Acta Astronautica 115 (2015) 376–383

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Performance model for space-based laser debris sweepers

Manuel Schmitz [a,*], Stefanos Fasoulas [b], Jens Utzmann [c]

[a] 6741 Marul 31, Austria
[b] University of Stuttgart, Institute of Space Systems, Pfaffenwaldring 29, 70569 Stuttgart, Germany
[c] Airbus Defence and Space, Claude-Dornier-Strasse, 88090 Immenstaad, Germany

ARTICLE INFO

Article history:
Received 9 February 2015
Received in revised form
1 May 2015
Accepted 24 May 2015
Available online 2 June 2015

Keywords:
Space debris
Laser ablation
Active debris removal
Parametric mission concept study
Space based
Low Earth orbit (LEO)

ABSTRACT

This paper outlines the capabilities of a space-based laser for active debris removal in the range of 1 cm to 10 cm. As for respective ground based solutions, the space based laser ablates small bits of mass from the targeted debris objects. When directed properly, the resulting propulsive effect will lower the targets' perigees and facilitate an expeditious burnup in the Earth's atmosphere.

For this particular type of space mission, a mission-specific performance model has been developed. It does not only deliver the removal performance (e.g. the number of objects removed) of a given mission scenario. It also provides insight into the relevant mechanisms that are driving the performance. The user can tell why a particular scenario is strong or weak, and iteratively tune the mission and the system parameters of the orbital debris sweeper. This parametric model allows case studies, parameter studies and optimizations of mission scenarios.

As an illustration of the application of this model, a baseline mission scenario for a debris sweeper mission is being presented. Three system driver quantities for the overall mission concept are highlighted, and a reasonable parameter range for each of them is presented. Finally, it is shown how a single laser could already reduce the debris density in low Earth orbit (LEO) by more than 20% in 10 years.

1. Introduction

Currently, there are more than 17,000 space objects in the catalogue of the United States Space Surveillance Network (SSN)[5], with only about 1200 of them being active satellites [32]. A collision threat emanates from all the other inactive and uncontrollable objects, the space debris.

Because most of these objects have a residual orbital lifetime in the range of several hundreds or thousands of years, it is possible that they will collide with another object or desintegrate because of other reasons. The collision probability depends on the number of objects. Each collision generates new objects (fragments), so that the growth of debris objects will eventually follow an exponential population dynamics ('snowball effect'). The number of individual debris objects would then keep on growing even if all the human space activity stopped tomorrow [7].

The examination of the phenomenon shows that one would have to remove more or less five large objects per year in order to stabilize the debris situation and keep the status quo [11]. This figure, however, assumes a nearly perfect compliance with the established post-mission disposal rules. Under the current circumstances, the removal of more than five objects per year could be necessary. The necessity and effectiveness of active debris removal have been affirmed by a majority of researchers [10,3].

However, at present no economically viable technical concept to perform such an active removal has been

E-mail address: port17@gmx.net (M. Schmitz).

http://dx.doi.org/10.1016/j.actaastro.2015.05.032
0094-5765/

developed. Never in the history of spaceflight, an uncooperative object was ever removed from orbit. Venturing into uncharted waters, engineers from all over the world are conceiving, evaluating and comparing different approaches and concepts: de-orbitation by a one-shot chaser [24], attachment of drag enhancement devices, electrodynamic tethers [15,14], booster packages [4] or expanding foam [1], ion-beam shepherds [2] or others. All of these concepts have one thing in common: they are targeting at the removal of individual large objects.

Less attention has been spent on the removal of medium-size debris, ranging from 1 cm to 10 cm in size. In this case, there are more than 20 times as many objects. These particles are particularly dangerous to active spacecraft because the collision probability is relatively high due to their large number. At the same time, one of these objects can destroy a space asset just as a larger object can. In fact, some researchers have argued that the immediate, short-term threat to satellites originates from these medium-size objects, while large-size debris will fragment into medium and small size debris in the future. Therefore, remediation efforts should not be exclusively directed towards large-object removal, but also consider the removal of medium-size debris [3].

Removal concepts for larger junk usually require rendezvous operations with each targeted object. This is costly in terms of fuel and time, and therefore impractical for dealing with a high number of smaller objects. The use of high-power lasers appears to be one of the (very) rare means by which medium-size debris can be addressed: when a high-intensity pulsed laser engages an object, it ablates small bits of mass. The resulting recoil will change the velocity of the targets and lower the perigees, so that the orbital lifetimes of the specific objects are reduced (Fig. 1). Sometimes, this interaction mechanism is called ‘Laser-ablation propulsion’ [19] or ‘Laser-impulse coupling’[20].

The debris-clearing laser can be placed on the ground or in orbit. So far, mainly ground-based concepts have been studied [21], most notably within the ORION study in the 1990s [22].

