_H^2, M_W^2) + \left(6 + \frac{M_H^2}{M_W^2}\right) J_2(q^2, M_H^2, M_W^2) \right]$$
 (70)

and

$$I_F = \sum_f 4Q_f^2 \left[ -J_1(q^2, M_H^2, M_f^2) + 4J_2(q^2, M_H^2, M_f^2) \right] . \tag{71}$$

Using the above coupling and comparing with the case  $H \to \gamma Z$ , Eqs. (52), (53) e (54), it is straightforward to obtain

$$\Gamma(H \to \gamma \gamma) = \frac{\alpha^3 M_H^3}{256\pi^2 \sin^2 \theta_W M_W^2} |I|^2 \tag{72}$$

in agreement with refs. [6, 8].

# 5 The 3-point functions $Z^* \to H\gamma$ and $\gamma^* \to H\gamma$

For some applications it is also important to know the related off–shell 3-point functions  $Z^* \to H\gamma$  and  $\gamma^* \to H\gamma$ . For completeness we collect them here.

<sup>&</sup>lt;sup>3</sup>We are assuming that only one photon is off-shell because this is case of interest.

#### 5.1 The $Z^* \to H\gamma$ 3-point function

We use the results of refs. [12, 13]. The amplitude can be written as<sup>4</sup>

$$i\mathcal{M} = i\epsilon_Z^{\nu}(q)\epsilon_A^{\mu}(k) \left(\frac{e^2g}{16\pi^2 M_W}\right) \left(g_{\mu\nu} k \cdot q - k_{\nu} q_{\mu}\right) A(q^2, M_H)$$
 (73)

where

$$A = A_{SM} + A_{BSM} \tag{74}$$

The standard model dimensionless amplitude  $A_{SM}$  is given by Eq. (49). The sign difference between Eq. (49) and Eq. (73) is due to the fact that in the first one q is an outgoing momentum and in the last one is incoming.

# 5.2 The $\gamma^* \to H\gamma$ 3-point function

Again using ref. [12] we have

$$i \mathcal{M} = -i \,\epsilon_A^{\nu}(q) \,\epsilon_A^{\mu}(k) \, \left(\frac{e^2 g}{16\pi^2 M_W}\right) \, (g_{\mu\nu} \, k \cdot q - k_{\nu} q_{\mu}) \, I(q^2, M_H)$$
 (75)

where

$$I = I_{SM} + I_{BSM} \tag{76}$$

The standard model dimensionless amplitude  $I_{SM}$  is given by Eq. (69).

#### 5.3 An Effective Lagrangian for the SM Couplings

We can write an effective Lagrangian that reproduces the couplings given in Eqs. (47), (67), (73), (75). This is specially useful if we want to add new physics, in addition to the SM, as we will show in the next section. We get

$$\mathcal{L}^{eff} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} H \, \mathcal{I}_{SM} + \frac{1}{2} A_{\mu\nu} Z^{\mu\nu} H \, \mathcal{A}_{SM}$$
 (77)

where we have defined

$$A_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \quad ; \quad Z_{\mu\nu} = \partial_{\mu}Z_{\nu} - \partial_{\nu}Z_{\mu} \tag{78}$$

and

$$\mathcal{I}_{SM} = \frac{e^2 g}{16\pi^2 M_W} I_{SM}(q^2, M_H)$$

<sup>&</sup>lt;sup>4</sup>Notice that our conventions here differ by a factor  $-1/\sin\theta_W$  with respect to ref. [13].

$$\mathcal{A}_{SM} = \frac{e^2 g}{16\pi^2 M_W} A_{SM}(q^2, M_H)$$
 (79)

The effective Lagrangian, Eq. (77), is valid for the case of one on–shell photon, the other photon (or  $Z^0$ ) can be either on–shell or off-shell.  $A_{SM}$  and  $I_{SM}$  are given in Eqs. (49) and (69).

# 6 An example of extension of the SM

A possible enhancement of the production and decay rates of the Higgs boson can be originated by an anomalous couplings of the Higgs boson to the vector bosons. These interactions can be described in terms of an effective dimension-six term in the interaction Lagrangian density

$$\mathcal{L}_{eff} = \sum_{i=1}^{7} \frac{f_i}{\Lambda^2} O_i \tag{80}$$

where the  $O_i$  are the operators which represent the anomalous couplings,  $\Lambda$  is the typical energy scale of the interaction and  $f_i$  are the constants which define the strength of each term [7, 8].

