CSCI 3202: Intro to Artificial Intelligence

Lecture 26: Markov Models

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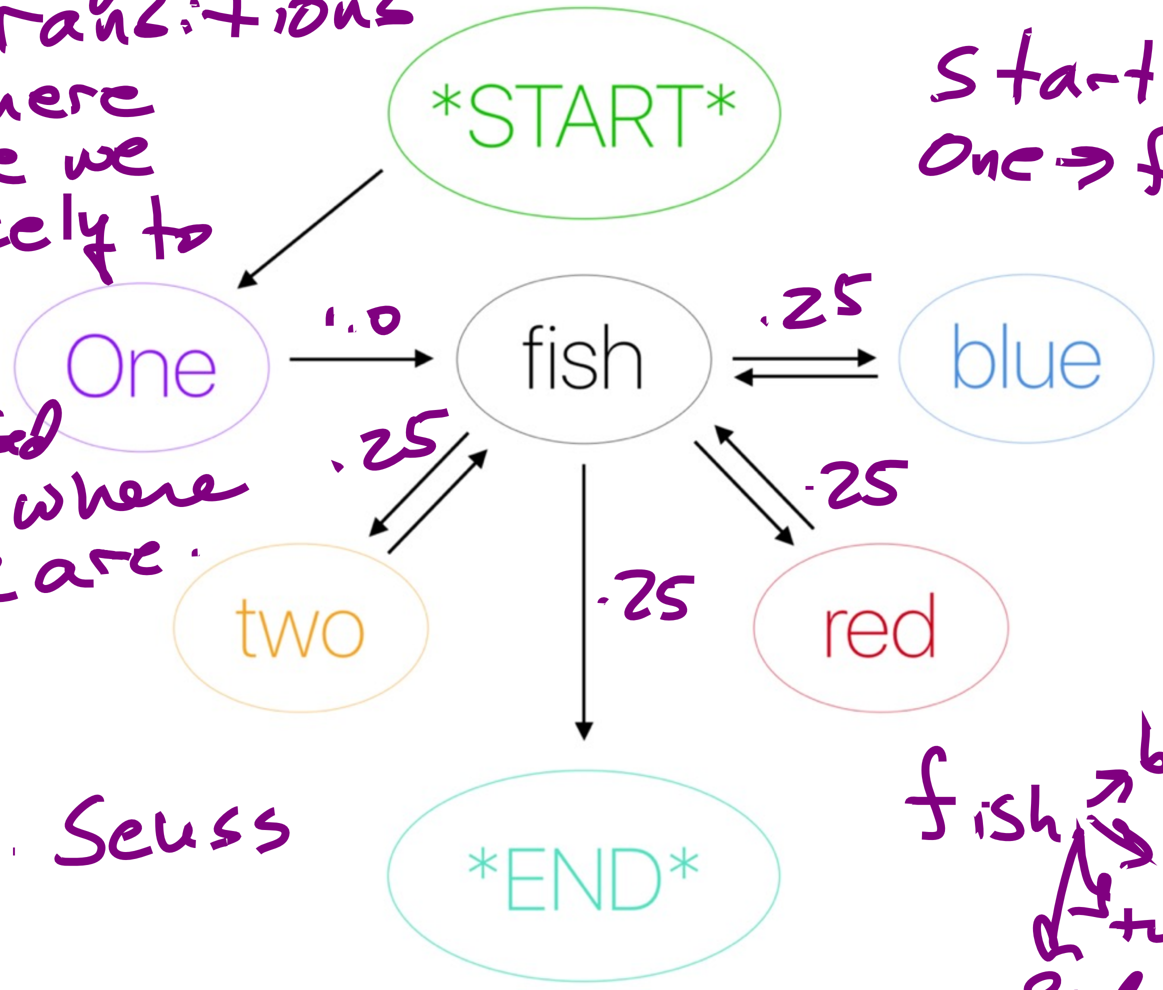
Sequence of random var.

Transitions

where are we likely to go based on where we are.

start → one

one → fish



Dr. Seuss

fish → blue
fish → red
fish → two
fish → end

One fish two fish red fish blue fish

Source

Bayesian Networks – Recap

The point of Bayes nets is to represent full joint probability distributions, and to encode an interrelated set of conditional independence/probability statements.

Prob. of single outcome

- Consists of nodes (events), and
- Conditional probability tables (CPTs), relating those events.

