# PLA

## Lista_2
## Sistemas Lineares

01/09/00

* Lista II - Sistemas Lineares

1 Resolva os seguintes sistemas de equações lineares:

$$S_1\begin{cases} x+4y+3z=1 \\ 2x+5y+4z=4 \\ x-3y-2z=5 \end{cases} \quad e$$

$$S_2\begin{cases} 2x_1+4x_2-x_3+2x_4=15 \\ x_1+2x_2-x_3+2x_4=11 \\ 3x_1-x_2+x_3+x_4=7 \\ x_1+x_2+x_3+x_4=5 \end{cases}$$

Utilizando os métodos de:

a) Cramer $S_1$

$$D=\begin{vmatrix} 1 & 4 & 3 \\ 2 & 5 & 4 \\ 1 & -3 & -2 \end{vmatrix} = -10+16-18-15+12+16=1$$

$$D_x=\begin{vmatrix} 1 & 4 & 3 \\ 4 & 5 & 4 \\ 5 & -3 & -2 \end{vmatrix} = -10+80-36-75+12+32=3$$

$$D_y=\begin{vmatrix} 1 & 1 & 3 \\ 2 & 4 & 4 \\ 1 & 5 & -2 \end{vmatrix} = -8+4+30-12-20+4=-2$$

/ /

$$D_z = \begin{vmatrix} 1 & 4 & 1 \\ 2 & 5 & 4 \\ 1 & -3 & 5 \end{vmatrix} = 25 + 16 - 6 - 5 - 40 + 12 = 2$$

$x = \dfrac{D_x}{D}$ ; $y = \dfrac{D_y}{D}$ ; $z = \dfrac{D_z}{D}$

$\boxed{S_1 = \{3, -2, 2\}}$

**$S_2$**

$$D = \begin{vmatrix} 2 & 4 & -1 & 2 \\ 1 & 2 & -1 & 2 \\ 3 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

$$D = (-1)^{1+1} \cdot 2 \cdot \begin{vmatrix} 2 & -1 & 2 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + (-1)^{1+2} \cdot 4 \cdot \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + (-1)^{1+3} \cdot (-1) \cdot \begin{vmatrix} 1 & 2 & 2 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + (-1)^{1+4} \cdot 2 \cdot \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$D = 2 \cdot [2 - 1 - 2 - (2 + 2 + 1)] - 4[1 - 1 + 6 - (2 + 1 - 3)] - [-1 + 2 +$
$+ 6 - (-2 + 1 + 6)] - 2[-1 + 2 - 3 - (1 + 1 + 6)]$

$D = 2(-6) - 4(6) - (2) - 2(-10) = -12 - 24 - 2 + 20$

$D = -18$

$$D_{x1} = \begin{vmatrix} 15 & 4 & -1 & 2 \\ 11 & 2 & -1 & 2 \\ 7 & -1 & 1 & 1 \\ 5 & 1 & 1 & 1 \end{vmatrix}$$

$$D_{x1} = (-1)^{1+1} \cdot 15 \cdot \begin{vmatrix} 2 & -1 & 2 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + (-1)^{1+2} \cdot 4 \cdot \begin{vmatrix} 11 & -1 & 2 \\ 7 & 1 & 1 \\ 5 & 1 & 1 \end{vmatrix} + (-1)^{1+3} \cdot (-1) \cdot \begin{vmatrix} 11 & 2 & 2 \\ 7 & -1 & 1 \\ 5 & 1 & 1 \end{vmatrix} + (-1)^{1+4} \cdot 2 \cdot \begin{vmatrix} 11 & 2 & -1 \\ 7 & -1 & 1 \\ 5 & 1 & 1 \end{vmatrix}$$

$$D_{x1} = 15[2-1-2-(2+2+1)] - 4[11-5+14-(10+11-7)] - [-11+10+14-(-10+11+14)] - 2[-11+10-7-(5+11+14)]$$

$$D_{x1} = 15(-6) - 4(6) - (-2) - 2 \cdot (-38)$$

$$D_{x1} = -90 - 24 + 2 + 76$$

$$D_{x1} = -36$$

$$D_{x2} = \begin{vmatrix} 2 & 15 & -1 & 2 \\ 1 & 11 & -1 & 2 \\ 3 & 7 & 1 & 1 \\ 1 & 5 & 1 & 1 \end{vmatrix}$$

$$D_{x2} = (-1)^{1+1} \cdot 2 \cdot \begin{vmatrix} 11 & -1 & 2 \\ 7 & 1 & 1 \\ 5 & 1 & 1 \end{vmatrix} + (-1)^{1+2} \cdot 15 \cdot \begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + (-1)^{1+3} \cdot (-1) \cdot \begin{vmatrix} 1 & 11 & 2 \\ 3 & 7 & 1 \\ 1 & 5 & 1 \end{vmatrix} + (-1)^{1+4} \cdot 2 \cdot \begin{vmatrix} 1 & 11 & -1 \\ 3 & 7 & 1 \\ 1 & 5 & 1 \end{vmatrix}$$

$$D_{x2} = 2 \cdot [11-5+14-(10+11-7)] - 15 \cdot [1-1+6-(2-3+1)] - [7+11+30-(14+33+5)] - 2[7+11-15-(-7+5+33)]$$

$$D_{x2} = 2 \cdot 6 - 15 \cdot 6 - (-4) - 2 \cdot (-28)$$

$$D_{x2} = 12 - 90 + 4 + 56$$

$$D_{x2} = -18$$

/ /

$$Dx_3 = \begin{vmatrix} 2 & 4 & 15 & 2 \\ 1 & 2 & 11 & 2 \\ 3 & -1 & 7 & 1 \\ 1 & 1 & 5 & 1 \end{vmatrix}$$

$$Dx_3 = (-1)^{1+1}.2.\begin{vmatrix} 2 & 11 & 2 \\ -1 & 7 & 1 \\ 1 & 5 & 1 \end{vmatrix} + (-1)^{1+2}.4.\begin{vmatrix} 1 & 11 & 2 \\ 3 & 7 & 1 \\ 1 & 5 & 1 \end{vmatrix} + (-1)^{1+3}.15.$$

$$.\begin{vmatrix} 1 & 2 & 2 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + (-1)^{1+4}.2.\begin{vmatrix} 1 & 2 & 11 \\ 3 & -1 & 7 \\ 1 & 1 & 5 \end{vmatrix}$$

$Dx_3 = 2.[14+11-10-(14+10-11)]-4[7+11+30-(14+5+33)]+$
$+15.[-1+2+6-(-2+1+6)]-2[-5+14+33-(-11+7+30)]$

$Dx_3 = 2.2-4.(-4)+15.2-2(16)$

$Dx_3 = 4+16+30-32$

$Dx_3 = 18$

$$Dx_4 = \begin{vmatrix} 2 & 4 & -1 & 15 \\ 1 & 2 & -1 & 11 \\ 3 & -1 & 1 & 7 \\ 1 & 1 & 1 & 5 \end{vmatrix}$$

$$Dx_4 = (-1)^{1+1}.2.\begin{vmatrix} 2 & -1 & 11 \\ -1 & 1 & 7 \\ 1 & 1 & 5 \end{vmatrix} + (-1)^{1+2}.4.\begin{vmatrix} 1 & -1 & 11 \\ 3 & 1 & 7 \\ 1 & 1 & 5 \end{vmatrix} + (-1)^{1+3}.(-1).$$

$$.\begin{vmatrix} 1 & 2 & 11 \\ 3 & -1 & 7 \\ 1 & 1 & 5 \end{vmatrix} + (-1)^{1+4}.15.\begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$D_{x4} = 2.[10-7-11-(11+14+5)]-4.[5-7+33-(-11+7-15)]-$
$-[-5+14+33-(-11+7+30)]-15.[-1+2-3-(1+6+1)]$

$D_{x4} = 2.(-38)-4.28-16-15.(-10)$

$D_{x4} = -76-112-16+150$

$D_{x4} = -54$

$x_1 = \frac{D_{x1}}{D}$

$x_1 = \frac{-36}{-18} = 2$

$x_2 = \frac{D_{x2}}{D}$

$x_2 = \frac{-18}{-18} = 1$

$x_3 = \frac{D_{x3}}{D}$

$x_3 = \frac{18}{-18} = -1$

$x_4 = \frac{D_{x4}}{D}$

$x_4 = \frac{-54}{-18} = 3$

$\boxed{S_2 = \{2, 1, -1, 3\}}$

**b)** Gauss

**$S_1$**

/ /

$$\left(\begin{array}{ccc|c} 1 & 4 & 3 & 1 \\ 2 & 5 & 4 & 4 \\ 1 & -3 & -2 & 5 \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 3 & 1 \\ 0 & -3 & -2 & 2 \\ 0 & -7 & -5 & 4 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(-7/3) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 3 & 1 \\ 0 & -3 & -2 & 2 \\ 0 & 0 & -1/3 & -2/3 \end{array}\right)$$

$$\begin{aligned} x + 4y + 3z &= 1 & &\rightarrow x = 3 \\ -3y - 2z &= 2 & &\rightarrow y = -2 \\ -\frac{1}{3}z &= -\frac{2}{3} & &\rightarrow z = 2 \end{aligned}$$

