This assignment is to help you prepare for questions on the new material on the Final Exam. It is NOT TO BE TURNED IN! Solutions will be posted on Piazza closer to the actual final.

1. Determine whether each of the following relations $R \subseteq A \times A$, where A is the set of all CU students, is reflexive, symmetric, transitive, and/or an equivalence relation. Briefly justify each conclusion.

   (a) $(a, b) \in R$ if and only if a shares at least one class with b.

   (b) $(a, b) \in R$ if and only if a has a higher GPA than b.

   (c) $(a, b) \in R$ if and only if a is roommates with b.

2. Consider the relation $R = \{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (2, 0), (2, 1), (2, 2), (3, 3)\}$, where $R \subseteq A \times A$, with $A = \{0, 1, 2, 3\}$.

   (a) Draw the graph of R. **Note**: If possible, it is good practice to organize your graph such that all directed edges are non-intersecting.

   (b) Is the relation R reflexive? Symmetric? Transitive? An equivalence relation? Fully justify your responses.

   (c) The **complement** of a relation $R \subseteq A \times A$ is defined as $\overline{R} = (A \times A) - R$.

      i. What is the set $\overline{R}$ for R as defined in this problem?

      ii. Draw the graph of $\overline{R}$. Again, organize your graph such that all directed edges are non-intersecting.

      iii. Is the following statement true or false? Briefly justify your conclusion. "A relation R is symmetric if and only if its complement $\overline{R}$ is symmetric."

3. For each of the degree sequences shown below, determine whether they represent a valid undirected graph with no self-loops. If they do, draw the graph. If they do not, explain why.

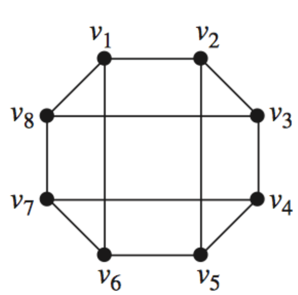
   (a) $4, 3, 2, 1, 0$

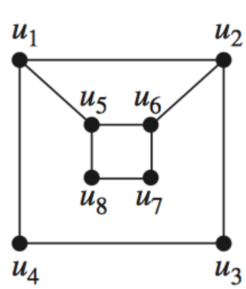
   (b) $2, 2, 2, 2, 2$

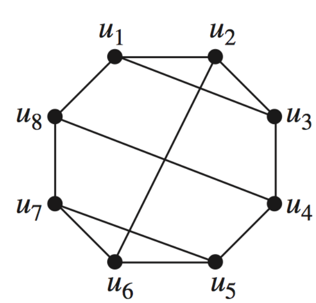
   (c) $1, 1, 1, 1, 1$

   (d) $4, 4, 3, 2, 1$

The next 2 questions involve the undirected graphs below.







4. For each of the graphs above, determine if the graph has an Eulerian Tour. If it does, give one such tour. If it does not, explain why. For graphs that do not contain Eulerian tours, can you **add** a small number of edges so that they do contain one?

5. For each of the graphs above, determine if the graph is Bipartite. If it is, specify a two-coloring of the vertices. If it is not, explain why. For graphs that are not bipartite, can you **remove** a small number of edges so that they become bipartite?

6. Use the Greedy Coloring Algorithm to find a coloring of the vertices of the graphs below. To start out, consider the vertices in alphabetical order. Then choose your own vertex order and repeat the coloring process.