  $P(A \mid B, E)$, $P(J \mid A)$

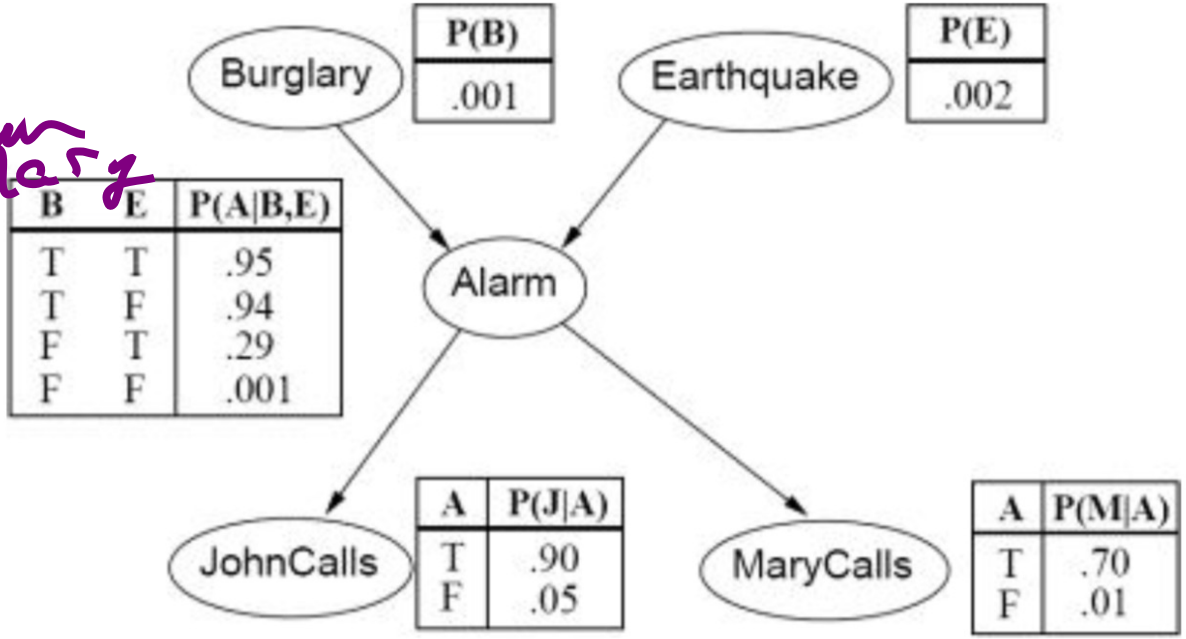
- Describes how variables interact locally

  Eg. relationship between alarm and burglary

- Chain together local interactions to estimate global, indirect interactions

  $P(J \mid B)$ $P(J, A \mid E)$

  $P(J \mid M)$ $P(B \mid J)$



P(B)
.001

P(E)
.002

B	E	P(A|B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.05

A	P(M|A)
T	.70
F	.01

Markov Models

Used to reason about a **sequence of events** (random variables) Events at time $t, t+1, t+2 \ldots$

Examples:

- Robot localization Maze navigation. Path planning
- Speech recognition P(word | previous word)
- Medical monitoring change in blood sugar at time t, given t-1
- Weather forecasting rain on day t if rain on t-1

➢ Need to **introduce time into our Bayesian Network** model

➢ Probability of an event happening as the next state, given the current state of the system

➢ Long-term behavior e.g. from individual transitions at t to t+1, get long term probability of rain, speech patterns, etc.

Markov Models

A **Markov model** is a chain-structured Bayesian network.

- The value of X at a time t is the state at time t X is random variable
- Stationary Markov model: All subsequent nodes have the same CPT (identically distributed) e.g. if rain on day t means 50% chance of rain on day t+1, for all values of t.

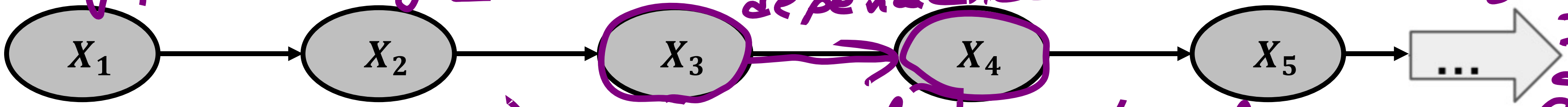
Example: Is it raining? Let X_t denote the event that it is raining on day t

X_t = rain

Day 1 Day 2

Conditional dependence between nodes, and dependence

Each edge is one unit of time passing. I'm using days, but that's in the problem def.



