CSCI 2824: Discrete Structures

Lecture 9: Rules of Inference (part 2)

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Moodle Quizlet 03 - Due Wed. at 8 pm

Moodle HW3 - Due Friday at Noon

Rules of Inference

$\neg(n > 3) \Longrightarrow n \leq 3$

Example: What valid argument form is present in the following?

If n is a real number with $n > 3$, then $n^2 > 9$.
Suppose that $n^2 \leq 9$. Therefore $n \leq 3$.

$$\text{If } \overbrace{n > 3}^{p}, \text{ then } \overbrace{n^2 > 9}^{q}$$

$$n^2 \leq 9 \qquad \neg q$$

$$\therefore n \leq 3 \qquad \neg p$$

$\boxed{\text{Modus tollens}}$

Rules of Inference

Example: What valid argument form is present in the following?

If $\underbrace{\sqrt{2} > \frac{3}{2}}_{p}$ then $\underbrace{(\sqrt{2})^2 > \left(\frac{3}{2}\right)^2}_{q}$. We know that $\underbrace{\sqrt{2} > \frac{3}{2}}_{p}$.

Consequently, $\underbrace{(\sqrt{2})^2 > \left(\frac{3}{2}\right)^2}_{q}$.

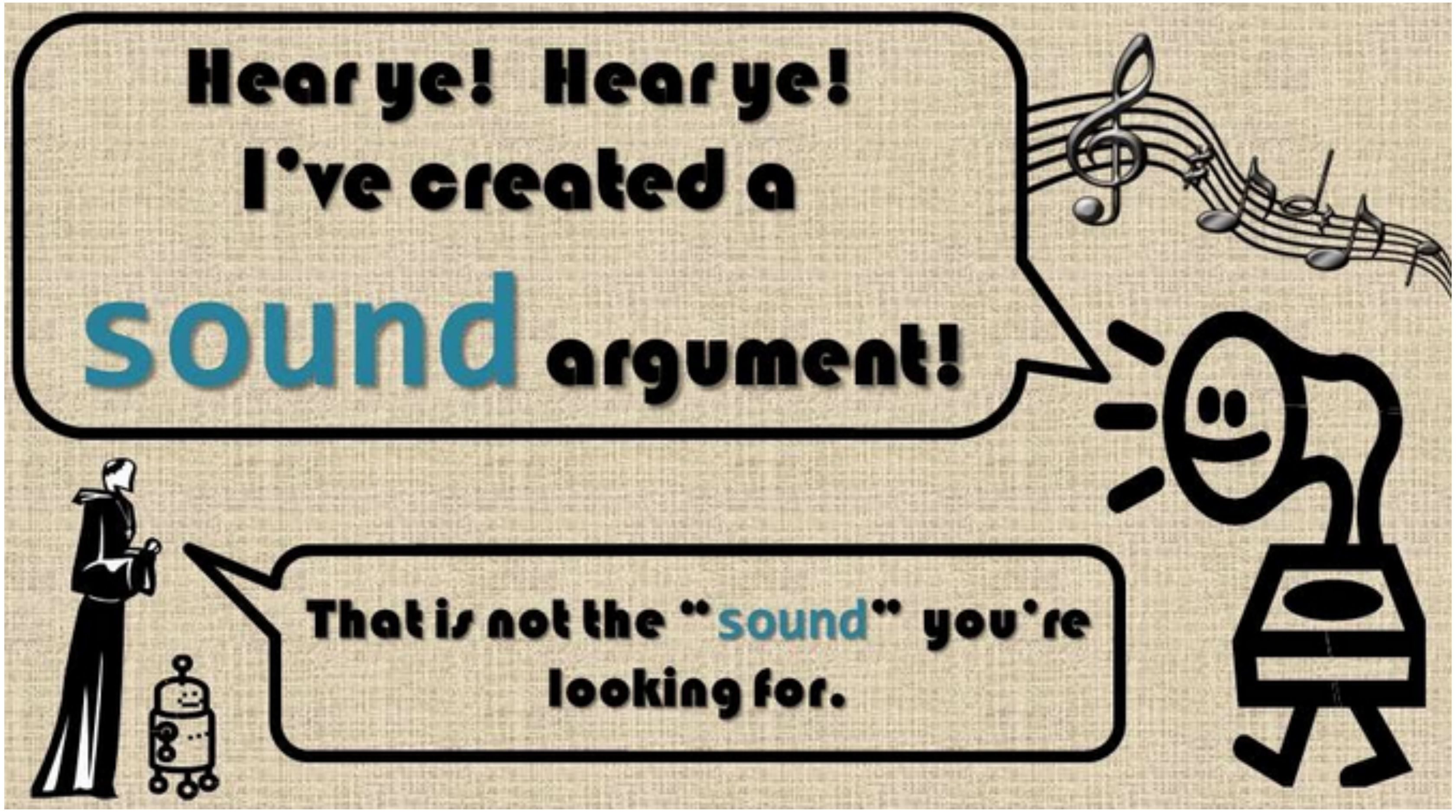
$\boxed{\text{modus ponens}}$

