CSCI 3202: Intro to Artificial Intelligence

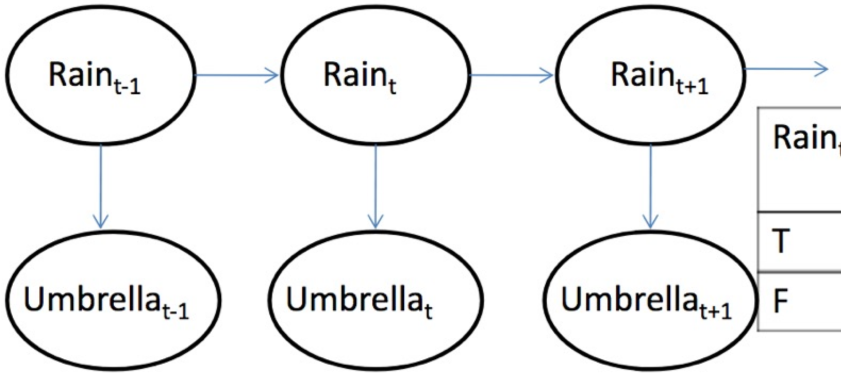
Lecture 27: Hidden Markov Models

Rhonda Hoenigman

Department of Computer Science

$Rain_{t-1}$	$P(Rain_t)$
T	0.7
F	0.3

$P(Rain_0 = T) = 0.5$



$Rain_t$	$P(Umbrella_t)$
T	0.9
F	0.2

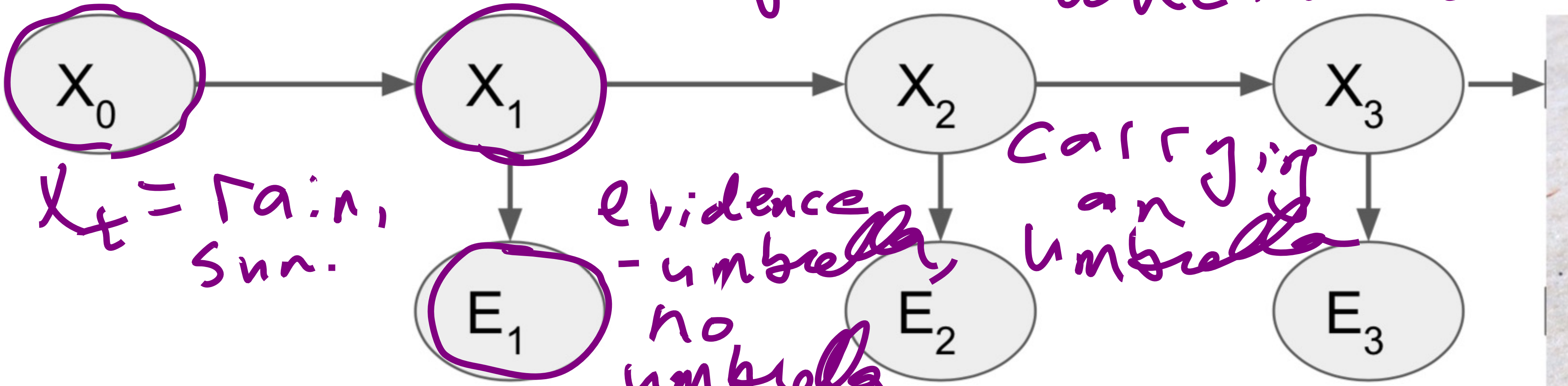
HMMs — Adapted from Russel and Norvig, Chapter 15.

Hidden Markov Models (HMMs)

Example: Suppose you are a graduate student in a basement office. You are writing your dissertation, so you don't get to leave very often.

You are curious if it is raining, and the only contact you have with the outside world is through your advisor. If it is raining, she brings her umbrella 90% of the time, and has it just in case on 20% of sunny days. You know that historically, 40% of rainy days were followed by another rainy day, and 30% of sunny days were followed by a rainy day.

