Tutorial proposal: Optimal Transport for Signal Processing

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1) Presenters

Laetitia Chapel (laetitia.chapel@irisa.fr). Prof. Chapel is a full professor in computer science at Institut Agro Rennes-Angers. She received a PhD in computer science in 2007 and a French habilitation to supervise research in computer science in 2022. Her research takes place within the OBELIX team of IRISA, a mixed research unit in computer science, signal and image processing, and robotics. Her main research topic is machine learning with a particular focus on structured data (such as time series, graphs, hierarchical representations) and with applications in remote sensing. She notably has worked in the field of computational optimal transport, devising several algorithms to make optimal transport more robust and tractable. She contributes to the Python Optimal Transport toolbox.
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[4] G. Mahey, **L. Chapel**, G. Gasso, C. Bonet, N. Courty, "Fast optimal transport through sliced generalized Wasserstein geodesics", *Advances in Neural Information Processing Systems*, 2023
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Felipe Tobar (ftobar@uchile.cl). Dr Tobar is an Associate Professor at Universidad de Chile and the Director of the Initiative for Data and Artificial Intelligence of the same Institution. He holds Researcher positions at the Center for Mathematical Modeling and the Advanced Center for Electrical and Electronic Engineering. Felipe was a Research Associate at the Machine Learning Group, University of Cambridge, during 2015 and he received a PhD in Signal Processing from Imperial College London in 2014. Felipe's research lies between Machine Learning and Statistical Signal Processing, including approximate inference, Bayesian nonparametrics, spectral estimation, optimal transport (OT) and Gaussian processes (GP). He is an author of the Multi-output GP Toolkit. From Oct 2024, Dr Tobar will be with the Department of Mathematics at Imperial College London as a Senior Lecturer in Machine Learning.
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[9] **F. Tobar**, E. Cazelles, T. de Wolff, "Computationally-efficient initialisation of GPs: The generalised variogram method", *Transactions of Machine Learning Research*, 2023
[10] J. Backhoff-Veraguas, J. Fontbona, G. Rios, **F. Tobar** , "Stochastic gradient descent in Wasserstein space", *Journal of Applied Probability*, (in press), 2024.

2) Presenter's previous tutorial delivery experience

Prof Chapel has delivered tutorials on OT at the CNRS Summer School on Geometry and Data (2023), and the Summer School on Statistical and Geometric Divergences for ML (2022). In 2024, she presented her work at the Kantorovich Initiative (USA), the International Centre for Theoretical Sciences (India), and at the Optimal Transport Workshop: From theory to applications (Berlin). She was a keynote speaker at the Optimal Transport and Machine Learning @NeurIPS workshop in 2023.
Dr Tobar has delivered tutorials on Gaussian processes at the Neural Information Processing Systems 2021, Gaussian Process Summer School 2021, and the IEEE CIS-Chile Summer School on Computational Intelligence 2021. He has also presented his work at Imperial College (LC2 Seminar & DataLearning Seminar), University of Cambridge (ML Group), University of Toulouse (IRIT, Signal Processing Group), Paris-Saclay (UQ Seminar), and Inria (CIMD).

3) Summary

The tutorial will be divided into two parts. First, we will introduce the usual formulations of OT, provide historical context, and discuss metric properties and computational considerations [2,4,8,10,12,14]. Second, we will show three applications to problems in signal processing: i) matching time series using dynamic time warping, ii) colour transfer via histogram transport, and iii) a novel distance between time series applying a Wassserstein-like distance to power spectra [1,3,5,6,7,9,11,13].

Contents

- Part I: An Introduction to Discrete Optimal Transport (45min)
  (a) History, Monge & Kantorovich formulations (15min)
  (b) Geometric properties, Wasserstein distance and barycenters (15min)
  (c) Entropic regularization, computation and examples (15min)
- Part II: Three applications to Signal Processing (45min)
  (a) Colour transfer via histogram matching (15min)
  (b) Dynamic time warping (15min)
  (c) The Wasserstein-Distance for time series (15min)

4) Importance, timeliness, and novelty

The proven success of the machine learning (ML) perspective to signal processing (SP) has paved the way for incorporating state-of-the-art mathematical methods into the theory and practice of time series and image processing. Optimal transport (OT) is one of those methods which provides a general-purpose framework to quantify the discrepancy between two probability distributions by lifting a distance defined on their support [12,14]. In the last decade, the impact of OT in ML cannot be overstated: current OT-powered ML methods include GANs [15], VAEs [16], distribution regression [17] and clustering [18] with applications on genomics, health, finance, audio, robotics, and astrophysics.

Though OT has been applied to time series [5,6,9,11,13], we claim that the full potential of OT for signal processing remains largely unexplored. Furthermore, due to the maturity reached by the theory and methods of OT [1-10], the incorporation of the OT toolbox into SP practice today is not only needed but is also more relevant and timely than ever. This tutorial introduces OT and shows recent successful case studies of OT-based signal processing (SP) applications, as well as pointing out novel research direction and open questions in the intersection of OT, ML and SP. All with the aim of encouraging the adoption of the OT toolbox by the MLSP community.

5) Statement of any previous or related versions of this tutorial

The proposed tutorial has not been presented before. Furthermore, to the best of our knowledge, no tutorial on Optimal Transport for Signal Processing has ever been presented to the academic community.

References (in addition to the presenters' references [1-10] above)

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