$P(X_1)$ $P(X_2|X_1)$

conditional indep. between other nodes. X_2 and X_4 are indep.

Markov Models

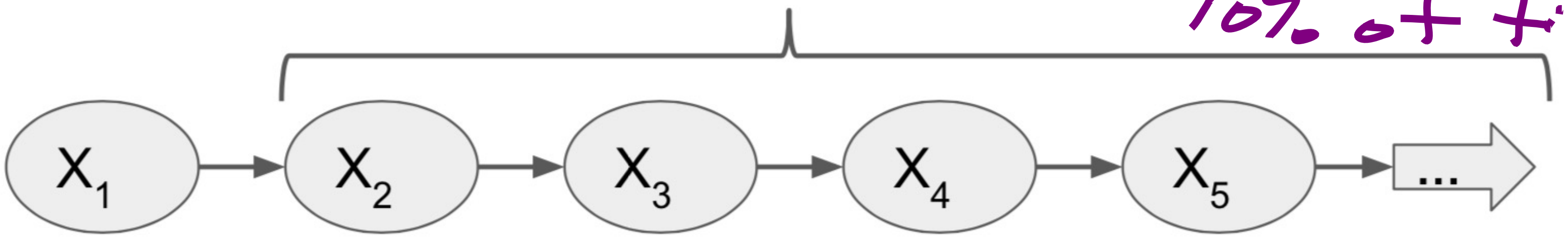
Recall: Causal chain Bayes net forms conditional independence

- Past and future are independent of the present. X_2 indep. of X_3
- State at $t+1$ only depends on state at t First-order Markov model
- The CPTs give the transition probabilities from one state to another for this.

If $X_t = T$, then $X_{t+1} = T$ 30% of time. If $X_t = F$, $X_{t+1} = T$ 10% of time.

X_t	$P(X_{t+1}=T \mid X_t)$
T	0.3
F	0.1

$P(X_1=T) = 0$



Markov Models

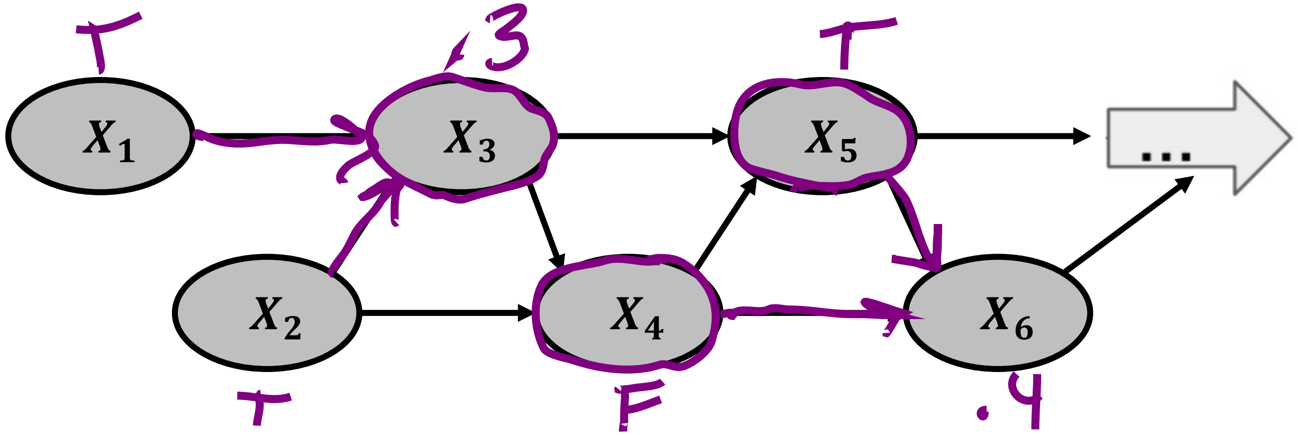
Recall: Causal chain Bayes net forms conditional independence

- Past and future are independent of the present.
- State at $t+1$ only depends on state at t
- The CPTs give the transition probabilities from one state to another for this.

