PLA

Lista_4

Método Simplex

Lista IV - Programação linear (Simplex)

1 Maximizar: $z = 5x_1 + 4x_2$

sujeito a:

$$\begin{cases} -3x_1 + 2x_2 \leq 10 \\ x_1 + 3x_2 \leq 29 \\ 2x_1 + 3x_2 \leq 37 \\ 3x_1 + 2x_2 \leq 43 \\ 2x_1 + x_2 \leq 28 \\ x_2 \leq 8 \\ x_1, x_2 \geq 0 \end{cases}$$

	VNB	VNB	VB	VB	VB	VB	VB	VB		TØ	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	Q	
	-3	2	1	0	0	0	0	0	10		$E_{15(3/2)}$
	1	3	0	1	0	0	0	0	29	29	$E_{25(-3/2)}$
	2	3	0	0	1	0	0	0	37	37/2 = 18,5	$E_{35(-1)}$
	3	2	0	0	0	1	0	0	43	43/3 = 14,3	$E_{45(-3/2)}$
	(2)	1	0	0	0	0	1	0	28	14	$E_{5(1/2)}$
	0	1	0	0	0	0	0	1	8		
z	-5	-4	0	0	0	0	0	0	0		$E_{75(5/2)}$

↑

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 10, 29, 37, 43, 28, 8)$

$z = 5x_1 + 4x_2$

$x_1 = x_2 = 0$

$z = 5.(0) + 4(0) = 0$

	VB		VB	VB	VB	VB		VB		T1	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	Q	
	0	7/2	1	0	0	0	3/2	0	52	104/7	$E_{14}(-7)$
	0	5/2	0	1	0	0	-1/2	0	15	6	$E_{24}(-5)$
	0	2	0	0	1	0	-1	0	9	9/2	$E_{34}(-4)$
	0	(1/2)	0	0	0	1	-3/2	0	1	2	/ $E_4(2)$
	1	1/2	0	0	0	0	1/2	0	14	28	$E_{54}(-1)$
	0	1	0	0	0	0	0	1	8	8	$E_{64}(-2)$
z	0	-3/2	0	0	0	0	5/2	0	70		$E_{74}(3)$

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$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (14, 0, 52, 15, 9, 1, 0, 8)$

$z = 5(14) + 4(0) = 70$

	VB	VB	VB	VB	VB			VB		T2	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	Q	
	0	0	1	0	0	-7	12	0	45	15/4	$E_{13}(-12/5)$
	0	0	0	1	0	-5	7	0	10	10/7	$E_{23}(-7/5)$
	0	0	0	0	1	-4	(5)	0	5	1	/ $E_3(1/5)$
	0	1	0	0	0	2	-3	0	2		$E_{43}(3/5)$
	1	0	0	0	0	-1	2	0	13	13/2	$E_{53}(-2/5)$
	0	0	0	0	0	-2	3	1	6	2	$E_{63}(-3/5)$
z	0	0	0	0	0	3	-2	0	73		$E_{73}(2/5)$

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$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (13, 2, 45, 10, 5, 0, 0, 6)$

$z = 5(13) + 4(2) = 73$

	VB	VB	VB	VB			VB	VB	T3
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b
	0	0	1	0	-12/5	13/5	0	0	33
	0	0	0	1	-7/5	3/5	0	0	3
	0	0	0	0	1/5	-4/5	1	0	1
	0	1	0	0	3/5	-2/5	0	0	5
	1	0	0	0	-2/5	3/5	0	0	11
	0	0	0	0	-3/5	2/5	0	1	3
z	0	0	0	0	2/5	7/5	0	0	75

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$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (11, 5, 33, 3, 0, 0, 1, 3)$

$z = 5.(11) + 4(5) = 75$

Solução: $(x_1, x_2) = (11, 5)$ e $z_{máx} = 75$

2 Maximizar: $z = 4x_1 + 5x_2 + 7x_3 - x_4$

sujeito a:
$$2x_1 - x_2 + 3x_3 + 4x_4 \leq 10$$
$$x_1 + x_2 + x_3 - x_4 \leq 5$$
$$x_1 + 2x_2 - 2x_3 + 4x_4 \leq 12$$
$$x_1, x_2, x_3, x_4 \geq 0$$

S-SI
$$2x_1 - x_2 + 3x_3 + 4x_4 + x_5 = 10$$
$$x_1 + x_2 + x_3 - x_4 + x_6 = 5$$
$$x_1 + 2x_2 - 2x_3 + 4x_4 + x_7 = 12$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	Q	
	2	-1	(3)	4	1	0	0	10	10/3	$E_1(1/3)$
	1	1	1	-1	0	1	0	5	5	$E_{21}(-1/3)$
	1	2	-2	4	0	0	1	12		$E_{31}(2/3)$
z	-4	-5	-7	1	0	0	0	0		$E_{41}(7/3)$

