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Abstract: *Most of the classical methods for edge detection are based on the first and second order derivatives of gray levels of the pixels of the original image utilizing 2D spatial convolution masks to approximate the derivative. In this paper we present an algorithm for edge detection in gray level images to solve the previous problem of traditional methods. The method is employs Renyi entropy measure of order α. The main advantages of the proposed method are its robustness and its flexibility. It has been tested on many different kinds of test images from the real-world and synthetic images.*

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1. Introduction

Edge detection can be defined as the boundary between two regions separated by two relatively distinct gray level properties. The causes of the region dissimilarity may be due to some factors such as the geometry of the scene, the radio metric characteristics of the surface, the illumination and so on. The detection results benefit applications such as image enhancement, morphing, compression, retrieval, watermarking, hiding, recognition, restoration, and registration etc [5, 8].
The traditional edge detection algorithms have been developed based on computation of the intensity gradient vector, which, in general, is sensitive to noise in the image. In order to suppress the noise, some spatial averaging may be combined with differentiation such as the Laplacian of Gaussian operators [11]. The Laplacian edge detection method has used a 2-D linear filter to approximate second-order derivative of pixel values of the image [13].

Canny [1, 3] derived analytically optimal step edge operators and showed that the first derivative of Gaussian filter is a good approximation of such operators. An alternative to gradient techniques is based on statistical approaches. The idea is to examine the distribution of intensity values in the neighbourhood of a given pixel and determine if the pixel is to be classified as an edge. In comparison with the differential approaches, less attention has been paid to statistical approaches. However, this method has been approached by some authors, e.g., Bovik et al. [2] and Yakimovsky [12].

The Laplacian generally is not used in its original form for edge detection for several reasons: As a second-order derivative, the Laplacian typically is unacceptably sensitive to noise. The magnitude of the Laplacian produces double edges, an undesirable effect because it complicates segmentation. For these reasons, the Laplacian is combined with smoothing as a precursor to finding edges via zero-crossings. Marr and Hildreth achieved this by using the Laplacian of a Gaussian (LOG) function as a filter [8].
To solve these problems, the study proposed a new technique based on information theory. Renyi entropy is an important measure among several measures of information. The proposed method is decrease the

computation time.

The remainder of the paper is organized as follows: Section (2) presents some fundamental concepts of the mathematical setting of the threshold selection and Renyi entropy. In section (3), we describe the newly proposed method of edge detection in gray level images using Renyi entropy. In section (4), the experimental result of many edge detections obtained when applied to some real-world and synthetic images, then we report the effectiveness of our method and finally in section (5) the discussion and conclusion about our method are given.

2. Selection of suitable threshold value

Entropy is used to measure the amount of information. Entropy is defined in terms of the probabilistic behaviour of a source of information. Given events $e_1, e_2, \ldots, e_k$ occurring with probabilities $p_1, p_2, \ldots, p_k$, being k the total number of states, $\sum p_i = 1$, i=1,2,…,k and $0 \leq p_i \leq 1$.

Shannon entropy is defined as [4, 10]:

$$S = -\sum_{i=1}^{k} p_i \ln(p_i) \qquad (1)$$

If we consider that a system can be decomposed in two statistical independent subsystems A and B, the Shannon entropy has the extensive property: $S(A+B) = S(A) + S(B)$. This formalism has been shown to be restricted to the Boltzmann-Gibbs-Shannon (BGS) statistics [10].

