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CSCI 2824: Discrete Structures

Lecture 2: Binary Arithmetic & Intro to Python

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Computer Science



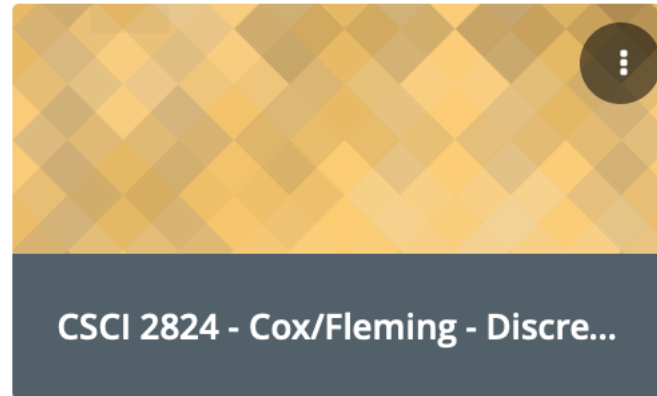
Adding Machine 1880-1920 computers.org

1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0 1 0 1 1 0 0 0 0 1 0 0 0

Course Logistics – Platforms

- 1) [Moodle](#) – Online Homework, Online Quizlets, Grades

CSCI 2824 - FALL 2019



- 2) [Piazza](#) – Class discussion forum

CSCI 2824 ▾ Q & A Resources Statistics Manage Class

University of Colorado at Boulder - Fall 2019

CSCI 2824: Discrete Structures

- 3) Gradescope – Submission of written homework



Course Logistics – Grading

Weekly Homework (30%)

- **Half Written, Half Online**

Quizlets (10%)

- **Online**

Two Midterms (20% each)

- **October 1st , November 5th**

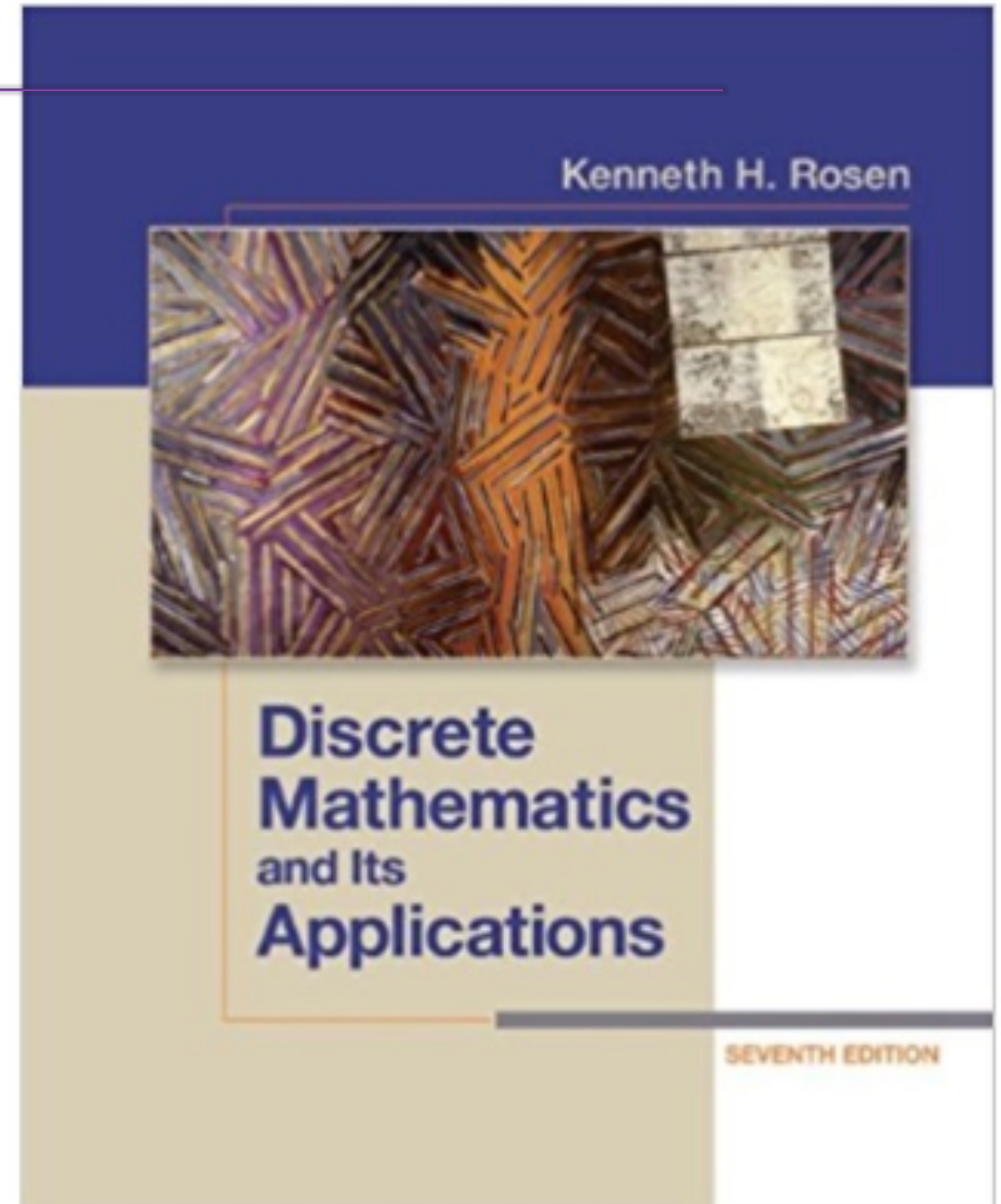
Final Exam (20%)

