Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

Counting

1. How many bitstrings of length 5 contain the sub-string 010 or 000?

2. Suppose that valid passwords must be strings of length 10 containing only numbers and letters.
   (a) How many passwords exist if uppercase and lowercase letters are indistinguishable from one another?
   (b) How many passwords exist if uppercase and lowercase letters are distinguishable?
   (c) How many passwords exist if passwords must contain at least one uppercase and at least one lowercase letter?

3. There are 10 identical computers that are to be distributed among 5 CAs. How many ways are there to distribute the computers?

4. Suppose you have 10 students taking an exam and you only have 4 dividers that separates the students. How many ways can the dividers be placed in?

5. How many different-appearing arrangements can be created using all the letters `SUBBOOKKEEPER`?

Pigeonhole Principle

6. Show that in any group of n people, there are two who have an identical number of friends within the group. That is, show that there are two people in the group who have exactly k number of friends within that group. Also assume that a person has at minimum at-least 1 friend.

Review

Matrix Operations

7. Find $\begin{bmatrix} 2 & 7 & -13 \\ 6 & -3 & -1 \end{bmatrix} - \begin{bmatrix} 4 & 5 & 9 \\ -4 & 8 & 20 \end{bmatrix}$

8. Find **AB** if:
   (a) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$

Set Operations

9. Suppose $A = \{9, 4, 6, 1\}$ and $B = \{11, 7, 1, 9\}$. Find:
   (a) $A \cup B$
   (b) $A \cap B$
   (c) $A - B$
   (d) $B - A$
   (e) $A \times B$

Induction

10. Which amounts of money can be formed from just two-dollar bills and five-dollar bills? Prove your answer using strong induction.

Recursion

As a reminder, there is Python code on the Workgroup Piazza to generate solvable recursive equations like those following problem.

11. Find a closed-form solution for the following recursive equations:
    (a) $a_n = 3a_{n-1} - 4 \mid a_0 = -10$
    (b) $a_n = 6a_{n-1} - 2 \mid a_0 = 4$
    (c) $a_n = 5a_{n-1} + 9 \mid a_0 = 7$
    (d) $a_n = 7(n-2)a_{n-1} \mid a_0 = 3$