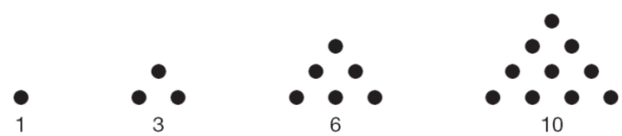
Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in. Since this worksheet contains a lot of problems, a good strategy would be to first skim the worksheet and then discuss and solve the problems which you think are difficult.

Recursion

1. Find $f(2)$, $f(3)$, $f(4)$, and $f(5)$ if f is defined recursively by $f(0) = -1$, $f(1) = 2$, and for $n = 1, 2, 3, ...$
   (a) $f(n+1) = f(n) + 3f(n-1)$
   (b) $f(n+1) = f(n)^2 \cdot f(n-1)$
   (c) $f(n+1) = 3f(n)^2 - 4f(n-1)$
   (d) $f(n+1) = \dfrac{f(n-1)}{f(n)}$

2. The sequence of **triangular numbers** counts the number of points in an equilateral triangle, if there are n rows of points in the triangle. The figure below shows the first four elements of the sequence, T_1, T_2, T_3 and T_4.



   Come up with a recurrence relation relating the n^{th} element of the sequence T_n to the previous elements of the sequence $T_{n-1}, T_{n-2}, \ldots$

3. Find a closed form for each of these recursively defined sequences.
   (a) $a_n = (n-1)a_{n-1}, \quad a_1 = 2$
   (b) $a_n = 2a_{n-1}, \quad a_0 = 3$
   (c) $a_n = a_{n-1} - n, \quad a_0 = 5$. *Hint:* $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

Induction

4. In the kingdom of Ioanatopia, currency comes in 3- and 7-rupee denominations. Prove, using induction, that you can create any amount of money of at least 12 rupees. Did you use strong or weak induction?

5. Suppose the amount of pizza that Aiden eats on any given day n is the sum of the amount of pizza that Aiden ate on the previous day $(n-1)$ and twice the amount that he ate two days before $(n-2)$. In this dream-come-true scenario, a recurrence relation to define how much pizza Aiden eats on any given day is:

$$a_n = a_{n-1} + 2a_{n-2}$$

   Suppose on day 0, Aiden eats no pizza, and on day 1, Aiden eats 1 pizza. These initial conditions are: $a_0 = 0$ and $a_1 = 1$

   Prove using **strong induction** that $a_n = \dfrac{1}{3} \cdot 2^n - \dfrac{1}{3} \cdot (-1)^n$ is a closed form solution to the above recurrence relation.

Counting

6. Suppose you are a minimalist and have only 4 shirts of colors - `blue, red, black, orange` and pants of colors - `blue, black and tan`.

   (a) How many combinations of shirts and pants are possible?

   (b) How many combinations of shirts and pants are possible if you are cannot wear a pant and a shirt of the same color?

7. *Practice coding problem*

   Write a Python function to calculate the n^{th} of the triangular number sequence from Problem 2.

   This can be done using a recursive programming approach, or using an iterative function, to calculate the n^{th} value of the sequence. The function takes only 1 argument, which is n. For example:

   `function_name(7) -> 21`