- **Section 001 – 9am: Wednesday December 18th , 1:30-4:00pm**
- **Section 002 – 11am: Sunday December 15th , 1:30-4:00pm**



Course Logistics – Book

Textbook – *Discrete Mathematics and Its Applications*, 7th Ed. by Kenneth H. Rosen



FRAC TIONS in Binary

Powers of 2:

$$2^{-1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{4}$$

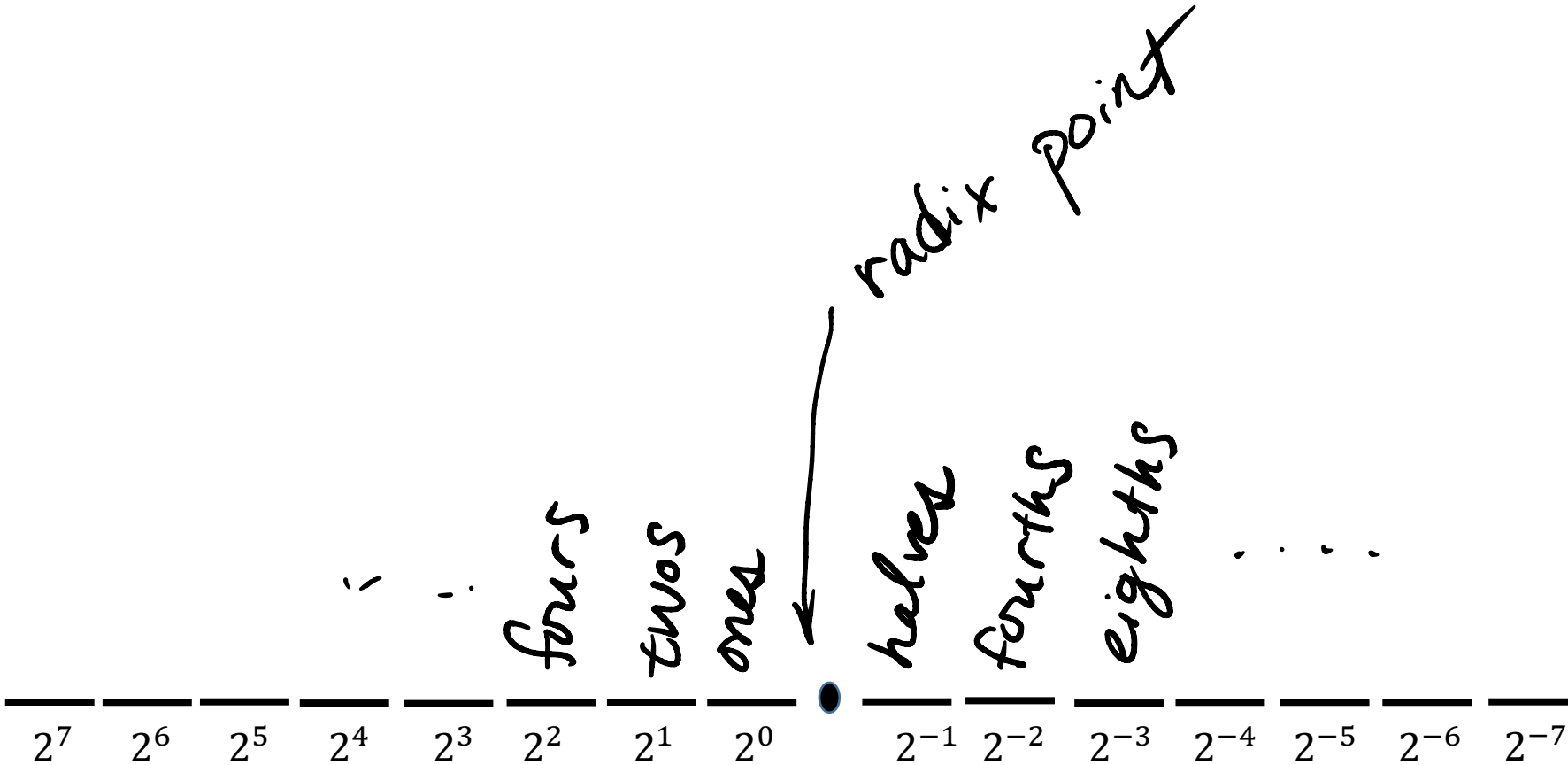
$$2^{-3} = \frac{1}{8}$$

$$2^{-4} = \frac{1}{16}$$

$$2^{-5} = \frac{1}{32}$$

$$2^{-6} = \frac{1}{64}$$

$$2^{-7} = \frac{1}{128}$$



Example: Convert 0.75 from decimal to binary

$$0.75 = 0.5 + 0.25$$

$$= \frac{1}{2} + \frac{1}{4}$$

$$= 2^{-1} + 2^{-2}$$

$$= 1 * 2^{-1} + 1 * 2^{-2}$$

Powers of 2:

$$2^{-1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{4}$$

$$2^{-3} = \frac{1}{8}$$

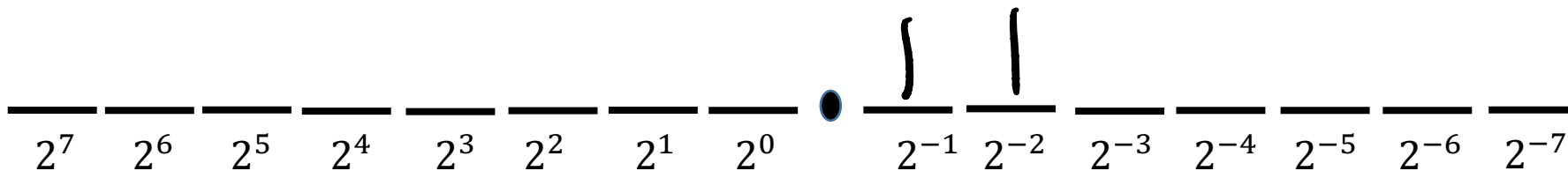
$$2^{-4} = \frac{1}{16}$$