This is valid but not true.

Note: $2 > \frac{9}{4} = 2.25$

- we are only guaranteed a true conclusion if the premises are true. $\sqrt{2} \not> \frac{3}{2}$

Rules of Inference



When the argument is valid AND the premises are true, we call the argument **sound**.

Rules of Inference

If Socrates is a man, then Socrates is mortal.
Socrates is a man.

Therefore, Socrates is mortal.

All men are mortal.
Socrates is a man.

Therefore, Socrates is mortal.

Rules of Inference

All men are mortal.
Socrates is a man.

Therefore, Socrates is mortal.

Let $M(x)$ be "x is a man" and $D(x)$ be "x is mortal."
Then this argument has the form:

$$\forall x\,\big(M(x) \rightarrow D(x)\big)$$

$$M(SOCRATES)$$

$$\therefore D(SOCRATES)$$

Rules of Inference – for Quantifiers

Universal Instantiation:

$$\frac{\forall x\, P(x)}{\therefore P(c)}$$

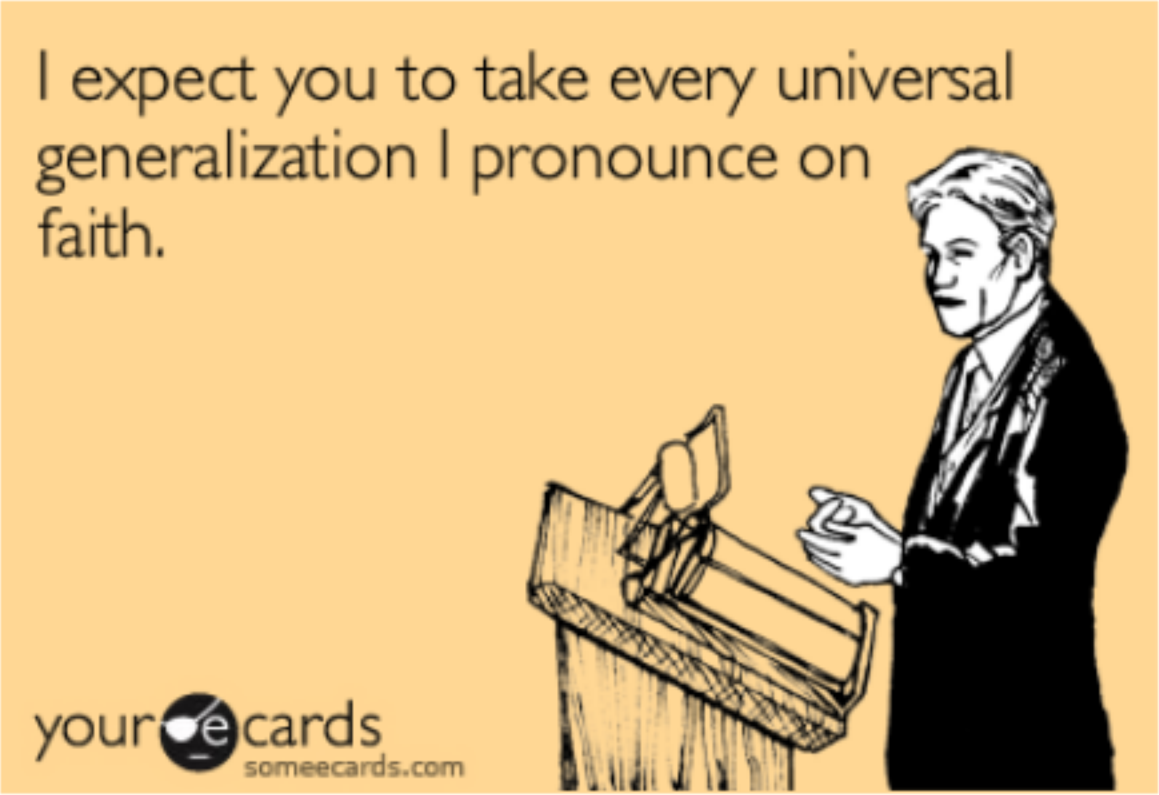
Intuition: If we know $P(x)$ is true for all x then we can insert any value of x and know that it's true.