Let $f(x, y)$ be the gray value of the pixel located at the point (x, y). In a digital image of size $M \times N$, $1 \leq x \leq M$ and $1 \leq y \leq N$. Let the histogram be $h(a)$ for $a \in G = \{0, 1, 2, \ldots, 255\}$ with f as the amplitude (brightness) of the image at the real coordinate position (x, y). Let t be a threshold value, the optimal threshold t^* is determined by optimizing a suitable criterion function obtained from the gray level distribution of the image and some other features of the image. If t^* is determined solely from the gray level of each pixel, the thresholding method is point dependent [7]. The result of thresholding an image function $f(x, y)$ at gray level t is a binary function $f_t(x, y)$ such that $f_t(x, y) = 0$ if $f_t(x, y) \leq t$ otherwise, $f_t(x, y) = 1$. In general, a thresholding method determines the value t^* of t based on a certain criterion function. The Rényi entropy is a one-parameter generalization of the Shannon entropy. There is extensive literature on the applications of the Rényi entropy in many fields from biology, medicine, genetics, linguistics, and economics to electrical engineering, computer science, geophysics, chemistry and physics. The Renyi's entropy measure of order α of an image, $H_\alpha(P)$ is defined as (see Refs. [6,9]):

$$H_\alpha(P) = \frac{1}{1-\alpha} \ln \sum_{i=0}^{255} (p_i)^\alpha \qquad (2)$$

where $\alpha \neq 1$ is a positive real parameter. Since Shannon entropy measure is a special case of the Renyi entropy for $\alpha \rightarrow 1$.

Let $p_i = p_0, p_1, \ldots, p_k$ be the probability distribution for an image with k gray-levels. From this distribution, we derive two probability distributions, one for the object (class A) and the other for the background (class B), given by:

$p_A:\ p_0/P_A,\ p_1/P_A,\ \ldots, p_t/P_A$ and $p_B:\ p_{t+1}/P_B,\ p_{t+2}/P_B,\ \ldots, p_k/P_B$.

where $P_A = \sum_{i=0}^{t} p_i$, $P_B = \sum_{i=t+1}^{k} p_i = 1 - P_A$. For the gray levels G, put k =255. The Renyi entropy of order α for each distribution is defined as:

$$H_\alpha^A(t) = \frac{1}{1-\alpha} \ln \sum_{i=0}^{t} (p_i / P_A)^\alpha \tag{3}$$

$$\text{and} \quad H_\alpha^B(t) = \frac{1}{1-\alpha} \ln \sum_{i=t+1}^{255} (p_i / P_B)^\alpha \tag{4}$$

$H_\alpha(t)$ is parametrically dependent upon the threshold value t for the foreground and background. We try to maximize the information measure between the two classes (object and background). When $H_\alpha(t)$ is maximized, the luminance level t that maximizes the function is considered to be the optimum threshold value.

$$t^*(\alpha) = Arg \quad \max_{t \in G} [H_\alpha^A(t) + H_\alpha^B(t)]. \tag{5}$$

The technique consists of treating each pixel of the original image and creating a new image, such that $f_t(x,y) = 0$ if $f_t(x,y) \le t^*(\alpha)$ otherwise, $f_t(x, y) = 1$ for every $1 \le x \le M$ and $1 \le y \le N$. Since Shannon entropy measure is a special case of the Renyi entropy. The following expression can be used as a criterion function to obtain the optimal threshold at $\alpha \to 1$.

$$t^*(1) = Arg \quad \max_{t \in G} [S^A(t) + S^B(t)]. \tag{6}$$

To select suitable threshold value t^* and α can now be described as follows in *RenyiThreshold* algorithm:

Algorithm *RenyiThreshold*:
  Input: A digital grayscale image A of size $M \times N$.
  Output: Threshold value t^* at the parameter α.
   Begin
    1. Let $f(x, y)$ be the original gray value of the pixel at the point (x, y), x=1..M, y=1..N .
    2. For all i = 0, 1,…, 255 , calculate p_i and P.
    3. For all $t \in \{0,1,\ldots,255\}$:
      i. Calculate P_A, P_B, p_A, and p_B.
      ii. Calculate $H_\alpha^A(t)$ and $H_\alpha^B(t)$.
      iii. For α>0, calculate $t^*(\alpha)$, using Equations (5, 6)
  End.
End algorithm.