Much less work has been done on investigating the possibility of placing the laser into space. In contrast to a ground-based laser, the space-based concept benefits from its vicinity to the targets, reducing the necessary laser size and most of the auxiliary systems by a factor of 100–1000. On top of that, atmospheric effects do not need to be taken into account. This leads to an even smaller laser, the saving

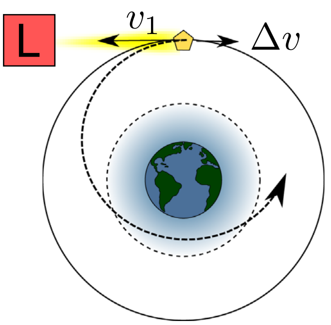

Fig. 1. The laser lowers the perigees of the targeted objects.

of a beam correction subsystem, and a larger range of options for the laser wavelength.

An early study in 1989 suggested a nuclear-powered laser platform, chasing individual objects or object clouds, with an electrical propulsion system being the system driver [13].

A later publication by Wolfgang Schall of the German Deutsches Zentrum für Luft- und Raumfahrt (DLR) assumed that the relative drift of the node lines of the debris objects (due to the Earth's oblateness) will bring most of the objects into the range of laser automatically. He introduced a number of assumptions and simplifications, and calculated that a 100 kW-laser could clean most of the debris in low Earth orbit (LEO) within a few years. [27]

The idea of putting a debris-clearing laser into space has had a renaissance lately, which reflects in the number of recent, on-topical publications [16,31]. In continuance of this work, a new parametric performance model for this mission type has been developed. Taking into account the scientific progress since 2002, some of Schall's assumptions can be replaced by certainties. Moreover, some of his calculation techniques have been replaced by more detailed models in the meantime. Most notably, a deterministic debris population based on the ESA MASTER model [6] has been used, and a refined laser–debris interaction model has been introduced.

This newly developed parametric performance model for space-based laser debris sweepers will be explained in the following pages. Then, it will be applied on to a baseline scenario as a case study. The model will be used to explore the general laser sweeper concept in greater depth, and trace out its limits: what are the boundary conditions under which the debris population can be reduced significantly? What are the top-level mission parameters such as orbit, laser operating range or mission duration?

Within the following sections, these questions will be answered in the reverse order: by determination of the necessary key mission parameters, it will be shown that the requirements of the missions can probably be met and that it is feasible.

2. Approach and outline

In a reasonable mission scenario, a meaningful share of the debris population will be de-orbited, so that the in-space debris density will be significantly reduced. The performance of the mission is therefore determined by the number of de-orbited debris objects.

A parametric mission performance model has been established, which allows parametric mission studies. It is the centerpiece of the mission concept analysis, and based on a debris population, which has been derived from the ESA MASTER-2009 model[6] at the May 2009 epoch. The mission performance model starts with the set of all medium-sized debris objects (Fig. 2), and (cascadingly) isolates subsets of debris objects using different necessary conditions for de-orbitation by the space-based laser. Conclusively, an object that satisfies all these conditions can be de-orbited within the particular mission scenario, and is an element of the set of removed objects. Finally, the magnitude of the set of removed objects indicates the mission performance.

Fig. 3 shows a schematic representation of this model: every branching stands for the isolation of a subset, using a specific necessary condition for de-orbitation. An object that satisfies all necessary conditions ends up in the rightmost branch (no. 8), which is the set of removed objects.

The different colours in Fig. 3 indicate different categories of conditions for de-orbitation: Yellow conditions or branches (no. 1 and 2) are a consequence of quality of the debris population only, and therefore a nearly constant part of the performance model. The red branching (no. 3) is the ‘reachability’ condition, the purple part (no. 4–6) describes the influence of the relative geometry between laser and target, and the blue part (no. 7) finally considers a laser–debris interaction model describing the laser-ablative effects.

In order to examine and quantify the relevant effects, a numerical trajectory propagation of the debris population along with the laser sweeper has been performed. A dedicated simulation tool for parametric mission studies has been developed. The next chapter will explain and discuss each branching in Fig. 3.

3. Theory and simulation

The performance model starts with the set of all objects from 1 cm to 10 cm size in the reference population, which are more than 700,000. Because only missions in LEO are being considered, all non-LEO objects can be excluded from further considerations in the first branching. This leaves around 570,000 objects, which are passing through altitudes below 2200 km.

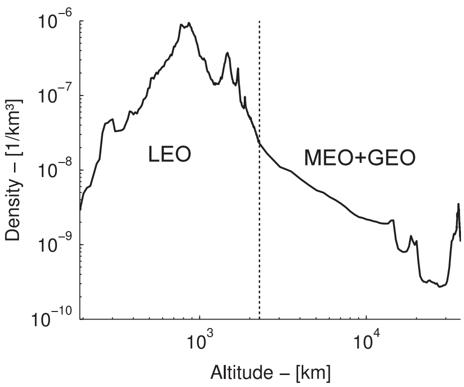

Fig. 2. Debris density over altitude (derived from the ESA MASTER model).

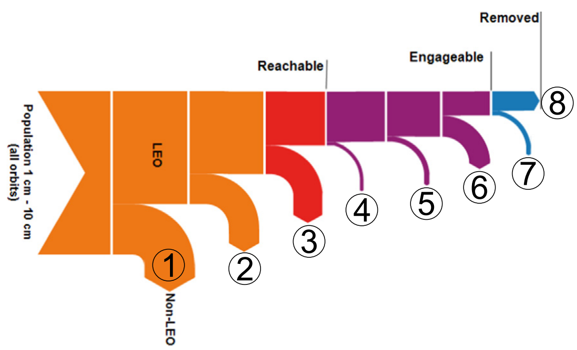

Fig. 3. Outline of the mission performance model.