Markov property
(first-order)

➢ We could also create models where the state at $t+1$ depends only on the state at t and $t-1$ (second-order Markov property), or higher …

$P(X_3 = T \mid X_2, X_1)$



X_{t-1}	X_t	$P(X_{t+1}=T \mid X_t, X_{t-1})$
T	T	0.3
T	F	0.1
F	T	0.4
F	F	0.1

Markov Models

Example: Suppose we want to forecast the weather. From historical data, we know that in our town, if the current day was sunny, then the following day was also sunny 90% of the time, and that if the current day was rainy, then the following day was also rainy 30% of the time.

Draw the state space graph and specify the CPT.

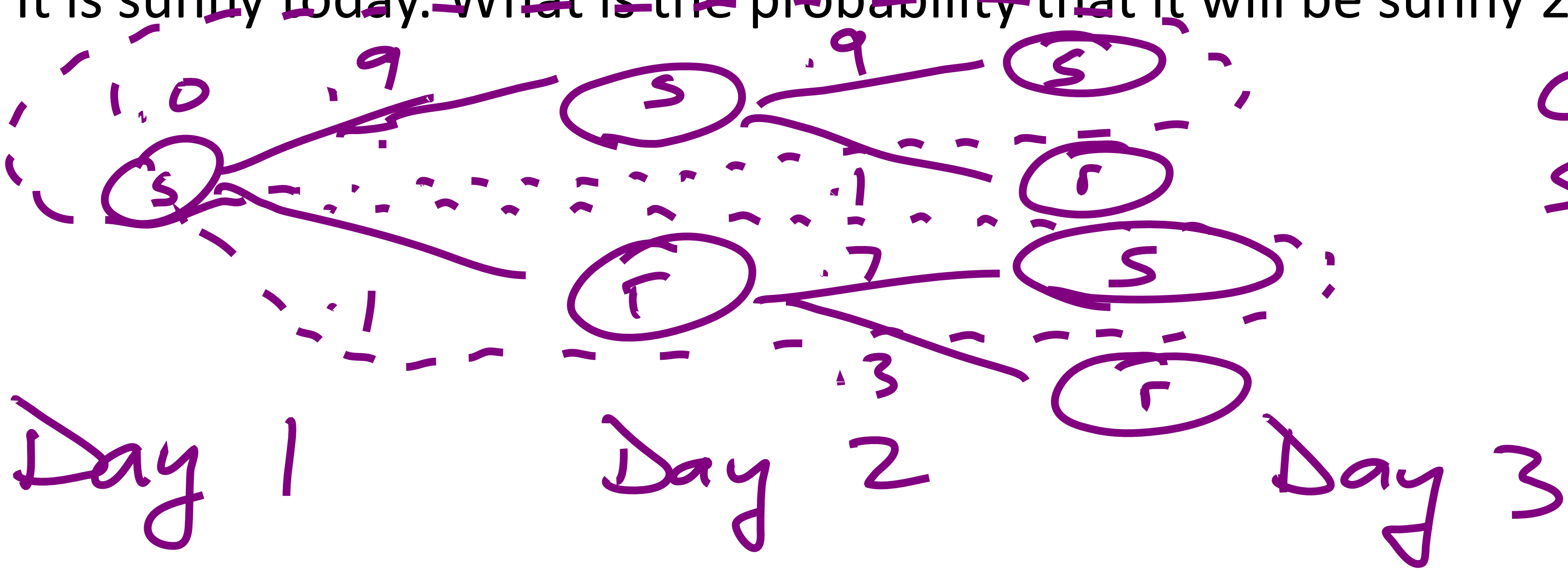
X_t	X_{t+1}	$P(X_{t+1} \mid X_t)$
sun	sun	.9 ← given
sun	rain	.1
rain	sun	.7
rain	rain	.3 ← given

State space graph.
Visual rep.
Draw states with transition probs as arrows

Markov Models

Example: Suppose we want to forecast the weather. From historical data, we know that in our town, if the current day was sunny, then the following day was also sunny 90% of the time, and that if the current day was rainy, then the following day was also rainy 30% of the time.

It is sunny today. What is the probability that it will be sunny 2 days from now?



Can be either Sun or rain on Day 2 and be sun on Day 3.

Mini-forward alg.

