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CSCI 3104, Algorithms
Problem Set 10 (50 points)

Due THURSDAY, APRIL 29, 2021
Spring 2021, CU-Boulder

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- The easiest way to access Gradescope is through our Canvas page. There is a Gradescope button in the left menu.
- Gradescope will only accept **.pdf** files.
- It is vital that you match each problem part with your work. Skip to 1:40 to just see the matching info.

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1. Let P_1, P_2 be two problems such that $P_1 \leq_p P_2$. That is, we have a polynomial-time reduction $r : P_1 \to P_2$. If we assume $P_2 \in P$, explain why this implies that $P_1 \in P$.

2. Recall the k-Colorability problem.

   - Input: Let G be a simple, undirected graph.
   - Decision: Can we color the vertices of G using exactly k colors, such that whenever u and v are adjacent vertices, u and v receive different colors?

   It is well known that k-Colorability belongs to NP, and 3-Colorability is NP-complete. Our goal is to show that the 4-Colorability problem is also NP-complete. To do so, we reduce the 3-Colorability problem to the 4-Colorability problem. We provide the reduction below. The questions following the reduction will ask you to verify its correctness.

   Reduction: Let G be a simple, undirected graph. We construct a new simple, undirected graph H by starting with a copy of G. We then add a new vertex t to H, and for each vertex $v \in V(G)$ we add the edge tv to $E(H)$.

   Your job is to verify the correctness of the reduction by completing parts (a)–(c) below, which completes the proof that 4-Colorability is NP-complete. For the rest of this problem, let G be a graph, and let H be the result of applying the reduction to G.

   (a) Suppose that G is colorable using exactly 3 colors. Argue that H is colorable using exactly 4 colors.

   (b) Suppose that H is colorable using exactly 4 colors. Argue that G is colorable using exactly 3 colors.

   (c) Let n be the number of vertices in G. Carefully explain why H can be constructed in time polynomial in n. [**Hint:** Count the number of vertices and edges we add to G in order to obtain H.]

Solutions

1. Assuming we have a P_2 that is solvable using the algorithm P. If we have another problem P_1 that is of similar nature to P_2, we can have a reduction of P_1 to P_2 by tranforming any input x of P_1 into the inputs of P_2 and running P. The results from P can be interpreted as the results of P_1. Therefore, we have that $P_2 \in P$, it implies that $P_1 \in P$.

2.

   - Assuming that G is colorable using 3 colors, H would be colorable using 4 colors if for every node, every neighbor have different colors.
   - Assuming that H is colorable using 4 colors, G would be colorable using 3 colors if for every node, every neighbor have different colors.
   - H can be constructed in timepolynomial in n, because ... I am really not sure how to solve for this part. My best guess would be that the reduction takes linear time and adds one single node. Therefore the other graph can be constructed using time in polynomial in n.