Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in. Since this worksheet contains a lot of problems, a good strategy would be to first skim the worksheet and then discuss and solve the problems which you think are difficult.

1. Let $A = \{cat, dog, \{bear\}, fox\}$. Determine whether each of the following are true or false, and explain why.
   (a) $\{bear\} \in A$
   (b) $cat \in A$
   (c) $\emptyset \in A$
   (d) $\{cat, dog, fox\} \subset A$
   (e) $\{cat, dog, bear\} \subset A$
   (f) $\{bear\} \subset A$
   (g) $\{cat, dog, fox\} \in A$
   (h) $cat \subset A$
   (i) $\{\{bear\}\} \subset A$

2. Let A, B, and C be sets. Show that:
   (a) $(A \cup B) \subseteq (A \cup B \cup C)$
   (b) $(A \cap B \cap C) \subseteq (A \cap B)$
   (c) $(A - B) - C \subseteq A - C$
   (d) $(A - C) \cap (C - B) = \emptyset$
   (e) $(B - A) \cup (C - A) = (B \cup C) - A$

3. Show that if A is a subset of the universal set U, then:
   (a) $A \oplus A = \emptyset$
   (b) $A \oplus \emptyset = A$
   (c) $A \oplus U = \bar{A}$
   (d) $A \oplus \bar{A} = U$

4. Prove that $\{(-8)^n \mid n \in \mathbb{Z}\} \subseteq \{(-2)^n \mid n \in \mathbb{Z}\}$, but $\{(-8)^n \mid n \in \mathbb{Z}\} \neq \{(-2)^n \mid n \in \mathbb{Z}\}$

5. Prove the following if A, B, C are sets.

   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, using set builder notation

   (b) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

6. Uncountable/countable sets

   (a) Prove directly from the definition of countable/uncountable that the set of natural numbers that are multiples of 3 or multiples of 4 is countable.

   (b) Prove that the set of real numbers in the interval $[4, 5]$ is uncountable.

   (c) Suppose A and B are both countable sets. Prove whether the Cartesian product $A \times B$ is countable or uncountable.

7. Show that if A, B, C, and D are sets with $|A| = |B|$ and $|C| = |D|$, then $|A \times C| = |B \times D|$.

8. Show that if A and B are sets and $A \subset B$ then $|A| \leq |B|$.

9. Prove or disprove each of these statements about the floor and ceiling functions

   (a) $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ for all real numbers x and y.

   (b) $\lfloor \sqrt{\lceil x \rceil} \rfloor = \lfloor \sqrt{x} \rfloor$ for all positive real numbers x.

10. Consider the following functions $f(n)$, where the domain for n is the set of integers, $\mathbb{Z}$. Fill in the table regarding whether or not each function is one-to-one and/or onto.

$f(n)$	One-to-One?	Onto?
$f(n) = n^2$		
$f(n) = n + 3$		
$f(n) = \lfloor \sqrt{n} \rfloor$		
$f(n) = \begin{cases} n-1 & n \text{ odd} \\ n+1 & n \text{ even} \end{cases}$		

11. Give an example of a function from $\mathbb{N}$ to $\mathbb{N}$ that is:

    (a) one-to-one, but not onto

    (b) onto, but not one-to-one

    (c) one-to-one and onto

    (d) neither one-to-one nor onto

12. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

    (a) $f(x) = -3x + 4$

    (b) $f(x) = x^2 + 1$

    (c) $f(x) = -3x^2 + 7$

    (d) $f(x) = x^5 + 1$