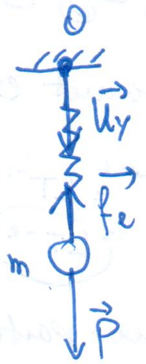
Corrigé CC1 – ~~Maths GP2~~ SP1 GP3 Méca point 1 12/11/2024.

Q1.

1)



$\vec{P} = mg\,\vec{u}_y$

$\vec{f_e} = -k(y - l_0)\,\vec{u}_y$

2) $\sum \vec{F} = m\vec{a} \Rightarrow mg\,\vec{u}_y - k(y-l_0)\vec{u}_y = m\vec{a}$

$\Rightarrow mg - k(y - l_0) = m\ddot{y}(t)$

$\Rightarrow \boxed{\ddot{y}(t) + \frac{k}{m} y = g + \frac{k}{m} l_0}$. (2)

3) $y_{eq} \Leftrightarrow \ddot{y}(t) = 0$

$\Rightarrow \frac{k}{m} y_{eq} = g + \frac{k}{m} l_0$

$\boxed{y_{eq} = \frac{gm}{k} + l_0}$

4) $y(t) = y_{eq} + A\cos\omega t + B\sin\omega t$

$\ddot{y}(t) = -\omega^2 (A\cos\omega t + B\sin\omega t)$

$= -\omega^2 (y(t) - y_{eq})$

en remplaçant en (2)

$-\omega^2 (y(t) - y_{eq}) + \frac{k}{m} y(t) = g + \frac{k}{m} l_0$

$\left(-\omega^2 + \frac{k}{m}\right) y(t) = -\omega^2 y_{eq} + \frac{k}{m}\underbrace{\left(l_0 + \frac{mg}{k}\right)}_{y_{eq}}$

$\left(-\omega^2 + \frac{k}{m}\right) y(t) = \left(-\omega^2 + \frac{k}{m}\right) y_{eq}$

$\Rightarrow \left(-\omega^2 + \frac{k}{m}\right)\left(y(t) - y_{eq}\right) = 0$

Si $y(t) \neq y_{eq}$, il faut choisir ω de façon approprié pour satisfaire l'EDO

5) $-\omega^2 + \frac{k}{m} = 0 \Rightarrow \omega = \sqrt{\frac{k}{m}}$

$y(0) = y_{eq}$, $\dot{y}(0) = V_0$ (conditions initiales)

$y(0) = y_{eq} + A\cos(0) + B\sin(0)$

$= y_{eq} + A \underset{\uparrow \text{condition.}}{=} y_{eq} \Rightarrow \boxed{A = 0}$ m.

$\dot{y}(t) = -\omega A \sin\omega t + \omega B \cos\omega t$

$\dot{y}(0) = -\omega A \cdot 0 + \omega B \cdot 1 = \omega B \underset{\text{condition}}{=} V_0$

$\Rightarrow \boxed{B = \frac{V_0}{\omega}}$

6) $k = 36$ N/m , $m = 1$ g $= 10^{-3}$ kg , $V_0 = 6$ m/s

$\omega = \sqrt{\frac{36}{10^{-3}}} = \sqrt{6^2 \cdot 10^2 \cdot 10}$

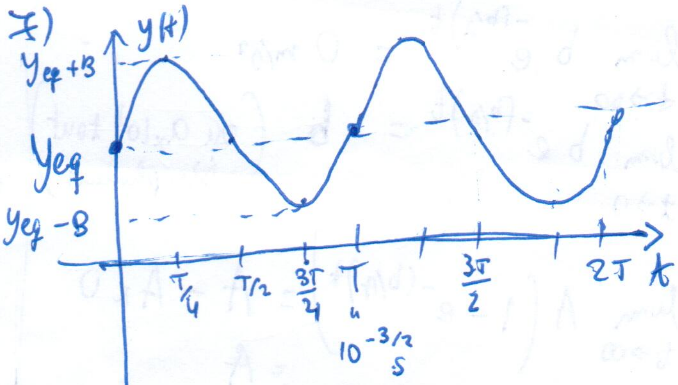
$= 6 \cdot 10\sqrt{10} = 60\sqrt{10}$ rad/s.