Markov Models

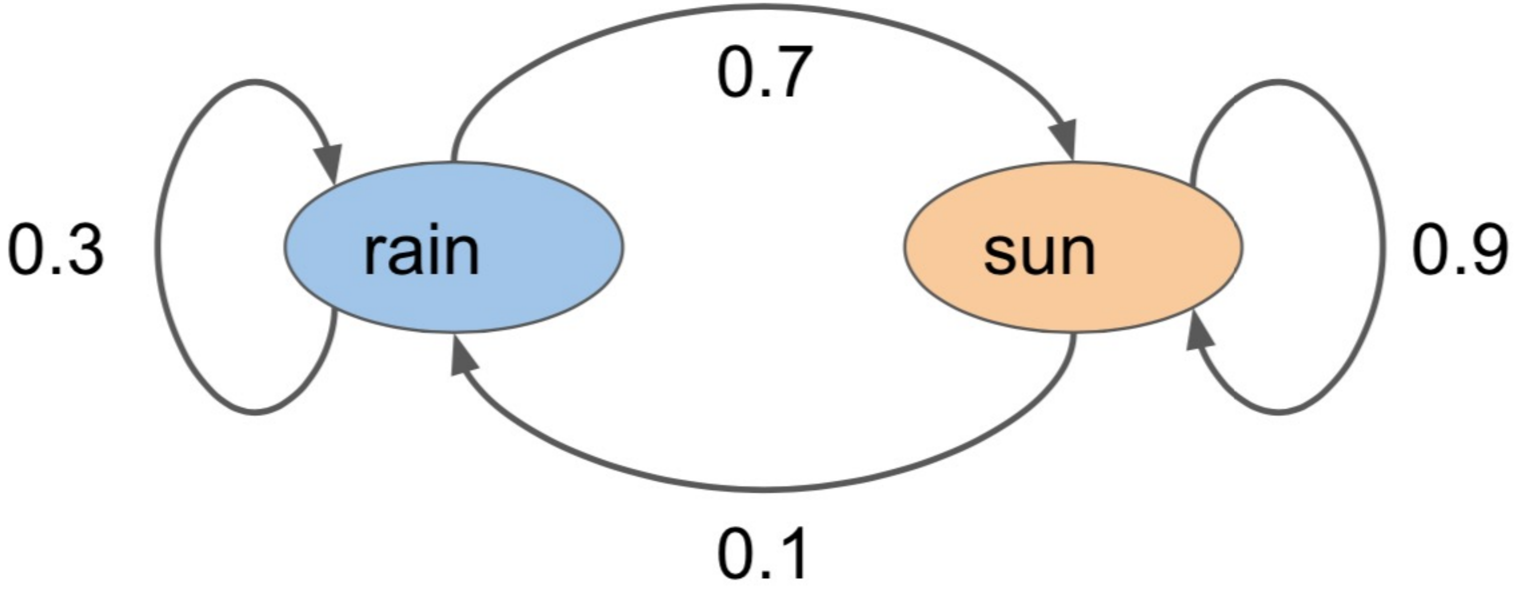
Mini-forward algorithm: incremental belief updating

This is what we did.

Suppose that x_1 is known. Day 1 is sunny

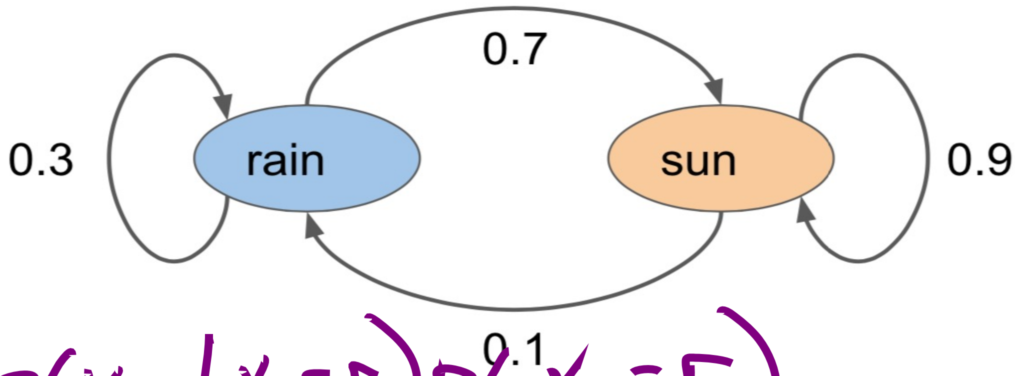
Then for $t = 2, 3, 4, \ldots P(x_t) = \sum_{x_{t-1}} P(x_t | x_{t-1}) P(x_{t-1})$