VB VB VB TØ

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (0, 0, 0, 0, 10, 5, 12)$

$z = 4.0 + 5.0 + 7.0 - 0 = 0$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	Q	
	2/3	-1/3	1	4/3	1/3	0	0	10/3		$E_{12}(1/3)$
	1/3	(4/3)	0	-7/3	-1/3	1	0	5/3	5/4	$E_2(3/4)$
	7/3	4/3	0	20/3	2/3	0	1	56/3	14	$E_{32}(-1)$
z	2/3	-22/3	0	31/3	7/3	0	0	70/3		$E_{42}(11/3)$

VB VB TI

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (0, 0, {}^{10}/_3, 0, 0, {}^{5}/_3, {}^{56}/_3)$

$z = 4.0 + 5.0 + 7.({}^{10}/_3) - 0 = {}^{70}/_3$

			VB	VB			VB	T2		
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	Q	
	${}^3/_4$	0	1	${}^3/_4$	${}^1/_4$	${}^1/_4$	0	${}^{15}/_4$	5	$E_{13}(-{}^1/_{12})$
	${}^1/_4$	1	0	$-{}^7/_4$	$-{}^1/_4$	${}^3/_4$	0	${}^5/_4$		$E_{23}({}^7/_{36})$
	2	0	0	(9)	1	−1	1	17	${}^{17}/_9$	$E_3({}^1/_9)$
z	${}^5/_2$	0	0	$-{}^5/_2$	${}^1/_2$	${}^{11}/_2$	0	${}^{65}/_2$		$E_{43}({}^5/_{18})$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (0, {}^5/_4, {}^{15}/_4, 0, 0, 0, 17)$

$z = 4.0 + 5.({}^5/_4) + 7({}^{15}/_4) - 0 = {}^{65}/_2$

		VB	VB	VB			T3	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
	${}^7/_{12}$	0	1	0	${}^1/_6$	${}^1/_3$	$-{}^1/_{12}$	${}^7/_3$
	${}^{23}/_{36}$	1	0	0	$-{}^1/_{18}$	${}^5/_9$	${}^7/_{36}$	${}^{41}/_9$
	${}^2/_9$	0	0	1	${}^1/_9$	$-{}^1/_9$	${}^1/_9$	${}^{17}/_9$
z	${}^{65}/_{18}$	0	0	0	${}^{13}/_{18}$	${}^{47}/_9$	${}^5/_{18}$	${}^{335}/_9$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (0, {}^{41}/_9, {}^7/_3, {}^{17}/_9, 0, 0, 0)$.

$z = 4.0 + 5.({}^{41}/_9) + 7({}^7/_3) - {}^{17}/_9 = {}^{335}/_9$

Solução: $(x_1, x_2, x_3, x_4) = (0, {}^{41}/_9, {}^7/_3, {}^{17}/_9)$ $\therefore z_{máx} = {}^{335}/_9$

3 Maximizar: $z = 2x_1 + 3x_2 + 3x_3$

sujeito a:
$$\begin{cases} 3x_1 + 2x_2 \leq 60 \\ -x_1 + x_2 + 4x_3 \leq 10 \\ 2x_1 - 2x_2 + 5x_3 \leq 50 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

Observação: o exercício em questão apresenta "empate no critério de entrada". Resolva-o das duas maneiras possíveis.

1 1

$$S-S_1 \begin{cases} 3x_1 + 2x_2 + x_4 = 60 \\ -x_1 + x_2 + 4x_3 + x_5 = 10 \\ 2x_1 - 2x_2 + 5x_3 + x_6 = 50 \end{cases}$$

				VB	VB	VB		TO	
	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	3	2	0	1	0	0	60	30	$E_{12}(-2)$
	-1	(1)	4	0	1	0	10	10	
	2	-2	5	0	0	1	50		$E_{32}(2)$
z	-2	-3	-3	0	0	0	0		$E_{02}(3)$

$(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 0, 60, 10, 50)$

$z = 2.0 + 3.0 + 3.0 = 0$

1ª Resolução

		VB		VB		VB		T1	
	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	(5)	0	-8	1	-2	0	40	8	$E_1(1/5)$
	-1	1	4	0	1	0	10		$E_{21}(1/5)$
	0	0	13	0	2	1	70		
z	-5	0	9	0	3	0	30		$E_{31}(1)$

$(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 10, 0, 40, 0, 70)$

$z = 2.0 + 3.10 + 3.0 = 30$

					T2		
	x_1	x_2	x_3	x_4	x_5	x_6	b
	1	0	$-8/5$	$1/5$	$-2/5$	0	8
	0	1	$12/5$	$1/5$	$3/5$	0	18
	0	0	13	0	2	1	70
z	0	0	1	1	1	0	70

$(x_1, x_2, x_3, x_4, x_5, x_6) = (8, 18, 0, 0, 0, 70)$

$z = 2.8 + 3.18 + 3.0 = 70$

Solução: $(x_1, x_2, x_3) = (8, 18, 0)$ e $z_{MÁX} = 70$

2ª Resolução

				VB	VB	VB		TØ	
	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	3	2	0	1	0	0	60		
	-1	1	(4)	0	1	0	10	5/2	$E_2(1/4)$
	2	-2	5	0	0	1	50	10	$E_{32}(-5/4)$
z	-2	-3	-3	0	0	0	0		$E_{42}(3/4)$

