

### 1. Exam Information

### 2. Number 2

$$2.1 \quad \frac{2}{8} = \boxed{\frac{1}{4}}$$

$$2.2 \quad P(H) = P(H|F) \cdot P(F) + P(H|BH) \cdot P(BH) + P(H|BT) \cdot P(BT)$$

$$P(H) = \frac{1}{2} \cdot \frac{2}{8} + \frac{3}{4} \cdot \frac{3}{8} + \frac{1}{4} \cdot \frac{3}{8}$$

$$P(H) = \frac{1}{8} + \frac{9}{32} + \frac{3}{32} = \frac{16}{32} = \boxed{\frac{1}{2}}$$

$$2.3 \quad P(F|TTH) = \frac{P(TTH|F) \cdot P(F)}{P(TTH|F) \cdot P(F) + P(TTH|BH) \cdot P(BH) + P(TTH|BT) \cdot P(BT)}$$

$$P(F|TTH) = \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{8}}{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{8} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{8} + \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{3}{8}} = \boxed{\frac{4}{13}}$$

$$2.4 \quad P(H) = P(H|F) \implies \frac{1}{2} = \frac{1}{2}, \text{ Therefore, they are } \boxed{\text{independent}}.$$

### 3. Varied questions

$$3.1 \quad P(A_2|A_1) = \boxed{\frac{3}{51}}$$

$$3.2 \quad \text{These events are independent, therefore } \boxed{\frac{4}{52}}$$

3.3 Let D: "Person has Disease", T: "Person tested positive", d: "Person does NOT have disease."

The probability that a person with the disease will test positive is 0.99

The probability that a person with the disease will test negative is 0.01

The probability that a person without the disease will test positive is 0.02

The probability that a person without the disease will test negative is 0.98

$$P(D|T) = \frac{P(T|D) \cdot P(D)}{P(T|D) \cdot P(D) + P(T|d) \cdot P(d)}$$

$$P(D|T) = \frac{(0.99) \cdot (\frac{1}{10,000})}{(0.99) \cdot (\frac{1}{10,000}) + (0.02) \cdot (1 - \frac{1}{10,000})} \approx \boxed{0.0049261084}$$

3.4 Sample Variance:  $\frac{\sum (x-\bar{x})^2}{n-1} = \boxed{2.2}$