Example: All dogs go to heaven. Therefore, Charlie B. Barkin is going to heaven.

Rules of Inference – for Quantifiers

Universal Generalization:

$$\begin{array}{l} P(c) \text{ for an arbitrary } c \\ \hline \therefore \forall x\, P(x) \end{array}$$



If you prove something about an arbitrary element of your domain, then you can make a universal statement.

It is NOT that you can prove something about a specific element of your domain and then make a universal statement.

Rules of Inference – for Quantifiers

Example: Is the following argument valid?

All dogs go to heaven.

Laddie is a dog.

If Laddie goes to heaven, then he walks through the pearly gates.

Consequently, all dogs walk through the pearly gates.

Example: What about this one?

All dogs go to heaven.

"A" is an arbitrary dog.

If "A" goes to heaven, then s/he walks through the pearly gates.

Consequently, all dogs walk through the pearly gates.

Rules of Inference – for Quantifiers

<u>Existential Instantiation:</u>

$$\frac{\exists x\ P(x)}{\therefore P(c) \text{ for some element } c}$$

<u>Example</u>: There exists a man that skydives and base jumps. We don't know who he is, so we'll call him John Doe.

Rules of Inference – for Quantifiers

Existential Generalization:

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x\, P(x)}$$

You can go from a statement about a specific element to an existential statement.

Example: Joe skydives and basejumps. Therefore, there exists a man who both skydives and base jumps.

