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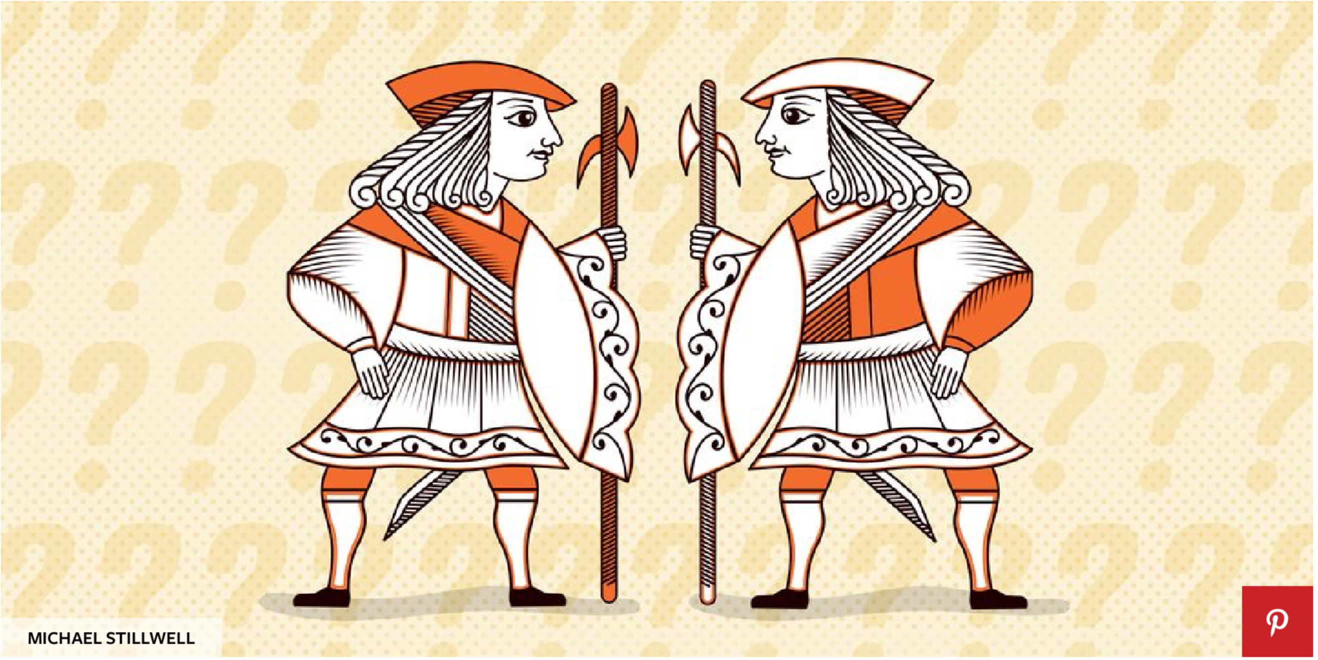
CSCI 2824: Discrete Structures

Lecture 3: Propositional Logic

Rachel Cox

Department of Computer Science



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Announcements & Reminders

- Quizlet 01 posted on Moodle. Due Wednesday Sept. 4th at 8am.
- HW1 posted on Moodle. Due Friday Sept. 6th at 12pm (Noon)
- Lecture slides, Office Hours } posted/linked on Moodle
- Youtube Channel! 'CSCI 2824 - Discrete Structures'

Warmup

Example: How many bits are needed to encode each lowercase letter of the English alphabet?

A - 1

B - 2

C - 3

D - 4

⋮

Z - 26

$(26)_{10} = (11010)_2$?

26	0
13	1
6	0
3	1
1	1

$\boxed{\text{5 bits are required}}$

Propositional Logic

Definition: The basic building block of logic is a **proposition**. A proposition is a declarative statement that is either true or false, but not both.

4 Types of Sentences

Declarative Sentence
- Tells something.
- Ends with a period. (.)

Interrogative Sentence
- Asks a question.
- Ends with a question mark. (?)

Exclamatory Sentence
- Shows strong feeling.
- Ends with a period. (!)

Imperative Sentence
- Gives a command.
- Ends with a period. (. or !)

Propositional Logic

Examples of Propositions

Boulder is a city in Colorado.

Golden is the capital of Colorado.

$2 + 2 = 5$

$1 + 2 = 3$

Examples of NOT Propositions

Don't do that.

Where are you going?

6

$x + 2 = 3$

$1 + 2 = 3$ True

$2 + 2 = 3$ False

Propositional Logic

Definition: The **truth value** of a proposition is **true** (denoted T) if the proposition is true; it is **false** (denoted F) if the proposition is false.

1. Boulder is a city in Colorado. True
2. 2+2 = 5 False

Propositional Logic

$\neg p$ "not p"

Definition: Let p be a proposition. The **negation** of p, denoted by $\neg p$, is the proposition "It is not the case that p". The truth value of $\neg p$ is the opposite of the truth value of p.

Example: Let p denote the proposition: "It is raining today."

Then, $\neg \boldsymbol{p}$ denotes: "It is **not the case that** it is raining today." or "It is **not** raining today."

Propositional Logic

Definition: It is convenient to tabulate of the possible truth values for the various configurations of the propositions. This is done using a **truth table**.