3. The edge detection

We will use the usual masks for detecting the edges [8]. A spatial filter mask may be defined as a matrix w of size $m \times n$. Assume that $m=2\mu+1$ and $n=2\rho+1$, where μ, ρ are nonzero positive integers. For this purpose, smallest meaningful size of the mask is 3×3. Such mask coefficients, showing coordinate arrangement as

Figure 1. Image region under the above mask is shown as Figure 2 .

w (-1,-1)	w (-1,0)	w (-1,1)
w (0,-1)	w (0,0)	w (0,1)
w (1,-1)	w (1,0)	w (1,1)

Figure 1

$f(x\text{-}1, y\text{-}1)$	$f(x\text{-}1,y)$	$f(x\text{-}1, y\text{+}1)$
$f(x, y\text{-}1)$	$f(x, y)$	$f(x, y\text{+}1)$
$f(x\text{+}1, y\text{-}1)$	$f(x\text{+}1,y)$	$f(x\text{+}1, y\text{+}1)$

Figure 2

In order to edge detection, firstly classification of all pixels that satisfy the criterion of homogeneousness, and detection of all pixels on the borders between different homogeneous areas. In the proposed scheme, first create a binary image by choosing a suitable threshold value using Renyi entropy. Window is applied on the binary image. Set all window coefficients equal to 1 except centre, centre equal to × as shown in Figure 3.

1	1	1
1	×	1
1	1	1

Figure 3

Move the window on the whole binary image and find the probability of each central pixel of image under the window. Then, the entropy of each central pixel of image under the window is calculated as $S(CPix) = - p_c \ln(p_c)$. Where, p_c is the probability of central pixel *CPix* of binary image under the window. When the probability of central pixel, p_c =1, then the entropy of this pixel is zero. Thus, if the gray level of all pixels under the window homogeneous, p_c= 1 and S =0. In this case, the central pixel is not an edge pixel. Other possibilities of entropy of central pixel under window are shown in Table 1.

Table 1: p and S of central under window.

p	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9
S	0.2441	0.3342	0.3662	0.3604	0.3265	0.2703	0.1955	0.1047

In cases sum = 8, p_c= 8/9, and sum=9, p_c=7/9 , the diversity for gray level of pixels under the window is low. So, in these cases, central pixel is not an edge pixel. In remaining cases, $sum \leq 6$ and $p_c \leq 6/9$, the diversity for gray level of pixels under the window is high. The complete *RenyiEdgeDetection* algorithm can now be described as follows:

Algorithm *RenyiEdgeDetection*;

Input: A digital grayscale image A of size $M \times N$.
Output: The edge detection image g of A.
Begin
1. Select suitable t^* and α, using *RenyiThreshold*.
2. Create a binary image: If $f(x, y) \le t^*(\alpha)$ then $f(x,y) = 0$ Else $f(x, y) = 1$.
3. Create a mask w, with dimensions $m \times n$: Normally, $m=n=3$. $\mu = (m-1)/2$ and $\rho=(n-1)/2$.
4. For all $1 \le x \le M$ and $1 \le y \le N$: find g an output image by set $g(x, y) = f(x, y)$.
5. For all $\rho+1 \le y \le N-\rho$ and $\mu+1 \le x \le M-\mu$, checking for edge pixels:
   i. $sum = 0$;
   ii. For all $-\rho \le k \le \rho$ and $-\mu \le j \le \mu$:
      If ($f(x, y) = f(x+j, y+k)$) Then $sum = sum+1$.
   iii. If ($sum > 6$) Then $g(x,y)=0$ Else $g(x,y)=1$.

End
End algorithm.