3.1. Reachability

An object can only be de-orbited by the laser, if the object enters the laser's operating range at least once during the mission. The laser's operating range is determined by three factors: the laser source, the beam focussing subsystem, and the target's ablation threshold. It can be approximated by

$$R_{\max} = \sqrt{\frac{\pi}{4}\left(\frac{d_L}{M^2\lambda}\right)^2 \frac{W_p}{\Phi_{th}}} \tag{1}$$

where W_p is the laser's pulse energy, d_L the primary focussing aperture, M^2 the beam quality of the laser, λ it's wavelength, and Φ_{th} the target's ablation threshold at the laser's pulse duration (τ). As a preliminary approximation, the following ablation threshold has been assumed for all targets:

$$\Phi_{th}(\tau) = 28\frac{kJ}{m^2}\sqrt{\frac{\tau}{10\ ns}} \tag{2}$$

One can imagine the laser's operating range as a sphere, which is moved along a mission orbit, and the distinct debris objects crossing the sphere are counted.

This has been done systematically for different mission altitudes, inclinations and laser operating ranges. Generic values have been chosen for all other necessary parameters. The simulated mission time was 1 year. The results of this parameter study are visualized in Fig. 4: it can be seen that most of the objects are reached at altitudes where the debris density is highest. Therefore, an altitude of 871 km has been chosen as a baseline value for further investigation.

At that altitude, the number of objects reached increases over the inclination until near-polar orbits. That is why only near-polar inclinations have been considered further.

Simulations revealed the same qualitative behaviour for all laser operating ranges between 2 km and 200 km,

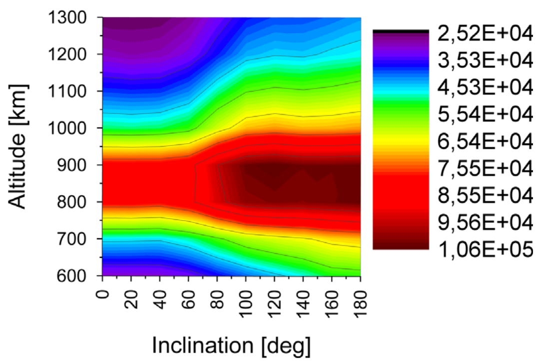

Fig. 4. Number of distinct reached objects for a laser operating range of 20 km within one year.

so that the selection of the mission altitude and inclination is valid for all laser ranges in this domain.

After it has been asserted that the laser's orbital (mean) altitude will be 871 km, all debris objects not crossing that altitude can be discarded from the performance model. An object cannot be de-orbited if

$$r_{peri,obj} > \underbrace{(871\text{ km}+R_e)(1+e_L)}_{r_{apo,L}} + R_{\max}$$
$$\vee \quad r_{apo,obj} < \underbrace{(871\text{ km}+R_e)(1-e_L)}_{r_{peri,L}} - R_{\max} \tag{3}$$

where $r_{peri,obj}$ and $r_{apo,obj}$ are the perigee and apogee radii of the debris object, $r_{peri,L}$ and $r_{apo,L}$ are the perigee and the apogee radii of the laser, R_e is the Earth's radius, and e_L is the orbital eccentricity of the laser. Although this branching formally depends on e_L and $R_{\max}$, their influence is usually small. This condition accounts for the second branching in Fig. 3.

Now that altitude and inclination have been selected, the number of objects actually coming into the range of laser depends only on two factors: the laser's operating range $R_{\max}$ and the mission duration. (The orbital eccentricity of the laser e_L also plays a role, but is negligible in a first approximation.) Again, a larger number of scenarios have been investigated in order to determine the connections between mission duration, operating range and reached objects. The results are summarized in a diagram (Fig. 5); the diagram represents the third red branching in Fig. 3.

From Eq. (1) follows that the laser operating range depends on the quality and power of the laser. At this point, it is clear that the operating range $R_{\max}$ is a driver quantity of the mission concept. It is legitimate to ask, how many additional objects can be reached by ‘investing’ an additional kilometer of laser range. Fig. 6 provides the answer: within a 10 years mission, with a laser range around 10 km it is possible to reach 33% of the 570,000 LEO objects (with little effort), extending the range to 20 km means that 50% of the LEO objects can be reached, and with a range of 50 km it is possible to reach almost all objects crossing the 871 km altitude. This means that it does not seem to make much sense to dimension the laser for the engagement of objects beyond a distance of 50 km.

