CSCI 2834-003/004: Discrete Structures Workgroup – Fall 2019 **Worksheet 14**

Get in small groups (about 4 students maximum) and work out these problems on the whiteboard. Ask one of the teaching assistants for help if your group gets stuck. You do **not** need to turn anything in.

1. (a) Draw the graph of the relation $R \subseteq A \times B$ where $R = \{(1, c), (1, d), (2, c), (2, d), (3, a), (3, d)\}$, and $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$.
   (b) Is this graph reflexive, transitive, and/or symmetric? If the graph does not meet the criteria of one of the terms, provide an example that breaks the criteria.

2. Construct five relations on the set a, b, c, d that are, respectively:
   (a) reflexive, symmetric, but not transitive.
   (b) antireflexive, symmetric, and transitive.
   (c) antireflexive, antisymmetric, and not transitive.
   (d) reflexive, neither symmetric nor antisymmetric, and transitive.
   (e) neither reflexive, antireflexive, symmetric, antisymmetric, nor transitive.

3. For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, antisymmetric, and/or transitive. Is the relation an equivalence relation? Why or why not?
   (a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
   (b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
   (c) $\{(2, 4), (4, 2)\}$
   (d) $\{(1, 2), (2, 3), (3, 4)\}$
   (e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
   (f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

4. How many relations are there on the set $\{a, b, c, d\}$?

5. How many relations are there on the set $\{a, b, c, d\}$ that contain the pair (a, a)?

6. Let A be the set of students at your school and B the set of books in the school library. Let $R1$ and $R2$ be the relations consisting of all ordered pairs (a, b), where student a is required to read book b in a course, and where student a has read book b, respectively. Describe the ordered pairs in each of these relations.
   (a) $R1 \cup R2$
   (b) $R1 \cap R2$
   (c) $R1 \oplus R2$
   (d) $R1 - R2$
   (e) $R2 - R1$

7. Which of these are partitions of the set $\mathbb{Z} \times \mathbb{Z}$ of ordered pairs of integers?
   (a) the set of pairs (x, y), where x or y is odd; the set of pairs (x, y), where x is even; and the set of pairs (x, y), where y is even
   (b) the set of pairs (x, y), where both x and y are odd; the set of pairs (x, y), where exactly one of x and y is odd; and the set of pairs (x, y), where both x and y are even
   (c) the set of pairs (x, y), where x is positive; the set of pairs (x, y), where y is positive; and the set of pairs (x, y), where both x and y are negative.
   (d) the set of pairs (x, y), where $x > 0$ and $y > 0$; the set of pairs (x, y), where $x > 0$ and $y \leq 0$; the set of pairs (x, y), where $x \leq 0$ and $y > 0$; and the set of pairs (x, y), where $x \leq 0$ and $y \leq 0$
   (e) the set of pairs (x, y), where $x \neq 0$ and $y \neq 0$; the set of pairs (x, y), where $x = 0$ and $y \neq 0$; and the set of pairs (x, y), where $x \neq 0$ and $y = 0$.

8. For each of the degree sequences shown below, determine whether they represent a valid undirected graph with no self-loops. If they do, draw the graph. If they do not, explain why.

   (a) 3, 3, 2, 2, 0

   (b) 4, 2, 2, 2, 1

   (c) 4, 2, 2, 2, 0

   (d) 3, 2, 1, 0, 2

   (e) 4, 4, 3, 2, 1

   (f) 0, 3, 3, 2, 4

9. Does there exist a single possible value of x (for part (a)) or y (for part (b)) in order to make the relation transitive? Or reflexive? Or symmetric? Or an equivalence relation?

   (a) $\{(1,2),(2,3),(1,4),(4,3),(1,x),(2,2),(3,3),(4,4),(2,1),(3,2),(4,1),(3,4),(3,1)\}$

   (b) $\{(1,1),(2,2),(3,3),(1,2),(2,3),(3,y),(2,1),(1,3),(3,2)\}$

10. Determine whether each of the following relations $R \subseteq A \times A$, where A is the set of all CU students, is reflexive, symmetric, transitive, and/or an equivalence relation. Briefly justify each conclusion.

   (a) $(a,b) \in R$ if and only if a is older than b.

   (b) $(a,b) \in R$ if and only if a is on a sports team with b.

   (c) $(a,b) \in R$ if and only if a is acquainted with b.