

Read these instructions!

This assignment is **due on Friday, November 15 to Gradescope by 12 PM noon**. You are expected to write up your solutions neatly, with **full explanations and justifications** when necessary. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own. **Important:** On the **FRONT** of your assignment clearly write your full name and the lecture section you belong to (001 or 002). You may upload your assignment as a pdf or as images of your work. Make sure that your images/scans are clear or you will lose points/possibly be given a 0. Additionally, please be sure to match the problems from the Gradescope outline to your uploaded images.

1. Suppose Ioana has 21 cookies to share with her friends Anna, Jeff, Walker, and Nicole.
 - (a) How many ways are there for Ioana to distribute the cookies among herself and her 4 friends?
 - (b) Suppose Ioana wants to keep at least 5 cookies for herself. Now, how many ways are there for Ioana to distribute the cookies?
2. Consider all length-12 strings of all uppercase letters. Letters may be repeated. Please do **not** simplify your answers.
 - (a) How many such strings are there?
 - (b) How many such strings contain the word NINJA?
 - (c) How many strings contain neither the word NINJA nor the word TURTLES?
3. Yahtzee is a game in which a player rolls five six-sided dice simultaneously. The actual rules are a bit more involved, but for the sake of simplicity, let us only consider the case of rolling all five dice at once.
 - (a) What is the probability of obtaining all unique outcomes on each of the five dice rolled? For example, $\{1, 2, 4, 5, 6\}$ is all unique outcomes, but $\{1, 2, 4, 5, 2\}$ repeats a 2, so those outcomes are not all unique.
 - (b) A “small straight” consists of four numbers all in a sequence, plus one other die that can be anything. For example, one such outcome could be $\{2, 3, 4, 5, 3\}$, and another could be $\{2, 3, 4, 5, 6\}$ (we are ignoring the distinction between a small and a large straight). What is the probability of rolling a small straight when you roll all five dice?
Caution: beware of counting specific outcomes multiple times!
 - (c) You roll all five dice but then your phone alerts you that you’ve caught a new Pokémon with your Fortnite. Nice! You become so engrossed in the video games that you don’t see what the outcome of the dice rolls is. Your kind friend tells you that the dice are all unique. Given this information, what then is the probability of that you have rolled a small straight?
 - (d) Are the events [roll a small straight] and [roll all unique outcomes] independent? Fully justify your answer **with math**.

4. Solve the following problems and be sure to show all your work.

- (a) What is the coefficient on $x^{57}y^{43}$ in the expansion of $(x + y)^{100}$? Do not simplify your answer.
- (b) What is the coefficient on $x^{57}y^{43}$ in the expansion of $(-x - 3y)^{100}$? Do not simplify your answer.
- (c) In the expansion of $(x + y)^{100}$, is there a term whose only x and y components are $x^{58}y^{43}$? Why or why not?
- (d) How many possible distinct rearrangements of the letters SPACESHIP are there? Do not simplify your answer.

Hint: Any two of the same letter are indistinguishable from one another. For example, the two Ps are indistinguishable.