$\boxed{S_1 = \{3, -2, 2\}}$

**$S_2$**

$$\left(\begin{array}{cccc|c} 2 & 4 & -1 & 2 & 15 \\ 1 & 2 & -1 & 2 & 11 \\ 3 & -1 & 1 & 1 & 7 \\ 1 & 1 & 1 & 1 & 5 \end{array}\right) \begin{array}{l} E_{12} \\ \\ \\ \\ \end{array} \quad \left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 2 & 4 & -1 & 2 & 15 \\ 3 & -1 & 1 & 1 & 7 \\ 1 & 1 & 1 & 1 & 5 \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-3) \\ E_{41}(-1) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & -7 & 4 & -5 & -26 \\ 0 & -1 & 2 & -1 & -6 \end{array}\right) \begin{array}{l} \\ E_{23} \\ \\ \\ \end{array}$$

tilibra

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 0 & -7 & 4 & -5 & -26 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & -1 & 2 & -1 & -6 \end{array}\right) E_{42}(-1/7)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 0 & -7 & 4 & -5 & -26 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & 0 & 10/7 & -2/7 & -16/7 \end{array}\right) E_{43}(-10/7)$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 0 & -7 & 4 & -5 & -26 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & 0 & 0 & 18/7 & 54/7 \end{array}\right)$$

$$\frac{18}{7}x_4 = \frac{54}{7} \rightarrow x_4 = 3$$

$$x_3 - 2(3) = -7 \rightarrow x_3 = -1$$

$$-7x_2 + 4(-1) - 5.3 = -26 \rightarrow x_2 = 1$$

$$x_1 + 2.(1) - (-1) + 2(3) = 11$$

$$x_1 + 2 + 1 + 6 = 11 \rightarrow x_1 = 2$$

$$\boxed{S_2 = \{2, 1, -1, 3\}}$$

**c)** Gauss-Jordan

**$S_1$**

$$\left(\begin{array}{ccc|c} 1 & 4 & 3 & 1 \\ 2 & 5 & 4 & 4 \\ 1 & -3 & -2 & 5 \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-1) \end{array}$$

/ /

$$\left(\begin{array}{ccc|c} 1 & 4 & 3 & 1 \\ 0 & -3 & -2 & 2 \\ 0 & -7 & -5 & 4 \end{array}\right) \begin{array}{l} \\ E_3(-1/3) \\ \\ \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 3 & 1 \\ 0 & 1 & 2/3 & -2/3 \\ 0 & -7 & -5 & 4 \end{array}\right) \begin{array}{l} E_{12}(-4) \\ \\ E_{32}(7) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1/3 & 11/3 \\ 0 & 1 & 2/3 & -2/3 \\ 0 & 0 & -1/3 & -2/3 \end{array}\right) \begin{array}{l} \\ \\ E_3(-3) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1/3 & 11/3 \\ 0 & 1 & 2/3 & -2/3 \\ 0 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} E_{13}(-1/3) \\ E_{23}(-2/3) \\ \\ \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$\boxed{S_1 = \{3, -2, 2\}}$

**$S_2$**

$$\left(\begin{array}{cccc|c} 2 & 4 & -1 & 2 & 15 \\ 1 & 2 & -1 & 2 & 11 \\ 3 & -1 & 1 & 1 & 7 \\ 1 & 1 & 1 & 1 & 5 \end{array}\right) \begin{array}{l} E_{12} \\ \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 2 & 4 & -1 & 2 & 15 \\ 3 & -1 & 1 & 1 & 7 \\ 1 & 1 & 1 & 1 & 5 \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-3) \\ E_{41}(-1) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & -7 & 4 & -5 & -26 \\ 0 & -1 & 2 & -1 & -6 \end{array}\right) \begin{array}{l} \\ E_{23} \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 0 & -7 & 4 & -5 & -26 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & -1 & 2 & -1 & -6 \end{array}\right) \begin{array}{l} \\ E_{2}(-1/7) \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 2 & 11 \\ 0 & 1 & -4/7 & 5/7 & 26/7 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & -1 & 2 & -1 & -6 \end{array}\right) \begin{array}{l} E_{12}(-2) \\ \\ \\ E_{42}(1) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1/7 & 4/7 & 25/7 \\ 0 & 1 & -4/7 & 5/7 & 26/7 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & 0 & 10/7 & -2/7 & -16/7 \end{array}\right) \begin{array}{l} E_{13}(-1/7) \\ E_{23}(4/7) \\ \\ E_{43}(-10/7) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 6/7 & 32/7 \\ 0 & 1 & 0 & -3/7 & -2/7 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & 0 & 0 & 18/7 & 54/7 \end{array}\right) \begin{array}{l} \\ \\ \\ E_{4}(7/18) \end{array}$$

$$\left(\begin{array}{ccccc} 1 & 0 & 0 & 5/7 & 32/7 \\ 0 & 1 & 0 & -3/7 & -2/7 \\ 0 & 0 & 1 & -2 & -7 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right) \begin{array}{l} E_{14}(-5/7) \\ E_{24}(3/7) \\ E_{34}(2) \\ \ \end{array}$$

$$\left(\begin{array}{ccccc} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right)$$

$x_1 = 2$

$x_2 = 1$

$x_3 = -1$

$x_4 = 3$

$\boxed{S = \{2, 1, -1, 3\}}$

**2** Utilize o método de Gauss-Jordan para inverter as seguintes matrizes:

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 5 & 4 \\ 1 & -3 & -2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 2 & 5 & 4 & 0 & 1 & 0 \\ 1 & -3 & -2 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \ \\ E_{21}(-2) \\ E_{31}(-1) \end{array}$$

$\underbrace{\qquad\qquad}_{A} \qquad \underbrace{\qquad\qquad}_{I_3}$

$$\left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & -3 & -2 & -2 & 1 & 0 \\ 0 & -7 & -5 & -1 & 0 & 1 \end{array}\right) \begin{array}{l} \\ E_2(-1/3) \\ \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2/3 & 2/3 & -1/3 & 0 \\ 0 & -7 & -5 & -1 & 0 & 1 \end{array}\right) \begin{array}{l} E_{12}(-4) \\ \\ E_{32}(7) \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1/3 & -5/3 & 4/3 & 0 \\ 0 & 1 & 2/3 & 2/3 & -1/3 & 0 \\ 0 & 0 & -1/3 & 11/3 & -7/3 & 1 \end{array}\right) \begin{array}{l} \\ \\ E_3(-3) \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1/3 & -5/3 & 4/3 & 0 \\ 0 & 1 & 2/3 & 2/3 & -1/3 & 0 \\ 0 & 0 & 1 & -11 & 7 & -3 \end{array}\right) \begin{array}{l} E_{13}(-1/3) \\ E_{23}(-2/3) \\ \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 8 & -5 & 2 \\ 0 & 0 & 1 & -11 & 7 & -3 \end{array}\right)$$

$$\underbrace{\phantom{1\ 0\ 0}}_{I_3} \qquad \underbrace{\phantom{2\ -1\ 1}}_{A^{-1}}$$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Temos que a matriz inversa será

$$\boxed{A^{-1} = \begin{pmatrix} 2 & -1 & 1 \\ 8 & -5 & 2 \\ -11 & 7 & -3 \end{pmatrix}}$$

/ /

$$B = \begin{pmatrix} 2 & 4 & -1 & 2 \\ 1 & 2 & -1 & 2 \\ 3 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 2 & 4 & -1 & 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & -1 & 2 & 0 & 1 & 0 & 0 \\ 3 & -1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \quad E_1(1/2)$$

$$\underbrace{\phantom{xxxxxxx}}_{B} \quad \underbrace{\phantom{xxxxxxx}}_{I_3}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & -1/2 & 1 & 1/2 & 0 & 0 & 0 \\ 1 & 2 & -1 & 2 & 0 & 1 & 0 & 0 \\ 3 & -1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} \\ E_{21}(-1) \\ E_{31}(-3) \\ E_{41}(-1) \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & -1/2 & 1 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & -1/2 & 1 & -1/2 & 1 & 0 & 0 \\ 0 & -7 & 5/2 & -2 & -3/2 & 0 & 1 & 0 \\ 0 & -1 & 3/2 & 0 & -1/2 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} E_{14}(2) \\ E_{24}(-1) \\ E_{34}(-7) \\ \\ \end{array}$$

