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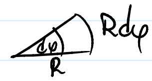
P1

cargado uniformemente $dq' = \lambda\, dl$
$= \lambda R\, d\varphi$



$$\vec{E}_{anillo} = \frac{1}{4\pi\varepsilon_0}\int dq' \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$\vec{r} = 2z\,\hat{k}$

$\vec{r}' = R(\cos\varphi + \sin\varphi)$

$\vec{r}-\vec{r}' = -R(\cos\varphi + \sin\varphi) + 2z\hat{k}$

$|\vec{r}-\vec{r}'| = \sqrt{R^2 + 4z^2}$

$$= \frac{1}{4\pi\varepsilon_0}\int dq' \frac{-R(\cos\varphi + \sin\varphi) + 2z\hat{k}}{(R^2 + 4z^2)^{3/2}}$$

$dq' = \lambda\, dl$
$= \lambda R\, d\varphi$

$$\vec{E}_{anillo} = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \lambda R \cdot \frac{-R(\cancel{\cos\varphi}^{\,0} + \cancel{\sin\varphi}^{\,0}) + 2z\hat{k}}{(R^2 + 4z^2)^{3/2}}\, d\varphi$$

$$\frac{\lambda R}{\cancel{4\pi}\varepsilon_0} \cdot \cancel{2\pi} \cdot \frac{\cancel{2}z}{(R^2 + 4z^2)^{3/2}} = \frac{\lambda R z}{\varepsilon_0 (R^2 + 4z^2)^{3/2}}\,\hat{k}$$

$$Q = \int dq' = \int \lambda\, dl$$
$$= \lambda \int dl = \lambda 2\pi R$$

$Q = \lambda 2\pi R$

$$\lambda = \frac{Q}{2\pi R}$$

a) $\dfrac{Q}{2\pi R} = \lambda$ // o en terminos del campo E

$$E = \frac{\lambda R z}{\varepsilon_0 (R^2 + 4z^2)^{3/2}} \Rightarrow \frac{E\cdot\varepsilon_0 (R^2 + 4z^2)^{3/2}}{Rz} = \lambda \text{ //}$$

$$E = \frac{\lambda R z}{\varepsilon_0 (R^2 + 4z^2)^{3/2}} \Rightarrow \frac{E \cdot \varepsilon_0 (R^2 + 4z^2)^{3/2}}{R z} = \lambda \quad //$$

b) $Q = \lambda 2\pi R$ /

c) $\vec{F} = q \vec{E}$

$$\vec{F} = q \cdot \frac{\lambda R z}{\varepsilon_0 (R^2 + 4z^2)^{3/2}} \hat{k} \quad [N]$$

d) $R \ll z$

$$F = q \cdot \frac{\lambda R z}{\varepsilon_0 \left(z^2 \left(\cancelto{0}{\frac{R^2}{z^2}} + 4 \right) \right)^{3/2}}$$

$$F = q \cdot \frac{\lambda R z}{\varepsilon_0 \left((2z)^{\cancel{2}} \right)^{3/2}} = \frac{q}{\varepsilon_0} \cdot \frac{\lambda R \cancel{z}}{8 z^{\cancel{3}2}} = q \cdot \frac{\lambda R}{8 \varepsilon_0 z^2} \cdot \frac{2\pi}{2\pi}$$

$$= \frac{1}{4\pi\varepsilon_0} \, q \, \frac{\overbrace{\lambda 2\pi R}^{Q}}{\underbrace{4 z^2}_{d^2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q Q}{d^2} \quad //$$

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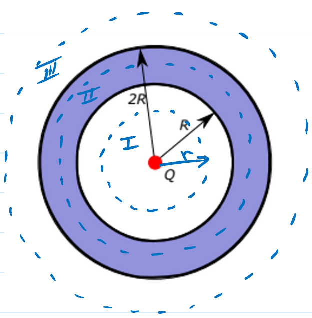
P2.