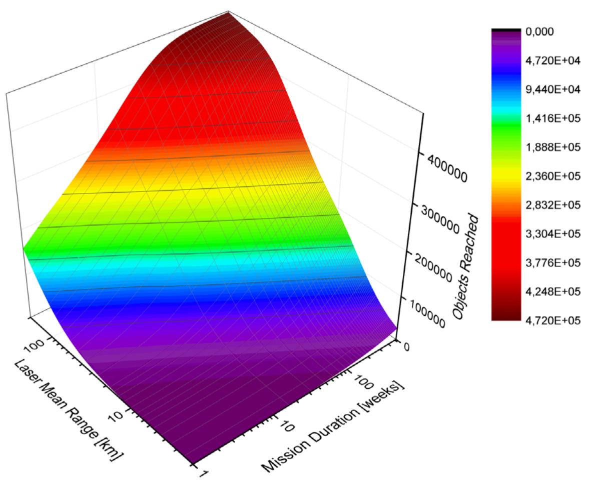

Fig. 5. Total number of distinct reached objects, depending on mission duration and the laser's operating range.

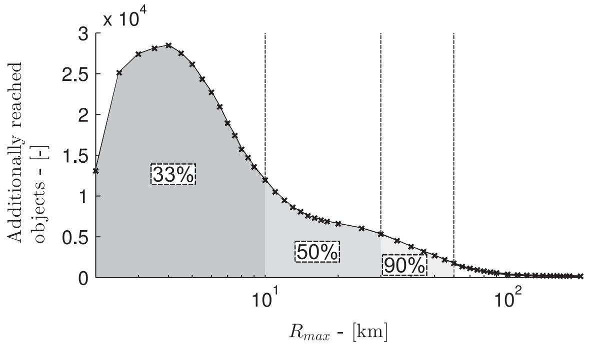

Fig. 6. Differential reachable objects curve for a 10 y mission: The surface under the curve is a measure for the total number of reached objects.

3.2. Relative geometry

After explaining the yellow and red branching in Fig. 3, this subsection will discuss the purple part (no. 4–6) of the performance model.

In the Introduction, it has been pointed out that objects with a perigee below 200 km have a short orbital life expectancy. This means that the laser does not need to engage these objects, which are usually 1–2% of the reached objects.

The second purple branching subtracts objects that never drift into a favourable position. Before providing a figure, a definition of ‘favourable’ in this context must be provided.

When a laser-ablative propulsion mechanism changes the velocity of a debris object, it is clear that the object's orbit can either be raised or lowered (or neither of the two). Which of the two, depends (in most cases) on the direction of the laser-ablative recoil on the target. For the sake of simplicity, it will be assumed that the direction of the recoil will always be in the direction of the laser beam. This relative position vector is determined by the kinematic states of laser and target at a given time index.

Now, the question is: how large must the absolute value of the imposed Δv be in order to lower the object's perigee altitude to $H_{Peri} = 200$ km? If there is no solution for a particular situation, the target cannot be moved to a short-lived orbit (200 km), and the current relative geometry between laser and object's position is ‘unfavourable’. Engaging the object in such a situation would probably raise the object's perigee. All other cases are considered ‘favourable’, although the necessary Δv to achieve the desired target perigee of 200 km may be very high in some cases.

Derived from the vis viva equation and the conservation of angular momentum, the following equation can be used to distinguish the favourable from the unfavourable situations:

$$a\Delta v^2 + b\Delta v + c = 0 \tag{4}$$

with

$$a = r_{Peri}^2 r - r^3(Y^2 + Z^2) \tag{5}$$

$$b = 2r_{Peri}^2 r(v_{1R}X + v_{1S}Y) - 2r^3 v_{1S}Y \tag{6}$$

$$c = r_{Peri}^2 r v_1^2 - 2\mu(r_{Peri}^2 - r r_{Peri}) - r^3 v_{1S}^2 \tag{7}$$

In this equation, r_{Peri} is $R_{Earth} + 200$ km, r is the object's current geocentric distance, μ is the Earth's gravitational parameter, v_1 is the object's orbital velocity at r, and $(X, Y, Z)^T$ is the direction vector of the laser-ablative thrust in the RSW-system (object-centered, X=radially outward, Y=along-track, Z=off-plane) (Fig. 7). (A derivation of the equation is provided in the appendix.)

If this quadratic equation has at least one positive, non-complex solution for Δv, the geometrical relative situation is favourable. For the analysis of an entire mission scenario, one can solve this equation for all relative situations, for all debris objects at all time indices. There are some debris objects that will never be encountered in a favourable relative position for the entire duration of the mission. Numerical simulation has shown that roughly 4% of the reachable objects (third, red branching in Fig. 3) are unfavourable.

The third purple branching in Fig. 3 (no. 6) is a question of relative velocity. In order to acquire and track the target object with the laser, the space-based removal system must either move parts of the laser and focussing subsystem, or rotate the entire spaceborne platform. This poses a limit to the 'engageability' of objects, because the laser cannot be pointed and moved at arbitrary velocities with the necessary precision due to technical constraints.

Objects which always exceed the maximum tracking velocity of the laser when they are in a favourable position are removed from further consideration in the last purple branching. Considering a 10 year reference mission, the lower boundary for the laser tracking velocity is $2.4^\circ/_s$. Below this threshold, less than 1% of the favourable objects can be engaged. An upper limit may be in the region of $25^\circ/_s$, where more than 90% of the favourable objects can be engaged. Both, tracking velocity and pointing accuracy are a second system driver quantity in addition to the laser range $R_{\max}$.

