CSCI 2824: Discrete Structures

Lecture 8: Rules of Inference (part 1)

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Rules of Inference

"Valid Arguments"

valid: the conclusion must follow from the truth of the preceding statements (premises)

argument: a sequence of statements that end with a conclusion

- Rules of Inference are the basic tools for establishing the truth of statements.

The Distinction between truth and validity	
TRUTH	**VALIDITY**
Concerned with **what is the case**	Concerned with whether **conclusions** follows from **premises**
	The validity of an argument is independent of the truth or falsity of the premises it contains.

Rules of Inference

An **argument** in propositional logic is a sequence of propositions. All but the final proposition in the argument are called premises and the final proposition is called the conclusion. An argument is valid if the truth of all its premises implies that the conclusion is true.

$$\begin{array}{ll} p \to q & \text{premise} \\ p & \text{premise} \\ \hline \therefore q & \text{conclusion} \end{array}$$

"stencil" of the argument form

An **argument form** in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is valid no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are true.

Rules of Inference

Example: “If you have a current password, then you can log onto the network.”
“You have the current password.”
“Therefore, you can log onto the network.”

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore\ q \end{array}$$

Example: ”If you have access to the network, then you can change your grade.”
“You have access to the network.”
”You can change your grade.”

Rules of Inference

Rules of Inference – Relatively simple argument forms

Question: What about Truth Tables to prove an argument form is valid?

Answer: You can always use a truth table to verify the validity of an argument form.
But what if there are 10 different propositional variables?

$$2^{10} = 1024 \text{ rows}$$

Rules of Inference

For our argument to be valid, it must be the case that there is no situation (i.e., truth values for *p* and *q*) in which the premises of the argument are true but the conclusion false.

We joined the premises with the conclusion in a conditional (as *premises* → *conclusion*)

- So for the argument to be valid, the conditional describing it must be **always true** (i.e., it needs to be a tautology)

Check with a truth table:

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore\ q \end{array}$$

premises

p	q	$p \to q$	$(p \to q) \land p$	$((p \to q) \land p) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Rules of Inference

Modus Ponens – Law of Detachment

- ➢ Latin for *mode that affirms*

$$\begin{array}{l} \quad p \\ \quad p \rightarrow q \\ \hline \therefore q \end{array}$$

- If a conditional statement and the hypothesis of the conditional statement are both true, then the conclusion must also be true.

Rules of Inference

Modus Tollens – Law of Detachment

- Latin for *mode that denies*

$$\begin{array}{l} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array}$$

E.g. If it is day, then it is not night.
But it is night.
Therefore, it is not day.

$p \rightarrow q$

$\neg q$

$\therefore \neg p$

- If a conditional statement is true, then so is its contrapositive.

Rules of Inference

Derivation of Modus Tollens:

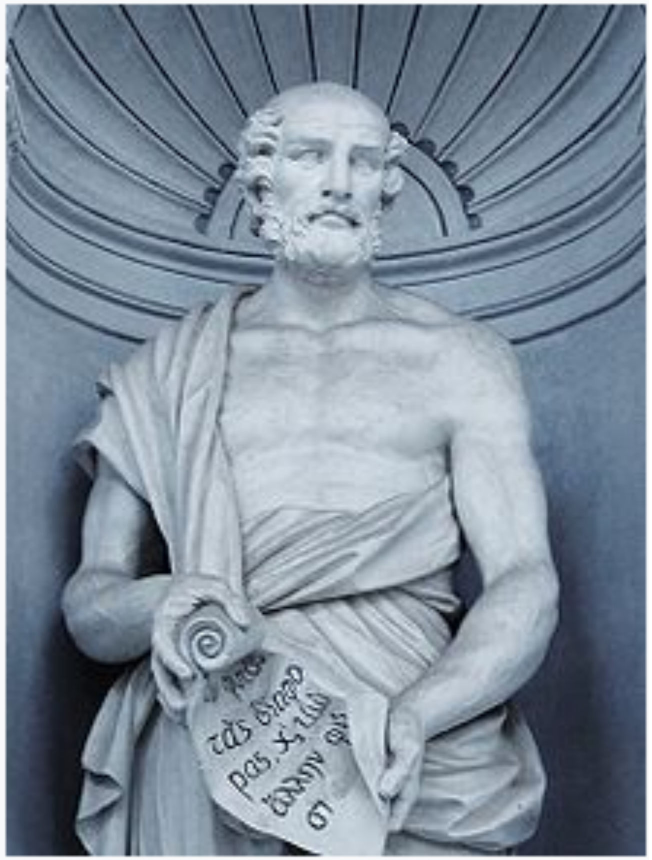
	Step	Justification
1.	$p \to q$	premise
2.	$\neg q$	premise
3.	$\neg q \to \neg p$	law of contraposition (1)
4.	$\therefore \neg p$	by modus ponens (2), (3)