Zona I $0 < r < R$

$$\Phi = \oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi r^2 \varepsilon_0}$$

a) $\dfrac{Q}{4\pi r^2 \varepsilon_0}$

Zona II $R < r < 2R$

$$\oint E \cdot ds = \frac{q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q + q_{esf}}{\varepsilon_0}$$

$$q_{esf} = \int dq = \int \rho(r)\, dV$$

$$= \int_R^r \frac{\alpha}{\cancel{r}} \cdot 4\pi r^{\cancel{2}} \cdot dr$$

$$= 4\pi\alpha \int^r r\, dr$$

$$= 4\pi\alpha \int_R^r r\,dr$$

$$4\pi\alpha \left.\frac{r^2}{2}\right|_R^r = 2\pi\alpha\left[r^2 - R^2\right]$$

$$E(4\pi r^2) = \frac{Q + 2\pi\alpha(r^2 - R^2)}{\varepsilon_0}$$

b) $\bar{E} = \dfrac{Q + 2\pi\alpha(r^2 - R^2)}{4\pi r^2 \varepsilon_0}$ //

Zona $\underline{III}$ $\quad r > 2R$

$$E(4\pi r^2) = \frac{Q_{total}}{\varepsilon_0}$$

$$Q_{total\ esfera} = \int dq$$

$$= \int \rho_{(r)}\, dv = \int \frac{\alpha}{\cancel{r}} \cdot 4\pi r^{\cancel{2}}\, dr$$

$$= 4\pi\alpha \int_R^{2R} r\, dr$$

$$= 4\pi\alpha \cdot \left.\frac{r^2}{2}\right|_R^{2R}$$

$$= 2\pi\alpha\left(4R^2 - R^2\right)$$

$$= 2\pi\alpha\, 3R^2$$

$$= 6\pi\alpha R^2$$

$$\bar{E}(4\pi r^2) = \frac{Q + 6\pi\alpha R^2}{\varepsilon_0}$$

c) $\bar{E} = \dfrac{Q + 6\pi\alpha R^2}{4\pi r^2 \varepsilon_0}$

d) en zona $\underline{II}$ $\quad R < r < 2R$

$$\bar{E}_{II} = \frac{Q + 2\pi\alpha(r^2 - R^2)}{4\pi r^2 \varepsilon_0}$$

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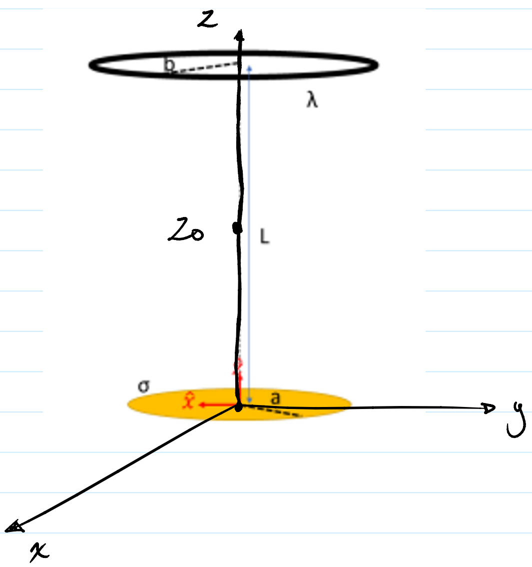
P3

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{|\vec{r}-\vec{r}'|}$$



Potencial del disco en Z_0

$$\frac{1}{4\pi\varepsilon_0}\int \frac{dq}{|\vec{r}-\vec{r}'|}$$

$\vec{r} = Z_0\hat{k}$

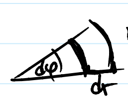
$\vec{r}' = r'(\cos\varphi + \sin\varphi)$

$\vec{r}-\vec{r}' = -r'(\cos\varphi + \sin\varphi) + Z_0\hat{k}$

$|\vec{r}-\vec{r}'| = \sqrt{r'^2 + Z_0^2}$

$$\frac{1}{4\pi\varepsilon_0}\int \frac{dq'}{\sqrt{r'^2+Z_0^2}}$$

$dq' = \sigma\, ds$



$r'dr'd\varphi$

$$\frac{1}{4\pi\varepsilon_0}\int_0^a\int_0^{2\pi} \frac{\sigma\, r'dr'd\varphi}{\sqrt{r'^2+Z_0^2}}$$

$$\frac{\sigma}{2\varepsilon_0}\int_0^a \frac{r'}{\sqrt{r'^2+Z_0^2}}\,dr'$$

sea $u^2 = r'^2 + Z_0^2$

$$\frac{\sigma}{2\varepsilon_0}\int_0^a \frac{r'}{\sqrt{r'^2+z_0^2}}\,dr' \qquad \text{sea } u^2 = r'^2 + z_0^2$$