$$\left(\begin{array}{cccccccc} 1 & 0 & 5/2 & 1 & -1/2 & 0 & 0 & 2 \\ 0 & 1 & -2 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & -8 & -2 & 2 & 0 & 1 & -7 \\ 0 & -1 & 3/2 & 0 & -1/2 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} \\ \\ E_3(-1/8) \\ E_{42}(1) \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 5/2 & 1 & -1/2 & 0 & 0 & 2 \\ 0 & 1 & -2 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1/4 & -1/4 & 0 & -1/8 & 7/8 \\ 0 & 0 & -1/2 & 1 & -1/2 & 1 & 0 & 0 \end{array}\right) \quad \begin{array}{l} E_{13}(-5/2) \\ E_{23}(2) \\ \\ E_{43}(1/2) \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 3/8 & 1/8 & 0 & 5/16 & -3/16 \\ 0 & 1 & 0 & 3/2 & -1/2 & 1 & -1/4 & 3/4 \\ 0 & 0 & 1 & 1/4 & -1/4 & 0 & -1/8 & 7/8 \\ 0 & 0 & 0 & 9/8 & -5/8 & 1 & -1/16 & 7/16 \end{array}\right) E_4(8/9)$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 3/8 & 1/8 & 0 & 5/16 & -3/16 \\ 0 & 1 & 0 & 3/2 & -1/2 & 1 & -1/4 & 3/4 \\ 0 & 0 & 1 & 1/4 & -1/4 & 0 & -1/8 & 7/8 \\ 0 & 0 & 0 & 1 & -5/9 & 8/9 & -1/18 & 7/18 \end{array}\right) \begin{array}{l} E_{14}(-3/8) \\ E_{24}(-3/2) \\ E_{34}(-1/4) \\ \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/3 & -1/3 & 1/3 & -1/3 \\ 0 & 1 & 0 & 0 & 1/3 & -1/3 & -1/6 & 1/6 \\ 0 & 0 & 1 & 0 & -1/9 & -2/9 & -1/9 & 7/9 \\ 0 & 0 & 0 & 1 & -5/9 & 8/9 & -1/18 & 7/18 \end{array}\right)$$

$$\underbrace{\phantom{1\ 0\ 0\ 0}}_{I_3} \qquad \underbrace{\phantom{1/3\ -1/3\ 1/3\ -1/3}}_{B^{-1}}$$

**3** Calcule os determinantes das matrizes A e B do exercício 2, utilizando o método de Gauss.

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 5 & 4 \\ 1 & -3 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 4 & 3 \\ 2 & 5 & 4 \\ 1 & -3 & -2 \end{pmatrix} \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-1) \end{array}$$

$$\begin{pmatrix} 1 & 4 & 3 \\ 0 & -3 & -2 \\ 0 & -7 & -5 \end{pmatrix} \begin{array}{l} \\ \\ E_{32}(-7/3) \end{array}$$

$$\begin{pmatrix} 1 & 4 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & -1/3 \end{pmatrix}$$

Multiplicando-se a diagonal principal temos que: det A = 1

$$B = \begin{array}{cc} \begin{pmatrix} 2 & 4 & -1 & 2 \\ 1 & 2 & -1 & 2 \\ 3 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} & \begin{array}{l} E_{12} \\ \\ \\ \\ \end{array} \end{array}$$

$$\begin{array}{cc} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 2 & 4 & -1 & 2 \\ 3 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} & \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-3) \\ E_{41}(-1) \end{array} \end{array}$$

$$\begin{array}{cc} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & -7 & 4 & -5 \\ 0 & -1 & 2 & -1 \end{pmatrix} & \begin{array}{l} \\ E_{23} \\ \\ \\ \end{array} \end{array}$$

$$\begin{array}{cc} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & -7 & 4 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & -1 & 2 & -1 \end{pmatrix} & \begin{array}{l} \\ \\ \\ E_{42}(-1/7) \end{array} \end{array}$$

$$\begin{array}{cc} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & -7 & 4 & 5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 10/7 & -2/7 \end{pmatrix} & \begin{array}{l} \\ \\ \\ E_{43}(-10/7) \end{array} \end{array}$$

$$\begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & -7 & 4 & 5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 18/7 \end{pmatrix}$$

Multiplicando-se a diagonal principal e trocando-se o sinal 2 vezes, teremos que:
det B = -18

**4** Determine a solução geral dos seguintes sistemas (já triangularizados):

$$S_1: \begin{cases} 4x_1 + 3x_2 + 2x_3 = 1 \\ \quad 5x_2 + 6x_3 = 7 \end{cases}$$

$$5x_2 = 7 - 6x_3$$

$$\boxed{x_2 = \frac{7-6x_3}{5}}$$

$$4x_1 + 3.\left(\frac{7-6x_3}{5}\right) + 2x_3 = 1$$

$$4x_1 + \frac{21}{5} - \frac{18x_3}{5} = 1 - 2x_3$$

$$4x_1 = 1 - 2x_3 + \frac{18x_3}{5} - \frac{21}{5}$$

$$4x_1 = -\frac{16}{5} + \frac{8x_3}{5}$$

$$\boxed{x_1 = -\frac{4}{5} + \frac{2x_3}{5}}$$

$$S = \left(\frac{-4+2x_3}{5}, \frac{7-6x_3}{5}, x_3\right) \quad \forall x_3 \in \mathbb{R}$$

/ /

$S_2$: $\begin{cases} x_1 - 3x_2 + x_3 - 2x_4 = 4 \\ \quad x_2 - x_3 + 3x_4 = 2 \end{cases}$

$x_2 = 2 + x_3 - 3x_4$

$x_1 - 3(2 + x_3 - 3x_4) + x_3 - 2x_4 = 4$

$x_1 - 6 - 3x_3 + 9x_4 + x_3 - 2x_4 = 4$

$x_1 = 10 + 2x_3 - 7x_4$

$\boxed{S = (10 + 2x_3 - 7x_4,\ 2 + x_3 - 3x_4,\ x_3,\ x_4) \quad \forall x_3, x_4 \in \mathbb{R}}$

**5** Utilize o método de Gauss para resolver os seguintes sistemas:

$S_1$: $\begin{cases} 2x_1 + 4x_2 = 16 \\ 5x_1 - 2x_2 = 4 \\ 10x_1 - 4x_2 = 3 \end{cases}$

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 5 & -2 & 4 \\ 10 & -4 & 3 \end{array}\right) \begin{array}{l} \\ E_{21}(-5/2) \\ E_{31}(-5) \end{array}$$

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 0 & -12 & -36 \\ 0 & -24 & -77 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(-2) \end{array}$$

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 0 & -12 & -36 \\ 0 & 0 & -5 \end{array}\right)$$

Como $0x_1 + 0x_2 = -5$, temos que o sistema é impossível.

$$S_2: \begin{cases} 2x_1+4x_2=16 \\ 5x_1-2x_2=4 \\ 3x_1+x_2=9 \\ 4x_1-5x_2=-7 \end{cases}$$

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 5 & -2 & 4 \\ 3 & 1 & 9 \\ 4 & -5 & -7 \end{array}\right) \begin{array}{l} \\ E_{21}(-5/2) \\ E_{31}(-3/2) \\ E_{41}(-2) \end{array}$$

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 0 & -12 & -36 \\ 0 & -5 & -15 \\ 0 & -13 & -39 \end{array}\right) \begin{array}{l} \\ E_{2}(-1/12) \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 0 & 1 & 3 \\ 0 & -5 & -15 \\ 0 & -13 & -39 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(5) \\ E_{42}(13) \end{array}$$

$$\left(\begin{array}{cc|c} 2 & 4 & 16 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$2x_1+4x_2=16$

$x_1+2x_2=8$

$x_2=3$

$x_1+2.3=8 \rightarrow x_1=2$

$\boxed{S_2=(2,3)}$

/ /

$$S_3: \begin{cases} x+2y-2z+3v=2 \\ 2x+4y-3z+4v=5 \\ 5x+10y-8z+11v=12 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -2 & 3 & 2 \\ 2 & 4 & -3 & 4 & 5 \\ 5 & 10 & -8 & 11 & 12 \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-5) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -2 & 3 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 2 & 4 & 2 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(-2) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -2 & 3 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$x+2y-2z+3v=2$$
$$z-2v=1$$
$$z=1+2v$$

$$x+2y-2-4v+3v=2$$
$$x=4-2y+v$$

$$\boxed{S_3: (4-2y+v,\ y,\ 1+2v,\ v)\ \forall y, v \in \mathbb{R}}$$

$$S_4: \begin{cases} 2x+4y-z+2v+2t=1 \\ 3x+6y+z-v+4t=-7 \\ 4x+8y+z+5v-t=3 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 2 & 4 & -1 & 2 & 2 & 1 \\ 3 & 6 & 1 & -1 & 4 & -7 \\ 4 & 8 & 1 & 5 & -1 & 3 \end{array}\right) \begin{array}{l} \\ E_{21}(-3/2) \\ E_{31}(-2) \end{array}$$

$$\left(\begin{array}{ccccc|c} 2 & 4 & -1 & 2 & 2 & 1 \\ 0 & 0 & 5/2 & -4 & 1 & -17/2 \\ 0 & 0 & 3 & 1 & -5 & 1 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(-6/5) \end{array}$$

$$\left(\begin{array}{ccccc|c} 2 & 4 & -1 & 2 & 2 & 1 \\ 0 & 0 & 5/2 & -4 & 1 & -17/2 \\ 0 & 0 & 0 & 29/5 & -31/5 & 56/5 \end{array}\right)$$