The anomalous couplings  $H\gamma\gamma$ , HZZ,  $HZ\gamma$  and HWW follow from the effective Lagrangian (80) and can be written in the unitary gauge [7, 8] as,

$$\mathcal{L}_{eff}^{HVV} = g \frac{m_W}{\Lambda^2} \left[ -\frac{s^2 (f_{BB} + f_{WW} - f_{BW})}{2} H A_{\mu\nu} A^{\mu\nu} + \frac{2m_W^2}{g^2} \frac{f_{\phi,1}}{c^2} H Z_{\mu} Z^{\mu} \right] 
+ \frac{c^2 f_W + s^2 f_B}{2c^2} Z_{\mu\nu} Z^{\mu} (\partial^{\nu} H) - \frac{s^4 f_{BB} + c^4 f_{WW} + s^2 c^2 f_{BW}}{2c^2} H Z_{\mu\nu} Z^{\mu\nu} 
+ \frac{s (f_W - f_B)}{2c} A_{\mu\nu} Z^{\mu} (\partial^{\nu} H) + \frac{s (2s^2 f_{BB} - 2c^2 f_{WW} + (c^2 - s^2) f_{BW})}{2c} H A_{\mu\nu} Z^{\mu\nu} 
+ \frac{f_W}{2} (W_{\mu\nu}^+ W^{-\mu} + W_{\mu\nu}^- W^{+\mu}) (\partial^{\nu} H) - f_{WW} H W_{\mu\nu}^+ W^{-\mu\nu} \right]$$
(81)

where  $X_{\mu\nu} = \partial_{\mu}X_{\nu} - \partial_{\nu}X_{\mu}$  with X = A, Z, W, and  $s(c) = \sin\theta_W(\cos\theta_W)$ , respectively.

Both  $f_{\phi,1}$  and  $f_{BW}$  are already severely constrained by precise measurements at low energy experiments, once they contribute to the  $Z^0$  mass and to the  $B-W^3$  mixing, respectively. In what follows these parameters will be assumed to be zero. Under this assumption, both HWW and HZZ have the same tensorial structure. With the convention  $H(p_H) \to V^{\mu}(p_1) + V^{\nu}(p_2)$  for the momenta, we have:

$$T_V^{\mu\nu} \equiv -A_V \left[ p_1^{\nu} p_H^{\mu} - (p_1 \cdot p_H) g^{\nu\mu} + p_H^{\nu} p_2^{\mu} - (p_2 \cdot p_H) g^{\nu\mu} \right] + B_V \left[ p_1^{\nu} p_2^{\mu} - (p_1 \cdot p_2) g^{\nu\mu} \right]$$
(82)

(V = Z, W), where:

$$A_{W} \equiv -\frac{1}{2} \frac{f_{W}}{\Lambda^{2}}$$

$$B_{W} \equiv -2 \frac{f_{WW}}{\Lambda^{2}}$$

$$A_{Z} \equiv -\frac{1}{2} \left( \frac{f_{B}}{\Lambda^{2}} \sin^{2} \theta_{W} + \frac{f_{W}}{\Lambda^{2}} \cos^{2} \theta_{W} \right)$$

$$B_{Z} \equiv -2 \left( \frac{f_{BB}}{\Lambda^{2}} \sin^{4} \theta_{W} + \frac{f_{WW}}{\Lambda^{2}} \cos^{4} \theta_{W} \right)$$
(83)

The value of  $X_V(p_1, p_2, M_H, T^V)$  as defined in Eq. (7) is, thus, given by:

$$X_{V} = 4 \left\{ A_{V} \left[ 4 \frac{p_{1}^{2} p_{2}^{2}}{M_{H}^{2}} - \frac{p_{1} \cdot p_{2}}{M_{H}^{4}} ( (p_{1}^{2} - p_{2}^{2})^{2} - (p_{1}^{2} + p_{2}^{2}) M_{H}^{2} ) \right] \right.$$

$$+ B_{V} \left[ -6 \frac{(p_{1} \cdot p_{2}) p_{1}^{2} p_{2}^{2}}{M_{H}^{4}} \right]$$

