Dual LQG-PID Control of a Highly Nonlinear Magnetic Levitation System

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Abstract —**A study of one-dimensional position control of a light iron ball levitated by one electromagnet is proposed in this work. The vertical position of the ball is the target of the proposed control strategy. It is assumed that the position measurement of the ball is inaccessible and thus, estimates of the position are utilized as the feedback signal to the controller. Control of the ball's position is carried out using LQG-based controller with the objective of manipulating the electromagnet current to drive the ball to steady state with minimal error. The integrity of the controller as well as accuracy of the system's model is evaluated via simulation.**

***Keywords*—System identification, active control, position control, magnetic levitation, LQG-based controller**

I. INTRODUCTION

Magnetic Levitation Systems (MLS) have gained considerable interest due to their great practical importance in many engineering fields. Since magnetic levitation does not use long reaching and joint parts, wear and maintenance problem caused by friction are completely eliminated.

Magnetic levitation has been used in magnetic bearings [1-5], vibration suspension [6-9] and position tracking [10]. Furthermore, different types of control techniques have been used in magnetic levitation systems. PID, fuzzy logic, robust and adaptive controllers are the most common techniques [11-14]. In 2002, Morita et al [15] proposed a micro-levitation system using motion control. The proposed System is composed of a permanent magnet, a piezoelectric actuator, a small iron ball, and position sensors. The diameter of the iron ball was 2.0 mm and the weight was 32.8 mg. The system balances the weight of the ball when the time constant is within 20ms. Although magnetic levitation of the ball is achieved, large overshoot amplitude was observed. In 2006, Khamesee et al [16] studied one-dimensional magnetic levitation of tiny magnet with the mass of 0.386 g in the air gap region of 290mm. Eddy current damping was proposed to improve the performance of the levitation system by placing a conductive plate close to the levitated object and analytical expression for damping coefficient was presented. The system has been linearized around multiple working points and proportional and derivate controller was used in the feedback path of the system. In this work, we propose a high precision levitation control of an iron ball. The vertical position of the small levitated magnet is the target of the proposed control strategy. The levitation system models are obtained from both transient response and black-box identification of experimental data. The LQG based controller is for position estimates of the levitated ball and estimates are fed back through a PID controller.

II. MODELING OF THE SYSTEM

Control of a magnetically levitated ball Shown in Figure 1, requires dynamic models relating (a) coil current to electromagnetic force, (b) ball displacement to electromagnetic force and (c) ball displacement to coil current. The first model is obtained from literature provided by maglev system manufacturer INTECO Krakow and expressed as;

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{F_{em}}{m} + g \\
\dot{x}_3 &= \frac{1}{f_i(x_1)}(k_i u + c_i - x_3) \\
F_{em} &= x_3^2 \frac{F_{emP1}}{F_{emP2}} \exp\left(-\frac{x_1}{F_{emP2}}\right) \\
f_i(x_1) &= \frac{F_{iP1}}{F_{iP2}} \exp\left(-\frac{x_1}{F_{iP2}}\right)
\end{aligned} \tag{1}$$

Parameter values of Eq. 1 are listed in Table 1. And the four equations combined represent the dynamic relation between force, displacement, current and coil voltages.

Ball displacement-electromagnetic force model is obtained from a force step response of the levitated ball as shown in Figure 2. Displacement-electromagnetic force data obtained experimentally from step response of the system is utilized to extract the equivalent stiffness and damping coefficient of the system. State space representation of the force-displacement model is expressed as;

$$As = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \quad Bs = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$
$$Cs = [1 \quad 0] \quad Ds = 0 \tag{2}$$

Where $\omega_n = 98.1834\ rad/s$, $k = 428.02\ N/m$, $b = 7.235\ kg/s$, with levitated ball mass m=0.0444 kg.

The last model relating ball displacement to coil current, which is the control feedback, is determined using black-box system identification toolbox in Matlab ® [17]. The model has been derived from experimental displacement-current data shown in Figure 3. Various methods have been used to determine the best model representing 5000 data points, as shown in Figure 4. It is clear from the latter figure that, Narx5 has the best fit with 87.51% match. Figure 5 shows the step response of the model obtained by Narx5, which is used to linearize the model at steady state values [17].