$$2x + 4y - z + 2v + 2t = 1$$

$$5/2\, z - 4v + t = -17/2$$

$$29v - 31t = 56$$

$$29v = 56 + 31t$$

$$v = \frac{56 + 31t}{29}$$

$$\frac{5}{2}z - 4\left(\frac{56+31t}{29}\right) + t = \frac{-17}{2}$$

$$\frac{5}{2}z - \frac{224}{29} - \frac{124t}{29} + t = \frac{-17}{2}$$

$$\frac{5}{2}z = \frac{-17}{2} + \frac{224}{29} + \frac{95t}{29}$$

$$\frac{5}{2}z = \frac{-45}{58} + \frac{95t}{29}$$

$$z = \frac{-9 + 38t}{29}$$

/ /

$$2x+4y+\frac{9-38t}{29}+2\left(\frac{56+31t}{29}\right)+2t=1$$

$$2x=-4y-\frac{9}{29}+\frac{38t}{29}-\frac{112}{29}-\frac{62t}{29}-2t+1$$

$$2x=-\frac{92}{29}-4y-\frac{82t}{29}$$

$$x=-\frac{46}{29}-2y-\frac{41t}{29}$$

$$\boxed{S_4=\left(-{}^{46}/_{29}-2y-{}^{41t}/_{29},\ y,\ \frac{-9+38t}{29},\ \frac{56+31t}{29},\ t\right)\ \forall\, y,t\in\mathbb{R}}$$

$$S_5:\begin{cases} x+2z-t+u=-1\\ -y+t-u=3\\ z+2t+u=4\end{cases}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 1 & -1\\ 0 & -1 & 0 & 1 & -1 & 3\\ 0 & 0 & 1 & 2 & 1 & 4\end{array}\right)$$

$$x+2z-t+u=-1$$
$$-y+t-u=3$$
$$z+2t+u=4$$
$$z=4-2t-u$$

$$-y+t-u=3$$
$$y=-3+t-u$$

$$x+2(4-2t-u)-t+u=-1$$
$$x+8-4t-2u-t+u=-1$$
$$x=-9+5t+u$$

$\boxed{S_5: (-9+5t+u,\ -3+t-u,\ 4-2t-u,\ t,\ u)\ \forall t,u \in \mathbb{R}}$

**6** Utilize o Método de Gauss para resolver os seguintes sistemas homogêneos:

$$S_1: \begin{cases} 2x+4y=0 \\ 16x-8y=0 \\ 12x-2y=0 \end{cases}$$

$$\left(\begin{array}{cc|c} 2 & 4 & 0 \\ 16 & -8 & 0 \\ 12 & -2 & 0 \end{array}\right) \begin{array}{l} \\ E_{21}(-8) \\ E_{31}(-6) \end{array}$$

$$\left(\begin{array}{cc|c} 2 & 4 & 0 \\ 0 & -40 & 0 \\ 0 & -26 & 0 \end{array}\right)$$

$$2x+4y=0$$
$$y=0 \therefore x=0$$

$\boxed{S_1: (0,0)}$

$$S_2: \begin{cases} 3x+6y-9z=0 \\ 2x+4y-6z=0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 3 & 6 & -9 & 0 \\ 2 & 4 & -6 & 0 \end{array}\right) \begin{array}{l} E_1(1/3) \\ \\ \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & 4 & -6 & 0 \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \end{array}$$

/ /

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

grau de liberdade: 2

variáveis livres: y, z

$x = -2y + 3z$

$\boxed{\text{Solução geral: } (-2y+3z, y, z), \forall y, z \in \mathbb{R}}$

$$S_3: \begin{cases} x - 2y - 3z = 0 \\ x - 4y - 13z = 0 \\ 3x - 5y - 4z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 1 & -4 & -13 & 0 \\ 3 & -5 & -4 & 0 \end{array}\right) \begin{array}{l} \\ E_{21}(-1) \\ E_{31}(-3) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & -2 & -10 & 0 \\ 0 & 1 & 5 & 0 \end{array}\right) E_{23}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & -2 & -10 & 0 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(2) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

grau de liberdade: 1
variável livre: $z$

$$\begin{cases} x - 2y = 3z & (1) \\ y = -5z & (2) \end{cases}$$

(1) $x - 2(-5z) = 3z$

$x = -7z$

$\boxed{\text{Solução geral: } (-7z, -5z, z), \forall z \in \mathbb{R}}$

$$S_4: \begin{cases} x + y + z + t = 0 \\ -x - y + 2z + t = 0 \\ -x - y - 2z + t = 0 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ -1 & -1 & 2 & 1 & 0 \\ -1 & -1 & -2 & 1 & 0 \end{array}\right) \begin{array}{l} \\ E_{21}(1) \\ E_{31}(1) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 2 & 0 \\ 0 & 0 & -1 & 2 & 0 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(1/3) \end{array}$$

$$\left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 8/3 & 0 \end{array}\right)$$

grau de liberdade: 1
variável livre: $y$

/ /

$$\begin{cases} x + y + z + t = 0 & (1) \\ 3z + 2t = 0 & (2) \\ 8/3\, t = 0 & (3) \end{cases}$$

(3) $t = 0$

(2) $z = 0$

(1) $x = -y$

$$\boxed{\text{Solução geral: } (-y, y, 0, 0), \forall y \in \mathbb{R}}$$

**$S_5$** : $\begin{cases} x + y + z = 0 \\ x - y - z = 0 \\ x + y - z = 0 \end{cases}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right) \begin{array}{l} \\ E_{21}(-1) \\ E_{31}(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right)$$

$$\begin{cases} x + y + z = 0 & (1) \\ -2y - 2z = 0 & (2) \\ -2z = 0 & (3) \end{cases}$$

(3) $z = 0$

(2) $y = 0$

(1) $x = 0$

$$\boxed{\text{Solução: } (0,0,0)}$$

7 Determine $m$ ($m \in \mathbb{R}$) de modo que o sistema:

$$S\begin{cases} mx + 2y = 6 \\ 3x - y = -2 \\ x + y = 0 \end{cases}$$

seja compatível (determinado ou indeterminado) e resolva-o.

$$\left(\begin{array}{cc|c} m & 2 & 6 \\ 3 & -1 & -2 \\ 1 & 1 & 0 \end{array}\right) E_{13}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 3 & -1 & -2 \\ m & 2 & 6 \end{array}\right) \begin{array}{l} \\ E_{21}(-3) \\ E_{31}(-m) \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -4 & -2 \\ 0 & -m+2 & 6 \end{array}\right) \begin{array}{l} \\ \\ E_{32}\left(\frac{-m+2}{4}\right) \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -4 & -2 \\ 0 & 0 & \frac{m+10}{2} \end{array}\right)$$

Para que seja compatível devemos impor:

$$\frac{m+10}{2} = 0$$

$$m = -10$$

$$\begin{cases} x + y = 0 & (1) \\ -4y = -2 & (2) \end{cases}$$

/ /

(2) $y = 1/2$

(1) $x + 1/2 = 0$

$x = -1/2$

Solução: para $m = -10$, teremos sistema compatível e determinado e $(x, y) = (-1/2; 1/2)$.

8 Idem para o sistema:

$$S: \begin{cases} x - y = 1 \\ 2x + y = 2 \\ 3y = m \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 0 & 3 & m \end{array}\right) E_{21}(-2)$$

$$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 3 & m \end{array}\right) E_{32}(-1)$$

$$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & m \end{array}\right)$$

Para que o sistema seja compatível devemos impor $m = 0$

$$\begin{cases} x - y = 1 & (1) \\ y = 0 & (2) \end{cases}$$

② $y = 0$
① $x = 1$

Solução: para $m = 0$, teremos sistema compatível e determinado e $(x, y) = (1, 0)$

**9** Determine os valores de $a$ e $b$ ($a, b \in \mathbb{R}$) que tornam o sistema:

$$S: \begin{cases} 3x - 7y = a \\ x + y = b \\ 5x + 3y = 5a + 2b \\ x + 2y = a + b - 1 \end{cases}$$

compatível determinado.

$$\left(\begin{array}{cc|c} 3 & -7 & a \\ 1 & 1 & b \\ 5 & 3 & 5a+2b \\ 1 & 2 & a+b-1 \end{array}\right) \quad E_{12}$$

$$\left(\begin{array}{cc|c} 1 & 1 & b \\ 3 & -7 & a \\ 5 & 3 & 5a+2b \\ 1 & 2 & a+b-1 \end{array}\right) \begin{array}{l} \\ E_{21}(-3) \\ E_{31}(-5) \\ E_{41}(-1) \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & b \\ 0 & -10 & a-3b \\ 0 & -2 & 5a-3b \\ 0 & 1 & a-1 \end{array}\right) \begin{array}{l} \\ E_{24} \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & b \\ 0 & 1 & a-1 \\ 0 & -2 & 5a-3b \\ 0 & -10 & a-3b \end{array}\right) \begin{array}{l} \\ \\ E_{32}(2) \\ E_{42}(10) \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & b \\ 0 & 1 & a-1 \\ 0 & 0 & 7a-3b-2 \\ 0 & 0 & 11a-3b-10 \end{array}\right)$$