$(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 0, 60, 10, 50)$

$z = 2.0 + 3.0 + 3.0 = 0$

			VB	VB		VB		T1	
	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	3	2	0	1	0	0	60	20	$E_{13}(-12/13)$
	-1/4	1/4	1	0	1/4	0	5/2		$E_{23}(1/13)$
	(13/4)	-13/4	0	0	-5/4	1	75/2	150/13	$E_3(4/13)$
z	-11/4	-9/4	0	0	3/4	0	15/2		$E_{43}(11/13)$

$(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 5/2, 60, 0, 75/2)$

$z = 2.0 + 3.0 + 3.5/2 = 15/2$

	VB			VB	VB			T2	
	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	0	(5)	0	1	15/13	-12/13	330/13	66/13	$E_1(1/5)$
	0	0	1	0	2/13	1/13	70/13		
	1	-1	0	0	-5/13	4/13	150/13		$E_{31}(1/5)$
z	0	-5	0	0	-4/13	11/13	510/13		$E_{41}(1)$

$(x_1, x_2, x_3, x_4, x_5, x_6) = (150/13, 0, 70/13, 320/13, 0, 0)$

$z = 2.(150/13) + 3.0 + 3.(70/13) = 510/13$

	VB x_1	VB x_2	VB x_3	x_4	x_5	x_6	b	Q	T3
	0	1	0	1/5	3/13	-12/65	66/13		$E_{12}(12/5)$
	0	0	1	0	2/13	(1/13)	70/13	70	$E_2(13)$
	1	0	0	1/5	-2/13	8/65	216/13	135	$E_{32}(-8/5)$
z	0	0	0	1	11/13	-1/13	840/13		$E_{42}(1)$

$(x_1, x_2, x_3, x_4, x_5, x_6) = (215/13, 66/13, 70/13, 0, 0, 0)$

$z = 2.(216/13) + 3(66/13) + 3.(70/13) = 840/13$

	VB x_1	VB x_2	x_3	x_4	x_5	VB x_6	T4 b
	0	1	12/5	1/5	3/5	0	18
	0	0	13	0	2	1	70
	1	0	-8/5	1/5	-1/5	0	8
z	0	0	1	1	1	0	70

$(x_1, x_2, x_3, x_4, x_5, x_6) = (8, 18, 0, 0, 0, 70)$

$z = 2.8 + 3.18 + 3.0 = 70$

Solução: $(x_1, x_2, x_3) = (8, 18, 0)$ e $z_{máx} = 70$

4 Maximizar: $z = x_1 + x_2 + x_3$

sujeito a:

$x_1 + x_2 \geq 3$

$x_1 + 2x_2 + x_3 \geq 4$

$2x_1 + x_2 + x_3 \leq 2$

$x_1, x_2, x_3 \geq 0$

$$S-S_1 \begin{cases} x_1 + x_2 - x_4 + x_7 = 3 \\ x_1 + 2x_2 + x_3 - x_5 + x_8 = 4 \\ 2x_1 + x_2 + x_3 + x_6 = 2 \end{cases}$$

Variáveis de folga: $x_4, x_5, x_8 \geq 0$

Variáveis artificiais: $x_7, x_8 \geq 0$

Função Artificial: $M = -x_7 - x_8 \leq 0$

Maximizar. M

sujeito a S_1

							VB	VB	VB	TΦ	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b		
	1	1	0	-1	0	0	1	0	3		
	1	2	1	0	-1	0	0	1	4		
	2	1	1	0	0	1	0	0	2		
Z	-1	-1	-1	0	0	0	0	0	0		
M	0	0	0	0	0	0	1	1	0	$E_{51}(-1) + E_{52}(-1)$	

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 0, 0, 0, 0, 3, 4)$

$M = -3 - 4 = -7 \neq 0$

							VB	VB	VB		T1	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	Q	
	1	1	0	-1	0	0	1	0	3	3	$E_{13}(-1)$
	1	2	1	0	-1	0	0	1	4	2	$E_{23}(-2)$
	2	(1)	1	0	0	1	0	0	2	2	
Z	-1	-1	-1	0	0	0	0	0	0		$E_{43}(1)$
M	-2	-3	-1	1	1	0	0	0	-7		$E_{53}(3)$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 0, 0, 0, 2, 3, 4)$

$M = -3 - 4 = -7$

	VB						VB	VB	T2
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b
	1	0	-1	-1	0	-1	1	0	1
	-3	0	-1	0	-1	-2	0	1	0
	2	1	1	0	0	1	0	0	2
z	1	0	0	0	0	1	0	0	2
M	4	0	2	1	1	3	0	0	-1

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 2, 0, 0, 0, 0, 1, 0)$

$M = -1 - 0 = -1$

Não é possível majorar M, de forma que $M = 0$. Logo, $\nexists$ solução.