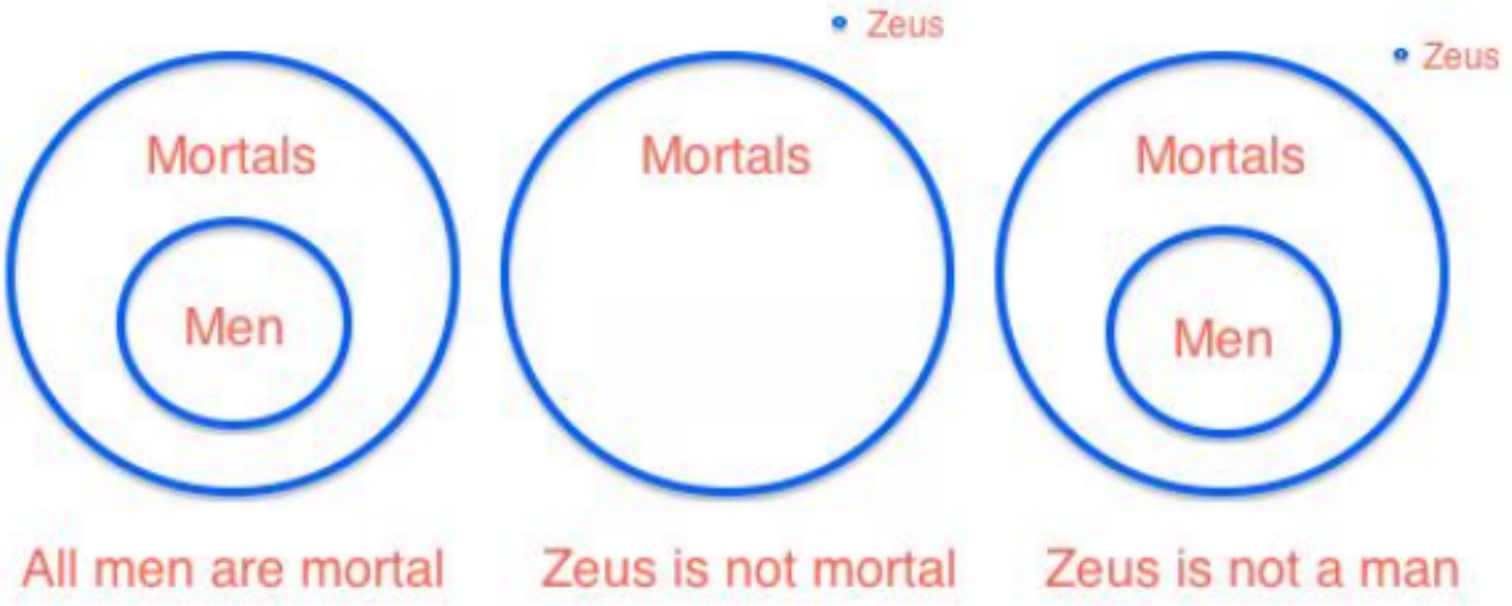
Rules of Inference – for Quantifiers

Universal Modus Tollens

$$\forall x\,(P(x) \rightarrow Q(x))$$

$\neg Q(a)$, where a is a particular element in the domain

$\therefore \neg P(a)$



Rules of Inference – for Quantifiers

Universal Modus Ponens

$$\forall x\big(P(x) \rightarrow Q(x)\big)$$

$P(a)$, where a is a particular element in the domain

$\therefore Q(a)$

Example:

All men are mortal.
Socrates is a man.

Therefore, Socrates is mortal.

	step	justification
1.	$\forall x\,(M(x) \rightarrow D(x))$	premise
2.	$M(SOCRATES)$	premise
3.	$M(SOCRATES) \rightarrow D(SOCRATES)$	universal inst. (1)
4.	$\therefore D(SOCRATES)$	mp (2) and (3)

Rules of Inference – for Quantifiers

Example: Translate the following argument using quantifiers. Show that the conclusion follows from the premises by logical inference.

All lions are fierce.

Some lions do not drink coffee.

Consequently, some fierce creatures do not drink coffee.

Domain: all creatures

$L(x)$: x is a lion.

$F(x)$: x is a fierce creature.

$C(x)$: x drinks coffee

$$\forall x\,(L(x) \rightarrow F(x))$$
$$\exists x\,(L(x) \wedge \neg C(x))$$
$$\overline{\exists x\,(F(x) \wedge \neg C(x))}$$

Rules of Inference – for Quantifiers

(Previous example continued)

Rules of Inference – for Quantifiers

Example: For the following argument, explain which rules of inference are used for each step.

Somebody in this class enjoys whale watching.

Every person who enjoys whale watching cares about ocean pollution.

Therefore, there is a person in this class who cares about ocean pollution.

Rules of Inference – for Quantifiers

(Previous example continued)

Rules of Inference – Fallacies

Fallacy of Affirming the Conclusion

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}$$

Note that is $p = F$ and $q = T$, then both the premises are true and the conclusion is false.

Example: If you travel to Thailand, you will eat fantastic Pad Thai. You ate fantastic Pad Thai. Therefore, you traveled to Thailand.

- People often make invalid arguments when conditionals are involved.

Rules of Inference – Fallacies

Fallacy of Denying the Hypothesis

$$\begin{array}{l} p \to q \\ \neg p \\ \hline \therefore \neg q \end{array}$$

Note that if $p = F$ and $q = T$, then both of the premises are true while the conclusion is false.