Para que o sistema seja compatível e determinado devemos impor:

$$\begin{cases} 7a-3b-2=0 \quad \times(-1) \\ 11a-3b-10=0 \end{cases}$$

$$\begin{cases} -7a+3b=-2 \\ 11a-3b=10 \end{cases}$$

$$4a = 8$$

$$a = 2$$

$$-7(2)+3b=2$$

$$b=4$$

Assim, teremos:

$$\left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 1 & 1 \end{array}\right)$$

$$\begin{cases} x+y=4 & (1) \\ y=1 & (2) \end{cases}$$

(2) $y=1$

(1) $x+1=4 \rightarrow x=3$

Solução: para $a = 2$ e $b = 4$ o sistema é compatível e determinado

**10** Determine o valor de $m$ ($m \in \mathbb{R}$) de modo que o sistema

$$S: \begin{cases} x + y + z = 0 \\ x - y + mz = 2 \\ mx + 2y + z = 1 \end{cases}$$

seja incompatível.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & m & 2 \\ m & 2 & 1 & 1 \end{array}\right) \begin{array}{l} \\ E_{21}(-1) \\ E_{31}(-m) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & m-1 & 2 \\ 0 & -m+2 & -m+1 & 1 \end{array}\right) \begin{array}{l} \\ \\ E_{32}\left(\frac{-m+2}{2}\right) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & m-1 & 2 \\ 0 & 0 & \frac{-m^2+m}{2} & -m+3 \end{array}\right)$$

Para que o sistema seja incompatível devemos impor

$\frac{-m^2+m}{2} = 0$ e $-m+3 \neq 0$

$m \neq 3$

$-m(m-1) = 0$

$m = 0 \lor$

$m = 1$

Solução: para $m = 0$ ou $m = 1$ o sistema será incompatível.

/ /

**11** Qual a relação que a, b e c devem satisfazer de modo que o sistema:

$$S: \begin{cases} x + 2y - 3z = a \\ 2x + 6y - 11z = b \\ x - 2y + 7z = c \end{cases}$$

tenha uma única solução?

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 6 & -11 & b \\ 1 & -2 & 7 & c \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 2 & -5 & -2a+b \\ 0 & -4 & 10 & -a+c \end{array}\right) \begin{array}{l} \\ \\ E_{32}(2) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 2 & -5 & -2a+b \\ 0 & 0 & 0 & -5a+2b+c \end{array}\right)$$

Como na última linha temos o coeficiente de z nulo, o sistema jamais admitirá uma única solução. Se $-5a+2b+c=0$, o sistema será indeterminado, senão se $-5a+2b+c \neq 0$, o sistema será incompatível.

**12** Determine o valor de K ($K \in \mathbb{R}$) de modo que o sistema:

$$S: \begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 - x_3 = 2 \\ x_1 + x_2 - x_3 = 3 \\ x_1 - x_2 + x_3 = K \end{cases}$$

seja compatível.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 2 \\ 1 & 1 & -1 & 3 \\ 1 & -1 & 1 & K \end{array}\right) \begin{array}{l} \\ E_{21}(-1) \\ E_{31}(-1) \\ E_{41}(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & -2 & 0 & K-1 \end{array}\right) \begin{array}{l} \\ \\ \\ E_{42}(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 2 & K-2 \end{array}\right) \begin{array}{l} \\ \\ \\ E_{43}(1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & K \end{array}\right)$$

Para que o sistema seja compatível devemos impor $K = 0$. Sendo assim, teremos:

$$\begin{cases} x_1 + x_2 + x_3 = 1 & (1) \\ \quad -2x_2 - 2x_3 = 1 & (2) \\ \qquad\quad -2x_3 = 2 & (3) \end{cases}$$

(3) $x_3 = -1$

(2) $x_2 = 1/2$

(1) $x_1 = 3/2$

/ /

**13** Determine o valor de K ($K \in \mathbb{R}$) de modo que o sistema:

$$S: \begin{cases} x - z = 0 \\ Kx + y + 3z = 0 \\ x + Ky + 3z = 1 \end{cases}$$

seja compatível determinado.

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ K & 1 & 3 & 0 \\ 1 & K & 3 & 1 \end{array}\right) \begin{array}{l} \\ E_{21}(-K) \\ E_{31}(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & K+3 & 0 \\ 0 & K & 4 & 1 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(-K) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & K+3 & 0 \\ 0 & 0 & -K^2-3K+4 & 1 \end{array}\right)$$

Para que o sistema seja compatível e determinado devemos impor

$-K^2 - 3K + 4 \neq 0 \therefore$

$K \neq -4$ e $K \neq 1$

**14** Estude o sistema:

$$S: \begin{cases} x - y + (K-1)z = K \\ 2x + Ky + (K+2)z = -2 \\ x + Ky + 2z = 4 \end{cases}$$

em função do parágrafo $K \in \mathbb{R}$.

$$\left(\begin{array}{ccc|c} 1 & -1 & K-1 & K \\ 2 & K & K+2 & -2 \\ 1 & K & 2 & 4 \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & K-1 & K \\ 0 & K+2 & -K+4 & -2K-2 \\ 0 & K+1 & -K+3 & -K+4 \end{array}\right) \begin{array}{l} \\ \\ E_{32}\left(-\frac{K+1}{K+2}\right) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & K-1 & K \\ 0 & K+2 & -K+4 & -2K-2 \\ 0 & 0 & \frac{-2K+2}{K+2} & K+5 \end{array}\right)$$

Teremos que:

$x \quad \frac{-2K+2}{K+2} = 0 \qquad (K \neq -2)$

$-2K+2 = 0$

$K = 1$

o sistema é incompatível, e se $K \neq 1$ o sistema é compatível e determinado

15 Determine $K (K \in \mathbb{R})$ de modo que a equação matricial:

$$\begin{pmatrix} 1 & 5 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = K\begin{pmatrix} x \\ y \end{pmatrix}$$

admita mais do que uma solução.

$$\begin{cases} x + 5y = Kx \rightarrow x - Kx + 5y = 0 \rightarrow (-K+1)x + 5y = 0 \\ 2x - y = Ky \rightarrow 2x - y - Ky = 0 \rightarrow \quad 2x - (K+1)y = 0 \end{cases}$$

$$\begin{cases} (-K+1)x + 5y = 0 \\ 2x - (K+1)y = 0 \end{cases}$$

$$\left(\begin{array}{cc|c} -K+1 & 5 & 0 \\ 2 & -K-1 & 0 \end{array}\right) E_{12}$$

$$\left(\begin{array}{cc|c} 2 & -K-1 & 0 \\ -K+1 & 5 & 0 \end{array}\right) E_{21}\left(\frac{K-1}{2}\right)$$

$$\left(\begin{array}{cc|c} 2 & -K-1 & 0 \\ 0 & -K^2+11 & 0 \end{array}\right)$$

Para que o sistema admita infinitas soluções, devemos impor que

$-K^2+11=0$

$K=\pm\sqrt{11}$

**16** Para que valores de p e q reais, o sistema:

$$S:\begin{cases} 3x+py+4z=0 \\ x+y+3z=-5 \\ 2x-3y+z=q \end{cases}$$

não admite solução.

$$\left(\begin{array}{ccc|c} 3 & p & 4 & 0 \\ 1 & 1 & 3 & -5 \\ 2 & -3 & 1 & q \end{array}\right) E_{12}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & -5 \\ 3 & p & 4 & 0 \\ 2 & -3 & 1 & q \end{array}\right) \begin{array}{l} \\ E_{21}(-3) \\ E_{31}(-2) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & -5 \\ 0 & p-3 & -5 & 15 \\ 0 & -5 & -5 & q+10 \end{array}\right) E_{23}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & -5 \\ 0 & -5 & -5 & q+10 \\ 0 & p-3 & -5 & 15 \end{array}\right) E_{32}\left(\frac{p-3}{5}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & -5 \\ 0 & -5 & -5 & q+10 \\ 0 & 0 & -p-2 & \frac{10p+pq-3q+45}{5} \end{array}\right)$$

Para que o sistema não admita soluções devemos impor que:

$-p-2=0$ e $\frac{10p+pq-3q+45}{5} \neq 0$

$p=-2$

$10.(-2)-2q-3q+45 \neq 0$

$-20-5q+45 \neq 0$

$q \neq 5$

Solução: para que o sistema seja incompatível devemos ter $p=-2$ e $q \neq 5$.

17 Estude o sistema:

$$S: \begin{cases} x+z+t=0 \\ x+Ky+K^2t=1 \\ x+(K+1)z+t=1 \\ x+z+Kt=2 \end{cases}$$

com relação ao parâmetro $K$ ($K \in \mathbb{R}$).