The three models obtained by the above are augmented to determine the LQG estimator-regulator gains. For clarity, this is done in two steps, first the Current-Electromagnetic force system with the quadruple ($A_{cf}, B_{cf}, C_{cf}, D_{cf}$) is augmented with force-displacement model found in Eq. 2, as follows,

$$Aa = \begin{bmatrix} Acf & \text{zeros(length(Acf), length(As)} \\ \text{Bs} * \text{Ccf} & \text{As} \end{bmatrix},$$
$$Ba = \begin{bmatrix} Bcf \\ Bs * Dcf \end{bmatrix}, Ca = [Ds * Ccf \quad Cs], \tag{3}$$
$$Da = [Ds * Dcf]$$

Finally, Eq. 3 is augmented with the linearized current-displacement model obtained by Narx5 that has the quadruple $A_{dc}, B_{dc}, C_{dc}, D_{dc}$, yielding a final state space representation of the model in the form;

$$Aa = \begin{bmatrix} Aa & \text{zeros(length(Aa), length(Adc)} \\ \text{Bdc} * \text{Ca} & \text{Adc} \end{bmatrix}$$
$$Ba = \begin{bmatrix} Ba \\ Bdc * Da \end{bmatrix} \quad Ca = [Ddc * Ca \quad Cdc]$$
$$Da = [Ddc * Da] \tag{4}$$

III. Simulation Results

Gains of the LQG-based Controller are determined from the solutions of the CARE and FARE (Eq. 5 and 6), respectively. The model state-space is the one given in Eq. 4, such that the performance index given in Eq. 7 is minimized.

$$Kc = B^T Sc, \quad A^T Sc + ScA - ScBB^T Sc + Q = 0 \tag{5}$$
$$Ke = SeC^T, ASe + SeA^T - SeC^T CSe + V = 0 \tag{6}$$
$$u = -Kc\hat{x}, \quad J^2 = E[\int_0^\infty (x^T Qx + u^T Ru)dt] \tag{7}$$

See [6] for further details on the solution of Eq. 5,6, and 7. The control scheme is shown in Figure 6, where displacement of the ball is estimated using LQG and fed back through a PID providing the control current needed to stabilize the ball when subjected to exogenous input such as a sine wave force.

Simulation of the effect of the control scheme in stabilizing the ball using only estimates of the position is carried out and the results are shown in Figures 7 and 8. It is clear from Figure 7 that the controller is able to reduce the transient overshoot and steady state error using only measurements of the coil current. The control current estimated values are shown in Figure 8. The latter shows that the estimated value of the control current is reasonable and stable yielding and effective manipulation of the ball position using only estimates of the position.

IV. Conclusion

A novel active control of a magnetic levitation system is presented in this paper. The levitation system models are obtained from both transient response and black-box identification of experimental data. The controller is LQG based and uses estimates of levitated ball position as feedback control signal rather than actual measurements. Estimates fed back through a PID controller are used to stabilize the ball when subjected to exogenous input. Simulation results have shown that estimates of the levitated ball can replace actual measurement for position control. Integrity of the control system is dependent on the relative accuracy of model used in constructing the appropriate LQG.

Acknowledgment

Authors acknowledge the support of the American University of Sharjah

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TABLE 1.
PARAMETERS OF EQUATION 1

Parameter	**Values**	**Units**
m	0.0444	kg
g	9.81	m/s^2
F_{em}	Function of x_1, x_3	N
F_{emP1}	1.7521×10^{-2}	H
F_{iP2}	4.5626×10^{-3}	m
k_i, c_i	2.5165, 0.0243	A
x_1, x_3		m, A respectively