The above procedures can be done together in the following MATLAB program:

```
A = imread('lena.tiff');
[M,N]=size(A);
if ~islogical(A)  A = im2uint8(A); end;
p = zeros(256,3);
for ii=1:256 p(ii,1)=ii-1; end;
p(:,2) = imhist(A);                % calculate histogram counts
p (p(:,2)==0,:) = [];              % remove zero entries in p
p(:,3) = p(:,2) ./ numel(A);       % normalize p so that sum(p) is one.

%%%%%%%%% Renyi Entropy to calculate the Threshold %%%%%%%%%
Alpha=0.1;
ArgMax=0;
for t=1 : size(p)
    PA= sum(p(1:t,3));
    PB= 1-PA ;
    p1=p(1:t,3)/PA;
    p2=p(t+1: size(p),3)/PB;
    Ha=log2( sum ((p1.^Alpha)) )/(1-Alpha);
    Hb=log2( sum ((p2.^Alpha)) )/(1-Alpha);
    Hab = Ha + Hb;
    if ( Hab > ArgMax )  t_opt=p(t,1); ArgMax=Hab; end;
end;
f = double (A);
for i=1:M;  for j=1:N; f(i,j)=0; if (A(i,j) >= t_opt) f(i,j)=1; end; end; end;

%%%%%%%%% Using Threshold_Renyi to find the edge detection of A %%%%%%%%%
m = 3;  n = 3;
a0 = (m-1)/2;
```

```
b0 = (n-1)/2;
g=f; % For all pixel coordinates, x and y, do Set g(x, y) = f(x, y)
% Checking for edge pixels:
for y = b0+1 : N-b0;
    for x = a0+1 : M-a0;
        sum1 = 0;
        for k=-b0:b0;
            for j=-a0:a0;
                if ( f(x,y) == f (x+j,y+k) ) sum1=sum1+1; end;
            end;
        end;
        if ( sum1>6 ) g(x,y)=0; else g(x,y)=1; end;
    end;
end;
figure;
imshow(g);
```

4. Analysis of Experimental results

The performance of the proposed scheme is evaluated through the simulation results using MATLAB for a many test images. For this purpose, first group, standard test images were taken from free image databases on-line: USC-SIPI Signal and Image Processing Institute in University of Southern California, and CMU. Carnegie Mellon University, Links to image databases. Second group, a test images from the real-world, and synthetic images. Here, we only are present a set of sixteen test images and the results of the proposed scheme are compared with the results of well-established edge detection operator on the same set of test images. Shannon method is chosen for comparison because both approaches are more similar in the technique of implementation, It is already pointed out in the introduction that the traditional methods give rise to the exponential increment of computational time. However, the method [8] is decrease the computation time with employ Shannon entropy of edge detection. Here, we have used in addition to the original gray level function $f(x, y)$, a function $g(x, y)$ that is the average gray level value in a 3×3 neighborhood around the pixel (x, y).

Our analysis is based on how much information is lost due to edge detection. In this analysis, given two edge detections of a same original image, we prefer the one which lost the least amount of information. The optimal threshold value was computed by the proposed method for these sixteen images. Table 1 lists the optimal threshold values t^* that are found for these images for α values equal to 0.01, 0.05, 0.1, 0.2, 0.4, 0.6, 0.8, 1, , 2, and 5, respectively. Some original images together with their edge images obtained by using the optimal threshold of some values t^* are displayed side by side in Figures 4–10.

Using the sixteen images files (see the first columns of Table 2), we conclude that when $0<\alpha<1$, our proposed method produced good edge detection with optimal threshold values. Moreover, the optimal threshold value does not change very much when the fractional α value changes a little. However, in most cases, when α was greater than 1, this proposed method did not produce good edge images and the threshold values produced were unacceptable. (See the last columns of Table 2).

This new method when $\alpha = 0.1$ performs better than Shannon method when the value of α was one on the test images. (see Figures 4–10).

We conclude that when α is 0.1 our proposed method produced the best optimal threshold values on the test images. From the results; it has again been observed that the performance of the proposed edge detection scheme is found to be satisfactory for all the test images and works well.

