CSCI 2824: Discrete Structures

Lecture 14: Exam 1 Review

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Time and Location:

6:30-8:00 PM on Tuesday October 1st

- BESC 185 – Last Names Abbasi - Finley
- EDUC 220 – Last Names Fitze - Mackillip
- HALE 270 – Last Names Mahre - Zuyus

Exam Rules:

- You are allowed to use a calculator. No smartphones or other devices that can store large amounts of data or access the internet.
- You are allowed one 3x5-inch notecard as a cheat sheet. You can write whatever you want on it and you can use both sides.
- You do not need to bring bluebooks or anything like that.
- Do bring your Buff OneCard
- Do bring multiple writing utensils.
- Get there early. If you arrive late, you will not receive extra time.

Exam format:

Some combination of (a) multiple choice, (b) short answer (brief justification type problems) and (c) free response (more involved problems; think along the lines of the written homework problems.)

Exam content:

Beginning of the semester through Proof Methods and Strategy (Lecture 11)

Special Accommodations:

If you have a documented special need for accommodations and have presented me with the requisite paperwork before the exam, then you can take the exam in **ECES 112 starting at 6:30 PM on Tuesday October 1st** and ending whenever your particular accommodation indicates. It is your responsibility to keep track of your time – the proctors are instructed not to bother you, since that's kind of the point.

This is a classroom with seating for about 20 people. If your particular needs require some different accommodation, just let me know in a private Piazza message (or email) and we'll sort it out.

Note: If you do not have a documented need through Disability Services then this doesn't apply to you, and you must take the exam in the regularly scheduled place/time.

Binary Representation of Numbers

Example: Represent 209 as a binary number $(209)_{10} = (11010001)_2$

$N = 209$	1	2^0	$N = \frac{209-1}{2} = 104$
104	0	2^1	$N = 52$
52	0	2^2	$N = 26$
26	0	$\vdots$	$N = 13$
13	1		$N = \frac{13-1}{2} = 6$
6	0		$N = 3$
3	1		$N = \frac{3-1}{2} = 1$
1	1		$N = 0$ Done.

Logical Equivalences

$p \to q \equiv \neg p \vee q$ Relation by Implication (RBI)

$p \to q \equiv \neg q \to \neg p$ Contraposition

$p \Leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ Definition of Biconditional

$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$ Alternate Definition of xor

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Rules of Inference

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Logical Inference

* practice show $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

Example: Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ is logically equivalent to $(p \vee q) \rightarrow r$

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$p \vee q$	$(p \vee q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

Truth Tables & Knights / Knaves

Example: The Island of Knights and Knaves has two types of inhabitants: Knights, who always tell the truth, and Knaves, who always lie. As you are exploring the Island of Knights and Knaves you encounter two people named A and B.

A tells you "Of B and myself, exactly one of us is a Knight."
B tells you "A is a Knave."

Determine the nature of A and B, if you can.

p: A is a Knight.
q: B is a Knight.

A: $p \oplus q$
B: $\neg p$

$p \Leftrightarrow p \oplus q \wedge q \Leftrightarrow \neg p$

p	q	$\neg p$	$p \oplus q$	(1) $p \Leftrightarrow p \oplus q$	(2) $q \Leftrightarrow \neg p$	(1) $\wedge$ (2)
T	T	F	F	F	F	F
T	F	F	T	T	T	T
F	T	T	T	F	T	F
F	F	T	F	T	F	T

A is a knight.
B is a knave.

Rules of Inference

p q r

Example: If it snows and it is dark out, then Tony will crash his bicycle. Suppose you see Tony and he has crashed his bicycle. What then, do you know must also be true?

a) It is snowing.
b) It is dark out.
c) It is snowing and it is dark out.
d) Nothing
e) It is not snowing and it is not dark out.

$$(p \wedge q) \to r$$
$$\underline{\quad r \quad}$$
$$\therefore$$

Fallacy of Affirming the conclusion

Valid Arguments

Example: Is the following argument valid?