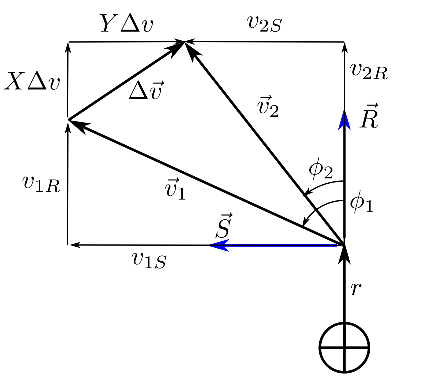

Fig. 7. Quantities used in Eq. 4

3.3. Engagements

So far, it has been determined how many objects can be engaged. Some of the objects will get a perigee below 200 km and burn up naturally within a few days or weeks. However, some of the objects may only be moved to lower orbits but above 200 km. This might happen, for example, because the objects are passing by so quickly and they are out of range before sufficient recoil has been generated. One must add the available interaction times to the considerations, which are determined by the kinematic relationships of debris and laser.

In order to determine the number of objects actually de-orbited, the engagements and laser firings have been simulated. While the laser is firing, the laser-ablative thrust is considered as an additional perturbing force in a numerical state propagator (4th order Runge–Kutta). The calculation of the force, however, requires a laser–debris interaction model.

The laser-ablative force is macroscopically modelled by the so-called coupling coefficient C_m:

$$F_{abl} = C_m P_{laser} \tag{8}$$

In this context P_{laser} is the laser power that reaches the target object. The scalar coupling C_m depends on a variety of parameters, but its qualitative trend remains nearly the same over large parameter ranges and for many lasers and materials. This trend is depicted in Fig. 8; the shape has been taken from [19], but the values have been scaled to account for the vacuum conditions in space. Approximate values from C_m in vacuum have been taken from [29] and [30].

A major unknown in the model is the shape of the debris targets, which are unknown but are expected to have an impact on the interaction [8,9]. One consequence of non-uniformly shaped targets is that their orientations and their angular velocities may influence the recoil vector, too. Because target shape effects are a subject of ongoing research, it has been accounted for in the model by an efficiency factor of 70%, which is being applied on C_m. Future, more sophisticated models may reproduce the effects more accurately.

Another concern is the mass loss of the target during de-orbitation: preliminary investigation showed that significant fractions of the targets can be evaporated, reducing the target's cross-section. On one hand, this would

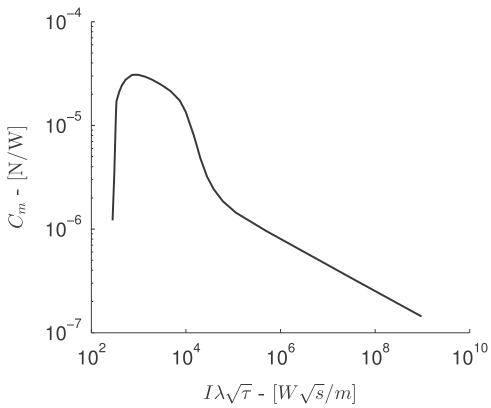

Fig. 8. Model for coupling coefficient.

reduce the absorbed energy per laser shot, but it would also reduce the inertia of the target. For now, the effect is neglected because the influence on the overall mission is considered to play a secondary role.

4. Results

Now that the performance model has been established, a baseline mission scenario will be discussed. It is meant to serve as reference case for ongoing and future studies.

For the laser–debris interaction model, a Nd:YAG laser is assumed. Its wavelength is 1064 nm, the pulse width is 10 ns, the beam quality is $M^2 = 12$, and the pulse repetition rate is 1000 shots per second. It has been chosen because the technology is known to have a development perspective towards high pulse energies, which are needed in order to achieve the necessary operating ranges (see Eq. 1). At the same time, the other laser parameters seem to be within reasonable limits.

Using the 2nd or 3rd harmonic of the laser could increase the efficiency. However, it would also increase the technical complexity. A frequency multiplication is usually performed by adding a crystal to the beamline. This crystal would be a potential source of failure. After carefully weighting the pros and cons, it has been decided that the risk outweighs the benefit. However, future studies may obsolete this provisional assumption.

The simulated baseline mission has a duration of 10 years, a laser operating range of 20 km, a laser pulse energy of 372 J and a beam tracking velocity limit of $15^{\circ}/_{s}$. Fig. 9 shows the debris density before and after the baseline mission: the debris density at the most critical altitude can be reduced by 23% with one laser in 10 years. The laser will engage many objects at least two times, and perform roughly 140,000 engagements in 10 years. This means that the average time between two firings will be between 30 min and 40 min.