5 Maximizar: $z = 20x_1 + 30x_2$

sujeito a:
$$\begin{cases} x_1 + x_2 \leq 60 \\ x_1 + 2x_2 \geq 100 \\ 2x_1 + x_2 \geq 110 \\ x_1, x_2 \geq 0 \end{cases}$$

$S \rightarrow S_1$
$$\begin{cases} x_1 + x_2 + x_3 = 60 \\ x_1 + 2x_2 - x_4 + x_6 = 100 \\ 2x_1 + x_2 - x_5 + x_7 = 110 \end{cases}$$

Variáveis de folga: $x_3, x_4, x_5 \geq 0$

Variáveis artificiais: $x_6, x_7 \geq 0$

Função artificial: $M = -x_6 - x_7 \leq 0$

Maximizar M

sujeito a S_1

			VB			VB	VB	TΦ	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	
	1	1	1	0	0	0	0	60	
	1	2	0	-1	0	1	0	100	
	2	1	0	0	-1	0	1	110	
Z	-20	-30	0	0	0	0	0	0	
M	0	0	0	0	0	1	1	0	$E_{52}(-1) + E_{53}(-1)$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (0, 0, 60, 0, 0, 100, 110)$

$M = -100 - 110 = -210$

Fazendo-se os ajustes temos:

			VB			VB	VB		T1	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	Q	
	1	1	1	0	0	0	0	60	60	$E_{43}(-1/2)$
	1	2	0	-1	0	1	0	100	100	$E_{23}(-1/2)$
	(2)	1	0	0	-1	0	1	110	55	$E_5(1/2)$
Z	-20	-30	0	0	0	0	0	0		$E_{45}(10)$
M	-3	-3	0	1	1	0	0	-210		$E_{53}(3/2)$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (0, 0, 60, 0, 0, 100, 110)$

$M = -100 - 110 = -210$

	VB		VB			VB			T2	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b	Q	
	0	(1/2)	1	0	1/2	0	-1/2	5	10	$E_1(2)$
	0	3/2	0	-1	1/2	1	-1/2	45	30	$E_{21}(-3)$
	1	1/2	0	0	-1/2	0	1/2	55	110	$E_{31}(-1)$
Z	0	-20	0	0	-10	0	10	1100		$E_{41}(40)$
M	0	-3/2	0	1	5/2	0	3/2	-45		$E_{51}(3)$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (55, 0, 5, 0, 0, 45, 0)$

$M = -45 - 0 = -45$

(1 / 1)

	VB					VB		TB
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
	0	1	2	0	1	0	-1	10
	0	0	-3	-1	-1	1	1	30
	1	0	-1	0	-1	0	-1	50
z	0	0	40	0	10	0	-10	3300
M	0	0	3	1	4	0	0	-30

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (50, 10, 0, 0, 0, 30, 0)$

$M = -30 - 0 = -30$

Não é possível majorar M, de forma que M = 0. Logo, ∄ solução.

6 Maximizar: $z = 40x_1 + 60x_2$

sujeito a:
$$\begin{cases} 2x_1 + x_2 \geq 70 \\ x_1 + x_2 \geq 40 \\ x_1 + 3x_2 \geq 90 \\ x_1, x_2 \geq 0 \end{cases}$$

$$S \to S_1 \begin{cases} 2x_1 + x_2 - x_5 + x_6 = 70 \\ x_1 + x_2 - x_4 + x_7 = 40 \\ x_1 + 3x_2 - x_5 + x_8 = 90 \end{cases}$$

Variáveis de folga: $x_3, x_4, x_5 \geq 0$

Variáveis artificiais: $x_6, x_7, x_8 \geq 0$

Função artificial: $M = -x_6 - x_7 - x_8 \leq 0$

Maximizar M

sujeito a S_1

FORON!

						VB	VB	VB	TØ	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	
	2	1	-1	0	0	1	0	0	70	
	1	1	0	-1	0	0	1	0	40	
	1	3	0	0	-1	0	0	1	90	
Z	-40	-60	0	0	0	0	0	0	0	
M	0	0	0	0	0	1	1	1	0	$E_{51}(-1) + E_{52}(-1) + E_{53}(-1)$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 0, 0, 0, 70, 40, 90)$

$M = -70 - 40 - 90 = -200$

Fazendo-se os ajustes, teremos:

						VB	VB	VB		T1	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	Q	
	2	1	-1	0	0	1	0	0	70	70	$E_{13}(-1/3)$
	1	1	0	-1	0	0	1	0	40	40	$E_{23}(-1/3)$
	1	(3)	0	0	-1	0	0	1	90	30	$E_3(1/3)$
Z	-40	-60	0	0	0	0	0	0	0		$E_{43}(20)$
M	-4	-5	1	1	1	0	0	0	-200		$E_{53}(5/3)$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 0, 0, 0, 70, 40, 90)$

$M = -200$

		VB				VB	VB			T2	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	Q	
	5/3	0	-1	0	1/3	1	0	-1/3	40	24	$E_{12}(-5/2)$
	(2/3)	0	0	-1	1/3	0	1	-1/3	10	15	$E_2(3/2)$
	1/3	1	0	0	-1/3	0	0	1/3	30	90	$E_{32}(-1/2)$
Z	-20	0	0	0	-20	0	0	20	1800		$E_{42}(60)$
M	-7/3	0	1	1	-2/3	0	0	5/3	-50		$E_{52}(7/2)$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 30, 0, 0, 0, 40, 10)$

