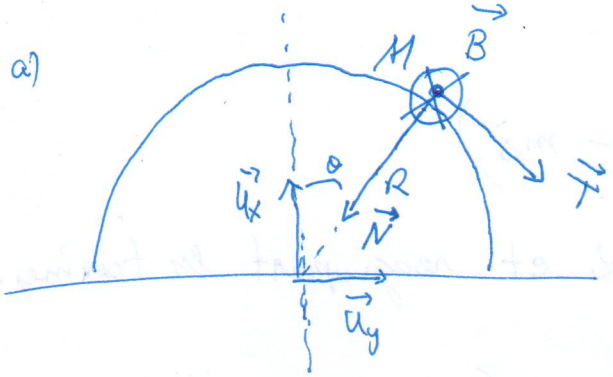
Corrigé CC1 - SPI Méca Point 2 — 13/03/2024

Q1 : Voir pdf

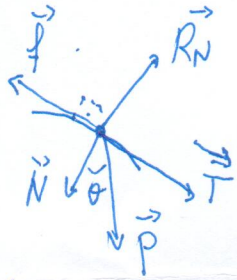
Q2 :

a)



$\dot{\theta} > 0$, donc $\vec{T}$ pointe dans le sens horaire selon la convention de θ qui croît de $\vec{u_x}$ vers $\vec{u_y}$

b) $\vec{P} = mg(\cos\theta\,\vec{N} + \sin\theta\,\vec{T})$, $\vec{R_N} = -R_N\vec{N}$, $\vec{f} = -f\vec{T}$



c) $\vec{M_O}(\vec{P}) = \vec{OM} \wedge \vec{P} = (-R\vec{N}) \wedge (\cos\theta\,\vec{N} + \sin\theta\,\vec{T})mg$

$= -mgR(\cos\theta\,\underbrace{\vec{N}\wedge\vec{N}}_{\vec{0}} + \sin\theta\,\underbrace{\vec{N}\wedge\vec{T}}_{-\vec{B}}) = mgR\sin\theta\,\vec{B}$

$\vec{M_O}(\vec{f}) = -R\vec{N} \wedge (-f\vec{T}) = Rf\,\vec{N}\wedge\vec{T} = -Rf\,\vec{B}$

$\vec{M_O}(\vec{R_N}) = -R\vec{N} \wedge R_N\vec{N} = -RR_N\,\vec{N}\wedge\vec{N} = \vec{0}$

d) $\vec{L_O} = \vec{OM} \wedge m\vec{v} = -R\vec{N} \wedge m\dot{s}\vec{T} = -Rm\dot{s}\,\vec{N}\wedge\vec{T} = Rm\dot{s}\,\vec{B}$

e) $\frac{d\vec{L_O}}{dt} = \sum \vec{M_O}(\vec{F}) \Rightarrow \frac{d}{dt}(Rm\dot{s}\,\vec{B}) = Rm\ddot{s}\,\vec{B} = (mgR\sin\theta - Rf)\vec{B}$

$\Rightarrow m\ddot{s} = mg\sin\theta - f$

f) $m\vec{a} = \sum \vec{F} \Rightarrow m\frac{\dot{s}^2}{R} = mg\cos\theta - R_N$