Example: If a Run. Hide. Fight email is sent out, then you go hide behind a tree. A Run. Hide. Fight email is not sent out. Therefore, you do not hide behind a tree.

Extra Practice

Example: Translate and show the argument is valid

All hummingbirds are richly colored

No large birds live on honey

Birds that do not live on honey are dull in color

Therefore, hummingbirds are small

Example: Translate the following argument using quantifiers. Then show that the conclusion follows from the premises by logical inference

Babies are illogical

Nobody is despised who can manage a crocodile

Illogical people are despised

Therefore, babies cannot manage crocodiles

Let $B(x)$ mean "x is a baby", $L(x)$ mean "x is logical", $C(x)$ mean "x can handle a crocodile", and $D(x)$ mean "x is despised"

Solutions

Example: Translate and show the argument is valid

All hummingbirds are richly colored

No large birds live on honey

Birds that do not live on honey are dull in color

Therefore, hummingbirds are small

This can be symbolized as

$$\begin{array}{rl} & \forall x\,(B(x) \rightarrow C(x)) \\ & \neg\exists x\,(L(x) \wedge H(x)) \\ & \forall x\,(\neg H(x) \rightarrow \neg C(x)) \\ \hline \therefore & \forall x\,(B(x) \rightarrow \neg L(x)) \end{array}$$

	step	justification
1.	$\forall x\,(B(x) \rightarrow C(x))$	premise
2.	$\neg\exists x\,(L(x) \land H(x))$	premise
3.	$\forall x\,(\neg H(x) \rightarrow \neg C(x))$	premise
4.	$B(a) \rightarrow C(a)$	universal inst. (1)
5.	$\forall x\,(\neg L(x) \lor \neg H(x))$	DeMorgan (2)
6.	$\neg L(a) \lor \neg H(a)$	universal inst. (5)
7.	$\neg H(a) \rightarrow \neg C(a)$	universal inst. (3)
8.	$C(a) \rightarrow H(a)$	contraposition (6)
9.	$B(a) \rightarrow H(a)$	hypothetical syll. (4), (9)
10.	$\neg H(a) \lor \neg L(a)$	commutation (6)
11.	$H(a) \rightarrow \neg L(a)$	that one rule (10)
12.	$B(a) \rightarrow \neg L(a)$	hypothetical syll. (9), (11)
13.	$\therefore \forall x\,(B(x) \rightarrow \neg L(x))$	universal gen. (12)

Note: Step 13 works b/c all instances of a were arbitrary

Example: Translate the following argument using quantifiers. Then show that the conclusion follows from the premises by logical inference

Babies are illogical

Nobody is despised who can manage a crocodile

Illogical people are despised

Therefore, babies cannot manage crocodiles

Let $B(x)$ mean "x is a baby", $L(x)$ mean "x is logical", $C(x)$ mean "x can handle a crocodile", and $D(x)$ mean "x is despised"

Then the argument we want to derive is as follows

$$\begin{array}{ll} & \forall x\,(B(x) \rightarrow \neg L(x)) \\ & \forall x\,(C(x) \rightarrow \neg D(x)) \\ & \forall x\,(\neg L(x) \rightarrow D(x)) \\ \hline \therefore & \forall x\,(B(x) \rightarrow \neg C(x)) \end{array}$$

	step	justification
1.	$\forall x\,(B(x) \to \neg L(x))$	premise
2.	$\forall x\,(C(x) \to \neg D(x))$	premise
3.	$\forall x\,(\neg L(x) \to D(x))$	premise
4.	$B(a) \to \neg L(a)$	universal inst. (1)
5.	$\neg L(a) \to D(a)$	universal inst. (3)
6.	$B(a) \to D(a)$	hypothetical syll.(4), (5)
7.	$C(a) \to \neg D(a)$	universal inst. (2)
8.	$D(a) \to \neg C(a)$	contraposition of (7)
9.	$B(a) \to \neg C(a)$	hypothetical syll. (6), (8)
10.	$\therefore\ \forall x\,(B(x) \to \neg C(x))$	universal gen. (9)