$M = -0 - 40 - 10 = -50$

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	VB x_1	VB x_2	x_3	x_4	x_5	x_6	x_7	x_8	b	T3 Q	
	0	0	-1	(5/2)	-1/2	1	-5/2	1/2	15	6	/ $E_1(2/5)$
	1	0	0	-3/2	1/2	0	3/2	-1/2	15		$E_{21}(3/5)$
	0	1	0	1/2	-1/2	0	-1/2	1/2	25	50	$E_{31}(-1/5)$
z	0	0	0	-30	-10	0	30	10	2100		$E_{41}(12)$
M	0	0	1	-5/2	1/2	0	7/2	1/2	-15		$E_{51}(1)$

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (15, 25, 0, 0, 0, 15, 0, 0)$

$M = -15 - 0 - 0 = -15$

	VB x_1	VB x_2	x_3	VB x_4	x_5	x_6	x_7	x_8	b	T4 Q	
	0	0	-2/5	1	-1/5	2/5	-1	1/5	6		$E_{13}(1)$
	1	0	-3/5	0	(1/5)	3/5	0	-1/5	24	120	/ $E_{23}(5)$
	0	1	1/5	0	-2/5	-1/5	0	2/5	22		$E_3(2)$
z	0	0	-12	0	-16	12	0	16	2280		$E_{43}(80)$
M	0	0	0	0	0	1	1	1	0		

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (24, 22, 0, 6, 0, 0, 0, 0)$

$M = 0$

$(x_1, x_2, x_3, x_4, x_5) = (24, 22, 0, 6, 0)$

$z = 40\,24 + 60.22 = 2280$

	VB x_1	VB x_2	x_3	VB x_4	VB x_5	b	T5 Q
	1	0	-1	1	0	30	
	5	0	-3	0	1	120	
	0	1	-1	0	0	70	
z	80	0	-60	0	0	4200	

Não é possível encontrar os valores na coluna Q. Concluímos, então, que z não é limitada superiormente.

FORDM

PLA Método Simplex

7 Maximizar: $z = 15x_1 + 25x_2 + 10x_3$

Sujeito a S: $\begin{cases} -6x_2 + 3x_3 \geq 12 \\ 3x_1 + 12x_2 + 6x_3 = 30 \\ x_1 \geq 0; x_2 \geq 0; x_3 \geq 0 \end{cases}$

Solução:

S → S1: $\begin{cases} -6x_2 + 3x_3 - x_4 + x_5 = 12 \\ 3x_1 + 12x_2 + 6x_3 + x_6 = 30 \\ x_1 \geq 0; x_2 \geq 0; x_3 \geq 0 \end{cases}$

Variável de Folga: x_4

Variáveis Artificiais: x_5 *e* x_6

Função Artificial : $M = -x_5 - x_6$

Ajuste: $(-1)L_1 + (-1)L_2 + L_4$

T_0	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	0	–6	3	–1	1	0	12	4	$E_1(1/3)$
	3	12	6	0	0	1	30	5	$E_{21}(-2)$
z	–15	–25	–10	0	0	0	0		$E_{31}(10/3)$
M	0	0	0	0	1	1	0		
M	–3	–6	–9	1	0	0	–42		$E_{41}(3)$

$(x_1, x_2, x_3, x_4, x_5, x_6) =$

$(0, 0, 0, 0, 12, 30)$

$M = -x_5 - x_6 \quad \boldsymbol{M = -42}$

T_1	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	0	–2	1	–1/3	1/3	0	4		$E_{12}(2/24)$
	3	24	0	2	–2	1	6	1/4	$E_2(1/24)$
z	–15	–45	0	-10/3	10/3	0	40		$E_{32}(45/24)$
M	–3	–24	0	–2	3	0	–6		

$(x_1, x_2, x_3, x_4, x_5, x_6) =$

$(0, 0, 4, 0, 0, 6)$

$M = -x_5 - x_6 \quad \boldsymbol{M = -6}$

T_2	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	1/4	0	1	–1/6	1/6	1/12	9/2	18	$E_{12}(-2)$
	1/8	1	0	1/12	–1/12	1/24	1/4	2	$E_2(8)$
z	–75/8	0	0	5/12	–5/12	45/24	205/4		$E_{32}(75)$
M	0	0	0	0	1	1	0		

