CSCI 2834-003/004: Discrete Structures Workgroup – Fall 2019 **Worksheet 12**

Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

1. Suppose that A and B are events with probabilities $P(A) = 2/3$ and $P(B) = 1/2$.

   (a) What is the largest $P(A \cap B)$ can be? What is the smallest it can be? Give examples for both extremes.

   (b) What is the largest $P(A \cup B)$ can be? What is the smallest it can be? Give examples for both extremes.

2. What is the probability that a 13 card hand contains no pairs?

3. Given a loaded die where 2 is twice as likely to come up as any other number, what is the probability that rolling this die would produce an odd number?

4. Suppose you obtain a second loaded die, where 3 is twice as likely to be rolled as any other number. Using the die form the previous problem and this one, what is the probability that rolling both dice will give a 5 as the resulting sum?

5. Your doctor gives you a test for swine flu, which for the purposes of this exercise we will say is suspected to affect 1 in 10,000 people in the United States. The test is 99% accurate, in that if someone has swine flu, the probability of testing positive is 99%. Suppose the probability of a false positive is 1% and you test positive. What is the probability that you have swine flu?

6. You and your group go to a clown conference, where 75% of the attendees are actually clowns. Naturally, there are also many red ties present - among the clowns, half of them wear red ties, and among the non-clown attendees, only one-fourth of them wear red ties. The keynote speaker at the conference is wearing a red tie. What is the probability that they are a clown?

7. You're a magician trying to figure out one of your competitor's most dazzling tricks. As a last ditch effort, you've made a shady deal with someone who looks suspiciously like Tucker in a back alley to steal your competitor's prop. He gives you a bag containing 6 regular coins and 4 two-faced coins (both sides are heads), leaving you to figure out the trick for yourself.

   (a) Suppose you choose a random coin from the bag and flip it. What is the probability of getting a heads?

   (b) Suppose you choose a random coin from the bag. Then, you flip this coin twice and both results are heads. What is the probability that the coin is two-faced?

   (c) Suppose you flip a particular coin n times and all of the results are heads. How many flips do you need to see come up heads before the probability that the coin is two-faced is greater than 90%?

8. Now suppose your hired thief switched the bag out with one filled with fair and unfair die. You heard that your competitor is using it in her next act. You ask the thief about the dice in the bag. He tells you that there are 10 fair dice and 7 unfair dice, where the unfair dice are four times as likely to roll a 6 as any other number. When you go to the magician's show, you are selected to roll the dice and, based on the outcome of the roll, must guess if the die is fair or unfair.

   (a) What is the probability of any given roll of a random die from the bag resulting in a 6?

   (b) You select a die and roll it 3 times. You get no sixes. What is the probability of this die being unfair?

   (c) How many rolls would you need to be 95% sure of the die being fair or unfair?

9. Suppose that Willem selects a ball by first picking one of two boxes at random and then selecting a ball from this box. The first box contains three orange balls and four black balls, and the second box contains five orange balls and six black balls. What is the probability that Willem picked a ball from the second box if he has selected an orange ball?

10. How would the 3 Doors Problem change if it were instead a 4 Doors Problem? Here, the game show host will still reveal one of the doors that you didn't pick and that does not have the prize behind it. There is still only one prize and three goats, and you are allowed to switch to one of the other two unopened doors.

11. You are given a number N. Write an algorithm to find the number of different ways to write the number as a sum of 1, 3 and 4.