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 1 & K & 0 & K^2 & 1 \\ 1 & 0 & K+1 & 1 & 1 \\ 1 & 0 & 1 & K & 2 \end{array}\right) \begin{array}{l} \\ E_{21}(-1) \\ E_{31}(-1) \\ E_{41}(-1) \end{array}$$

/ /

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & K & -1 & K^2-1 & 1 \\ 0 & 0 & K & 0 & 1 \\ 0 & 0 & 0 & K-1 & 2 \end{array}\right)$$

Se $K-1=0$; $K=1$ ou $K=0$, o sistema é incompatível.
Se $K \neq 1$ e $K \neq 0$, o sistema é determinado

**18** Resolva a seguinte equação matricial:

$$\begin{pmatrix} 1 & 4 & 5 \\ 3 & -1 & 7 \\ 1 & -22 & -11 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x + 4y + 5z = 0 \\ 3x - y + 7z = 0 \\ x - 22y - 11z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 3 & -1 & 7 & 0 \\ 1 & -22 & -11 & 0 \end{array}\right) \begin{array}{l} \\ E_{21}(-3) \\ E_{31}(-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & -13 & -8 & 0 \\ 0 & -26 & -16 & 0 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(-2) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & -13 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

grau de liberdade: 1

variável livre: $z$

$$\begin{cases} x + 4y = -5z & (1) \\ -13y = 8z & (2) \end{cases}$$

(2) $y = -8/13\, z$

(1) $x + 4(-8/13\, z) = -5z$

$$x = \frac{32}{13}z - 5z = \frac{32z - 65z}{13}$$

$$x = -\frac{33}{13}z$$

Solução geral: $\left(-33/13\, z, -8/13\, z, z\right), \forall z \in \mathbb{R}$.

**19** Discuta e resolva os seguintes sistemas:

$$S_1: \begin{cases} x + 3y = 2 \\ 3x - 2y = 1 \qquad (a \in \mathbb{R}) \\ 2x - ay = -1 \end{cases}$$

$$\left(\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & -2 & 1 \\ 2 & -a & -1 \end{array}\right) \begin{array}{l} \\ E_{21}(-3) \\ E_{31}(-2) \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & -11 & -5 \\ 0 & -a-6 & -5 \end{array}\right) \begin{array}{l} \\ \\ E_{32}\left(\frac{-a-6}{11}\right) \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & -11 & -5 \\ 0 & 0 & \frac{5a-25}{11} \end{array}\right)$$

/ /

O sistema será incompatível se:

$$\frac{5a-25}{11} \neq 0$$

$a \neq 5$

Se $a = 5$, o sistema será determinado e então, teremos:

$$\begin{cases} x + 3y = 2 & (1) \\ -11y = -5 & (2) \end{cases}$$

(2) $y = 5/11$

(1) $x + 15/11 = 2$

$x = 7/11$

$$S_2: \begin{cases} x - y + z = 0 \\ 2x - 3y + 2z = 0 \\ 4x + 3y + wz = 0 \end{cases} \quad (w \in \mathbb{R})$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 4 & 3 & w & 0 \end{array}\right) \begin{array}{l} \\ E_{21}(-2) \\ E_{31}(-4) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 7 & w-4 & 0 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(7) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & w-4 & 0 \end{array}\right)$$

O sistema será indeterminado se:

$w - 4 = 0$

$w = 4$

Teremos então:

grau de liberdade: 1

variável livre: $z$

$$\begin{cases} x - y = -z & (1) \\ -y = 0 \rightarrow y = 0 & (2) \end{cases}$$

(2) $y = 0$

(1) $x = -z$

Solução geral: $(-z, 0, z), \forall z \in \mathbb{R}$

O sistema será determinado se:

$w \neq 4$.

Teremos então

$$\begin{cases} x - y + z = 0 & (1) \\ -y = 0 & (2) \\ (w-4)z = 0 & (3) \end{cases}$$

(3) $z = 0$

(2) $y = 0$

(1) $x = 0$

$\boxed{\text{Solução: } (0,0,0).}$

$$S_3: \begin{cases} x_1 - 2x_2 + 3x_3 = -4 \\ 5x_1 - 6x_2 + 7x_3 = -8 \\ 6x_1 + 8x_2 + ax_3 = b \end{cases} \quad (a, b \in \mathbb{R})$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 5 & -6 & 7 & -8 \\ 6 & 8 & a & b \end{array}\right) \begin{array}{l} \\ E_{21}(-5) \\ E_{31}(-6) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 4 & -8 & 12 \\ 0 & 20 & a-18 & b+24 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(-5) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -4 \\ 0 & 4 & -8 & 12 \\ 0 & 0 & a+22 & b-36 \end{array}\right)$$

se $a \neq -22$, o sistema será determinado, $\forall b \in \mathbb{R}$
se $a = -22$ e $b \neq 36$, o sistema será incompatível
se $a = -22$ e $b = 36$, o sistema será indeterminado e teremos:

grau de liberdade: 1
variável livre: $x_3$

$$\begin{cases} x_1 - 2x_2 = -4 - 3x_3 & (1) \\ 4x_2 = 12 + 8x_3 & (2) \end{cases}$$

(2) $x_2 = 3 + 2x_3$
(1) $x_1 - 2(3 + 2x_3) = -4 - 3x_3$
$x_1 = 6 - 4 - 3x_3 + 4x_3$
$x_1 = 2 + x_3$

$\boxed{\text{Solução geral: } (2 + x_3,\ 3 + 2x_3,\ x_3),\ \forall x_3 \in \mathbb{R}}$

$S_4: \begin{cases} 3x + py - 4z = -5 \\ x + y + 3z = 0 \\ 2x - 3y + z = q \end{cases} \quad (p, q \in \mathbb{R})$

$$\left(\begin{array}{ccc|c} 3 & p & -4 & -5 \\ 1 & 1 & 3 & 0 \\ 2 & -3 & 1 & q \end{array}\right) E_{12}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 3 & p & -4 & -5 \\ 2 & -3 & 1 & q \end{array}\right) \begin{array}{l} \\ E_{21}(-3) \\ E_{31}(-2) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & p-3 & -13 & -5 \\ 0 & -5 & -5 & q \end{array}\right) E_{23}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -5 & -5 & q \\ 0 & p-3 & -13 & -5 \end{array}\right) E_{32}\left(\frac{p-3}{5}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -5 & -5 & q \\ 0 & 0 & -p-10 & \frac{pq-3q-25}{5} \end{array}\right)$$

$-p-10 = 0 \rightarrow p = -10$

$\frac{pq - 3q - 25}{5} = 0 \rightarrow -10q - 3q - 25 = 0 \rightarrow q = \frac{-25}{13}$

se $p \neq -10$, o sistema será determinado, $\forall q \in \mathbb{R}$.

se $p = -10$ e $q \neq {-25}/{13}$, o sistema será incompatível

/ /

se $p = -10$ e $q = -25/13$, o sistema será indeterminado e então teremos:

grau de liberdade: 1
variável livre: $z$

$$\begin{cases} x + y = -3z & (1) \\ -5y = -25/13 + 5z & (2) \end{cases}$$

(2) $y = 5/13 - z$

(1) $x = -3z - \frac{5}{13} + z$

$x = -5/13 - 2z$

$\boxed{\text{Solução geral: } (-5/13 - 2z,\ 5/13 - z,\ z),\ \forall z \in \mathbb{R}}$

**20** Dados os sistemas:

$$S_1: \begin{cases} x + Ky = 1 \\ -x + (K+1)z = K+6 \\ x - (K^2+K-2)t = 1-K \\ x + (K+1)z - (K^2+K-2)t = 2 \end{cases}$$

$$S_2: \begin{cases} x + (K+1)y = K+3 \\ -x + Kz = 1 \\ x + (K^2-K)t = K+5 \\ -x + (K+1)y + Kz = K+2 \end{cases}$$

$$S_3: \begin{cases} (K-1)x_1 = K+2 \\ K(K-2)x_2 = K \\ (K^2-1)x_3 = K+1 \end{cases}$$

**S4**: $\begin{cases} x + Ky = -K \\ x + (K+1)z = K-1 \\ -x + (K+2)t = K-2 \\ -x - Ky - Kt = K \end{cases}$

**S5**: $\begin{cases} x + (K+1)y = 1 \\ -x + Kz = 2 \\ -x + (K^2-4)t = 10 \\ x - Kz - (K^2-4)t = -10 \end{cases}$

Pede-se:

a) Discutir os mesmos em função do parâmetro $K \in \mathbb{R}$;

b) Dar a solução geral nos casos indeterminados;

c) Encontre, se existir, o valor de K para o qual se tem um sistema determinado com:

$y = 4$ $(S_1)$

$z = 3$ $(S_2)$

$x_2 = 1$ $(S_3)$

$z = 1$ $(S_4)$

$y = 1$ $(S_5)$

**S1**

a) $\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & 1 \\ -1 & 0 & K+1 & 0 & K+6 \\ 1 & 0 & 0 & -(K^2+K-2) & 1-K \\ 1 & 0 & K+1 & -(K^2+K-2) & 2 \end{array}\right) \begin{array}{l} \\ E_{21}(1) \\ E_{31}(-1) \\ E_{41}(-1) \end{array}$