$(x_1, x_2, x_3, x_4, x_5, x_6) =$

$\left(0, \frac{1}{4}, \frac{9}{2}, 0, 0, 0\right)$

$M = -x_5 - x_6 \quad \boldsymbol{M = 0}$

$z = 15x_1 + 25x_2 + 10x_3$

$z = 15 \cdot 0 + 25 \cdot \frac{1}{4} + 10 \cdot \frac{9}{2}$

$\boldsymbol{z = \frac{205}{5}}$

T_3	x_1	x_2	x_3	x_4	b	Q	
	0	–2	1	–1/3	4		
	1	8	0	2/3	2		
z	0	75	0	20/3	70		

$(x_1, x_2, x_3, x_4, x_5, x_6) =$

$(2, 0, 4, 0, 0, 0)$

$z = 15 \cdot 2 + 25 \cdot 0 + 10 \cdot 4$

$\boldsymbol{z = 70}$

Resposta: $\boldsymbol{z_{Max}(2, 0, 4) = 70}$

8 Maximizar: $z = 2x_1 + x_2$

sujeito a: $\begin{cases} 3x_1 + 4x_2 \leq 12 \\ 4x_1 + 2x_2 \leq 10 \\ x_1, x_2 \geq 0 \end{cases}$

$S \rightarrow S_1 \begin{cases} 3x_1 + 4x_2 + x_3 = 12 \\ 4x_1 + 2x_2 + x_4 = 10 \end{cases}$

Variáveis de folga: $x_3, x_4 \geq 0$

			VB x_3	VB x_4		TØ	
	x_1	x_2	x_3	x_4	b	Q	
	3	4	1	0	12	4	$E_{12}(-3/4)$
	(4)	2	0	1	10	5/2	$E_2(1/4)$
z	-2	-1	0	0	0		$E_{02}(1/2)$

$(x_1, x_2, x_3, x_4) = (0, 0, 12, 10)$

$z = 0$.

FOROM!

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	VB x_1	VB x_2	x_3	x_4	T1 b	
	0	5/2	1	-3/4	9/2	$E_1(2/5)$
	1	1/2	0	1/4	5/2	$E_{21}(-1/5)$
z	0	0	0	1/2	5	

$(x_1, x_2, x_3, x_4) = (5/2, 0, 9/2, 0)$

$z = 2.(5/2) + 0 = 5$

	VB x_1	VB x_2	x_3	x_4	T2 b
	0	1	2/5	-3/10	9/5
	1	0	-1/5	2/5	8/5
z	0	0	0	1/2	5

$(x_1, x_2, x_3, x_4) = (8/5, 9/5)$

$z = 2.(8/5) + (9/5) = 5$

Solução: Todos os pontos de extremos $(8/5; 9/5)$ e $(5/2, 0)$, $z_{máx} = 5$.
Se o problema exigir soluções inteiras, temos $(x_1, x_2) = (2, 1)$ e $z_{MÁX} = 5$.

9 Maximizar: $z = x_1 + x_2 + x_3$

sujeito a:
$$\begin{cases} x_1 + x_2 \leq 30 \\ x_3 \leq 30 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

$$S \to S_1 \begin{cases} x_1 + x_2 + x_4 = 30 \\ + x_3 + x_5 = 30 \end{cases}$$

Variáveis de folga: $x_4, x_5 \geq 0$

VB VB TØ

	x_1	x_2	x_3	x_4	x_5	b	Q	
	(1)	1	0	1	0	30	30	/
	0	0	1	0	1	30		
Z	-1	-1	-1	0	0	0		$E_{s1(1)}$

/

$(x_1, x_2, x_3, x_4, x_5) = (0, 0, 0, 30, 30)$

$z = 0$

VB VB T1

	x_1	x_2	x_3	x_4	x_5	b	Q	
	1	1	0	1	0	30		
	0	0	(1)	0	1	30	30	/
Z	0	0	-1	1	0	30		$E_{s2(1)}$

↗

$(x_1, x_2, x_3, x_4, x_5) = (30, 0, 0, 0, 30)$

$z = 30$

VB VB T2

	x_1	x_2	x_3	x_4	x_5	b
	1	1	0	1	0	30
	0	0	1	0	1	30
Z	0	0	0	1	0	60

$(x_1, x_2, x_3, x_4, x_5) = (30, 0, 30, 0, 0)$

$z = 30 + 0 + 30 = 60$

VB VB T3

	x_1	x_2	x_3	x_4	x_5	b
	1	1	0	1	0	30
	0	0	1	0	1	30
Z	0	0	0	1	0	60

$(x_1, x_2, x_3, x_4, x_5) = (0, 30, 30, 0, 0)$

$z = 0 + 30 + 30 = 60$

FORONI

Solução: Todos os pontos de extremos (30,0,30) = (0,30,30) e $z_{máx} = 60$.

10 Minimizar: $z = 10x_1 + 5x_2$,

sujeito a:
$$\begin{cases} 20x_1 + 50x_2 \geq 200, \\ 50x_1 + 10x_2 \geq 150, \\ 30x_1 + 30x_2 \geq 210 \\ x_1, x_2 \geq 0 \end{cases}$$

Solução:

Maximizar: $w = 200y_1 + 150y_2 + 210y_3$

sujeito a:
$$\begin{cases} 20y_1 + 50y_2 + 30y_3 \leq 10 \\ 50y_1 + 10y_2 + 30y_3 \leq 5 \\ y_1, y_2, y_3 \geq 0 \end{cases}$$

$$S - S_1 \begin{cases} 20y_1 + 50y_2 + 30y_3 + y_4 = 10 \\ 50y_1 + 10y_2 + 30y_3 + y_5 = 5 \end{cases}$$

