PLA

Lista_1

Matrizes

18/08/00

Lista I - Matrizes

1 Dada a matriz $A = [a_{ij}]_{2\times3}$ determine o valor de $a_{22}.a_{13} - a_{12}.a_{21}$ sabendo que:

$$a_{ij} = \begin{cases} 3i+j & \text{se } i<j \\ 7 & \text{se } i=j \\ i^2+j & \text{se } i>j \end{cases}$$

Solução:

$$A = \begin{pmatrix} 7 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix}$$

$$a_{22}.a_{13} - a_{12}.a_{21} = 7.6 - 5.5 = 42 - 25 = \boxed{17}$$

$S = \{17\}$

2 Determine x e y reais sabendo que:

$$\begin{pmatrix} x^2 & y^3 \\ x^2 & y^2 \end{pmatrix} + \begin{pmatrix} 3x & -y \\ 4x & 2y \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 5 & -1 \end{pmatrix}$$

Solução:

$$\begin{cases} x^2 + 3x = 4 \\ x^2 + 4x = 5 \quad (-1) \end{cases}$$

$$\begin{cases} y^3 - y = 0 \quad (2) \\ y^2 + 2y = -1 \end{cases}$$

$y(y^2 - 1) = 0$; $y = 0$; $y = \pm 1$

$y^2 + 2y + 1 = 0$

$\Delta = 4 - 4 = 0$

$y = \frac{-2}{2} =$

$$\begin{cases} x^2 + 3x = 4 \\ -x^2 - 4x = -5 \end{cases}$$

$-x = -1$

$\boxed{x = 1}$

$x^2 + 3x = 4$

$x^2 + 3x - 4 = 0$

$\Delta = 9 + 16 = 25$

$x = \frac{-3 \pm 5}{2}$; 1 ; -4

$x^2 + 4x - 5 = 0$

$\Delta = 16 + 20$

$x = \frac{-4 \pm 6}{2}$; 1 ; -5

$$\begin{cases} 2y^3 \quad - \cancel{2y} = 0 \\ \quad y^2 + \cancel{2y} = -1 \end{cases}$$

$2y^3 + y^2 = -1$

$y^2(2y+1) = -1$

$y^2 = -1 \notin \mathbb{R}$

$2y+1 = -1$

$2y = -2$

$\boxed{y = -1}$

$S = \{x, y \in \mathbb{R} \mid x = 1 \wedge y = -1\}$

3 Determine a matriz $X \in M_2(\mathbb{R})$ sabendo que

$$\frac{X-A}{2} = \frac{B+X}{3} + C$$

São dados: $A = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$; $B = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$; $C = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$

Solução:

$$\frac{X-A}{2} = \frac{B+X}{3} + C$$

$$\frac{X-A}{2} = \frac{B+X+3C}{3}$$

$$3X - 3A = 2B + 2X + 6C$$

$$X = 3A + 2B + 6C$$

$$X = 3\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix} + 2\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} + 6\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 6 & 9 \\ 3 & -3 \end{pmatrix} + \begin{pmatrix} -2 & 4 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 24 & -6 \\ 12 & 6 \end{pmatrix}$$

$$\boxed{X = \begin{pmatrix} 28 & 7 \\ 17 & 3 \end{pmatrix}}$$

4 Dadas as matrizes, $A = (1,0,0)$, $B = (0,1,0)$ e $C = (0,0,1)$, determine as matrizes X, Y e $Z \in M_{1\times3}(\mathbb{R})$ tais que:

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$$\begin{cases} 2X - Y + Z = A \\ X - 2Y + Z = B \\ 3X + Y - Z = C \end{cases}$$

(Sugestão: utilize a regra de Cramer)

Solução:

$$D = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -2 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 4 - 3 + 1 + 6 - 1 - 2 = 5$$