Is it raining outside? We only know whether advisor is carrying an umbrella



X_t = rain, sun.

evidence - umbrella, no umbrella

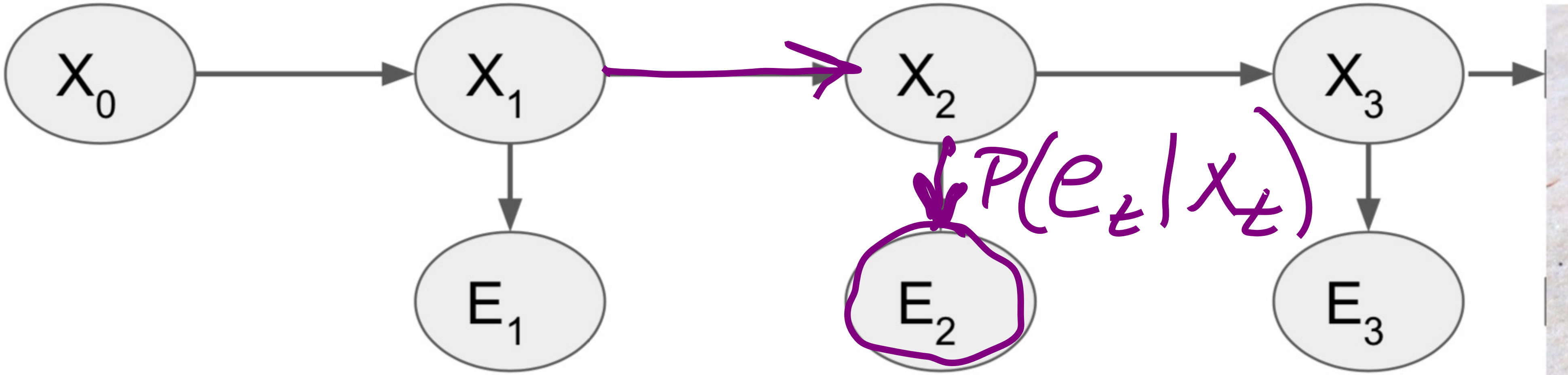
Hidden Markov Models (HMMs)

Example: Suppose you are a graduate student in a basement office. You are writing your dissertation, so you don't get to leave very often.

Evidence: advisor carrying an umbrella
State: Rain or sun

Can you predict the state X_t from the evidence E_t?

$P(X_t \mid X_{t-1})$



Hidden Markov Models (HMMs)

Example: People in a small village are either Healthy or have Fever. A doctor diagnoses Fever by asking people how they feel, and they say either normal, cold, or dizzy. In the village, 60% of people are Healthy and 40% have Fever. If a person reports feeling dizzy, there is a 10% chance they are healthy, a 40% chance if they report feeling cold, and a 50% chance if they report feeling normal.

A patient visits the doctor two days in a row. What is P(Fever) on each day if they report feeling normal on day 1 and dizzy on day 2?

Evidence: How the patient feels. cold, dizzy, normal

State: Healthy or Fever

Hidden Markov Models (HMMs)

- Type of Bayes net where we use a Markov process to reason about current state given the evidence
- True state is unknown
- Determine most likely state given the evidence

- Each state has:
  - a CPT that describes transition probabilities: $P(X_t \mid X_{t-1})$

    Markov
  - a CPT for the evidence, also called emission probabilities: $P(E_t \mid X_t)$

    Probability of observing the evidence given the state
  - Inference – probability of being in a state given the evidence: $P(X_t \mid E_t)$

    Bayes

- Initial probability distribution $P(X_1)$

Hidden Markov Models (HMMs) – inference

- **Filtering:** Type of inference. Compute the belief state given the evidence observed so far.
  - $P(X_t \mid e_{1:t})$ t = 3, incorporate evidence for days 1, 2, 3
  - Examples:
    - What is probability of rain today given the observation of the umbrella every day, including today?
    - What is the probability of Fever given the observation of cold, dizzy, cold evidence?