$\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & 1 \\ 0 & K & K+1 & 0 & K+7 \\ 0 & -K & 0 & -(K^2+K-2) & -K \\ 0 & -K & K+1 & -(K^2+K-2) & 1 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(1) \\ E_{42}(1) \end{array}$

$$\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & 1 \\ 0 & K & K+1 & 0 & K+7 \\ 0 & 0 & K+1 & -(K^2+K-2) & 7 \\ 0 & 0 & 2K+2 & -(K^2+K-2) & K+8 \end{array}\right) \quad E_{43}(-2)$$

$$\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & 1 \\ 0 & K & K+1 & 0 & K+7 \\ 0 & 0 & K+1 & -(K^2+K-2) & 7 \\ 0 & 0 & 0 & K^2+K-2 & K-6 \end{array}\right)$$

$K^2+K-2=0 \rightarrow K=-2 \vee K=1$

$K-6=0 \rightarrow K=6$

$K+1=0 \rightarrow K=-1$

$K=0$

para $K=-2$

$$\left(\begin{array}{cccc|c} 1 & -2 & 0 & 0 & 1 \\ 0 & -2 & -1 & 0 & 5 \\ 0 & 0 & -1 & 0 & 7 \\ 0 & 0 & 0 & 0 & -8 \end{array}\right)$$

Da ultima linha, temos que
$0x+0y+0z+0t=-8$ ∴ o sistema é incompatível para $K=-2$.
O mesmo ocorrerá para $K=1$.

para $K=0$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & -2 & -6 \end{array}\right) \quad E_{32}(-1)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 & -6 \end{array}\right) \quad E_{43}(1)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -6 \end{array}\right)$$

Da última linha, teremos que
$0x + 0y + 0z + 0t = -6$ ∴ o sistema é incompatível para $K=0$.

para $K=-1$

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & -2 & -7 \end{array}\right)$$

grau de liberdade: 1
variável livre: $z$

$$\begin{cases} x - y = 1 & (1) \\ -y = 6 & (2) \\ -2t = -7 & (3) \end{cases}$$

(3) $t = 7/2$
(2) $y = -6$
(1) $x - (-6) = 1 \rightarrow x = -5$

**b)** Solução geral: $(-5, -6, z, 7/2)$, $\forall z \in \mathbb{R}$

**c)** $y = 4$

/ /

$$\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & 1 \\ 0 & K & K+1 & 0 & K+7 \\ 0 & 0 & K+1 & -(K^2+K-2) & 7 \\ 0 & 0 & 0 & K^2+K-2 & K-6 \end{array}\right)$$

$(K^2+K-2)t = K-6$

$$t = \frac{K-6}{(K^2+K-2)}$$

$$(K+1)z - (K^2+K-2)\cdot\frac{(K-6)}{(K^2+K-2)} = 7$$

$(K+1)z = 7+K-6$

$$z = \frac{K+1}{K+1} = 1$$

$Ky + (K+1)z = K+7 \rightarrow y = 4$ e $z = 1$

$4K + (K+1)\cdot 1 = K+7$

$4K + \cancel{K} + 1 = \cancel{K}+7$

$$K = \frac{6}{4} = \frac{3}{2}$$

$x + Ky = 1$

$x + \frac{3}{2}\cdot 4 = 1$

$x = -5$

$$t = \frac{\frac{3}{2}-6}{\left(\frac{3}{2}\right)^2+\frac{3}{2}-2} = \frac{-\frac{9}{2}}{\frac{9}{4}+\frac{3}{2}-2} = \frac{-\frac{9}{2}}{\frac{9+6-8}{4}} = \frac{-\frac{9}{2}}{\frac{7}{4_2}} = -\frac{18}{7}$$

Solução:

Tilibra

Se $k = 0, 1, -2$ o sistema é incompatível.
Se $k \neq \pm 1$ e $k \neq -2$, o sistema é determinado. ($k \neq 0$)
Se $k = -1$, o sistema é indeterminado e possui solução geral $\left(-5, -6, z, \frac{7}{2}\right)$, $\forall z \in \mathbb{R}$.
Para $y = 4$, teremos $k = \frac{3}{2}$ e $x = -5$, $z = 1$ e $t = -\frac{18}{7}$

$S_2$

a)

$$\left(\begin{array}{cccc|c} 1 & (k+1) & 0 & 0 & k+3 \\ -1 & 0 & k & 0 & 1 \\ 1 & 0 & 0 & k^2-k & k+5 \\ -1 & (k+1) & k & 0 & k+2 \end{array}\right) \begin{array}{l} E_{12} \\ \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} -1 & 0 & k & 0 & 1 \\ 1 & (k+1) & 0 & 0 & k+3 \\ 1 & 0 & 0 & k^2-k & k+5 \\ -1 & (k+1) & k & 0 & k+2 \end{array}\right) \begin{array}{l} \\ E_{21}(1) \\ E_{31}(1) \\ E_{41}(-1) \end{array}$$

$$\left(\begin{array}{cccc|c} -1 & 0 & k & 0 & 1 \\ 0 & (k+1) & k & 0 & k+4 \\ 0 & 0 & k & k^2-k & k+6 \\ 0 & (k+1) & 0 & 0 & k+1 \end{array}\right) \begin{array}{l} \\ \\ \\ E_{42}(-1) \end{array}$$

$$\left(\begin{array}{cccc|c} -1 & 0 & k & 0 & 1 \\ 0 & k+1 & k & 0 & k+4 \\ 0 & 0 & k & k^2-k & k+6 \\ 0 & 0 & -k & 0 & -3 \end{array}\right) \begin{array}{l} \\ \\ \\ E_{43}(1) \end{array}$$

$$\left(\begin{array}{cccc|c} -1 & 0 & k & 0 & 1 \\ 0 & k+1 & k & 0 & k+4 \\ 0 & 0 & k & k^2-k & k+6 \\ 0 & 0 & 0 & k^2-k & k+3 \end{array}\right)$$

( / / )

$K^2 - K = 0 \rightarrow K(K-1) = 0 \rightarrow K=0 \vee K=1$

$K+3=0 \rightarrow K=-3$

$K=0$

$K+1=0 \rightarrow K=-1$

Para $K=0$

$$\left(\begin{array}{cccc|c} -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 3 \end{array}\right)$$

Da última linha teremos que:
$0x + 0y + 0z + 0t = 3$ ∴ o sistema é incompatível para $K=0$.
O mesmo ocorre para $K=1$.

Para $K=-1$

$$\left(\begin{array}{cccc|c} -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 2 & 5 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right) E_{32}(-1)$$

$$\left(\begin{array}{cccc|c} -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right) E_{43}(-1)$$

$$\left(\begin{array}{cccc|c} -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} -x - z = 1 & (1) \\ -z = 3 & (2) \\ 2t = 2 & (3) \end{cases}$$

(3) $t = 1$

(2) $z = -3$

(1) $-x - (-3) = 1 \rightarrow -x = -2 \rightarrow x = 2$

grau de liberdade: 1

variável livre: $y$

**b)** Solução geral $(2, y, -3, 1), \forall y \in \mathbb{R}$

**c)** $z = 3$

$$\left(\begin{array}{cccc|c} -1 & 0 & K & 0 & 1 \\ 0 & K+1 & K & 0 & K+4 \\ 0 & 0 & K & K^2-K & K+6 \\ 0 & 0 & 0 & K^2-K & K+3 \end{array}\right)$$

$(K^2 - K)t = K+3$

$t = \dfrac{K+3}{K^2-K}$

$Kz + (K^2-K)t = K+6$

$Kz + (K^2-K)\dfrac{K+3}{(K^2-K)} = K+6 \qquad z = 3$

$3K = 3$

$K = 1$

/ /

$(K+1).y + Kz = K+4$

$2.y + 3 = 5$

$y = 1$

$-x + K.z = 1$

$-x + 1.3 = 1$

$x = 2$

$t = \frac{K+3}{K^2-K}$

$t = \frac{1+3}{1^2-1} = \frac{4}{0} \notin \mathbb{R} \therefore \nexists K$ para $z = 3$

Solução:
Se $K = 0 \vee K = 1$, o sistema é incompatível
Se $K \neq 0$ e $K \neq \pm 1$, o sistema é determinado.
Se $K = -1$, o sistema é indeterminado e terá solução geral $(2, y, -3, 1)$ $\forall y \in \mathbb{R}$.
Para $z = 3$, $\nexists K$ que torne o sistema determinado.

**S$_3$**

**a)**

$$\left(\begin{array}{ccc|c} K-1 & 0 & 0 & K+2 \\ 0 & K(K-2) & 0 & K \\ 0 & 0 & (K^2-1) & K+1 \end{array}\right)$$

$K^2 - 1 = 0 \rightarrow K = \pm 1$

$K + 1 = 0 \rightarrow K = -1$

$K(K-2) = 0 \rightarrow K = 0 \vee K = 2$

$K - 1 = 0 \rightarrow K = 1$

tilibra

Para $K = -1$

$$\left(\begin{array}{ccc|c} -2 & 0 & 0 & 1 \\ 0 & 3 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} -2x_1 = 1 \rightarrow x_1 = -1/2 \\ 3x_2 = -1 \rightarrow x_2 = -1/3 \end{cases}$$

grau de liberdade: 1
variável livre: $x_3$

b) Solução geral: $(-1/2, -1/3, x_3)$, $\forall x_3 \in \mathbb{R}$.