Table 2: The optimal threshold values t^* for various values of α

α / Image	$t^*(.01)$	$t^*(.05)$	$t^*(0.1)$	$t^*(0.2)$	$t^*(0.4)$	$t^*(0.6)$	$t^*(0.8)$	$t^*(1)$	$t^*(2)$	$t^*(5)$
baboon.tif	113	112	112	112	112	97	97	97	97	98
biomedical.tif	141	148	148	148	148	148	148	140	82	68
brain.tif	121	121	132	133	141	155	164	172	180	164
cameraman.tif	130	128	125	123	192	192	192	192	197	197
circuit.tif	99	98	96	95	92	89	89	88	83	69
composed.tif	121	121	121	121	121	58	58	58	51	51
eggspindle.tif	90	91	97	115	115	121	121	121	128	128
fluorescein.tif	121	121	121	121	113	106	98	98	82	82
fruits.tif	121	120	119	117	108	107	106	106	101	96
ic.tif	126	125	125	92	84	84	83	83	88	92
head.tif	133	148	148	148	148	164	164	164	164	164
lena.tif	127	127	127	126	124	124	123	123	121	164
peppers.tif	120	120	120	120	120	120	121	121	132	132
Saturn.tif	127	131	134	141	158	164	165	164	2	0
spine.tif	121	121	113	113	105	82	82	67	37	17
synthetic.tif	121	121	121	121	120	113	113	113	164	172

original image   gray level histogram   proposed method $t^*(0.1)=112$ Shannon method $t^*(1)=97$

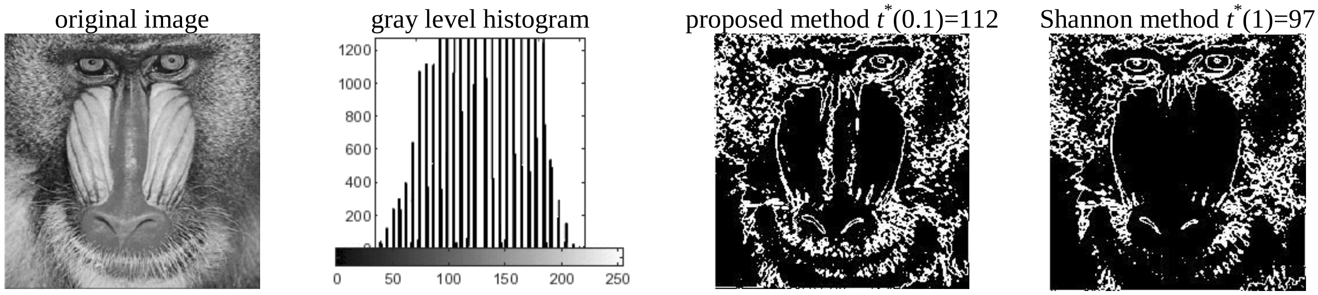

Figure 4: baboon images with 512×512 pixel.

original image   gray level histogram   proposed method $t^*(0.1)=125$ Shannon method $t^*(1)=192$

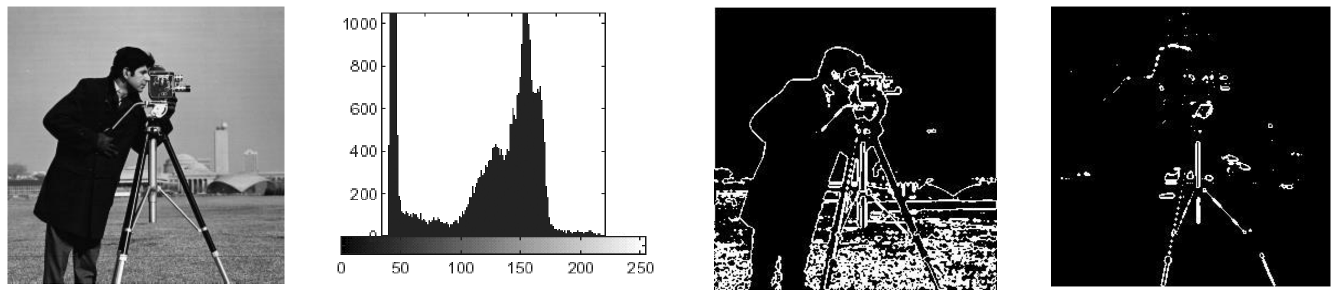

Figure 5: cameraman image with 256×256 pixel.

original image gray level histogram proposed method $t^{*}(0.1)=121$ Shannon method $t^{*}(1)=58$