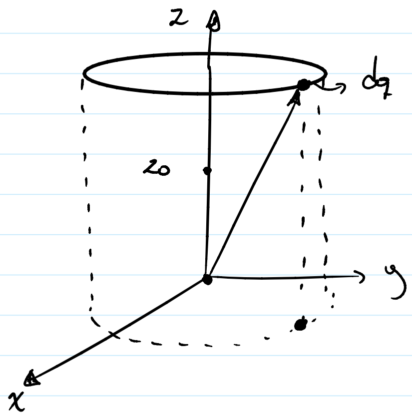
$$\cancel{2}u\,du = \cancel{2}r'dr'$$

$$\frac{\sigma}{2\varepsilon_0}\cdot\int\frac{u\,du}{\sqrt{u^2}} \to \int du \qquad u=\sqrt{r'^2+z_0^2}$$

$$\frac{\sigma}{2\varepsilon_0}\cdot u\Big|_0^a = \frac{\sigma}{2\varepsilon_0}\cdot\sqrt{r'^2+z_0^2}\Big|_0^a$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+z_0^2} - z_0\right]$$

Potencial del anillo en z_0



$$\vec{r} = z_0\hat{k}$$

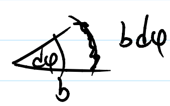
$$\vec{r}' = b(\cos\varphi + \sin\varphi) + L\hat{k}$$

$$\vec{r}-\vec{r}' = -b(\cos\varphi+\sin\varphi) - L\hat{k} + z_0\hat{k}$$

$$= -b(\cos\varphi+\sin\varphi) + (z_0 - L)\hat{k}$$

$$|\vec{r}-\vec{r}'| = \sqrt{b^2 + (z_0-L)^2}$$

$$\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{b^2+(z_0-L)^2}} \qquad dq = \lambda\,dl$$



$$\frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{\lambda b\,d\varphi}{\sqrt{b^2+(z_0-L)^2}} = \frac{1}{{}_2\cancel{4\pi}\varepsilon_0}\cancel{2\pi}\cdot\frac{\lambda b}{(b^2+(z_0-L)^2)^{1/2}}$$

$$= \frac{\lambda b}{2\varepsilon_0\,(b^2+(z_0-L)^2)^{1/2}}$$

a)

$$\frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+z_0^2} - z_0\right] + \frac{\lambda b}{2\varepsilon_0\,(b^2+(z_0-L)^2)^{1/2}} \;/\!/$$

b)

en $z = L/2 \Rightarrow V = 0$

b)

en $z = L/2 \Rightarrow V = 0$

$b = 2a$; $L = 4a$

$$\frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+\left(\frac{L}{2}\right)^2} - \frac{L}{2}\right] + \frac{\lambda b}{2\varepsilon_0 \left(b^2 + \left(\frac{L}{2}-L\right)^2\right)^{1/2}}$$

$$\frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+\frac{\cancel{16}^{4} a^2}{\cancel{4}}} - \frac{\cancel{4}^{2} a}{\cancel{2}}\right] + \frac{\lambda b}{2\varepsilon_0}\cdot\frac{1}{\left(4a^2+\frac{\cancel{16}^{4} a^2}{\cancel{4}}\right)^{1/2}}$$

$$\frac{\sigma}{2\varepsilon_0}\left(a\sqrt{5} - 2a\right) + \frac{\lambda \cancel{2a}}{\cancel{2}\varepsilon_0}\cdot\frac{1}{\cancel{a}\sqrt{8}}$$

$$\frac{\sigma}{2\varepsilon_0} a(\sqrt{5}-2) + \frac{\lambda}{\varepsilon_0\sqrt{8}} = 0$$

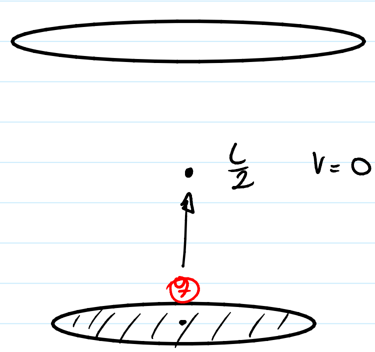
$$\frac{\sigma}{2\cancel{\varepsilon_0}} a(\sqrt{5}-2) = -\frac{\lambda}{\cancel{\varepsilon_0}\sqrt{8}}$$

$\sqrt{5} > \sqrt{4}$

b) $\frac{(-\sigma)a(\overbrace{\sqrt{5}-2}^{+})\sqrt{8}}{2} = \lambda$ //

el alambre y el disco deben tener cargas opuestas.

c)



en $z=0$

$V = \frac{\sigma a}{2\varepsilon_0}$ ...