$$+ A_{V}^{2} \left[ p_{1}^{2} p_{2}^{2} + \frac{(p_{1}^{2} + p_{2}^{2}) (4p_{1}^{2} p_{2}^{2} - (M_{H}^{2} - (p_{1}^{2} + p_{2}^{2}))^{2})}{4M_{H}^{2}} \right.$$

$$+ \frac{(M_{H}^{4} - (p_{1}^{2} - p_{2}^{2})^{2}) (4p_{1}^{2} p_{2}^{2} + M_{H}^{2} (p_{1}^{2} + p_{2}^{2}) - (p_{1}^{2} + p_{2}^{2})^{2})}{4M_{H}^{4}} \right]$$

$$+ A_{V} B_{V} \left[ -2 \frac{p_{1}^{2} p_{2}^{2} (M_{H}^{2} - (p_{1}^{2} + p_{2}^{2}))}{M_{H}^{2}} + \frac{p_{1}^{2} p_{2}^{2} ((p_{1}^{2} - p_{2}^{2})^{2} - M_{H}^{2} (p_{1}^{2} + p_{2}^{2}))}{M_{H}^{2}} \right]$$

$$+ B_{V}^{2} \left[ \frac{p_{1}^{2} p_{2}^{2}}{2M_{H}^{4}} ((M_{H}^{2} - (p_{1}^{2} + p_{2}^{2}))^{2} + 2p_{1}^{2} p_{2}^{2}) \right] \right\}$$

$$(84)$$

This expression can then be used in Eqs. (12), (31) and (44) to evaluate the decay widths. We have verified that if we use Eq. (84) with the definitions of Eq. (83) into Eq. (12) for the decay into two real vector bosons we recover the results of ref. [8]. However our expressions extend those results for the off-shell case.

The decays  $H \rightarrow \gamma Z$  and  $H \rightarrow \gamma \gamma$  appear at tree-level, the corresponding form factors, Eq. (47) and Eq. (67), are:

$$A_{BSM} \equiv \frac{2\pi M_W^2 \tan \theta_W}{\alpha} \left[ \frac{f_W}{\Lambda^2} - \frac{f_B}{\Lambda^2} + 4 \left( \frac{f_{BB}}{\Lambda^2} \sin \theta_W^2 - \frac{f_{WW}}{\Lambda^2} \cos \theta_W^2 \right) \right]$$
(85)

$$I_{BSM} \equiv \frac{8\pi M_W^2 \sin \theta_W^2}{\alpha} \left( \frac{f_{BB}}{\Lambda^2} + \frac{f_{WW}}{\Lambda^2} \right)$$
 (86)

With these variables it is possible to compute the various Higgs decay widths, including the interference of the new terms with the Standard Model, and allowing for decays to virtual gauge bosons.

In this model the Branching Ratios to  $\gamma\gamma$  and  $\gamma Z$  increase and these decays may become dominant for some region of parameters. For  $H{\to}WW$  and  $H{\to}ZZ$  the new contributions

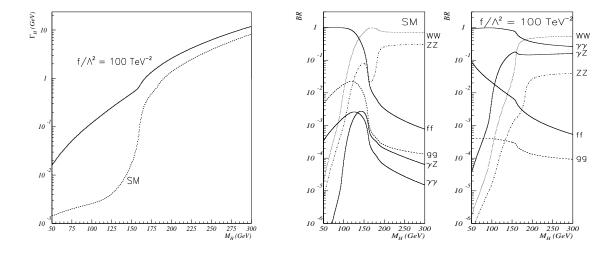


Figure 8: Higgs Width and Branching Ratios as a function of its mass

may interfere constructively or destructively with the Standard Model terms. In Fig. (8) the width and branching ratios of the Higgs as a function of its mass are displayed for the Standard Model and with the new contributions where all the non-zero  $f_i$  are assumed equal and  $f_i/\Lambda^2 = 100 \text{ TeV}^{-2}$ .

The variation of the total width and branching ratios with  $f/\Lambda^2$  is shown in Fig. (9), for a Higgs boson mass of 150 GeV. In Fig. (10) all  $f_i$  except the ones contributing directly to the H decay to  $\gamma\gamma$  are set to 0. The variation with  $f_{BB}/\Lambda^2$  and  $f_{WW}/\Lambda^2$  is displayed for two different masses: 85 GeV and 150 GeV.