Variáveis de folga: $y_4, y_5 \geq 0$

				VS y_4	VB y_5	b	TØ Q	
	y_1	y_2	y_3					
	20	50	30	1	0	10	1/3	$E_{12}(-1)$
	50	10	(30)	0	1	5	1/6	$E_2(1/30)$
w	-200	-150	-210	0	0	0		$E_{32}(7)$

$(y_1, y_2, y_3, y_4, y_5) = (0,0,0,10,5)$

$w = 0$

			VB	VB			T1	
	y_1	y_2	y_3	y_4	y_5	b	Q	
	-30	(40)	0	1	-1	5	1/8	$E_1(1/40)$
	5/3	1/3	1	0	1/30	1/6	1/2	$E_{21}(-1/120)$
w	150	-80	0	0	7	35	.	$E_{31}(2)$

$(y_1, y_2, y_3, y_4, y_5) = (0, 0, 1/6, 5, 0)$

$w = 200.0 + 150.0 + 210.(1/6) = 35$

	VB	VB			T2	
	y_1	y_2	y_3	y_4	y_5	b
	-3/4	1	0	1/40	-1/40	1/8
	23/12	0	1	-1/120	1/24	1/8
w	90	0	0	2	5	45

$(y_1, y_2, y_3, y_4, y_5) = (0, 1/8, 1/8, 0, 0)$

$w = 200.0 + 150.(1/8) + 210.(1/8) = 45$

Solução: $(x_1, x_2) = (2, 5)$ e $z_{MÍN} = 45$

11 Minimizar: $w = y_1 + y_2$

sujeito a:
$$\begin{cases} 2y_1 + y_2 \geq 2 \\ y_1 + 2y_2 \geq 2 \\ 6y_1 + y_2 \geq 3 \\ y_1, y_2 \geq 0 \end{cases}$$

Maximizar: $z = 2x_1 + 2x_2 + 3x_3$

sujeito a:
$$\begin{cases} 2x_1 + x_2 + 6x_3 \leq 1 \\ x_1 + 2x_2 + x_3 \leq 1 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

S – S:
$$\begin{cases} 2x_1 + x_2 + 6x_3 + x_4 = 1 \\ x_1 + 2x_2 + x_3 + x_5 = 1 \end{cases}$$

Variáveis de folga: $x_4, x_5 \geq 0$

				VB x_4	VB x_5		TØ	
	x_1	x_2	x_3	x_4	x_5	b	Q	
	2	1	(6)	1	0	1	1/6	$E_1(1/6)$
	1	2	1	0	1	1	1	$E_{21}(-1/6)$
z	-2	-2	-3	0	0	0		$E_{31}(1/2)$

$(x_1, x_2, x_3, x_4, x_5) = (0, 0, 0, 1, 1)$

$z = 0$

			VB x_3		VB x_5		T1	
	x_1	x_2	x_3	x_4	x_5	b	Q	
	1/3	1/6	1	1/6	0	1/6	1	$E_{12}(-1/11)$
	2/3	(11/6)	0	-1/6	1	5/6	5/11	$E_2(6/11)$
z	-1	-3/2	0	1/2	0	1/2		$E_{32}(9/11)$

$(x_1, x_2, x_3, x_4, x_5) = (0, 0, 1/6, 0, 5/6)$

$z = 2.0 + 2.0 + 3(1/6) = 1/2$

	VB x_1	VB x_2					T2	
	x_1	x_2	x_3	x_4	x_5	b	Q	
	(3/11)	0	1	2/11	-1/11	1/11	1/3	$E_1(11/3)$
	4/11	1	0	-1/11	6/11	5/11	5/4	$E_{21}(-4/3)$
z	-5/11	0	0	4/11	9/11	13/11		$E_{31}(5/3)$

$(x_1, x_2, x_3, x_4, x_5) = (0, 5/11, 1/11, 0, 0)$

$z = 2.0 + 2.(5/11) + 3.(1/11) = 13/11$

	VB x_1	VB x_2				T3
	x_1	x_2	x_3	x_4	x_5	b
	1	0	11/3	2/3	-1/3	1/3
	0	1	-4/3	-1/3	2/3	1/3
z	0	0	5/3	2/3	2/3	4/3

$(x_1, x_2, x_3, x_4, x_5) = (1/3, 1/3, 0, 0, 0)$

$z = 2.(1/3) + 2.(1/3) + 3.0 = 4/3$

Solução: $(y_1, y_2) = (2/3, 2/3)$, $z_{MÍN} = 4/3$

12 Minimizar: $z = 4x_1 + 5x_2 + x_3$

sujeito a:
$$\begin{cases} x_1 + 2x_2 \geq 4 \\ x_1 + x_2 + x_3 \geq 3 \\ -2x_1 - 2x_2 + x_3 \leq -2 \times (-1) \rightarrow -2x_1 + 2x_2 - x_3 \geq 2 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