Rules of Inference

Hypothetical Syllogism

- Greek : *Συλλογισμος* – *conclusion or inference*

$$
\begin{array}{r}
p \to q \\
q \to r \\
\hline
\therefore p \to r
\end{array}
$$



Θεοφραστος

- sometimes called chain rule or transitivity of implication

e.g from math

$a < b$ and $b < c \Rightarrow a < c$

Rules of Inference

Disjunctive Syllogism

- aka *modus tollendo ponens* – mode that affirms by denying

$$\begin{array}{l} \quad p \vee q \\ \neg p \\ \hline \therefore\ q \end{array}$$

E.g. Jacob is having either spaghetti or chicken
soup for dinner.
He doesn't have spaghetti.
Therefore, he must be having chicken soup.

- also sometimes called disjunction elimination or elimination

Rules of Inference

Derivation of Disjunctive Syllogism:

	Step	Justification
1.	$p \vee q$	premise
2.	$\neg p$	premise
3.	$\neg p \to q$	RBI of (1)
4.	q	modus ponens of (2), (3)

Rules of Inference

Addition

$$\frac{p}{\therefore p \lor q}$$

Simplification

$$\frac{p \land q}{\therefore p}$$

Conjunction

$$\frac{\begin{array}{c} p \\ q \end{array}}{\therefore p \land q}$$

Resolution

$$\frac{\begin{array}{c} p \lor q \\ \neg p \lor r \end{array}}{\therefore q \lor r}$$

- These rule of inference can also be found in Table 1 on page 72 of our textbook.

Rules of Inference

Derivation of Resolution:

	Step	Justification
1.	$p \vee q$	premise
2.	$\neg p \vee r$	premise
3.	$\neg p \rightarrow q$	RBI of (1)
4.	$p \rightarrow r$	RBI of (2)
5.	$\neg q \rightarrow p$	contrapositive of (3)
6.	$\neg q \rightarrow r$	hypothetical syllogism (5), (4)
7.	$q \vee r$	RBI of (6)

$\neg(p \wedge \neg r)$

Rules of Inference

Example: Use rules of inference to show that the following argument is valid.

$(p \vee q) \rightarrow \neg r$

$\neg r \rightarrow s$

p

$\therefore\ s$

	steps	justifications
1.	$(p \vee q) \rightarrow \neg r$	premise
2.	$\neg r \rightarrow s$	premise
3.	p	premise
4.	$(p \vee q) \rightarrow s$	hypothetical syllogism (1), (2)
5.	$p \vee q$	Addition (3)
	$\therefore\ s$	modus ponens (4), (5)

Rules of Inference

Example: Which argument form is used by the following:

Socrates is immortal (p) or Socrates is a man (q).
Socrates is not immortal. $\neg p$

Therefore, Socrates is a man. q

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Disjunctive syllogism

Rules of Inference

Example: What could you conclude from the following?

If it is sunny outside, I will go for a hike.
If I go for a hike, then I will hike on Mount Sanitas.

hypothetical syllogism

∴ If it is sunny outside, then I will hike on Mt. Sanitas.

Extra Practice

Ex. 1 Show that $p \wedge q, p \to \neg r, q \to \neg s$, therefore $\neg r \wedge \neg s$ is a valid argument.

Solution

Ex. 1 Show that $p \wedge q, p \rightarrow \neg r, q \rightarrow \neg s$, therefore $\neg r \wedge \neg s$ is a valid argument.

	step	justification
1.	$p \wedge q$	premise
2.	$p \rightarrow \neg r$	premise
3.	$q \rightarrow \neg s$	premise
4.	p	simplification of (1)
5.	$\neg r$	modus ponens (2), (4)
6.	q	simplification of (1)
7.	$\neg s$	modus ponens (3), (6)
6.	$\therefore \neg r \wedge \neg s$	conjunction (5) and (7)