Hidden Markov Models (HMMs) – inference

- **Prediction:** Type of inference. Compute the distribution over the future state given all evidence up to current state.

  - *$P(X_{t+k} \mid e_{1:t})$, where $k > 0$. (We typically use $k = 1$)*

  - Examples:
    - What is probability of rain tomorrow, or in three days, given the umbrella observations up to today?

- What will happen tomorrow given what we've observed through today.

Hidden Markov Models (HMMs) – smoothing

- **Smoothing:** Compute the belief state given the evidence over a previous state using all evidence up to current state.

  - $P(X_k \mid e_{1:t})$, where $0 <= k < t$

  - Examples:
    - What is probability that it rained two days ago given our umbrella observations up through today?
    - What is the probability that a person was Healthy two days ago given our observations up through today?

- We don't have complete information about the past

Hidden Markov Models (HMMs) – most likely explanation

- Given a sequence of observations, find the most likely state sequence that generated those observations.

  - $argmax_{\mathbf{x}_{1:t}} P(\mathbf{X}_{1:t} \mid \mathbf{e}_{1:t})$

  - Examples:
    - Umbrella appears three days in a row and then doesn’t appear on the fourth day. The most likely explanation is that it rained on the first three days and then not on the fourth day.

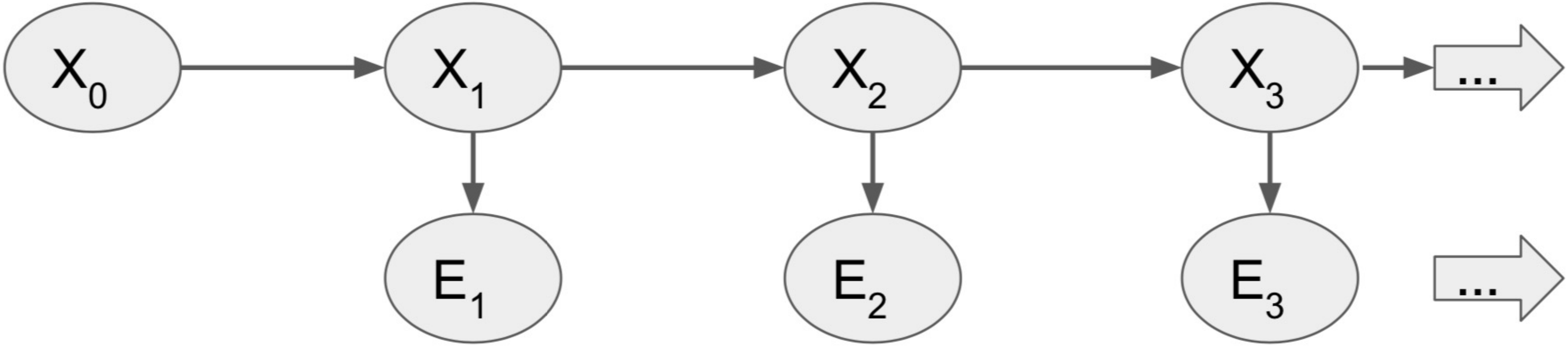
Hidden Markov Models (HMMs)

Notation:

$$X_{\{0:t\}} = [X_0, X_1, X_2, \dots, X_t]$$

Sensor Markov Assumption: Measurement E_t conditionally independent of all previous measurements and states, given the state X_t

$$P(E_t \mid X_{\{o:t\}}, \boxed{E_{\{1:t-1\}})} = P(E_t \mid X_t)$$



HMM example

Example: People in a small village are either Healthy or have Fever. A patient visits three days in a row and reports feeling normal, dizzy, then normal. What is the P(Fever) on each day?