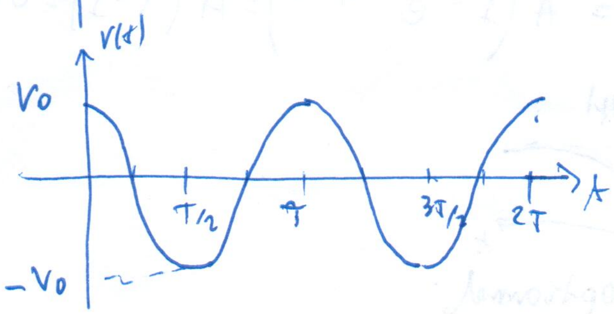
$A = 0$ m

$B = \frac{6}{60\sqrt{10}} = \frac{10^{-1}}{\sqrt{10}} = 10^{-1} \times 10^{-1/2} = 10^{-3/2}$ m.

$T = \frac{2\pi}{\omega} = \frac{2 \times 3}{60\sqrt{10}} = \frac{10^{-1}}{\sqrt{10}} = 10^{-3/2}$ s

7)





Q2

1) $V_x(t) = A\left(1 - e^{-(b/A)t}\right)$

$a_x(t) = \frac{dV_x}{dt} = A\left(0 - \left(\frac{-b}{A}\right)e^{-(b/A)t}\right)$

$= \frac{b}{A} \times A \times e^{-(b/A)t} = b\, e^{-(b/A)t}$

$x(t) = \int V_x(t)\,dt = \int A\left(1 - e^{(-b/A)t}\right)dt + k$

$= A\left(t - \frac{1}{\left(\frac{-b}{A}\right)} e^{-\frac{b}{A}t}\right) + k$

$= A\left(t + \frac{A}{b} e^{-b/A\, t}\right) + k$

$x(0) = A\left(0 + \frac{A}{b} \cancel{e^{0}}^{1}\right) + k = \frac{A^2}{b} + k = 0$

$k = -\frac{A^2}{b}$.

$x(t) = At + \frac{A^2}{b} e^{(-b/A)t} + \frac{A^2}{b}$

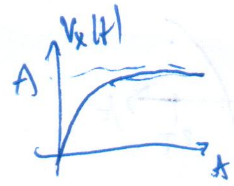
$= At + \frac{A^2}{b}\left(1 + e^{-b/A\, t}\right)$.

2) $\lim_{t\to\infty} b\, e^{-(b/A)t} = 0$ m/s²

$\lim_{t\to 0} b\, e^{-(b/A)t} = b$ (ou $a_x(0)$ tout simplement)

3) $\lim_{t\to\infty} A\left(1 - e^{-(b/A)t}\right) = A - A \times 0 = A$

$V_x(0) = A\left(1 - e^{(-b/A)\times 0}\right) = A(1-1) = 0$ m/s.



optionnel.

4)

$[A] = [V_x] = LT^{-1}$ car $[1 - e^{(-b/A)t}] = 1$

~~pour b : [-b/A] = ... (car l'argument~~

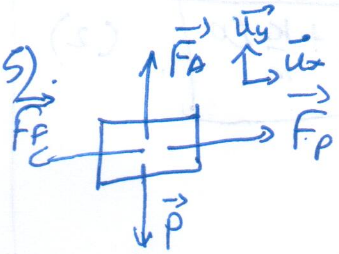
pour b : $\left[\frac{-b}{A} t\right] = 1$ (argument expo

$\frac{[b]}{[A]} T = 1 \Rightarrow [b] = [A]T^{-1} = LT^{-1}T^{-1} = LT^{-2}$

A : vitesse physiquement, en particul la vitesse maximale.

b : Accélération, en particulier l'accélération initiale.

5)



$\vec{F_A}$: archimède

$\vec{P}$: poids.

$\sum \vec{F} = m\vec{a} \Rightarrow \begin{cases} x: & F_P - F_f = m a_x(t) \\ y: & F_A - P = m a_y(t) \end{cases}$ ~~équilibre~~

6)

$F_A - P = m \times \underbrace{a_y}_{0 \text{ équilibre}} = 0$

$F_A = P = m \times g = 80 \times 10 = 800$ N.

7) $\vec{F_f} = -B\vec{V}(t)$ $\quad [\vec{F_f}] = [B][\vec{V}]$

$F_f = -BV_x$ $\quad MLT^{-2} = [B]\,LT^{-1}$

$[B] = MT^{-1}$

du PFD :

$F_P = F_f + m a_x(t) = -BV_x(t) + m a_x(t)$

$= -BA\left(1 - e^{-\frac{b}{A}t}\right) + m \times b\, e^{(-b/A)t}$

$= -BA + (BA + mb)\, e^{(-b/A)t}$.

8) À l'infini car la l'accélération est nulle.

Alternative : $F_f \xrightarrow{t\to\infty} -BA$, $F_P \xrightarrow{t\to\infty} -BA$) = la même chose