#### 7 Discussion

In this paper we derive the complete set of formulas for the decay widths of the Higgs boson in vector bosons. The formulas are valid both for the Standard Model (SM) and for any arbitrary extension. For the case of the decay into the  $W^{\pm}$  and  $Z^0$  the formulas are also valid for off-shell decays. This is important for Higgs boson masses close to the threshold of the production of one or two real vector bosons. As many of these results have appeared before in the literature [5, 6, 7, 8, 9, 10], sometimes for particular cases, we will now comment on the comparison of our results with those.

For the on–shell decay  $H \to VV$  our final expression Eq. (12), is in agreement with ref. [10]. There is a factor 2 difference with respect to ref. [9]. For the off-shell decay  $H \to VV^*$  our final expression, Eq. (34) in agreement with ref. [10]. We are also in agreement

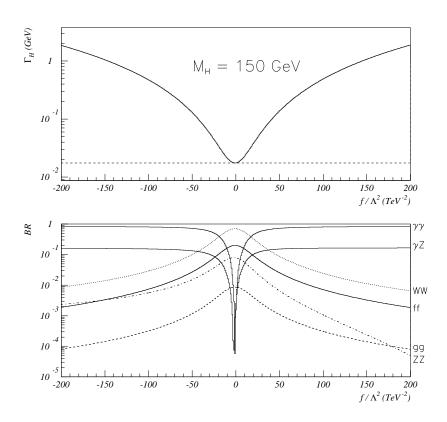


Figure 9: Higgs width as a function of  $f/\Lambda^2$ 

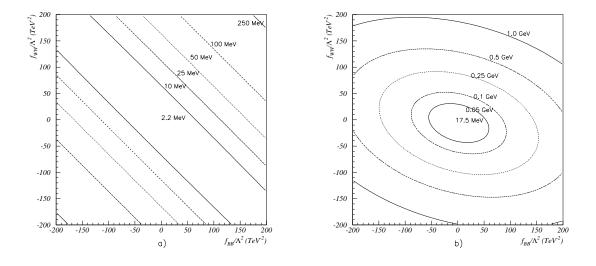


Figure 10: Constant Higgs' Width lines as a function of  $f_{BB}/\Lambda^2$  and  $f_{WW}/\Lambda^2$  for: a)  $M_H=85$  GeV and b)  $M_H=150$  GeV

with Eq. (6) of ref. [5] in the zero width limit. Eqs. (9-10) of ref. [5] are also consistent with our results and in agreement with ref. [9]. For the off-shell decay  $H \to V^*V^*$  our result, Eq. (44), is in agreement with ref. [10] except for the factor  $\delta_V$ . For the on-shell decay  $H \to \gamma \gamma$  we are in agreement with refs. [6, 8], while for the on-shell decay  $H \to \gamma Z$  we agree with ref. [6] but have a factor of two difference with respect to ref. [8]. The formulas for the off-shell decays  $H \to \gamma \gamma^*$  and  $H \to \gamma Z^*$  are either new, or in agreement with refs. [12, 13].

As our main contribution is to extend the formulas for an arbitrary extension of the SM, including off-shell decays we studied, as an example, the case of the gauge–invariant effective Lagrangian models of ref. [7, 8]. Our expressions reproduce the results of ref. [8] for the on–shell decays and extend them for the region of the Higgs boson mass close to the two W's threshold where the off–shell decays have to be considered. This region is important for the studies done at the Tevatron and at LEPII where these models have been considered [16, 17].

## Appendix A: Standard Model Feynman Rules

Because of the interference terms between the Standard Model (SM) and possible extensions, it is important that we state our conventions for the SM. We follow the conventions of ref. [6]. These differ in some signs from the conventions used in refs. [12, 13]. For the convenience of the reader we collect the most important Feynman rules here.

$$Z_{\mu} - i \frac{g}{\cos \theta_W} \gamma^{\mu} \left( g_V^f - g_A^f \gamma_5 \right) \tag{89}$$

$$V_{\rho} \qquad i g_{V} \left[ g^{\mu\nu} (p_{2} - p_{3})^{\rho} + g^{\nu\rho} (p_{3} - p_{1})^{\mu} + g^{\rho\mu} (p_{1} - p_{2})^{\nu} \right] \qquad (90)$$

$$W_{\mu}^{-}$$

for V=A,Z with  $g_A=e,\,g_Z=g\cos\theta_W$  and

$$g_V^f = \frac{1}{2} T_3^f - Q_f \sin^2 \theta_W \quad ; \quad g_A^f = \frac{1}{2} T_3^f \tag{91}$$

where  $Q_f$  is the charge of fermion f in units of |e|.