The model of the mission does not only compute an estimation of the post-mission debris density but it can also extract the information on the scheduling of engagements, and it can create a power profile and predict requirements for the power supply system. From the pulse energy and the repetition rate follows that the average output power of the laser is 372 kW. For an overall laser efficiency of 7%, the electrical input would therefore be 5.31 MW. However, the average engagement is about 2 s long, and the required energy is ≈ 3 kWh per engagement. In the idle phase of roughly 30 min, which follows the engagement phase, the storage is replenished with a rate of about 6 kW. (The real computation of the mission power profile is more complex than shown here. A more detailed view, which includes the distributions of the engagement durations and the idle phase durations, will be provided in another publication.)

The apparent key quantity is not the source of the electrical energy, but the storage capacity and the rate of discharge. It must be kept in mind that these values are highly sensitive to some input parameters of the model. An analysis of the sensitivities has shown that the power supply requirements can easily shrink by a factor of 100, just by moderately improving the laser. For example, Soulard et al. have recently proposed a nearly diffraction limited laser for a debris sweeper [31]. According to Eq. (1), this alone reduces the pulse energy from 372 J to 2.6 J, and scales down all power requirements by a factor of $(1/12)^2 \approx 0.007$.

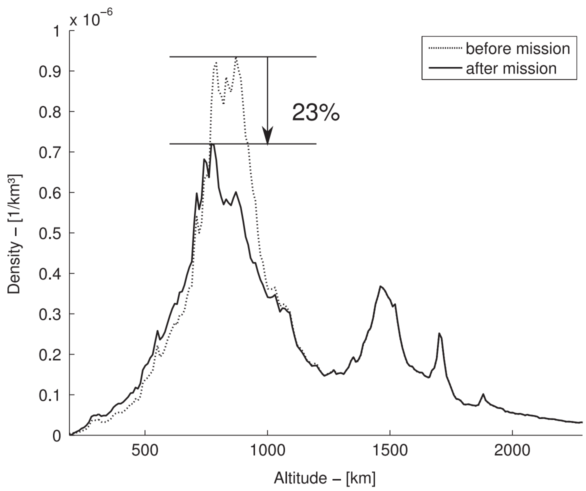

Fig. 9. Orbital debris density after the 10 year baseline mission.

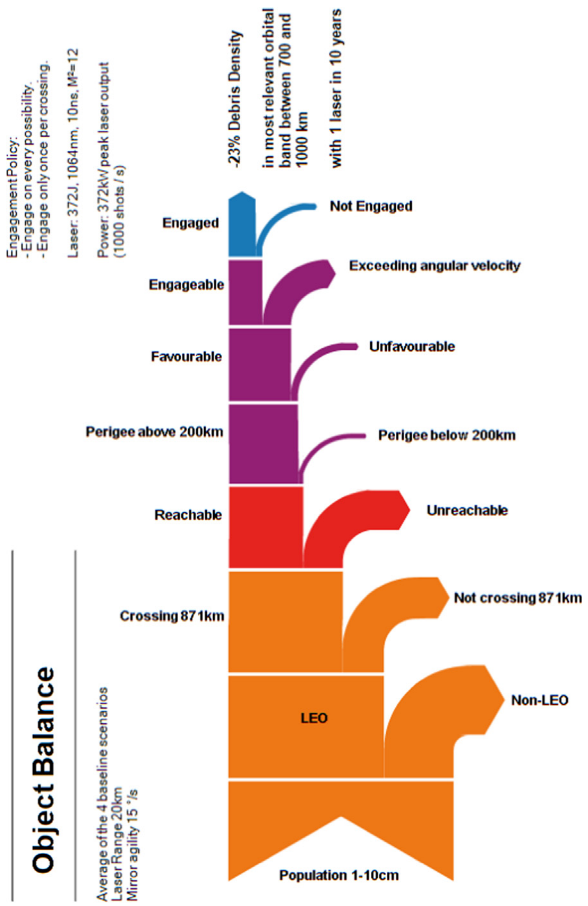

Fig. 10. Object balance of baseline scenario.

The object balance (Fig. 10) reveals the optimization potentials of the scenario: many objects are not removed because they can either not be reached, or they are too fast. Increasing R_{max} or the maximum tracking velocity will yield an improvement of the mission's removal performance.

5. Conclusion

Summing up, a space-based debris removal mission would take place at the altitudes where the debris density is highest, and the laser would have a near-polar inclination. A small orbital eccentricity of the laser, in the range of 0.01–0.02, has been found to enhance the mission performance.

Three system drivers have been identified: laser maximum operating range, beam tracking velocity or mirror agility, and power supply. Laser ranges lower than 10 km would be insufficient, because too few objects will cross the influence sphere. On the other hand, laser ranges beyond 50 km do not seem to be worth the effort. Conclusively, the vicinity to the targets really is an advantage of the removal concept. The primary mirror or platform agility should be in the range of 2.4 $^{\circ}/_{s}$ to 25 $^{\circ}/_{s}$, and the electrical power system must be able to provide peak powers of at least 100 kW for few seconds.

Like the ground-based sister concepts, the space-based laser will probably require a catalogue of the targeted debris population. Buildup, maintenance and accuracy of such a catalogue will have to be addressed. Since it is unlikely that the catalogue will provide sufficient accuracy for laser pointing, the removal system will require some form of cueing subsystem for agile and high-precision laser beam guidance.