$$2^{-5} = \frac{1}{32}$$

$$2^{-6} = \frac{1}{64}$$

$$2^{-7} = \frac{1}{128}$$

$$(0.75)_{10} = (.11)_2$$



Example: $(0.\underline{0}\underline{0}\underline{0}\underline{1})_2 = \underline{0} * 2^{-1} + \underline{0} * 2^{-2} + \underline{0} * 2^{-3} + \underline{1} * 2^{-4}$

\uparrow halves
 \uparrow fourths

$$= 0 + 0 + 0 + \frac{1}{16}$$

$$= (0.0625)_{10}$$

Example: $(0.10101)_2 = 1 * 2^{-1} + 0 * 2^{-2} + 1 * 2^{-3} + 0 * 2^{-4} + 1 * 2^{-5}$

$$= \frac{1}{2} + 0 + \frac{1}{8} + 0 + \frac{1}{32}$$

$$= \frac{16}{32} + \frac{4}{32} + \frac{1}{32}$$

$$= \underline{\underline{\frac{21}{32}}} = (.65625)_{10}$$

Converting Decimal Fractions to Binary

1. Let m be a number less than 1
2. Move left to right (from radix point)
3. Multiply m by 2. Set bit to the value in the ones place.
4. Reset m to the stuff after the decimal place.
5. Continue until $m=0$