$D \neq 0$: sistema compatível e determinado

$$D_x = \begin{vmatrix} A & -1 & 1 \\ B & -2 & 1 \\ C & 1 & -1 \end{vmatrix} = 2A - C + B + 2C - B - A = A + C$$

$$X = \frac{D_x}{D} = \frac{A+C}{5} = \frac{(1\ 0\ 0) + (0\ 0\ 1)}{5} = \frac{(1\ 0\ 1)}{5}$$

$$\boxed{X = (1/5\quad 0\quad 1/5)}$$

$$D_y = \begin{vmatrix} 2 & A & 1 \\ 1 & B & 1 \\ 3 & C & -1 \end{vmatrix} = -2B + 3A + C - 3B + A - 2C = 4A - 5B - C$$

$$Y = \frac{D_y}{D} = \frac{4A - 5B - C}{5} = \frac{4(1\ 0\ 0) - 5(0\ 1\ 0) - (0\ 0\ 1)}{5}$$

$$Y = \frac{(4\ 0\ 0) - (0\ 5\ 0) - (0\ 0\ 1)}{5} = \frac{(4\ \ -5\ \ -1)}{5}$$

$$\boxed{Y = (4/5\quad -1\quad -1/5)}$$

$$D_z = \begin{vmatrix} 2 & -1 & A \\ 1 & -2 & B \\ 3 & 1 & C \end{vmatrix} = -4C - 3B + A + 6A + C - 2B = 7A - 5B - 3C$$

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$$Z = \frac{D_z}{D} = \frac{7A-5B-3C}{5} = \frac{7(1\ 0\ 0)-5(0\ 1\ 0)-3(0\ 0\ 1)}{5}$$

$$Z = \frac{(7\ 0\ 0)-(0\ 5\ 0)-(0\ 0\ 3)}{5} = \frac{(7\ \ -5\ \ -3)}{5}$$

$$\boxed{Z = (7/5\ \ -1\ \ -3/5)}$$

5 Para cada $\alpha \in \mathbb{R}$ consideremos a matriz de rotação:

$$T_\alpha = \begin{pmatrix} \cos\alpha & -\operatorname{sen}\alpha \\ \operatorname{sen}\alpha & \cos\alpha \end{pmatrix}$$

Mostre que:

a) $T_\alpha . T_\beta = T_{\alpha+\beta}$

Solução:

$$T_\alpha . T_\beta = \begin{pmatrix} \cos\alpha & -\operatorname{sen}\alpha \\ \operatorname{sen}\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\beta & -\operatorname{sen}\beta \\ \operatorname{sen}\beta & \cos\beta \end{pmatrix}$$

$$T_\alpha . T_\beta = \begin{pmatrix} \cos\alpha.\cos\beta - \operatorname{sen}\alpha.\operatorname{sen}\beta & \cos\alpha.(-\operatorname{sen}\beta) - \operatorname{sen}\alpha.\cos\beta \\ \operatorname{sen}\alpha.\cos\beta + \cos\alpha.\operatorname{sen}\beta & \operatorname{sen}\alpha(-\operatorname{sen}\beta) + \cos\alpha.\cos\beta \end{pmatrix}$$

$$T_\alpha . T_\beta = \begin{pmatrix} \cos\alpha.\cos\beta - \operatorname{sen}\alpha.\operatorname{sen}\beta & -(\operatorname{sen}\alpha.\cos\beta + \operatorname{sen}\beta.\cos\alpha) \\ \operatorname{sen}\alpha.\cos\beta + \operatorname{sen}\beta.\cos\alpha & \cos\alpha.\cos\beta - \operatorname{sen}\alpha.\operatorname{sen}\beta \end{pmatrix}$$

Pela Trigonometria Temos que:

$\cos(a+b) = \cos a.\cos b - \operatorname{sen} a.\operatorname{sen} b$

$\cos(a-b) = \cos a.\cos b + \operatorname{sen} a.\operatorname{sen} b$

$\operatorname{sen}(a+b) = \operatorname{sen} a.\cos b + \operatorname{sen} b.\cos a$

$\operatorname{sen}(a-b) = \operatorname{sen} a.\cos b - \operatorname{sen} b.\cos a$

$$\boxed{T_\alpha . T_\beta = \begin{pmatrix} \cos(\alpha+\beta) & -\operatorname{sen}(\alpha+\beta) \\ \operatorname{sen}(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix}}$$