Additionally, the energy consumption and associated waste heat profile are to be analyzed, based on the mission simulation discussed herein.

Fig. 10 shows that many LEO objects still do not cross the laser's altitudes around 871 km. There could be a major optimization potential for the mission concept, by allowing the laser to change its altitude over time, or by using a constellation of lasers.

However, the next major step would be the improvement of the laser–debris interaction model. Only a broader campaign of experiments can verify the elaborate theoretical models, and create sufficient credibility and proof that the laser-ablative propulsion mechanism performs as expected when used with space debris targets. In view of the promising performance figures of the concept, $>20\%$ debris density reduction in 10 years, it seems justified to further pursue the development of the concept.

Appendix A. Derivation of Eq. (4)

A debris object is on an initial orbit with a perigee larger than r_{Peri} (e.g. R_e+200 km). It is encountered by the orbital laser at a given Earth-centered radius r, when the object's velocity is $\vec{v}_1$. The task is to change the object trajectory, so that the perigee becomes r_{Peri}. No doubt, the manoeuvre can be described by a single (vectorial) velocity change.

Furthermore, there is a constraint that a velocity change Δv can only be applied into a given direction $\vec{F}_{Dir}=(X,Y,Z)^T$. This vector is given by the relative position of the object w.r.t. the laser.

(The nomenclature and symbols are consistent with reference [12].)

The square of the specific orbital angular momentum of the target orbit (index 2) is

$$h_2^2=r_{Peri}^2 v_{Peri}^2=r^2 v_2^2 \sin^2\phi_2 \tag{A.1}$$

where v_{Peri} will be the velocity of the object at r_{Peri}, and v_2 is the velocity of the object immediately after the engagement. The conservation of energy (Vis-Viva) between the same two orbital points is

$$\frac{1}{2}v_{Peri}^2-\frac{\mu}{r_{Peri}}=\frac{1}{2}v_2^2-\frac{\mu}{r} \tag{A.2}$$

Together, one can write

$$\frac{r_{Peri}^2}{r^2}\left(1-\frac{2\mu}{v_2^2}\left(\frac{1}{r}-\frac{1}{r_{Peri}}\right)\right)-\sin^2\phi_2=0 \tag{A.3}$$

In the next step, Δv, $\overrightarrow{F_{Dir}}$ and v_1 are introduced into the equation, substituting ϕ_2 and v_2. Exploiting geometrical relationships (Fig. 7), one can find

$$v_2^2=(\vec{v_1}+\Delta v\overrightarrow{F_{Dir}})^2 \tag{A.4}$$

and

$$\sin^2\phi_2=\frac{(v_{1S}+\Delta vY)^2+\Delta v^2Z^2}{v_2^2} \tag{A.5}$$

Both relationships are put into Eq. (A.3). Together with the binomial theorem for $(\vec{v_1}+\Delta v\overrightarrow{F_{Dir}})^2$, Eq. (4) is the result.

References

[1] M. Andrenucci, P. Pergola, A. Ruggiero, Active Removal of Space Debris—Expanding Foam Application for Active Debris Removal, ESA ESTEC, Noordwijk, 2011.

[2] Claudio Bombardelli, Jesus Pelaez, Ion beam shepherd for contactless space debris removal, J. Guidance Control Dyn. 34 (3) (2011) 916–920.

[3] Christophe Bonnal, Jean-Marc Ruault, Marie-Christine Desjean, Active debris removal: recent progress and current trends, Acta Astronaut. 85 (2013) 51–60.

[4] Marco M. Castronuovo, Active space debris removal—a preliminary mission analysis and design, Acta Astronaut. 69 (9–10) (2011) 848–859.

[5] D. Shoots, Orbital Debris Quaterly News, vol. 18, Issue 4, in: Debi Shoots (Ed.), National Aeronautics and Space Administration, October 2014.

[6] S. Flegel, J. Gelhaus, M. M̈ockel, C. Wiedemann, and D. Kempf, Final report: maintenance of the ESA master model, Rev. 1.1, Tech. Univ. Braunschw. 2011.

[7] Donald J. Kessler, Burton G. Cour-Palais, Collision frequency of artificial satellites: the creation of a debris belt, J. Geophys. Res. 83 (A6) (1978) 2637.

[8] D.A. Liedahl, S.B. Libby, A.M. Rubenchik, and C.R. Phipps, Momentum transfer by laser ablation of irregularly shaped space debris, in: International Symposium on High Power Laser Ablation 2010, 2010, pp. 772–779.

[9] A. Liedahl, Alexander M. Rubenchik, Stephen B. Libby, S. Nikolaev, Claude R. Phipps, Pulsed laser interactions with space debris: target shape effects, Adv. Space Res. 52 (5) (2013) 895–915.

[10] J.-C. Liou, An active debris removal parametric study for LEO environment remediation, Adv. Space Res. 47 (11) (2011) 1865–1876.

[11] J.-C. Liou, N.L. Johnson, N.M. Hill, Controlling the growth of future LEO debris populations with active debris removal, Acta Astronaut. 66 (5–6) (2010) 648–653.