To calculate $P(X_t)$, need to know all paths through X_{t-1}



Mini_forward Algorithm

Example: Using the mini-forward algorithm, find $P(X_3 = s)$



$P(X_3 = s)$

Assume $P(X_1 = sun)$

$$P(X_t) = \sum_{t-1} P(X_t \mid X_{t-1})\, P(X_{t-1})$$

$$P(X_2) = P(X_2 \mid X_1 = s)\, P(X_1 = s) + P(X_2 \mid X_1 = r)\, P(X_1 = r)$$

$$= P(X_2 \mid X_1 = s) = \begin{pmatrix} .9 \\ .1 \end{pmatrix} \begin{matrix} P(s \mid s) \\ P(r \mid s) \end{matrix}$$

$$P(X_3) = \sum_{x_2} P(X_3 \mid X_2)\, P(X_2)$$

$$P(X_3 = s) = P(X_3 = s \mid X_2 = s)\, P(X_2 = s) + P(X_3 = s \mid X_2 = r)\, P(X_2 = r)$$

$$.9 \times .9 + .7 \times .1 = .88$$

$$P(X_3 = r) = P(X_3 = r \mid X_2 = s)\, P(X_2 = s) + P(X_3 = r \mid X_2 = r)\, P(X_2 = r)$$

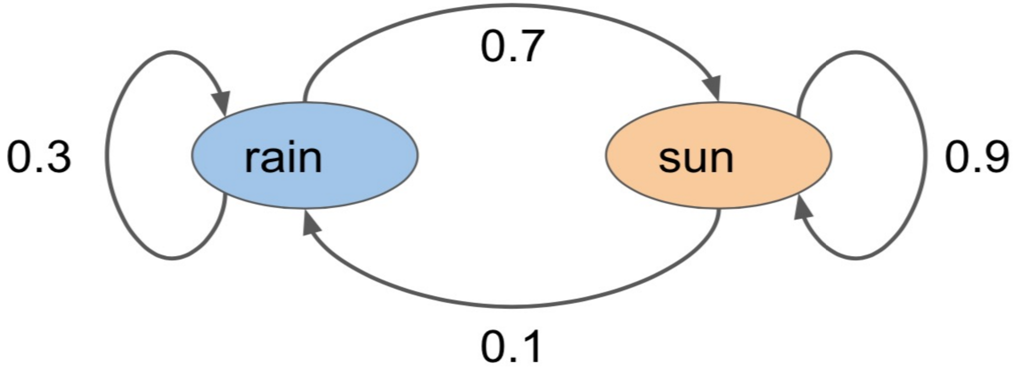
Markov Models

$[.1 \times .9] + [.3 \times .1] = .12$

Example: What is the long-run probability that it will be sunny?

Run the mini-forward alg many, many times.

On any given day, what is $P(X_t = rain)$



Markov Models

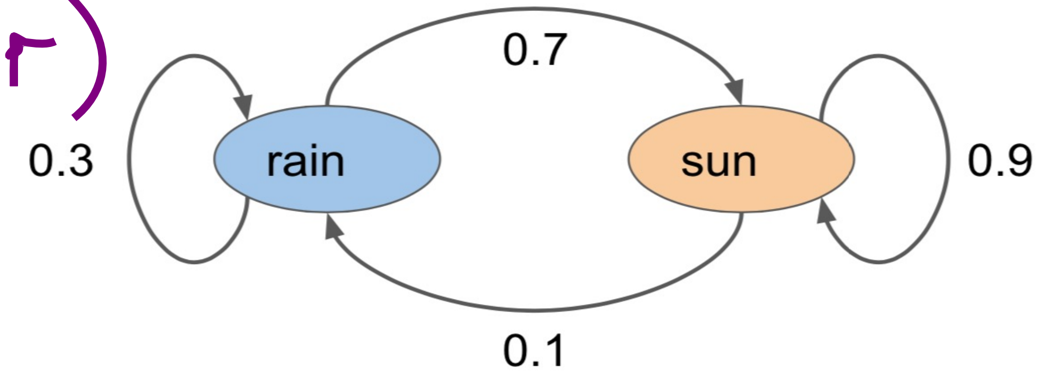
Define the transition matrix T such that T_{ij} = probability of moving from state i to j

Say X_1 = sun, X_2 = rain. Then: $T = \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix}$

$$T = \begin{array}{c} \\ \text{sun} \\ \text{rain} \end{array} \!\!\! \begin{array}{c} \begin{array}{cc} \text{sun} & \text{rain} \end{array} \\ \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix} \end{array}$$

P(r | s)

P(r | r)



Markov Models

It turns out that the transition matrix also gives us a short-cut for calculating multi-step transition probabilities.

