CSCI 2824 – Fall 2019 **Homework 12**

This assignment is due on Friday, December 6th to Gradescope by Noon. You are expected to write or type up your solutions neatly. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: Make sure to clearly write your full name, the lecture section you belong to (001 or 002), and your student ID number at the top of your assignment. You may **neatly** type your solutions in LaTex for +1 extra credit on the assignment.

1. (a) Find the closed form solution to the non-homogeneous recurrence relation $a_n = 4a_{n-1} + 9$, with the initial condition $a_0 = 3$. Use the solution techniques from class that involve finding the characteristic equation, general solution, etc. or use the formula that we derived.

   (b) Now consider the linear homogeneous recurrence relation $a_n = 6a_{n-1} - 8a_{n-2}$, with the initial conditions $a_0 = 1, a_1 = 6$. Find the closed form solution using the technique described in class that involves finding the characteristic polynomial, general solution, etc.

2. Consider the recurrence relation $a_n = -3a_{n-1} + 10a_{n-2} + 18n^2$ with initial conditions $a_0 = 0$ and $a_1 = 7$.

   Find a closed form solution for the given recurrence relation. In your solution, put a box around each of the following, and clearly label them:

   (a) the characteristic polynomial

   (b) the solution to the associated homogeneous recurrence relation $(a_n^{(h)})$

   (c) the full particular solution **guess** that you are plugging into the full nonhomogeneous recurrence relation $(a_n^{(p)})$

   (d) the full general solution (with unknown coefficients still) (a_n)

   (e) set up the two equations needed to solve for the remaining unknown coefficients. You do not need to solve these.

3. Find the closed form solution to the recurrence relation $a_n = -3a_{n-1} + 10a_{n-2} + 2^n$, $a_0 = 1$, $a_1 = \frac{11}{7}$.

4. Popeye and Olive Oyl frequently send each other text messages that are just contiguous strings of the three emojis 😃, 😉, and 😑. For instance, one particular length-5 emoji string might be 😃😉😃😉😑.

   (a) Find a recurrence relation for the number of possible length-n emoji strings that do not contain two consecutive winkey emojis, 😉😉.

   (b) What are the initial conditions for the recurrence relation?

   (c) Find a closed-form solution to the recurrence relation you found in part (a) by solving for the roots of the characteristic polynomial and then using initial conditions to determine the constants.

   (d) Use your closed form expression to determine the number of length-7 emoji strings that do not contain 2 consecutive winkey emojis. [Note: You may use Python to help you answer this question.]

5. Consider the relation $R = \{(1,1),(2,2),(3,3),(3,1),(3,4),(4,4),(4,1),(4,3)\}$, where $R \subseteq A \times A$, with $A = \{1,2,3,4\}$.

   (a) Draw the graph of R. **Note**: If possible, it is good practice to organize your graph such that all directed edges are non-intersecting.

   (b) Is the relation R reflexive? Symmetric? Transitive? An equivalence relation? Fully justify your responses.

   (c) The **complement** of a relation $R \subseteq A \times A$ is defined as $\overline{R} = (A \times A) - R$.

      i. What is the set $\overline{R}$ for R as defined in this problem?

      ii. Draw the graph of $\overline{R}$. Again, organize your graph such that all directed edges are non-intersecting.

      iii. Is the following statement true or false? Briefly justify your conclusion. "A relation R is symmetric **if and only if** its complement $\overline{R}$ is symmetric."