This assignment is due on Friday October 25 to Gradescope by Noon. You are expected to write or type up your solutions neatly. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: Make sure to clearly write your full name, the lecture section you belong to (001 or 002), and your student ID number at the top of your assignment. You may **neatly** LaTex your solutions for +1 extra credit on the assignment.

1. Consider the function $f(n) = 35n^3 + 2n^3 \log(n) - 2n^2 \log(n^2)$ which represents the complexity of some algorithm.

   (a) Find a tight big-**O** bound of the form $g(n) = n^p$ for the given function f with some natural number p. What are the constants C and k from the big-**O** definition?

   (b) Find a tight big-$\boldsymbol{\Omega}$ bound of the form $g(n) = n^p$ for the given function f with some natural number p. What are the constants C and k from the big-$\boldsymbol{\Omega}$ definition?

   (c) Can we conclude that f is big-$\boldsymbol{\Theta}(n^p)$ for some natural number p?

2. There is a long line of trick-or-treaters outside of Rachel's house, and with good reason! Word has gotten around that she will give out 3^k pieces of candy to the k^{th} trick-or-treater to arrive. Children love her, dentists despise her.

   (a) Expressed in summation notation (using a Σ), what is c_n, the total amount of candy that Rachel should buy to accommodate n trick-or-treaters total?

   (b) Use induction to prove that the total amount of candy that Rachel needs is given by the closed-form solution:

   $$c_n = \frac{3^{n+1} - 3}{2}$$

   Be sure to clearly state your induction hypothesis, and state whether you're using weak induction or strong induction.

3. Consider the following matrices.

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$$

For this problem, you must calculate the product $P = ABC$. But wait! Don't start yet! Matrix multiplication is associative, which means we can calculate the product P by first computing the matrix (AB), then multiplying this by C to obtain $P = (AB)C$. Or we could first compute the matrix (BC), then multiply it by A to obtain $P = A(BC)$. Furthermore, to multiply an $m \times n$ matrix by an $n \times k$ matrix requires $m \times n \times k$ multiplications. **Note:** we are only counting multiplications here.

(a) Suppose A is $m \times n$, B is $n \times k$ and C is $k \times p$. How many multiplications are needed to calculate P in the order $(AB)C$? Do not just write down an expression; show your work/justification!

(b) For the same matrix dimensions specified in (a), how many multiplications are needed to calculate P in the order $A(BC)$? Again, do not just write down an expression.

(c) Based on the specific dimensions of A, B, and C in the problem description, which multiplication order would be the most efficient?

(d) Calculate $P = ABC$ using whichever order you specified in part (c).

4. Let the sequence T_n be defined by $T_1 = T_2 = T_3 = 1$ and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$. Use induction to prove that

$$T_n < 2^n \quad \text{for} \quad n \geq 4$$

Be sure to clearly state your induction hypothesis, and state whether you're using weak induction or strong induction for each part.

5. Find the closed form solutions of the following recurrence relations with given initial conditions. Use **forward substitution** or **backward substitution** as described in Example 10 in the text.

(a) $a_n = -a_{n-1}$, $a_0 = 5$

(b) $a_n = a_{n-1} + 3$, $a_0 = 1$

(c) $a_n = a_{n-1} - n$, $a_0 = 4$

(d) $a_n = 2na_{n-1}$, $a_0 = 3$

(e) $a_n = -a_{n-1} + n - 1$, $a_0 = 7$