Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

1. Define the following terms in English. Also specify whether the conditions are satisfiable or unsatisfiable.
   (a) tautology
   (b) contradiction
   (c) contingency

2. Translate these English sentences into 2 unique propositions for each sentence. Additionally, use truth tables to show that the 2 propositions you made are logically equivalent.
   (a) Tucker and Rachel like chocolate, but neither Ioana nor Willem do.
   (b) If it is raining, then Aiden only brings an umbrella if he doesn't wear a jacket.

3. Reduce the following expressions to their simplest form. Also mention whether the following the statements are a tautology, contradiction or contingency
   (a) $p \wedge (p \vee \neg p)$
   (b) $p \vee \neg(p \wedge \neg p)$
   (c) $(p \vee (q \wedge \neg q)) \wedge (q \wedge (p \vee \neg p))$

4. Determine whether each of these compound propositions is satisfiable - *Hint:* First reduce the propositions into a simple expression and then use a truth table.
   (a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
   (b) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$
   (c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

5. Suppose p, q and r are all propositions. Prove that the following are logically equivalent. Use either logical equivalences and a truth table. When using logical equivalences, be sure to cite which rules you use.
$$p \wedge (q \to r) \equiv \neg((p \to q) \wedge (p \to \neg r))$$

6. Consider the domain to be all people at CU Boulder. Let $S(x)$ be: x is a student at CU Boulder. Let $P(x)$ be: x uses public transportation to get to Boulder. Let $W(x)$ be: x works at CU Boulder. Determine the truth of each statement. If the statement is false, give an example of someone (faculty, student, supporting staff, etc.) in the domain who is not covered by the statement.
   (a) $\exists x\ (\neg S(x) \to W(x))$
   (b) $\forall x\ (S(x) \vee P(x) \vee W(x))$
   (c) $\forall x\ (\neg S(x) \wedge \neg P(x))$
   (d) $\exists x\ (((S(x) \wedge P(x)) \to W(x))$

7. Translate these sentences into predicates that include nested quantifiers. Be sure to define any variables and propositional functions that you use.

   (a) All engineering students are tired

   (b) Not every person in Boulder enjoys quinoa

   (c) You can't finish all of your homework all of the time

   (d) Some CSCI 2830 students attend every session

8. Consider the domain of all real numbers. Let $P(x, y)$ be: $x + y = 0$; Let $Q(x, y)$ be: $x^2 - 2x + 1 = y$. Determine the truth of each statement.

   (a) $\forall x \exists y \; P(x, y)$

   (b) $\exists y \forall x \; Q(x, y)$

   (c) $\forall y \exists x \; (P(x, y) \wedge Q(x, y))$

   (d) $\forall y \exists x \; P(x, y) \wedge Q(x, y)$