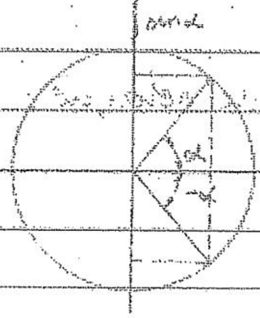
$$\boxed{T_{\alpha+\beta} = \begin{pmatrix} \cos(\alpha+\beta) & -\operatorname{sen}(\alpha+\beta) \\ \operatorname{sen}(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix}}$$

$\therefore T_\alpha . T_\beta = T_{\alpha+\beta}$ c.q.d.

b) $T_{-\alpha} = T_{\alpha}^{t}$

$$T_{-\alpha} = \begin{pmatrix} \cos(-\alpha) & -\operatorname{sen}(-\alpha) \\ \operatorname{sen}(-\alpha) & \cos(-\alpha) \end{pmatrix}$$

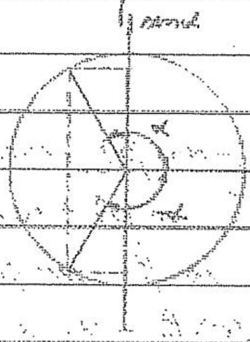
α no 1º quadrante



$\operatorname{sen}(-\alpha) = -\operatorname{sen}\alpha$

$\cos(-\alpha) = \cos\alpha$

α no 2º quadrante



$\operatorname{sen}(-\alpha) = -\operatorname{sen}\alpha$

$\cos(-\alpha) = \cos\alpha$

Independentemente do valor de α, teremos

$$\boxed{T_{-\alpha} = \begin{pmatrix} \cos\alpha & \operatorname{sen}\alpha \\ -\operatorname{sen}\alpha & \cos\alpha \end{pmatrix}}$$

$$T_{\alpha} = \begin{pmatrix} \cos\alpha & -\operatorname{sen}\alpha \\ \operatorname{sen}\alpha & \cos\alpha \end{pmatrix}$$

Para obtermos a matriz transposta, basta posicionarmos as linhas como colunas. Então, teremos:

$$\boxed{T_{\alpha}^{t} = \begin{pmatrix} \cos\alpha & \operatorname{sen}\alpha \\ -\operatorname{sen}\alpha & \cos\alpha \end{pmatrix}}$$

$\therefore T_{-\alpha} = T_{\alpha}^{t}$ c.q.d.

6 Determine todas as matrizes $A \in M_2(\mathbb{R})$, não nulas, tais que $A^2 = 0$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{cases} a^2 + bc = 0 \\ ab + bd = 0 \\ ca + dc = 0 \\ cb + d^2 = 0 \end{cases}$$

$$\begin{cases} a^2 + bc = 0 & \rightarrow c = -a^2/b \\ b(a+d) = 0 & \rightarrow b = 0 \vee a = -d \\ c(a+d) = 0 & \rightarrow c = 0 \vee a = -d \\ bc + d^2 = 0 & \rightarrow bc + a^2 \rightarrow c = -a^2/b \end{cases}$$

$S_1 = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$ $\quad b \neq 0$

$S_2 = \begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix}$ $\quad c \neq 0$

$S_3 = \begin{pmatrix} a & b \\ -a^2/b & -a \end{pmatrix}$ $\quad b \neq 0$

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7 Uma matriz $A \in M_n(\mathbb{R})$ inversível, se diz ortogonal, se e somente se, $A \cdot A^t = A^t \cdot A = I_n$

a) Verifique se a matriz $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix}$ é ortogonal

$$A^t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}$$

$$A \cdot A^t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \operatorname{sen}\alpha \\ 0 & -\operatorname{sen}\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\operatorname{sen}\alpha \\ 0 & \operatorname{sen}\alpha & \cos\alpha \end{pmatrix}$$

$$A \cdot A^t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos^2\alpha + \operatorname{sen}^2\alpha & 0 \\ 0 & 0 & \operatorname{sen}^2\alpha + \cos^2\alpha \end{pmatrix}$$

Substituindo o $\operatorname{sen}^2\alpha + \cos^2\alpha$ por 1, teremos

$$A \cdot A^t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3$$

$$A^t \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\operatorname{sen}\alpha \\ 0 & \operatorname{sen}\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \operatorname{sen}\alpha \\ 0 & -\operatorname{sen}\alpha & \cos\alpha \end{pmatrix}$$

$$A^t \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos^2\alpha + \operatorname{sen}^2\alpha & 0 \\ 0 & 0 & \operatorname{sen}^2\alpha + \cos^2\alpha \end{pmatrix}$$

$$A^t \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I_3$$

$\therefore$ $A \cdot A^t = A^t \cdot A = I_3$, então a matriz é ortogonal.

