MEC 2403 · OTIMIZAÇÃO

Lista 0: Felipe da Costa Pereira

① $f(x_1,x_2,x_3) = 3x_1^3x_2^2x_3 - 6x_1\log(x_2)x_3^4 + x_1^{-1}x_2^3 - x_1^2\sqrt{x_2}$

ⓐ gradiante: $\nabla f(x_1,x_2,x_3) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial x_3}\right)$

$\frac{\partial f}{\partial x_1} = 9x_1^2x_2^2x_3 - 6\log(x_2)x_3^4 - x_1^{-2}x_2^3 - 2x_1\sqrt{x_2}$

$\frac{\partial f}{\partial x_2} = 6x_1^3x_2x_3 - 6\frac{x_1x_3^4}{x_2} - x_1^2 \cdot \frac{1}{2\sqrt{x_2}} + 3x_1^{-1}x_2^2$

$\frac{\partial f}{\partial x_3} = 3x_1^3x_2^2 - 24x_1\ln(x_2)x_3^3$

$\nabla f(x_1,x_2,x_3) = \left(9x_1^2x_2^2x_3 - 6\log(x_2)x_3^4 - x_1^{-2}x_2^3 - 2x_1\sqrt{x_2},\right.$
$6x_1^3x_2x_3 - 6x_1x_2^{-1}x_3^4 + 3x_1^{-1}x_2^2 - x_1^2/2\sqrt{x_2},$
$\left.3x_1^3x_2^2 - 24x_1\log(x_2)x_3^3\right)$

ⓑ Matriz Hessiana: $H = \begin{bmatrix} \partial^2 f/\partial x_1^2 & \partial^2 f/\partial x_1\partial x_2 & \partial^2 f/\partial x_1\partial x_3 \\ \partial^2 f/\partial x_2\partial x_1 & \partial^2 f/\partial x_2^2 & \partial^2 f/\partial x_2\partial x_3 \\ \partial^2 f/\partial x_3\partial x_1 & \partial^2 f/\partial x_3\partial x_2 & \partial^2 f/\partial x_3^2 \end{bmatrix}$

(+) $\frac{\partial^2 f}{\partial x_1^2} = 18x_1x_2^2x_3 + 2\frac{x_2^3}{x_1^3} - 2\sqrt{x_2}$

(+) $\frac{\partial^2 f}{\partial x_2\partial x_1} = \frac{\partial}{\partial x_2}\left[\frac{\partial f}{\partial x_1}\right] = 18x_1^2x_2x_3 - \frac{6x_3^4}{x_2} - 3\frac{x_2^2}{x_1^2} - \frac{1\cancel{2}}{\cancel{2}}\frac{x_1}{\sqrt{x_2}}$

(+) $\frac{\partial^2 f}{\partial x_3\partial x_1} = 9x_1^2x_2^2 - 24\log(x_2)x_3^3$

(+) $\frac{\partial^2 f}{\partial x_3\partial x_2} = 6x_1^3x_2 - 24\frac{x_1}{x_2}\cdot x_3^3$

$$\frac{\partial^2 f}{\partial x_2^2} = 6x_1^3x_3 + \frac{6x_1x_3^4}{x_2^2} - \frac{x_1^2}{2}\frac{\partial}{\partial x_2}\left[x_2^{-1/2}\right] + \frac{6x_2}{x_1}$$

$$= 6x_1^3x_3 + \frac{6x_1x_3^4}{x_2^2} - \frac{x_1^2}{2}\cdot\left(-\frac{1}{2}\right)x_2^{-3/2} + \frac{6x_2}{x_1}$$

$$= 6x_1^3x_3 + \frac{6x_1x_3^4}{x_2^2} + \frac{x_1^2}{4}x_2^{-3/2} + \frac{6x_2}{x_1}$$

$$\frac{\partial^2 f}{\partial x_3^2} = -72x_1\log(x_2)x_3^2$$

... e ... $\frac{\partial^2 f}{\partial x_2\partial x_1} = \frac{\partial^2 f}{\partial x_1\partial x_2}$, $\frac{\partial^2 f}{\partial x_3\partial x_1} = \frac{\partial^2 f}{\partial x_1\partial x_3}$

e $\frac{\partial^2 f}{\partial x_3\partial x_2} = \frac{\partial^2 f}{\partial x_2\partial x_3}$, logo H:

$$H = \begin{bmatrix} 18x_1x_2^2x_3 + 2x_2^3/x_1^3 - 2\sqrt{x_2} & \to H_{21} & H_{31} \\ \underbrace{18x_1^2x_2x_3 - 6x_3^4/x_2 - 3x_2^2/x_1^2 - x_1/\sqrt{x_2}}_{H_{21}} & 6x_1^3x_3 + \frac{6x_1x_3^4}{x_2^2} + \frac{x_1^2}{4}x_2^{-3/2} + 6x_2/x_1 & H_{32} \\ \underbrace{9x_1^2x_2^2 - 24\log(x_2)x_3^3}_{H_{31}} & \underbrace{6x_1^3x_2 - \frac{24x_1x_3^3}{x_2}}_{H_{32}} & -72x_1\log x_2 x_3^2 \end{bmatrix}$$

(2) Classificar quanto à sua positividade:

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

Calculando os sub-determinantes de A

$A_1 = [2] \to \det(A_1) = 2 > 0$

$A_2 = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \to \det(A_2) = 2 - 1 = 1 > 0$

$A_3 = A \qquad \det(A) = 2 + 4 + \cancel{4} - \cancel{4} - 1 - 8$

$= -3 < 0$

$\det(A_3) < 0$

Logo, A não é definida positiva nem negativa
→ A é indefinida.

(3) $f(x) = e^x \qquad x_0 = 1 \qquad$ expansão em série de Taylor.
em torno de $x = 1$

$$S(x) = f(x_0) + \frac{1}{1!}\frac{df}{dx}\bigg|_{x=x_0} \cdot (x - x_0) + \frac{1}{2!}\frac{\partial^2 f}{\partial x^2}\bigg|_{x=x_0} (x - x_0)^2 + \dots$$
$$+ \frac{1}{3!}\frac{\partial^3 f}{\partial x^3}\bigg|_{x=x_0} (x - x_0)^3 + \dots + \frac{1}{n!}\frac{\partial^n f}{\partial x^n}\bigg|_{x=x_0} (x - x_0)^n.$$

$f(x) = e^x$:

$$S(x) = e^{x_0} + e^{x_0} \cdot (x - x_0) + \frac{1}{2} e^{x_0} \cdot (x - x_0)^2 + \frac{1}{6} e^{x_0} (x - x_0)^3$$
$$+ \dots + 1/n! \cdot e^{x_0} (x - x_0)^n$$

Polinômios de Taylor: $x_0 = 1$

$$\begin{cases} p_0(x) = e^{x_0} = e & p_1(x) = e + e(x-1) \\ p_2(x) = e + e(x-1) + \frac{e}{2}(x-1)^2 \\ p_3(x) = e + e(x-1) + \frac{e}{2}(x-1)^2 + \frac{e}{6}(x-1)^3 \end{cases}$$