## Appendix B: The $J_1$ and $J_2$ functions

The explicit expressions for the functions  $J_1$  and  $J_2$  introduced in Section 3, are [12, 13]

$$J_{1}(q^{2}, M_{H}^{2}, M_{X}^{2}) = -M_{W}^{2} C_{0}(q^{2}, 0, M_{H}^{2}, M_{X}^{2}, M_{X}^{2}, M_{X}^{2})$$

$$J_{2}(q^{2}, M_{H}^{2}, M_{X}^{2}) = \frac{1}{2} \frac{M_{X}^{2}}{q^{2} - M_{H}^{2}} \left[ 1 + 2M_{X}^{2} C_{0}(q^{2}, 0, M_{H}^{2}, M_{X}^{2}, M_{X}^{2}, M_{X}^{2}) + \frac{q^{2}}{q^{2} - M_{H}^{2}} \left( B_{0}(q^{2}, M_{X}^{2}, M_{X}^{2}) - B_{0}(M_{H}^{2}, M_{X}^{2}, M_{X}^{2}) \right) \right]$$
(92)

where  $B_0$  and  $C_0$  are the Passarino-Veltman functions[14] and  $M_X$  is the mass of the particle in the loop. These functions are related to the functions  $I_1$  and  $I_2$  of ref. [6] by the following relations

$$J_1(q^2, M_H^2, M_X^2) = I_2(\tau_X, \lambda_X)$$

$$J_2(q^2, M_H^2, M_X^2) = \frac{1}{4} I_1(\tau_X, \lambda_X)$$
(93)

with

$$\tau_X = \frac{4M_X^2}{M_H^2} \quad ; \quad \lambda_X = \frac{4M_X^2}{q^2}$$
(94)

With these relations it is easy to verify that Eq. (49) is in agreement with Eq. (2.22) of ref. [6]. To verify the equivalence of Eq. (69) with Eqs. (2.16) and (2.17) of ref. [6] for the on–shell decay  $H \to \gamma \gamma$  one has to note that

$$J_1(0, M_H^2, M_X) = I_2(\tau_X, \infty) = \frac{\tau_X}{2} f(\tau_X)$$

$$J_2(0, M_H^2, M_X) = \frac{1}{4} I_1(\tau_X, \infty) = -\frac{\tau_X}{8} + \frac{\tau_X^2}{8} f(\tau_X)$$
(95)

where  $f(\tau)$  is defined in Eq. (2.19) of ref. [6]. Then we get for the W contribution

$$I_{W} = -4 \left[ -4J_{1}(0, M_{H}^{2}, M_{W}^{2}) + \left(6 + \frac{M_{H}^{2}}{M_{W}^{2}}\right) J_{2}(0, M_{H}^{2}, M_{W}^{2}) \right]$$

$$= 16J_{1}(0, M_{H}^{2}, M_{W}^{2}) - \left(24 + \frac{16}{\tau_{W}}\right) J_{2}(0, M_{H}^{2}, M_{W}^{2})$$

$$= 2 + 3\tau_{W} + 3\tau_{W}(2 - \tau_{W}) f(\tau_{W})$$
(96)

and for a fermion of charge  $Q_f$ 

$$I_{F} = 4 Q_{f}^{2} \left[ -J_{1}(0, M_{H}^{2}, M_{W}^{2}) + 4J_{2}(0, M_{H}^{2}, M_{W}^{2}) \right]$$

$$= Q_{f}^{2} \left[ -2 \tau_{f} f(\tau_{f}) - 2 \tau_{f} + 2 \tau_{f}^{2} f(\tau_{f}) \right]$$

$$= Q_{f}^{2} \left[ -2 \tau_{f} \left( 1 + (1 - \tau_{f}) f(\tau_{f}) \right) \right]$$
(97)

in agreement with Eq. (2.17) of ref. [6].

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