b) Determine, se possível, x e y reais tais que a matriz $\begin{bmatrix} \sqrt{2} & x \\ y & \sqrt{2} \end{bmatrix}$ seja ortogonal.

$$A = \begin{pmatrix} \sqrt{2} & x \\ y & \sqrt{2} \end{pmatrix}$$

$$A^t = \begin{pmatrix} \sqrt{2} & y \\ x & \sqrt{2} \end{pmatrix}$$

Para que a matriz seja ortogonal devemos ter:

$A.A^t = A^t.A = I_2$

$$\begin{pmatrix} \sqrt{2} & x \\ y & \sqrt{2} \end{pmatrix}\begin{pmatrix} \sqrt{2} & y \\ x & \sqrt{2} \end{pmatrix} = \begin{pmatrix} \sqrt{2} & y \\ x & \sqrt{2} \end{pmatrix}\begin{pmatrix} \sqrt{2} & x \\ y & \sqrt{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} 2 + x^2 = 1 \\ \sqrt{2}y + \sqrt{2}x = 0 \\ \sqrt{2}y + \sqrt{2}x = 0 \\ y^2 + 2 = 1 \end{cases} \qquad \begin{cases} 2 + y^2 = 1 \\ \sqrt{2}x + \sqrt{2}y = 0 \\ \sqrt{2}x + \sqrt{2}y = 0 \\ x^2 + 2 = 1 \end{cases}$$

$$\begin{cases} x^2 = -1 \notin \mathbb{R} \\ \sqrt{2}(x+y) = 0 \\ y^2 = -1 \notin \mathbb{R} \end{cases}$$

$\sqrt{2}(x+y) = 0$

$x + y = 0$

$\therefore y = -x$

S: É impossível a determinação dos valores de x e de y.

8 Seja $B \in M_2(\mathbb{R})$ uma matriz que comuta com a matriz

$A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$.

Mostre que existem $a, b \in \mathbb{R}$ tais que $B = a.A + b.I_2$

$A.B = B.A$

$$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & y \\ z & t \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$x + 2z = x \qquad 3z = z$

$y + 2t = 2x + 3y \qquad 3t = 2z + 3t \rightarrow z = 0$

$$B = \begin{pmatrix} x & t-x \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} x & t-x \\ 0 & t \end{pmatrix} = a \cdot \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} + b \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x & t-x \\ 0 & t \end{pmatrix} = \begin{pmatrix} a & 2a \\ 0 & 3a \end{pmatrix} + \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix}$$

$$\begin{pmatrix} x & t-x \\ 0 & t \end{pmatrix} = \begin{pmatrix} a+b & 2a \\ 0 & 3a+b \end{pmatrix}$$

$$\begin{cases} x = a+b \\ t-x = 2a \rightarrow a = \frac{t-x}{2} \\ t = 3a+b \rightarrow t = \frac{3t-3x}{2} + b \rightarrow 2t = 3t - 3x + 2b \rightarrow b = \frac{3x-t}{2} \end{cases}$$

$$S = \left\{ \exists\, a, b \in \mathbb{R} \mid a = \frac{t-x}{2} \wedge b = \frac{3x-t}{2} \right\}$$

9 Dada a matriz $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, determine A^n. (ob. $A^n = \underbrace{A.A...A}_{n \text{ vezes}}$)

Se n for par, $A^n = I_2$. Se n for ímpar, $A^n = A$.

10 Dada a matriz $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ utilize o princípio de indução finita para determinar A^n.

para $n = 1$

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

para $n = 2$

$$A^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

para $n = 3$

$$A^3 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

Teremos então:

$$A^n = \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\cdots\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}}_{n \text{ vezes}} = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \text{ para } n \geq 1.$$

1º) caso inicial $n=1$

$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, logo a propriedade vale para $n=1$.