Maximizar: $w = 4y_1 + 3y_2 + 2y_3$

sujeito a:
$$\begin{cases} y_1 + y_2 + 2y_3 \leq 4 \\ 2y_1 + y_2 + 2y_3 \leq 5 \\ y_2 - y_3 \leq 1 \\ y_1, y_2, y_3 \geq 0 \end{cases}$$

$S \rightarrow S_1$
$$\begin{cases} y_1 + y_2 + 2y_3 + y_4 = 4 \\ 2y_1 + y_2 + 2y_3 + y_5 = 5 \\ y_2 - y_3 + y_6 = 1 \end{cases}$$

Variáveis de folga: $y_4, y_5, y_6 \geq 0$

	y_1	y_2	y_3	y_4 (VB)	y_5 (VB)	y_6 (VB)	b	Q (TD)	
	1	1	2	1	0	0	4	4	$E_{42}(-1/2)$
	(2)	1	2	0	1	0	5	5/2	$E_2(1/2)$
	0	1	-1	0	0	1	1		
w	-4	-3	-2	0	0	0	0		$E_{42}(2)$

$(y_1, y_2, y_3, y_4, y_5) = (0, 0, 0, 4, 5, 1)$

$w = 0$

1 1

	VB			VB		VB		T1	
	y_1	y_2	y_3	y_4	y_5	y_6	b	Q	
	0	1/2	1	4	-1/2	0	3/2	3	$E_{13}(-1/2)$
	1	1/2	1	0	1/2	0	5/2	5	$E_{23}(-1/2)$
	0	(1)	-1	0	0	1	1	1	
w	0	-1	2	0	2	0	10		$E_{43}(1)$

$(y_1, y_2, y_3, y_4, y_5, y_6) = (5/2, 0, 0, 3/2, 0, 1)$
$w = 4 \cdot (5/2) + 3 \cdot 0 + 2 \cdot 0 = 10$

	VB	VB		VB			T2
	y_1	y_2	y_3	y_4	y_5	y_6	b
	0	0	3/2	4	-1/2	-1/2	1
	1	0	3/2	0	1/2	-1/2	2
	0	1	-1	0	0	1	1
w	0	0	1	0	2	1	11

$(y_1, y_2, y_3, y_4, y_5, y_6) = (2, 1, 0, 1, 0, 0)$
$w = 4 \cdot 2 + 3 \cdot 1 + 0 = 11$

Solução: $(x_1, x_2, x_3) = (0, 2, 1)$ e $z_{MÍN} = 11$

13 Minimizar: $z = x_1 - 2x_2 - x_3$

sujeito a: S

$$x_1 + 2x_2 \leq 1$$
$$2x_1 + x_2 + x_3 \leq 2$$
$$x_1 + x_2 + 2x_3 \leq 3$$
$$x_1, x_2, x_3 \geq 0$$

Maximizar: $z_1 = -x_1 + 2x_2 + x_3$
sujeito a S

$$S \rightarrow S_1 \begin{cases} x_1 + 2x_2 + x_4 = 1 \\ 2x_1 + x_2 + x_3 + x_5 = 2 \\ x_1 + x_2 + 2x_3 + x_6 = 3 \end{cases}$$

Variáveis de folga: $x_4, x_5, x_6 \geq 0$

				VB x_4	VB x_5	VB x_6		TΦ	
	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	1	(2)	0	1	0	0	1	1/2	/ $E_1(1/2)$
	2	1	1	0	1	0	2	2	$E_{21}(-1/2)$
	1	1	2	0	0	1	3	3	$E_{31}(-1/2)$
z_1	1	-2	-1	0	0	0	0		$E_{A1}(1)$

/

$(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 0, 1, 2, 3)$

$z_1 = 0$

	VB x_1	VB x_2			VB x_5	VB x_6		T1	
	x_1	x_2	x_3	x_4	x_5	x_6	b	Q	
	1/2	1	0	1/2	0	0	1/2		
	3/2	0	1	-1/2	1	0	3/2	3/2	$E_{23}(-1/2)$
	1/2	0	(2)	-1/2	0	1	5/2	5/4	/ $E_3(1/2)$
z_1	2	0	-1	1	0	0	1		$E_{43}(1/2)$

/

$(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 1/2, 0, 0, 3/2, 5/2)$

$z_1 = -0 + 2(1/2) + 0 = 1$

	VB x_1	VB x_2			VB x_5		T2
	x_1	x_2	x_3	x_4	x_5	x_6	b
	1/2	1	0	1/2	0	0	1/2
	5/4	0	0	-1/4	1	-1/2	1/4
	1/4	0	1	-1/4	0	1/2	5/4
z_1	9/4	0	0	3/4	0	1/2	9/4

$(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 1/2, 5/4, 0, 1/4, 0)$

$z_1 = -0 + 2(1/2) + 5/4 = 9/4$

$z_{1MÁX} = z_1(0, 1/2, 5/4) = 9/4$

$z_{MÍN} = z(0, 1/2, 5/4) = -9/4$.

Solução: $(x_1, x_2, x_3) = (0, 1/2, 5/4)$ e $z_{MÍN} = -9/4$