$$.88000 = .88$$

Example: Convert 0.84375 from decimal to binary

$$m = 0.84375 = (.11011)_2$$

1. Let m be a number less than 1
2. Move left to right (from radix point)

$$2m = 1.6875$$

$$\text{set the } 2^{-1} \text{ bit} = 1 \quad m = .6875$$

$$2m = 1.375$$

$$\text{set the } 2^{-2} \text{ bit} = 1 \quad m = .375$$

$$2m = 0.75$$

$$\text{set the } 2^{-3} \text{ bit} = 0 \quad m = .75$$

$$2m = 1.5$$

$$\text{set the } 2^{-4} \text{ bit} = 1 \quad m = 0.5$$

$$2m = 1.0$$

$$\text{set the } 2^{-5} \text{ bit} = 1 \quad m = 0.0$$

$$2m = 0.0$$

$$\text{set the } 2^{-6} \text{ bit} = 0 \quad m = 0.0$$

3. Multiply m by 2. Set bit to the value in the ones place.
4. Reset m to the stuff after the decimal place.
5. Continue until $m=0$

⋮

Example: Convert 161.84375 from decimal to binary

- Convert the integer part and the fractional part separately, then stick them together with a radix point.

$$(161)_{10} = (10100001)_2$$

from last lecture

$$(.84375)_{10} = (.11011)_2$$

$$(161.84375)_{10} = (10100001.11011)_2$$

Binary

ARITHMETIC

$$\begin{array}{r} 110 \\ 98 \\ \hline 208 \end{array} \quad 30 + 2 = 32$$

Example: What is 30 in binary? What is 2 in binary?
What is 32 in binary?

Adding two numbers in binary:

Proceed right to left

30
15
7
3
1

0
1
1
1
1

$$N = 15$$

$$N = \frac{15-1}{2} = 7$$

$$N = \frac{7-1}{2} = 3$$

$$N = \frac{3-1}{2} = 1$$

$$N = \frac{1-1}{2} = 0$$

$$30 = (11110)_2$$

$$2 = (10)_2$$

$$32 = (100000)_2$$

$\overline{2^5} \overline{2^4} \overline{2^3} \overline{2^2} \overline{2^1} \overline{2^0}$

In decimal: If the column exceeds 10, we carry a 1 to the left

In binary: If our column exceeds 2, we carry a 1 to the left

Example: Add $(11110)_2 + (10)_2$

$$\begin{array}{r} 1 1 1 0 \\ + 0 0 1 0 \\ \hline 1 0 0 0 0 \end{array}$$

✓

$$2_{10} = (10)_2$$

this is the
binary representation
of 32, as expected.

Example: Subtract $(11101)_2 - (110)_2$

$$\begin{array}{r}
 \begin{array}{ccccc}
 & 0 & 1 & & \\
 1 & \cancel{1} & \cancel{1} & 1 & 0 \\
 - & & & 1 & 1 \\
 \hline
 1 & 0 & 1 & 1 & 1
 \end{array}
 \end{array}$$

$$\left. \begin{array}{c} 1 \\ 2 \\ 4 \\ 16 \end{array} \right\} = 23$$

$$11101 = 1 + 4 + 8 + 16 = 29$$

$$110 = 6$$

$$29 - 6 = 23$$

Subtracting two numbers in binary:

Proceed right to left.

In decimal: We can take 1 from the column to the left, and bring 10 to the right.

In binary: We can take 1 from the column to the left, and bring 2 to the right.

Intro to Python (Python 3)

Easiest way to get Python: <https://www.anaconda.com/download>

Good Practice: <https://www.hackerrank.com/domains/python>



More Examples IF WE HAVE TIME

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**"This calendar has 11 days a week, 9 weeks a month,
and 17 months a year. I bought it at the
Time Management Seminar!"**

Example: Convert 0.2 from decimal to binary

Example: **Truncation error.**

$$f(a, b) = 333.75b^6 + a^2(11a^2b^2 - b^6 - 121b^4 - 2) + 5.5b^8 + \frac{a}{2b}$$

$$a = 77617$$

$$b = 33096$$

If this is run on a 64-bit machine,

$$f(a, b) = -44450695952321879337122922496.000$$

OR

$$f(a, b) = -1.180592e + 21$$

Real Answer: $f(a, b) \approx -0.82739605$