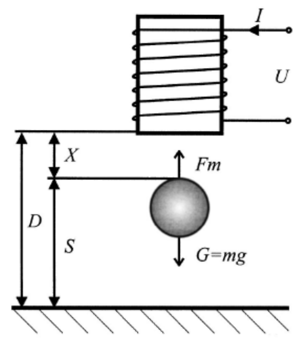

Figure 1: Magnetically levitated ball

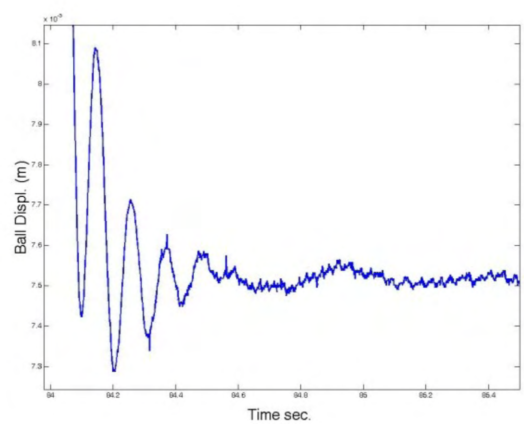

Figure 2: Step response of levitated ball

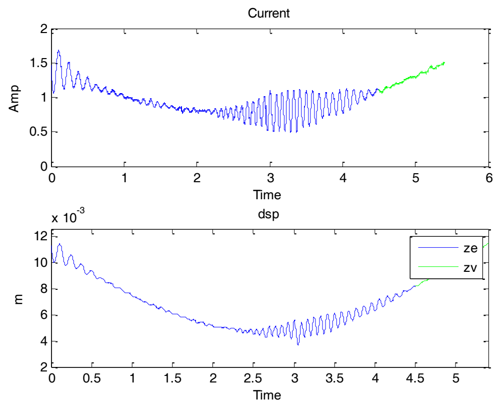

Figure 3: experimental data of current and displacement

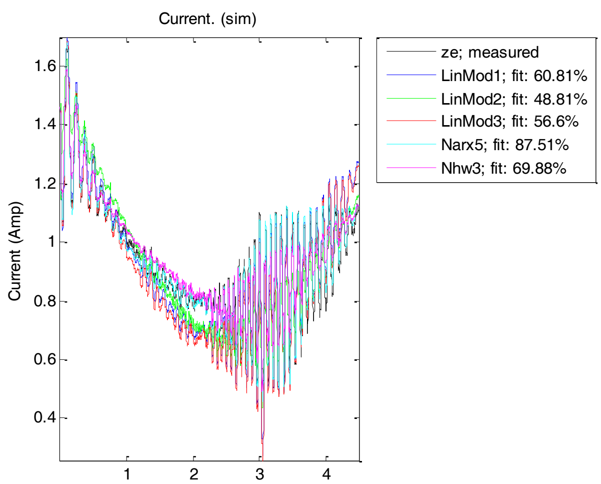

Figure 4: Nonlinear models constructed from data

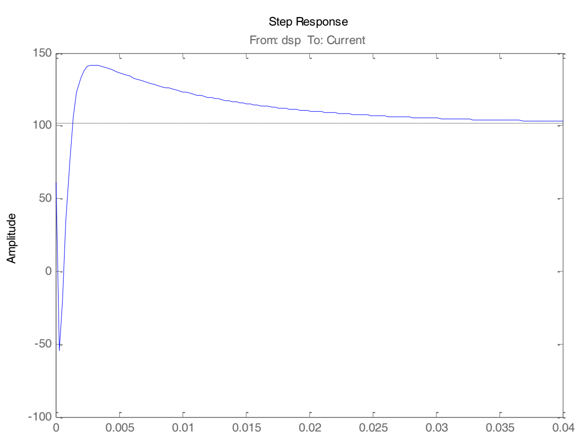

Figure 5: step response of the displacement-current model

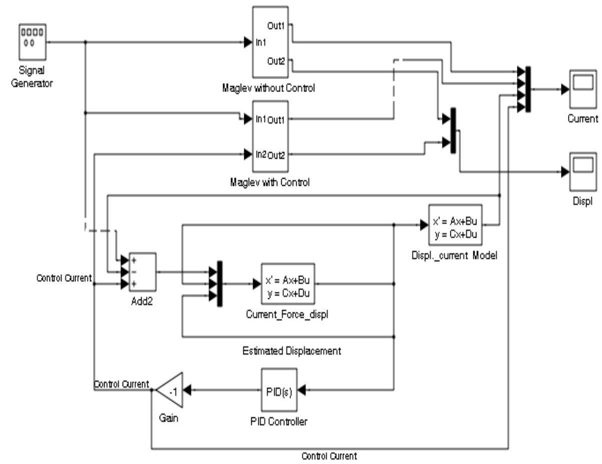

Figure 6: Control scheme implemented on Maglev system shown in Figure 1

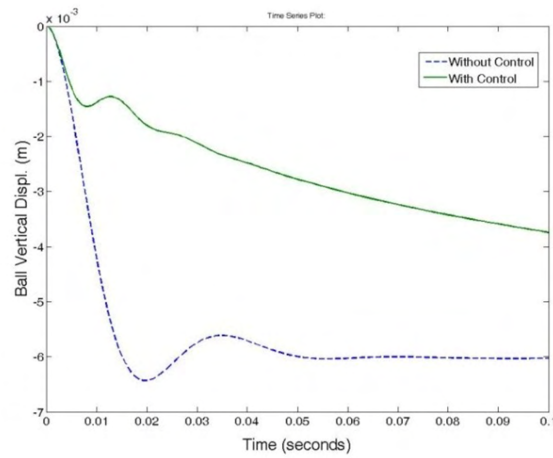

Figure 7: Displacement of levitated ball

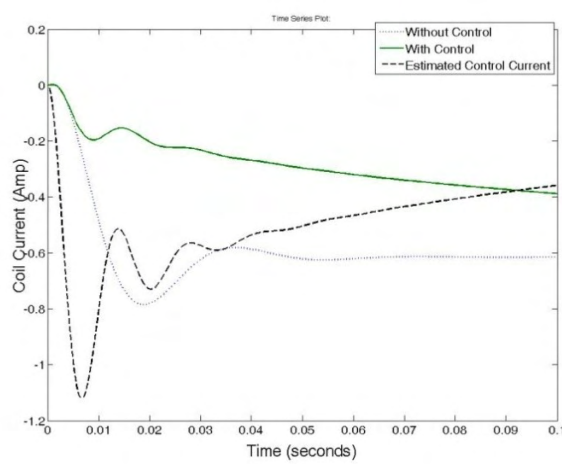

Figure 8: Maglev system coil current