2º) admitindo que a propriedade vale para $n=p$, isto é,

$$A^p = \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\cdots\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}}_{p \text{ vezes}} = \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}$$

provemos que a propriedade vale para $n=p+1$, ou seja,

$$A^{p+1} = \overbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\cdots\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}}^{A^p}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & p+1 \\ 0 & 1 \end{pmatrix}$$

$$A^{p+1} = \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1+p \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & p+1 \\ 0 & 1 \end{pmatrix}$$

e assim, a propriedade é válida para todo $n \in \mathbb{N}^*$

Portanto, $A^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$

11 Se a matriz $A = \begin{pmatrix} 2 & 1 & -1 \\ x^2 & 0 & 1-y \\ x & y-3 & 1 \end{pmatrix}$ é simétrica, determine $x+y$.

$\left.\begin{matrix} x^2 = 1 \\ x = -1 \end{matrix}\right\} \Rightarrow x = -1$

$y-3 = 1-y \rightarrow 2y = 4 \rightarrow y = 2 \therefore x+y = 2-1 = 1$

12 Determine a matriz $A \in M_2(\mathbb{R})$ sabendo que $A \cdot A^t = 0$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$a^2 + b^2 = 0 \rightarrow a = b = 0$

$ac + bd = 0$

$c^2 + d^2 = 0 \rightarrow c = d = 0$

$\therefore A = 0$

13 Verifique se são verdadeiras ou falsas as afirmações abaixo:

a) Se A e B são matrizes do mesmo tipo, então existem AB e BA;

$$\underset{A}{\begin{pmatrix} 3 & 2 \\ 1 & -4 \end{pmatrix}}\underset{B}{\begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}} = \begin{pmatrix} 2 & 11 \\ -4 & -5 \end{pmatrix}$$

$$\underset{B}{\begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}}\underset{A}{\begin{pmatrix} 3 & 2 \\ 1 & -4 \end{pmatrix}} = \begin{pmatrix} 3 & -12 \\ 5 & -6 \end{pmatrix}$$

R: Falsa

b) Podemos ter $AB = 0$ com $A \neq 0$ e $B \neq 0$;

$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$; $B = \begin{pmatrix} 4 & 2 \\ -2 & -1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 4 & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

R: Verdadeira

c) Se $A \neq 0$ podemos ter $AB = AC$ sem que B e C sejam iguais;

$$A=\begin{pmatrix}1&0\\5&0\end{pmatrix}\quad B=\begin{pmatrix}1&2\\3&4\end{pmatrix}\quad C=\begin{pmatrix}1&2\\0&8\end{pmatrix}$$

$$AB=\begin{pmatrix}1&0\\5&0\end{pmatrix}\begin{pmatrix}1&2\\3&4\end{pmatrix}=\begin{pmatrix}1&2\\5&10\end{pmatrix}$$

$$AC=\begin{pmatrix}1&0\\5&0\end{pmatrix}\begin{pmatrix}1&2\\0&8\end{pmatrix}=\begin{pmatrix}1&2\\5&10\end{pmatrix}$$

$\therefore AB=AC$

R: Verdadeira

d) Se $A^2=0$ então $A=0$

$$A=\begin{pmatrix}0&b\\0&0\end{pmatrix}$$

$$A^2=\begin{pmatrix}0&b\\0&0\end{pmatrix}\begin{pmatrix}0&b\\0&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

R: Falsa

14 Sendo $A=\begin{pmatrix}\sqrt{2}&-1&1\\\sqrt{2}&1&-1\\0&\sqrt{2}&\sqrt{2}\end{pmatrix}$, verifique se a matriz

$X=\begin{pmatrix}\sqrt{2}&\sqrt{2}&0\\-1&1&\sqrt{2}\\1&-1&\sqrt{2}\end{pmatrix}$ é uma solução da equação:

$(A+X)^2=A^2+X^2+2AX^{(*)}$ ①

(Sugestão: desenvolva (*) antes de iniciar verificação.)

