

Dear editor,

We would like to thank the referee for the detailed comments and interest to our paper, which helped us to improve our paper. We deeply apologise for the delay with our reply, and hope that the new version of the paper is suitable for the publication in PRD. Our reply and summary of modifications is below.

Sincerely,

Alexander Belyaev on behalf of authors

(1) Referee comment:

In Ref. [29], they claim that the perturbativity and unitarity is more reliable by using the code 2HDMC than Ref. [51] and [60], especially for HHHH vertex. However, since the author does not use 2HDMC instead of the conditions presented in Ref. [51] and [60], I suggest the authors shall also check whether is there some parameter space not properly tested.

Response: We have explicitly checked that the limit from perturbative unitarity and perturbativity given by Eq.(13)-(16) we are using in our study is consistent with that implemented in the 2HDMC code[68]. We have added the respective reference and sentence to an updated version of the paper. Moreover, we would like to note that in our paper we present the limit on λ_2 (or λ_{HHHH} – notation from [29]) as a function of λ_{345} pictorially presented in Figure 2 which goes beyond findings in [29].

(2) Referee comment:

"A related question is whether the model can be better probed by indirect detection (ID) experiments, i.e. the detection of energetic cosmic rays like positron, gamma ray, and antiproton, which may be created by the annihilation of $h1$ pairs. ..., but the bounds are not competitive with those coming from DD."

Recently, the new AMS02 antiproton data seems to have some interesting behavior, see Ref. [1610.03071] and [1610.03840]. This could put some significant impact on i2HDM parameter space. I suggest the authors can add some discussion and estimation about this.

Response:

We have checked that the strongest bounds on the i2HDM parameter space coming from such experiments are set by gamma ray telescopes: both the Fermi- LAT gamma-ray space telescope [84] as well as ground based telescopes. Fermi-LAT is sensitive to gamma rays particularly in the low mass range up to $O(100 \text{ GeV})$, but the bounds are not competitive with those coming from DD. This conclusion is also confirmed by studies in Ref. [60]. The paper was updated respectively with this sentence.

(3) Referee comment: *The new allowed region (almost $mh1$ $mh2$ and $la_{345} 0$) discovered by the authors shall be identified and more emphasized.*

I was interesting in this region and trying to see how this region can be survived but in the end I only found the BM1 at the Table 1. Since this is the new region claimed by the authors, some more information shall be given.

I suspect the relic density of most this region are always too low from PLANCK data. For example, the BM1 has the relic density 0.092 and χ^2 based on Eq. (29) are 484 which is not good

for thermal relic scenario! If my guess is true, the authors shall mention whether is this new region owing to the non-thermal relic assumption?

Response: In the new version of the paper we have provided the requested details on this point.

First of all, we made clear in the end of page 14 that in our analysis we have assumed 10% theoretical uncertainty on the DM relic density prediction since its based on the tree-level calculation. This uncertainty is the dominant one in comparison to about 1% uncertainty on DM fit from the latest PLANCK results given above and relax the DM relic density limit to the following one at 95% CL:

$$\Omega_{\text{DM}}^{\text{limit}} h^2 = 0.1184 \pm 2 \times 0.1184/10 \simeq 0.118 \pm 2 \times 0.012 \quad (1)$$

Then in section 3.2.1 we discuss all details of this region, pointing to: a) that the width of this strip is defined by the maximum allowed value of $\Delta M = M_{h_2} - M_{h_1} = 8$ GeV, above which the parameter space is excluded by LEP di-lepton searches b) we state that in this allowed region DM relic density is never above PLANCK limit given by Eq. 1; c) that the maximum value of Ωh_{DM}^2 reaches the value of about 0.11 for $M_{h_2} - M_{h_1} \simeq 8$ GeV and $\lambda_{345} \simeq 0$, when the only $h_1 - h_2$ co-annihilation takes place; d) for $\Delta M < 8$ GeV and $M_{h_1} < 54$ GeV, the Ωh^2 drops below the $0.118 - 2 \times 0.012$ limit which use in our study because $h_1 - h_2$ co-annihilation via Z-boson increases with the decrease of h_1, h_2 masses; e) the upper edge at 73 GeV is defined by the rapid increase of the $h_1 h_1 \rightarrow WW^*$ contribution, which does not require co-annihilation above this mass. The typical $M_{h_2} - M_{h_1}$ mass split in the co-annihilation region is 7-8 GeV, is required to make the relic density consistent with the PLANCK limit.

To conclude on this point, taking into account theoretical uncertainty on the DM relic density prediction, the relic density in this region is consistent with PLANCK limit and reaches 0.11 value at maximum. This region therefore does not necessarily require non-thermal scenario, which in case of weak interactions would be a bit contrived.

(4) Referee comment: *Again, I was so looking forward to seeing the result about the life time of h^+ and the limit of delta M in the last paragraph of Section 3. However, the authors disappoint me by only using one paragraph with a lot of hand-waving. I think the authors shall present this part with useful figure(s). Otherwise, it is hard to drive me trust the sentence written in the conclusion:*

"At the same time, the potential of the LHC using a search for disappearing charged tracks is quite impressive in probing M_{h1} masses up to about 500 GeV already at 8 TeV with 19.5 fb⁻¹ luminosity as we have found in our study."

Response: In the updated version of the paper have provided all requested details in section 3.2.2:

- a) details of the evolution of the h^+ decay width in case of $h^+ - h_1$ degeneracy, which include effective $W - \text{pion}$ mixing;
- b) we provide the formula for the decay width of h^+ ;
- c) we provide plots for decay width and the charged track length for h^+ ; (Fig.10)
- d) we give the cross section for h^+ production and the relic density for the relevant 400-600 GeV mass range

(5) Referee comment:

Other suggestions:

(i) The text is well organized and very enjoyably readable only before section 3

but it becomes messy and hard to read after section 4. Especially, in the result section, many information/concepts/reasons are given scatteringly and all the figures are listed in the back. I

believe that the authors should be able to make it much clear in the presentation. For example, they can use bullets, items, and tables for classification and comparison.

In addition, I strongly suggest the authors to locate their figures before the text. The figure location can also help reader. Some less important figures can be integrated into an appendix.

(ii) Missing the space (Fig.20, Fig.21)

Response: We appreciate very much the Referee remark about the structure of the paper and completely agree with this point. In the new version of the paper we have done the the following changes:

- a) we have moved many figures on the scan of the parameter space into appendix, this making paper easier to read;
- b) we have produced simplified version of figures – Fig.6-8 for the section 3, aiming the same purpose – to make it easier readable.

To conclude – we agree with all remarks of the Referee and implemented the respective changes which include more details on the long-lived h^+ case as well as restructuring the paper. We believe these changes improve the quality of the paper and thank referee for this. We hope that the new version of the paper is now suitable for the publication in PRD.