[12] Ernst Messerschmid, Stefanos Fasoulas, Raumfahrtsysteme: einführung mitübungen and lösungen, 4., neu bearb. Aufl, Springer, Heidelberg, UA, 2011.
[13] John D. Metzger, Rene J. Leclaire, Steven D. Howe, Karen C. Burgin, Nuclear powered space debris sweeper, J. Propuls. Pow. 5 (5) (1989) 582–590.
[14] Shin-Ichiro Nishida, Satomi Kawamoto, Yasushi Okawa, Fuyuto Terui, Shoji Kitamura, Space debris removal system using a small satellite, Acta Astronaut. 65 (1–2) (2009) 95–102.
[15] Carmen Pardini, Toshiya Hanada, Paula H. Krisko, Benefits and risks of using electrodynamic tethers to de-orbit spacecraft, Acta Astronaut. 64 (5–6) (2009) 571–588.
[16] Claude R. Phipps, LADROIT—a spaceborne ultraviolet laser system for space debris clearing, Acta Astronaut. 104 (1) (2014) 243–255.
[17] Claude R. Phipps, Kevin L. Baker, Stephen B. Libby, Duane A. Liedahl, Scot S. Olivier, Lyn D. Pleasance, Alexander M. Rubenchik, James E. Trebes, E. Victor George, Bogdan Marcovici, James P. Reilly, and Michael T. Valley, Removing orbital debris with pulsed lasers, in: International Symposium on High Power Laser Ablation 2012, 2012, pp. 468–480.
[18] Claude R. Phipps, Kevin L. Baker, Stephen B. Libby, Duane A. Liedahl, Scot S. Olivier, Lyn D. Pleasance, Alexander M. Rubenchik, James E. Trebes, E. Victor George, Bogdan Marcovici, James P. Reilly, Michael T. Valley, Removing orbital debris with lasers, Adv. Space Res. 49 (9) (2012) 1283–1300.
[19] Claude R. Phipps, Mitat Birkan, Willy L. Bohn, Hans-Albert Eckel, Hideyuki Horisawa, Thomas Lippert, Max Michaelis, Yuri Rezunkov, Akihiro Sasoh, Wolfgang O. Schall, Stefan Scharring, John Sinko, Review: laser-ablation propulsion, J. Propuls. Pow. 26 (4) (2010) 609–637.
[20] Claude R. Phipps, James R. Luke, D. Funk, D. Moore, J. Glownia, Thomas Lippert, Laser impulse coupling at 130 fs, Appl. Surf. Sci. 252 (13) (2006) 4838–4844.
[21] Claude R. Phipps and M. M. Michaelis, Neo-lisp: deflecting near-earth objects using high average power, repetitively pulsed lasers, in: 23rd European Conference on Laser Interaction with Matter, 1994.
[22] Claude R. Phipps and James P. Reilly, Orion: clearing near-earth space debris in two years using a 30-kW repetitively-pulsed laser, in: XI International Symposium on Gas Flow and Chemical Lasers and High Power Laser Conference, 1997, pp. 728–731.
[23] Claude R. Phipps and John Sinko, Applying new laser interaction models to the orion problem, in: International Symposium on High Power Laser Ablation 2010, 2010, pp. 492–501.
[24] D. Reintsema, J. Thaeter, A. Rathke, W. Naumann, P. Rank, and J. Sommer, DEOS—the german robotics approach to secure and de-orbit malfunctioned satellites from low earth orbits, in: International Symposium on Artificial Intelligence, Robotics and Automation in Space (i-SAIRAS), 2010.
[25] Alexander M. Rubenchik, C.P. J. Barty, R.J. Beach, A.C. Erlandson, J.A. Caird, and Claude R. Phipps, Laser systems for orbital debris removal, in: International Symposium on High Power Laser Ablation 2010, 2010, pp. 347–353.
[26] Alexander M. Rubenchik, A.C. Erlandson, and Duane A. Liedahl, Laser system for space debris cleaning, in: International Symposium on High Power Laser Ablation 2012, 2012, pp. 448–455.
[27] Wolfgang O. Schall, Removal of small space debris with orbiting lasers, in: High-power laser ablation, 1998, pp. 564–574.
[28] Wolfgang O. Schall, Laser radiation for cleaning space debris from lower earth orbits, J. Spacecr. Rockets 39 (1) (2002) 81–91.
[29] Wolfgang O. Schall, Hans-Albert Eckel, Willy L. Bohn, Laser propulsion thrusters for space transportation, Laser Ablation and its Applications, 2007, pp. 435–454.
[30] Stefan Scharring, Johannes M.F. Peter, Daniel J. Förster, and Hans-Albert Eckel, Modellierungskonzepte zur laser-induzierten ablation f̈ur den microlas-antrieb, Workshop mikroantriebe, 2013.
[31] Rémi Soulard, Mark N. Quinn, Toshiki Tajima, géRARD Mourou, ICAN: a novel laser architecture for space debris removal, Acta Astronaut. 105 (1) (2014) 192–200.
[32] Union of Concerned Scientists, UCS Satellite Database: Includes Launches Through 7/31/14, 2014.