Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

1. What is wrong with this proof?

   Theorem: If n^2 is positive, then n is positive.

   Proof: Suppose that n^2 is positive. Because the conditional statement "If n is positive, then n^2 is positive" is true, we can conclude that n is positive.

2. Suppose A is a proposition that does not depend on the variable x, but the propositional function $P(x)$ does depend on x. Establish these logical equivalences, using logical equivalences for compound propositions and quantifiers.

   (a) $\forall x\ (P(x) \rightarrow A) \equiv \exists x\ P(x) \rightarrow A$
   (b) $\exists x\ (P(x) \rightarrow A) \equiv \forall x\ P(x) \rightarrow A$

   Hint: Recall that $\forall x\ P(x) \equiv P(a) \wedge P(b) \wedge \ldots$ and $\exists x\ P(x) \equiv P(a) \vee P(b) \vee \ldots$, if the domain for x is $a, b, \ldots$

3. Prove that the following rules of inference are tautologies. Try to name the rules of inference or logical equivalences without using any reference material.

   (a) $(p \wedge (p \rightarrow q)) \rightarrow q$
   (b) $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
   (c) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
   (d) $((p \vee q) \wedge \neg p) \rightarrow q$
   (e) $p \rightarrow (p \vee q)$
   (f) $(p \wedge q) \rightarrow p$
   (g) $((p) \wedge (q)) \rightarrow (p \wedge q)$
   (h) $((p \vee q))$

4. Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of

   (a) the positive real numbers.
   (b) the integers
   (c) the nonzero real numbers.

5. Use direct proofs to prove the following:

   (a) Show that the square of an even number is an even number using a direct proof.
   (b) Show that the product of two rational numbers is rational.
   (c) Show that every odd integer can be written as the difference of two squares.

6. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

   (a) a proof by contraposition.
   (b) a proof by contradiction.

7. Prove the following using a proof by contraposition:

   (a) Let $x \in \mathbb{Z}$; If $x^2 - 6x + 5$ is even, then x is odd.
   (b) If a product of two positive real numbers is greater than 100, then at least one of the number is greater than 10.

8. For each of the following proof blocks:

   (a) Identify the domain and translate the English statements into logical statements.

   (b) Use rules of inference and logical equivalences to prove that the given conclusion follows from the associated premises. Be sure to indicate whether you are using a Direct Proof, a Contrapositive Proof, a Proof by Cases, or a Proof by Contradiction. If you use a Proof by Cases, be sure to indicate what each case is, and in each case, which type of proof you are using.

      i. A. *Premise 1*: All students like coffee or like sleep.
         B. *Premise 2*: All students do not like sleep or like getting good grades.
         C. *Premise 3*: All students who are good students like getting good grades.
         D. *Premise 4*: Some students do not like coffee.
         E. *Conclusion*: Some students are not good students.

      ii. A. *Premise 1*: All university employees who are TA's teach students and like teaching.
          B. *Premise 2*: All university employees are TA's and run marathons.
          C. *Conclusion*: All university employees like teaching and run marathons.

      iii. A. *Premise 1*: All CSCI 2824 students either do homework or study.
           B. *Premise 2*: All CSCI 2824 students who study and do not do homework will get good grades.
           C. *Conclusion*: All CSCI 2824 students who get bad grades do homework.

9. *Take home coding problem (this week's concept is fall-through cases)*

   Tony is looking for someone who can code up a basic function that analyzes the truth values of an input. With your python coding expertise you step-up to that challenge. Write a function that takes 2 inputs, the first one is a string argument that represents what logical operation you need to perform. The valid strings are `AND`, `OR`, `CONDITIONAL`, `BICONDITIONAL`. Your second argument is a list of n truth values that you have to evaluate to a final `True` or `False` value which you will return. 2 example test cases are given below -

   (a) `function_name('AND', [True,False]) -> False`

   (b) `function_name('OR', [True,False,False]) -> True`

   For the biconditional case you have to check to make sure there are only 2 values in the list. Return `False` along with an error message if otherwise.