Example: It is sunny today. What is the probability that it will be sunny 2 days from now?

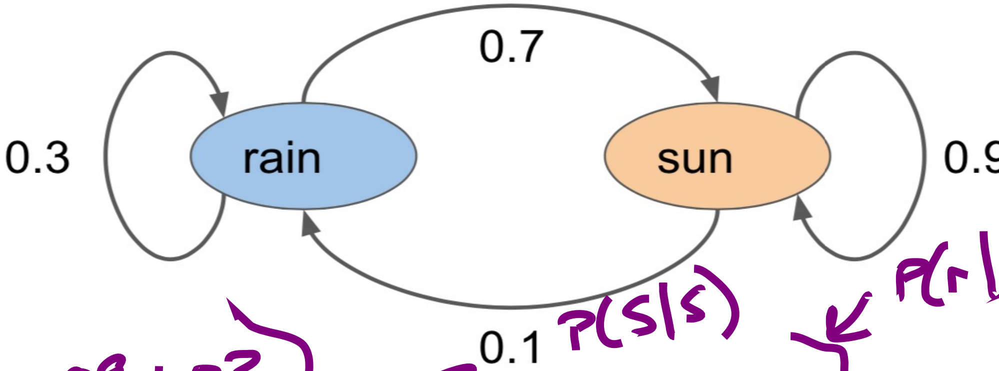
$x_1 \rightarrow x_2 \rightarrow x_3$

Sunny

$P(s \mid x_1 = s)$

Apply T for timestep $x_1 \Rightarrow x_2$

Apply T again for $x_2 \Rightarrow x_3$



$$T^2 = \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix} \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix} =$$

$$\begin{bmatrix} .9 \times .9 + .1 \times .7 & .9 \times .1 + .1 \times .3 \\ .9 \times .7 + .3 \times .7 & .7 \times .1 + .3 \times .3 \end{bmatrix} = \begin{bmatrix} .81 + .07 & .09 + .03 \\ .63 + .21 & .07 + .09 \end{bmatrix} = \begin{bmatrix} .88 & .12 \\ .84 & .16 \end{bmatrix}$$

$P(s \mid s)$ $P(r \mid s)$ $P(r \mid r)$ $P(s \mid r)$

Markov Models

Example: How could we use the transition matrix to find $P(X_\infty = s)$.

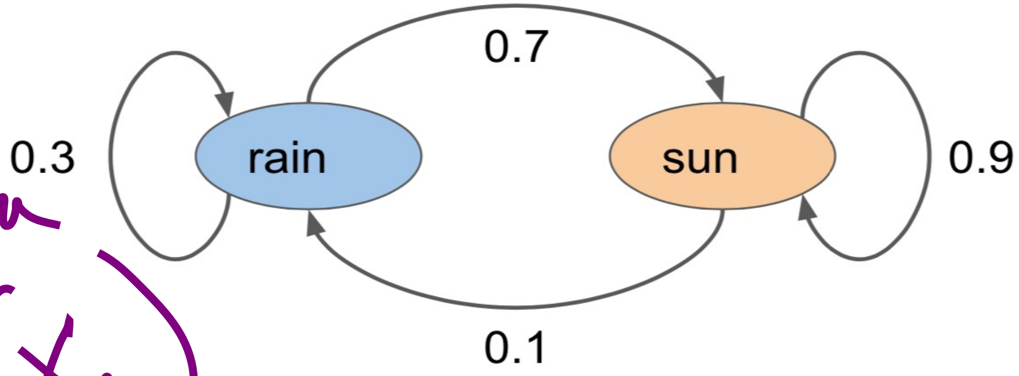
```
transition_matrix = np.array([[0.9, 0.1],[0.7, 0.3]])

new = transition_matrix

for k in range(1,13):
    new = np.matmul(new, transition_matrix)
    print('T**{} = \n{}'.format(k+1, new))
```

when $T^{t+1} = T^t$

we've computed long-term

Prob of sun

$P(X_{t+1}) = P(X_t)$



Next Time

- *Markov Models Notebook Day*