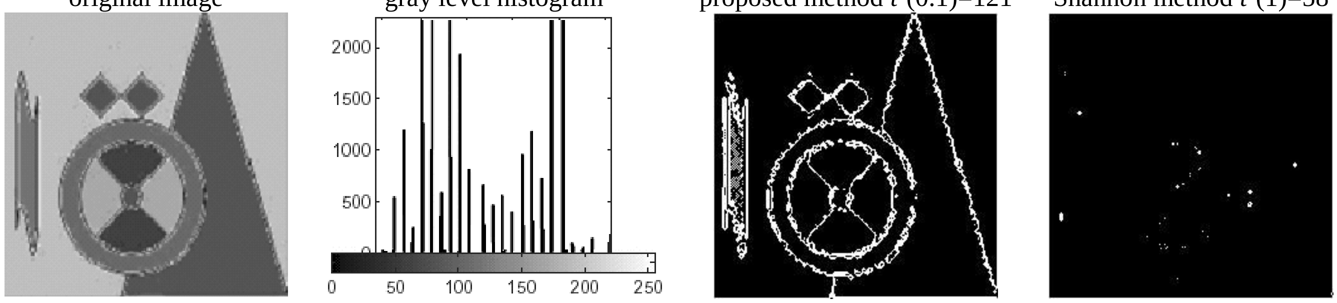

Figure 6: composed image with 226×218 pixel .

original image gray level histogram proposed method $t^{*}(0.1)=125$ Shannon method $t^{*}(1)=83$

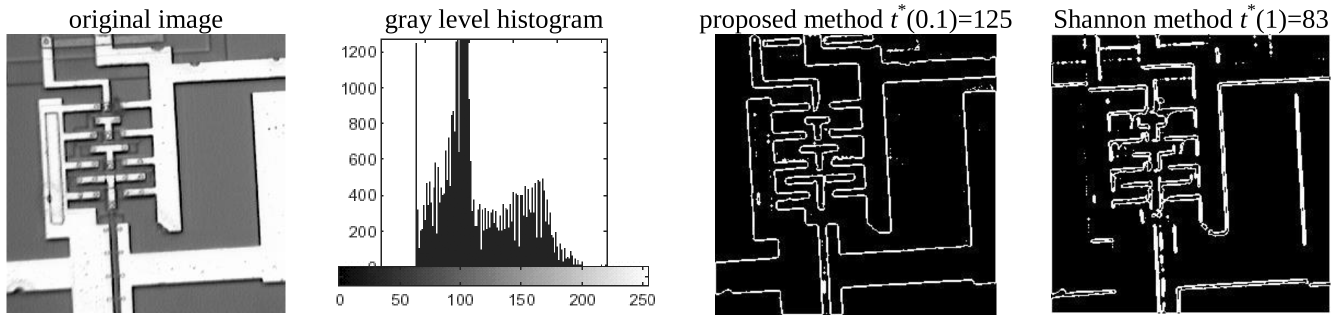

Figure 7: ic image with 256×256 pixel.

original image gray level histogram proposed method $t^{*}(0.1)=148$ Shannon method $t^{*}(1)=164$

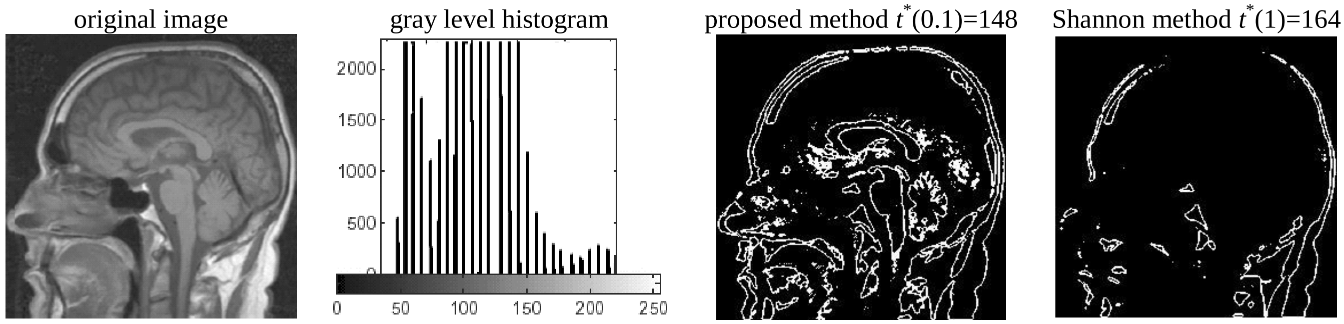

Figure 8: head image with 256×256 pixel.