$(A+X)^2=(A+X)(A+X)=(A+X).A+(A+X).X$

$=A+XA+AX+X^2$ ②

① = ②

$\cancel{A^2}+\cancel{X^2}+2AX=\cancel{A}+XA+\cancel{AX}+\cancel{X^2}$

$AX=XA$

Assim, X é solução da equação dada, desde que a-

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já satisfeita a propriedade comutativa

$$\underbrace{\begin{pmatrix} \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \\ 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \end{pmatrix}}_{X} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \end{pmatrix}}_{X} \underbrace{\begin{pmatrix} \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \\ 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}}_{A} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$AX = XA \therefore X$ é solução da equação $(A+X)^2$

15 Determine as matrizes $A = (a_{ij}) \in M_{2\times2}(\mathbb{R})$ com $a_{ij} = \begin{cases} x & \text{se } i = j \\ y & \text{se } i \neq j \end{cases}$ tais que $A^{-1} = A$.

$$A = \begin{pmatrix} x & y \\ y & x \end{pmatrix}$$

$A \cdot A^{-1} = I_2$

$$\begin{pmatrix} x & y \\ y & x \end{pmatrix}\begin{pmatrix} x & y \\ y & x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} x^2 + y^2 = 1 \\ xy + x^2 = 0 \rightarrow x(y+x) = 0 \rightarrow x = 0 \vee y = -x \\ xy + xy = 0 \rightarrow 2xy = 0 \rightarrow x = 0 \vee y = 0 \end{cases}$$

$x = 0 \therefore$

$y^2 = 1 \rightarrow y = \pm 1$

p/ $x=0$

$A=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \vee A=\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

p/ $y=0$

$A=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \vee A=\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

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16 Seja a matriz $A=(a_{ij}) \in M_{2\times3}(\mathbb{R})$. Determine os elementos de A sabendo que:

$a_{ij} = \begin{cases} (-i)^j & \text{se } i=j \\ 3^{|i-j|} & \text{se } i \neq j \end{cases}$

A seguir determine a matriz $B=(b_{ij})$ sabendo que $b_{ij} = \log \frac{i}{j}$ se $i=j$ e que $AB=I$.

Finalmente verifique se as matrizes A e B são comutáveis com relação à operação de multiplicação.

$A=\begin{pmatrix} -1 & 3 & 9 \\ 3 & 4 & 3 \end{pmatrix}$

$B=\begin{pmatrix} \log 1 & a \\ b & \log 1 \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & a \\ b & 0 \\ c & d \end{pmatrix}$

$AB = I_2$

$\begin{pmatrix} -1 & 3 & 9 \\ 3 & 4 & 3 \end{pmatrix} \begin{pmatrix} 0 & a \\ b & 0 \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 3b+9c & -a+9d \\ 4b+3c & 3a+3d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$3b+9c=1$

$-a+9d=0$

$4b+3c=0$
$3a+3d=1$

$$\begin{cases} 3b+9c=1 & :(-3) \\ 4b+3c=0 \end{cases}$$

$$\begin{cases} -b-3c=-1/3 \\ 4b+3c=0 \end{cases} \quad 3b=-\frac{1}{3} \rightarrow b=-\frac{1}{9} \text{ e } c=\frac{4}{27}$$

$$\begin{cases} -a+9d=0 \\ 3a+3d=1 & :(3) \end{cases}$$

$$\begin{cases} -a+9d=0 \\ a+d=1/3 \end{cases} \rightarrow 10d=\frac{1}{3} \rightarrow d=\frac{1}{30} \text{ e } a=\frac{9}{30}=\frac{3}{10}$$

$$B=\begin{pmatrix} 0 & 3/10 \\ -1/9 & 0 \\ 4/27 & 1/30 \end{pmatrix}$$

A e B não são comutativas, pois AB gera uma matriz quadrada de ordem 2 e BA, uma matriz quadrada de ordem 3.

17 Determine a família de matrizes que comuta com a matriz $A=\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$.

$AB=BA$

$$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} a+2c & b+2d \\ 3c & 3d \end{pmatrix} = \begin{pmatrix} a & 2a+3b \\ c & 2c+3d \end{pmatrix}$$

$a+2c=a$
$b+2d=2a+3b$
$3c=c$

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$3d = 2c + 3d$

$$\begin{cases} c = 0 \\ -2b + 2d = 2a \rightarrow -b + d = a \rightarrow b = d - a \end{cases}$$

$B = \begin{pmatrix} a & d-a \\ 0 & d \end{pmatrix} \quad \forall a, d \in \mathbb{R}$