Para $K = 1$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 3 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right)$$

Da última linha, temos que:
$0x_1 + 0x_2 + 0x_3 = 2$ ∴ o sistema é incompatível para $K = 1$.

Para $K = 0$

$$\left(\begin{array}{ccc|c} -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{array}\right)$$

$$\begin{cases} -x_1 = 2 \rightarrow x_1 = -2 \\ -x_3 = 1 \rightarrow x_3 = -1 \end{cases}$$

/ /

grau de liberdade: 1
variável livre: $x_2$

b) Solução geral: $(-2, x_2, -1)$; $\forall x_2 \in \mathbb{R}$

Para $K = 2$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 3 \end{array}\right)$$

Da segunda linha, teremos que
$0x_1 + 0x_2 + 0x_3 = 2$ $\therefore$ o sistema é incompatível para $K=2$

c) $x_2 = 1$

$$\left(\begin{array}{ccc|c} K-1 & 0 & 0 & K+2 \\ 0 & K(K-2) & 0 & K \\ 0 & 0 & (K^2-1) & K+1 \end{array}\right)$$

$$(K^2+1)x_3 = K+1$$
$$x_3 = \frac{K+1}{K^2+1}$$

$K(K-2)x_2 = K \qquad x_2 = 1$

$K(K-2) \cdot 1 = K$

$K^2 - 2K = K$

$K^2 - 3K = 0$

$K(K-3) = 0$

$K = 0 \lor K = 3$

Para $K=0$, conforme resolução anterior, teremos $x_1=-2$ e $x_3=-1$.

Para $K=3$:

$(K-1)x_1 = K+2$

$2x_1 = 5$

$x_1 = 5/2$

$(K^2-1)x_3 = K+1$

$8x_3 = 4$

$x_3 = 1/2$

Solução:

Se $K=1$ e $K=2$, o sistema é incompatível.

Se $K \neq -1, 0, 1, 2$, o sistema é determinado.

Se $K=-1$, o sistema é indeterminado e terá solução geral: $(-1/2, -1/3, x_3), \forall x_3 \in \mathbb{R}$

Se $K=0$, o sistema é indeterminado e terá solução geral: $(-2, x_2, -1), \forall x_2 \in \mathbb{R}$.

Para $x_2=1$, teremos $\begin{cases} K=0 \text{ e } x_1=-2 \text{ e } x_3=-1 \\ K=3 \text{ e } x_1=5/2 \text{ e } x_3=1/2 \end{cases}$

**S4**

**a)**

$$\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & -K \\ 1 & 0 & K+1 & 0 & K-1 \\ -1 & 0 & 0 & K+2 & K-2 \\ -1 & -K & 0 & -K & K \end{array}\right) \begin{array}{l} \\ E_{21}(-1) \\ E_{31}(1) \\ E_{41}(1) \end{array}$$

/ /

$$\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & -K \\ 0 & -K & K+1 & 0 & 2K-1 \\ 0 & K & 0 & K+2 & -2 \\ 0 & 0 & 0 & -K & 0 \end{array}\right) E_{32}(1)$$

$$\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & -K \\ 0 & -K & K+1 & 0 & 2K-1 \\ 0 & 0 & K+1 & K+2 & 2K-3 \\ 0 & 0 & 0 & -K & 0 \end{array}\right)$$

$K = 0$

$K+1 = 0 \rightarrow K = -1$

Para $K = 0$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x = 0 & (1) \\ z = -1 & (2) \\ z + 2t = -3 & (3) \end{cases}$$

(3) $-1 + 2t = -3$

$2t = -2$

$t = -1$

grau de liberdade: 1

variável livre: $y$

Tilibra

b) Solução geral: $(0, y, -1, -1), \forall y \in \mathbb{R}$

Para $K = -1$

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) E_{43}(-1)$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 5 \end{array}\right)$$

Da última linha, teremos que

$0x + 0y + 0z + 0t = 5 \therefore$ o sistema é incompatível para $K = -1$.

c) $z = 1$

$$\left(\begin{array}{cccc|c} 1 & K & 0 & 0 & -K \\ 0 & -K & K+1 & 0 & 2K-1 \\ 0 & 0 & K+1 & K+2 & 2K-3 \\ 0 & 0 & 0 & -K & 0 \end{array}\right)$$

$-Kt = 0 \rightarrow t = 0$

$(K+1)z + (K+2)t = 2K-3$ $\qquad z = 1$

$K+1+0 = 2K-3$

$K = 4$

/ /

$-Ky + (K+1)z = 2K-1$

$-4y + 5 = 7$

$y = -1/2$

$x + Ky = -K$

$x - 2 = -4$

$x = -2$

Solução:

Se $K=-1$, o sistema é incompatível.

Se $K\neq 0$ e $K\neq -1$, o sistema é determinado.

Se $K=0$, o sistema é indeterminado e terá solução geral: $(0, y, -1, -1)$, $\forall y \in \mathbb{R}$.

Para $z=1$, teremos $K=4$ e $x=-2$, $y=-1/2$ e $t=0$.

**$S_5$.**

**a)**

$$\begin{array}{c} \left(\begin{array}{cccc|c} 1 & K+1 & 0 & 0 & 1 \\ -1 & 0 & K & 0 & 2 \\ -1 & 0 & 0 & K^2-4 & 10 \\ 1 & 0 & K & -(K^2-4) & -10 \end{array}\right) \begin{array}{l} \\ E_{21}(1) \\ E_{31}(1) \\ E_{41}(-1) \end{array} \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & K+1 & 0 & 0 & 1 \\ 0 & K+1 & K & 0 & 3 \\ 0 & K+1 & 0 & K^2-4 & 11 \\ 0 & -(K+1) & K & -(K^2-4) & -11 \end{array}\right) \begin{array}{l} \\ \\ E_{32}(-1) \\ E_{42}(1) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & K+1 & 0 & 0 & 1 \\ 0 & K+1 & K & 0 & 3 \\ 0 & 0 & -K & K^2-4 & 8 \\ 0 & 0 & 2K & -(K^2-4) & -8 \end{array}\right) \begin{array}{l} \\ \\ \\ E_{43}(2) \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & K+1 & 0 & 0 & 1 \\ 0 & K+1 & K & 0 & 3 \\ 0 & 0 & -K & K^2-4 & 8 \\ 0 & 0 & 0 & K^2-4 & 8 \end{array}\right)$$

$K^2-4=0 \rightarrow K=\pm 2$

$K=0$

$K+1=0 \rightarrow K=-1$

Para $K=2$

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 1 \\ 0 & 3 & 2 & 0 & 3 \\ 0 & 0 & -2 & 0 & 8 \\ 0 & 0 & 0 & 0 & 8 \end{array}\right)$$

Da última linha, temos que:

$0x+0y+0z+0t=8$ ∴ o sistema é incompatível para $K=2$.

O mesmo ocorre para $K=-2$.

Para $K=0$

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & -4 & 8 \\ 0 & 0 & 0 & -4 & 8 \end{array}\right) E_{43}(-1) \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & -4 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x+y=1 \rightarrow x=1-3 \rightarrow x=-2 \\ y=3 \\ -4t=8 \rightarrow t=-2 \end{cases}$$

/ /

grau de liberdade: 1
variável livre: $z$

**b)** Solução geral: $(-2, 3, z, -2), \forall z \in \mathbb{R}$.

Para $K = -1$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & -1 & -3 & 8 \\ 0 & 0 & 0 & -3 & 8 \end{array}\right) E_{32}(+)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & -3 & 11 \\ 0 & 0 & 0 & -3 & 8 \end{array}\right) E_{43}(-1)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & -3 & 11 \\ 0 & 0 & 0 & 0 & -3 \end{array}\right)$$

Da última linha, temos que:
$0x + 0y + 0z + 0t = -3$ ∴ o sistema é incompatível para $K = -1$.

**c)** $y = 1$

$$\left(\begin{array}{cccc|c} 1 & K+1 & 0 & 0 & 1 \\ 0 & K+1 & K & 0 & 3 \\ 0 & 0 & -K & K^2-4 & 8 \\ 0 & 0 & 0 & K^2-4 & 8 \end{array}\right)$$

$(K^2-4)t = 8$

$t = \frac{8}{K^2-4}$ $\quad K \neq \pm 2$

$-Kz + (K^2-4)t = 8$

$-Kz + (K^2-4) \cdot \frac{8}{K^2-4} = 8$

$-Kz = 0 \rightarrow z = 0$

$(K+1)y + Kz = 3$ $\quad y = 1$ e $z = 0$

$K+1 = 3$

$K = 2$

$t = \frac{8}{0} \notin \mathbb{R}$ . $\nexists K$ para que o sistema seja determinado.

Solução:

Se $K = -2, -1, 2$, o sistema é incompatível.

Se $K \neq -2, -1, 0, 2$, o sistema é determinado.

Se $K = 0$, o sistema é indeterminado e terá solução geral $(-2, 3, z, -2)$, $\forall z \in \mathbb{R}$.

Para $y = 1$, $\nexists K$ que torne o sistema determinado.