States? = [Healthy, Fever]

Evidence? = {normal, cold, dizzy}

HMM example

Example: People in a small village are either Healthy or have Fever. A patient visits three days in a row and reports feeling normal, dizzy, then normal. What is the P(Fever) on each day?

Initial distribution [Healthy = 0.6, Fever = 0.4]

Transition CPT Healthy ⇄ fever

X_{t-1}	X_t	$P(X_t \mid X_{t-1})$
h	h	.7
h	f	.3
f	h	.4
f	f	.6

Emission CPT $P(E_t \mid X_t)$

Healthy	$P(E_t \mid Healthy_t)$
dizzy	0.1
cold	0.4
normal	0.5

Fever	$P(e \mid fever)$
dizzy	0.6
cold	0.3
normal	0.1

HMM Example

Track current state estimate and update it as new evidence becomes available

We want:

$$P(X_t | E_{1:t}) = \sum_{x_{t-1}} P(X_t | X_{t-1}) P(X_{t-1})$$

incorporate evidence

$$B(X_t) = P(X_t | e_{1:t}) = \underline{\alpha P(e_{1:t} | X_t) P(X_t)}$$

Bayes Rule

$$P(X_t | e_{1:t}) = \alpha \frac{P(e_t | X_t) P(X_t)}{\cancel{P(e_{1:t})}} \leftarrow \text{unknown}$$

HMM Example – Day 1

Initial dist $= [0.6, 0.4]$

$$P(X_1) = \sum_{x_0} P(X_1 \mid X_0) P(X_0)$$

$$= [h \to h + f \to h,\ h \to f + f \to f]$$

$$= [0.7 \times 0.6 + 0.4 \times 0.4,\ 0.3 \times 0.6 + 0.6 \times 0.4]$$

$$= [.58, .42]$$

$$\boxed{P(X_1)}$$

incorporate evidence

$$P(X_1 \mid normal_1) = \alpha P(normal_1 \mid X_1) P(X_1)$$

$$= \alpha [.5 \times .58,\ .1 \times .42]$$

$$= \alpha [.29,\ .042] \qquad \alpha = .29 + .042$$

$$= [.87,\ .13]$$

probability of being healthy on day 1 given feeling normal is .87

HMM Example – Day 2

- Patient feels dizzy

Our belief about the state of the patient influences our belief on day 2.

$P(X_2 \mid normal_1, dizzy_2)$

$[.87, .13]$

$$P(X_2 \mid normal_1) = \sum_{x_1} P(X_2 \mid x_1) P(x_1 \mid normal_1)$$

$$= [.7 \times .87 + .4 \times .13, \; .3 \times .87 + .6 \times .13]$$

$$= [.661, .339]$$

Evidence

$$P(X_2 \mid n_1, dizzy_2) = \alpha P(d_2 \mid X_2) P(X_2 \mid normal_1)$$

$$= \alpha [.1 \times .661, .6 \times .339]$$

$$= [.245, .755]$$

HMM Example – Day 3

What happens next?

We'll pick up here after the break.

HMM filtering example

Example: Back to the rain and the grad student stuck in the basement. If it is raining, the advisor brings her umbrella 90% of the time, and has it just in case 20% of sunny days. You know that historically, 40% of rainy days were followed by another rainy day, and 30% of sunny days were followed by a rainy day.

If you observe an umbrella two days in a row, what is the probability of rain on each day?

What is the evidence model and the transition model?

HMM filtering example – Day 1

Example: Back to the rain and the grad student stuck in the basement. If it is raining, the advisor brings her umbrella 90% of the time, and has it just in case 20% of sunny days. You know that historically, 40% of rainy days were followed by another rainy day, and 30% of sunny days were followed by a rainy day.

HMM filtering example – Day 2

Example: Back to the rain and the grad student stuck in the basement. If it is raining, the advisor brings her umbrella 90% of the time, and has it just in case 20% of sunny days. You know that historically, 40% of rainy days were followed by another rainy day, and 30% of sunny days were followed by a rainy day.