1. If an animal is a tapir, then it has short legs.
2. This animal is not a tapir.
3. Therefore, this animal does not have short legs.

Not Valid!

Fallacy of Denying the Hypothesis.

Quantifiers

Select the quantifier translation that best matches the following English statement.

$\forall x\,(C(x) \to L(x, pizza)) \wedge \exists x\,(C(x) \wedge L(x, soda)) \wedge \neg\exists x\,(C(x) \wedge L(x, asparagus))$

All computer scientists like pizza, and some computer scientists like soda, but no computer scientists like asparagus.

$\forall x\,(C(x) \to L(x, pizza)) \wedge \exists x\,(C(x) \wedge L(x, soda)) \wedge \forall x\, \neg(C(x) \wedge L(x, asp))$

Let $L(x, y)$ denote "x *likes* y", let $C(x)$ denote "x *is a computer scientist*" and suppose the domain for x is all people.

$\forall x\,(C(x) \to L(x, pizza)) \wedge \exists x\,(C(x) \wedge L(x, soda)) \wedge \forall x\,(\neg C(x) \vee \neg L(x, asp.))$

Select one:

$\forall x\,(C(x) \to L(x, pizza)) \wedge \forall x\,(C(x) \to \neg L(x, asparagus)) \wedge \exists x\,(C(x) \wedge L(x, soda))$

$\forall x\,(C(x) \to \neg L(x, aspar.))$

★ a. $\forall x\ \underbrace{[C(x) \to (L(x, pizza) \wedge \neg L(x, asparagus))]} \wedge \exists x\ (C(x) \wedge L(x, soda))$

b. $\forall x\ (C(x) \wedge L(x, pizza) \wedge \neg L(x, asparagus)) \wedge \exists x\ (C(x) \wedge L(x, soda))$

c. $\neg\exists x\ [(C(x) \wedge \neg L(x, pizza)) \wedge (C(x) \wedge L(x, asparagus))] \wedge \exists x\ (C(x) \wedge L(x, soda))$

d. $\forall x\ [C(x) \to (L(x, pizza) \wedge \neg L(x, asparagus))] \wedge \neg\forall x\ (C(x) \wedge \neg L(x, soda))$

Proofs

Example: Prove that a solution to $x^2 - 2x + 1 = 0$ exists.

quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)} = \frac{+2 \pm \sqrt{4-4}}{2}$$

$$= \frac{2}{2} = 1 \qquad \boxed{x=1}$$

Example: Prove that a solution to $2x + 4 = 0$ is unique.

Suppose x_1 and x_2 are both solutions.

$\Rightarrow \quad 2x_1 + 4 = 0 \quad$ and $\quad 2x_2 + 4 = 0$

$$2x_1 + 4 = 2x_2 + 4$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

Proofs

Example: If n is an integer and 3n+2 is even, then n is even.

Proof by Contradiction

Pf: Assume n is an integer and 3n+2 is even. $p \wedge \neg q$

For a contradiction proof, let's assume that n is odd.

$n = 2k+1$ for some integer k.

$3n+2 = 3(2k+1) + 2$

$= 6k + 3 + 2$

$= 6k + 4 + 1$

$= 2(3k+2) + 1$

contradiction

3n+2 can't be both even + odd. $\Rightarrow\Leftarrow$

Thus it must be that 3n+2 even ⇒ n is even. ∎

Proof by Contraposition $\neg q \rightarrow \neg p$

Pf: Assume that n is odd. $\neg q$

$n = 2k+1$ for some k

$3n+2 = 3(2k+1)+2$

$= 6k + 3 + 2$

$= 6k + 5$

$= 6k + 4 + 1$

$= 2(3k+2) + 1$

⇒ 3n+2 is odd. $\}\ \neg p$

By contraposition, we've proven that 3n+2 even ⇒ n even ∎

Happy Studying!

(and good luck!)