Given simple propositions p and q, the truth table allows us to enumerate all possible truth values of combinations of p and q.

Example: Give the truth table for p and $\neg p$

p	$\neg p$
T	F
F	T

Propositional Logic – Connectives, Logical Operators

With connectives, we can begin combining propositions to make compound and far more complex statements.

conjunction:	“and”	denoted $\wedge$
disjunction:	“or”	denoted $\vee$
conditional:	“if-then”	denoted $\rightarrow$
biconditional:	“if and only if”	denoted $\Leftrightarrow$

Propositional Logic – Connectives, Logical Operators

Definition: Let p and q be propositions. The **conjunction** of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ has the truth value T if both p and q are T and is F otherwise.

Example: Let p = "it is dark outside" and q = "my house is haunted"

p $\wedge$ q

- It is dark out side ***and*** my house is haunted. $p \wedge q$

$\neg p$ $\wedge$ q

It is light outside, but my house is still haunted. $\neg p \wedge q$

Truth Table for a Conjunction: $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- A conjunction is true when both p <u>and</u> q are true.
- False otherwise

Propositional Logic – Connectives, Logical Operators

"p or q"

Definition: Let p and q be propositions. The **disjunction** of p and q, denoted by $p \vee q$, is the proposition "p or q". The disjunction $p \vee q$ has the truth value T if either p or q are T and is F otherwise.

Example: Let p = "it is dark outside" and q = "my house is haunted"

It is dark out side ***or*** my house is haunted. $p \vee q$

This is an "inclusive or". So one proposition could be true, or they both could be true and the overall statement would be true.

Truth Table for a Disjunction: $p \lor q$

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

- False when p, q are both false
- True otherwise

Propositional Logic – Connectives, Logical Operators

Definition: Let p and q be propositions. The **exclusive or** of p and q, denoted $p \oplus q$, is the proposition that is true when exactly one of p or q is true, and false otherwise.

- Also abbreviated as "xor" sometimes

Example: Let p = "It is daytime." and q = "It is nighttime."

It is daytime ***or*** it is nighttime.

"Exclusive or" means that we could have either one of the propositions be true, but not both.

Truth Table for an Exclusive Or: $p \oplus q$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

- True when one and only one of the p and q are true.

Propositional Logic – Connectives, Logical Operators

Definition: Let p and q be two propositions. The **conditional** "if p then q", denoted by $p \to q$, is false when p is true but q is false, and true otherwise.

- The conditional describes an *if-then* relationship between the two propositions.
- Think of the conditional $p \to q$ as defining a rule. What are the cases where the rule holds or where the rule is broken.

Example: If you get a 100% on the final, then you will get an A.

Other ways to express $p \longrightarrow q$:

- If p, then q.
- If p, q.
- p is sufficient for q.
- q if p.
- q when p.
- q unless $\neg p$.
- p implies q
- p only if q $\neg q$ then $\neg p$
- A sufficient condition for q is p.
- q whenever p
- q is necessary for p
- q follows from p

Truth Table for a Conditional: $p \longrightarrow q$

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Propositional Logic – Connectives, Logical Operators

Definition: Let p and q be two propositions. The **biconditional** "p if and only if q", denoted by $p \Leftrightarrow q$, or p **iff** q, is true when p and q have the same truth value, and false otherwise.

- The conditional describes an *if-and-only-if* relationship between the two propositions.

Example: Let p = "A polygon has 3 sides." and q = "It is a triangle."

Truth Table for a biconditional: $p \Leftrightarrow q$

p	q	$p \Leftrightarrow q$

Propositional Logic – Compound Propositions

Compound propositions are constructed by linking together multiple simple propositions using connectives.

Example: Construct a truth table for $(p \longrightarrow q) \land (\neg p \longrightarrow r)$

Order of Operations/Precedence of Logical Operators

1. Negation
2. Conjunction over disjunction $p \land q \lor r$ means $(p \land q) \lor r$
3. Conditionals, biconditionals $p \lor q \longrightarrow r$ means $(p \lor q) \longrightarrow r$

Propositional Logic – Compound Propositions

Example: Construct a truth table for $(p \longrightarrow q) \wedge (\neg p \longrightarrow r)$

p	q	r	$\neg p$	$p \longrightarrow q$	$\neg p \longrightarrow r$	$(p \longrightarrow q) \wedge (\neg p \longrightarrow r)$

Propositional Logic – Knights and Knaves

Example: The island of Knights and Knaves. Suppose you are on an island where there are two types of people: Knights always tell the truth, and Knaves always lie.

Suppose on this island, you encounter two people, Alfred and Batman. Let's call them A and B for short. Suppose A tells you "I am a Knave or B is a Knight." Use a truth table to determine what kind of people A and B are.

p: Alfred is a Knight.

q: Batman is a Knight.

A's statement: $\neg p \vee q$

Propositional Logic – Knights and Knaves

$$\boldsymbol{p} \Leftrightarrow (\neg \boldsymbol{p} \vee \boldsymbol{q})$$

It must be True on this island that is A is a Knight, then his statement is true and if A's statement is true, then he must be a knight.

p	q			
T	T			
T	F			
F	T			
F	F			