Felipe Lima 001 Homework 10 Fri Nov 15

1.(a) Distribution of r items for n choices
$r \downarrow 21$, $n \downarrow 5$

r-combinations with repetition $\frac{(n+r-1)!}{r!(n-1)!}$

$\frac{(21+5-1)!}{21!(5-1)!} = \frac{25!}{21!4!} = 12650$ There are 12650 way for Isaac to distribute the cookies.

(b) Now the distribution is of $r = 21-5 = 16$ items for $n=5$ choices

$\frac{(16+5-1)!}{16!(5-1)!} = \frac{20!}{16!4!} = 4845$ There are 4845 ways for Iona to distribute the cookies.

2.(a) r-permutations with the repetition n^r
$n = 26$ letters in the alphabet, $r =$ length 12

$26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26$

26^{12} such strings

(b) _ _ _ _ _ _ _ _ _ _ _ _

5 letters in NINJA. $12-5=7$ letter available 8 places NINJA can be

$\therefore$

$26^7 \cdot 8$ such strings

(c) Number of strings containing NINJA $= 26^7 \cdot 8$
Number of strings containing TURTLES $= 26^5 \cdot 6$
Number of strings containing NINJA and TURTLES $= 2$
Number of strings containing neither NINJA nor TURTLES

$$26^{12} - [(26^7 \cdot 8) + (26^5 \cdot 6) - 2]$$

3.(a) # of possible outcomes $= 6^5$

# of possible all unique outcomes $= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 6!$

$p(E) = \frac{|E|}{|S|} = \frac{6!}{6^5} = \frac{720}{7776} = \frac{5}{54}$ → probability of obtaining all unique outcomes

(b) x 1234 = 1234 x
x 2345 = 2345 x
x 4567 = 3456 x

6 options for each x

∴ $|E| = 6 \cdot 3 = 18$

$|S| = 6^5$

$p(E) = \frac{18}{6^5} = \boxed{\frac{1}{432}}$

(c) $p(E|F) = \frac{p(E \cap F)}{p(F)}$

E = small straight $p(E) = \frac{1}{432}$ E = 18

F = all unique $p(F) = \frac{5}{54}$ F = 720

$(E \cap F) = 6$

1234 x
2345 x
3456 x

2 options for each x $= 2 \cdot 3 = 6$

$\frac{p(E \cap F)}{p(F)} = \frac{6}{6^5} \cdot \frac{54}{5} = \boxed{\frac{1}{120}}$

or

if all unique S = 720

unique and straight = 6

$\frac{6}{720} = \frac{1}{120}$.

(d) Events E and F are independent when $p(E|F) = p(E)$

From previous questions $p(E|F) = \frac{1}{120} \neq p(E) = \frac{1}{432}$

∴ the events are not independent.

$p(E|F) = \frac{6}{6^5} \neq p(E) \cdot p(F) = \frac{2160}{54}$

4. (a) $x^{57}y^{43}$ in $(x+y)^{100}$

thm $(x+y)^{100} = \sum_{k=0}^{100} \binom{100}{k} x^{100-k} y^{k}$ | The coefficient: $\binom{100}{43} x^{57} y^{43}$

$$\boxed{\frac{100!}{57!\,43!}}$$

(b) $x^{57}y^{43}$ in $(-x-3y)^{100}$

$$(-x-3y)^{100} = \sum_{n=0}^{100} \binom{100}{43} (-x)^{100-43} (-3y)^{43}$$

$$\binom{100}{43} (-1)^{57} (-3)^{43} = \frac{100!}{57!\,43!} (-1)(-3)^{43}$$

(c) No. there won't be a term whose only x and y components are $x^{58}y^{43}$ since $58+43 = 101 \neq 100$ since $(x+y)^{100}$ $n = 100$ and the power of x and y should add up to $n = 100$

(d) SPACESHIP has 9 letter $2 \times S$ and $2 \times P$

$$\frac{9!}{2!\,2!} = 90720 \text{ ways.}$$