original image gray level histogram proposed method $t^{*}(0.1)=127$ Shannon method $t^{*}(1)=123$

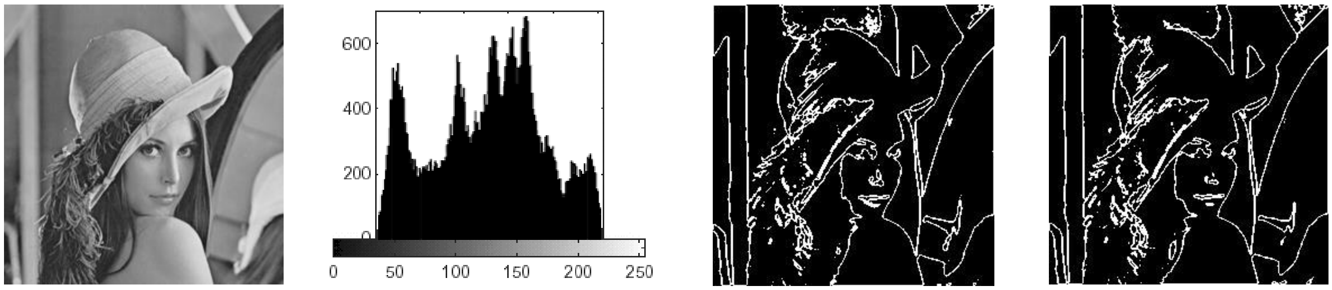

Figure 9: lena image with 256×256 pixel.

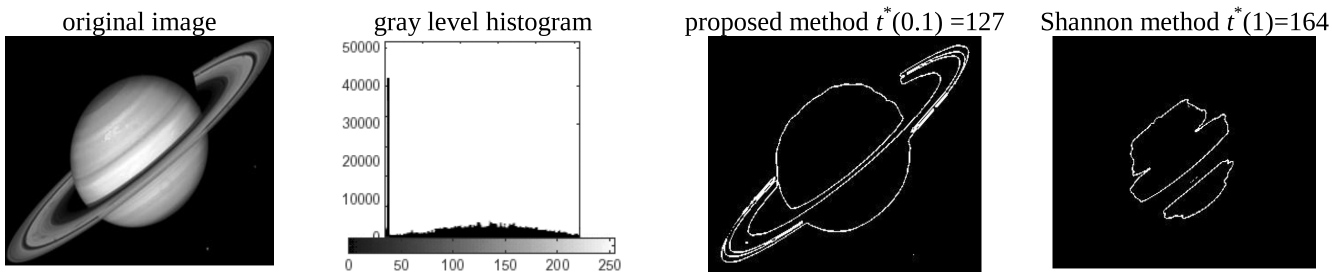

Figure 10: Saturn image with 512×512 pixel.

5. Conclusions

We developed a method of edge detection that employs Renyi's entropy of order α. It is already pointed out in the introduction that the traditional methods give rise to the exponential increment of computational time. However, the proposed method is decrease the computation time with generate high quality of edge detection. Experiment results have demonstrated that the proposed scheme for edge detection works satisfactorily for different gray level digital images. The work is under further progress to examine the performance of the proposed edge detector for different gray level images affected with different kinds of noise.The theoretical principles and systematic development of the algorithm for the proposed versatile edge detector is described in detail. The order α can be used as an adjustable value and can play an important role as a tuning parameter in the image detection chain for the same class of images. This can be an advantage when the image processing tasks depend on an automatic thresholding.

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