

Get in small groups (about 4 students maximum) and work out these problems on paper/the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

- Given one die where 1 is twice as likely to be rolled as any other number and another die where 5 is twice as likely to be rolled as any other number, if you roll both, what is the probability of their sum being 6?
- In the Mushroom Kingdom, currency comes in denominations of 1, 3, 5, and 7 coins. After purchasing a fire flower, Toad owes you 25 coins in change. If he gives you the change **one coin at a time**, how many ways are there for him to make your change?
- Find $f(n)$ with $n = 1, 2, 3, 4, 5$ for the following recurrences:
 - $f(n+1) = 3f(n)$
 - $f(n+1) = 2^{f(n)}$
 - $f(n)^2 - 2f(n) - 2$
 - $3^{f(n)/3}$
- Find closed-form solutions for each of the recurrence relations in the previous problem.
- Using the following table, find $P(\text{spam}|\text{cruise})$ and $P(\text{ham}|\text{cruise})$:

Spam	Ham	Ham	Spam	Spam
free	fly	cruise	money	pills
cruise	wedding	sister	free	deal
deal	cruise	trip	fly	money
trip	gift	deal	nigeria	free

Should mail with the word "cruise" be classified as ham or spam?

- Given the recurrence relation $a_n = 7a_{n-1} - 2a_{n-2}$:
 - What is the guess you plug in?
 - Find the characteristic polynomial
- For the following recurrence relations, identify them as either linear or non-linear and either homogeneous or non-homogeneous, as well as giving their degree:
 - $a_n = 2a_{n-1} - 3a_{n-3}$
 - $a_n = 5a_{n-2} + n$
 - $a_n = 7(a_{n-1})^2 + 9a_{n-2}$
- Find a solution to the recurrence relation $a_n = 4a_{n-1}$ with initial condition $a_1 = 12$
- Find a solution to the recurrence relation $a_n = a_{n-1} + 12a_{n-2}$ with initial conditions $a_0 = 2$ and $a_1 = -13$
- Find a solution to the recurrence relation $a_n = 10a_{n-1} - 25a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 15$
- Find a general solution to the recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$

12. **Review Problem** - Translate the following into propositions and connectives. State the converse, contrapositive, and inverse of each of these conditional statements:

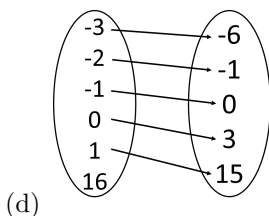
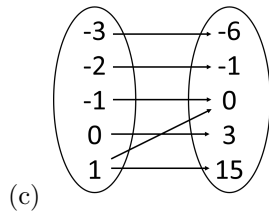
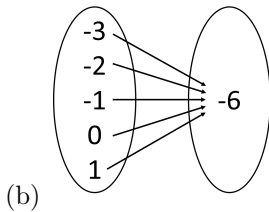
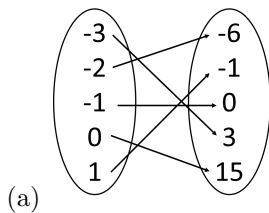
- (a) If it snows today, I will ski tomorrow.
- (b) I come to class whenever there is going to be a quiz.
- (c) A positive integer is a prime only if it has no divisors other than 1 and itself.

13. **Review Problem** - Suppose that a Bayesian spam filter is trained on a set of 10,000 spam messages and 5000 messages that are not spam. The word "enhancement" appears in 1500 spam messages and 20 messages that are not spam, while the word "herbal" appears in 800 spam messages and 200 messages that are not spam. Estimate the probability that a received message containing both the words "enhancement" and "herbal" is spam. Will the message be rejected as spam if the threshold for rejecting spam is 0.9?

14. **Recurrence Relations** -

- (a) Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5$, $a_1 = -9$, and $a_2 = 15$
- (b) Find the solution of the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2} + 2n + n + 3$ with $a_0 = 1$ and $a_1 = 4$.
- (c) Find all solutions of the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2n$.

15. Classify the following as a **function**, a **relation**, both a **function** and a **relation**, or neither a **function** nor a **relation**.



16. Write a python function that calculates the N^{th} term of a K-Fibonacci series. A K-Fibonacci series is defined as a sequence in which the first K -terms are 1 and the subsequent terms are the sum of the previous K terms. The function should 2 parameters, N and K and it should return the N^{th} term of the sequence.
- (a) Implement this function both recursively and dynamically.
 - (b) Time the functions and analyze which function is faster.
17. **Challenge Problem:** Solve Problem 2 on this worksheet for the case where Toad gives you the change all at once.