nova base

autovetores

antigo

Comp. principal

$$y_i = U^T x_i$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \vdash v_1 \dashv \\ \vdash v_2 \dashv \\ \vdash v_3 \dashv \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$v_i = [v_{i1} \; v_{i2} \; v_{i3}]$$

$$= \begin{bmatrix} v_1^T x \\ v_2^T x \\ v_3^T x \end{bmatrix}$$

λ_3 λ_2 λ_1

$$y = \begin{bmatrix} v_{11} x_1 + v_{12} x_2 + v_{13} x_3 \\ v_{21} x_1 + v_{22} x_2 + v_{23} x_3 \\ v_{31} x_1 + v_{32} x_2 + v_{33} x_3 \end{bmatrix}$$

CP $\downarrow$ Feature $\downarrow$

$$y_1 = v_{11} \underline{x_1} + v_{12} \underline{x_2} + v_{13} \underline{x_3}$$

$$v_1 = [v_{11}, v_{12}, v_{13}]$$

autovetor de Σ (cov)

λ_i não é importância de x_i

(variância da comp. i)

(feature i)

comp

comb. linear das feat

$$\underbrace{y_1}_{\text{comp}} = \overbrace{v_{11} x_1 + v_{12} x_2 + v_{13} x_3}^{\text{comb. linear das feat}}$$

$$v_1 = [v_{11}, v_{12}, v_{13}]$$

autovetor de Σ (cov)

1º comp

+ direção de maior variação.

$$y_1 = 1{,}5 \cdot \text{Peso} + 0.1 \cdot \text{Alt} + 0.3 \cdot \text{Salári}$$

$1{,}5 \gg 0{,}1,\ 0.3$

y_1, y_2 — AP + A + AS

— 0P + 0A + □S

perder interpretação

Posso usar o peso das features no 1º componente

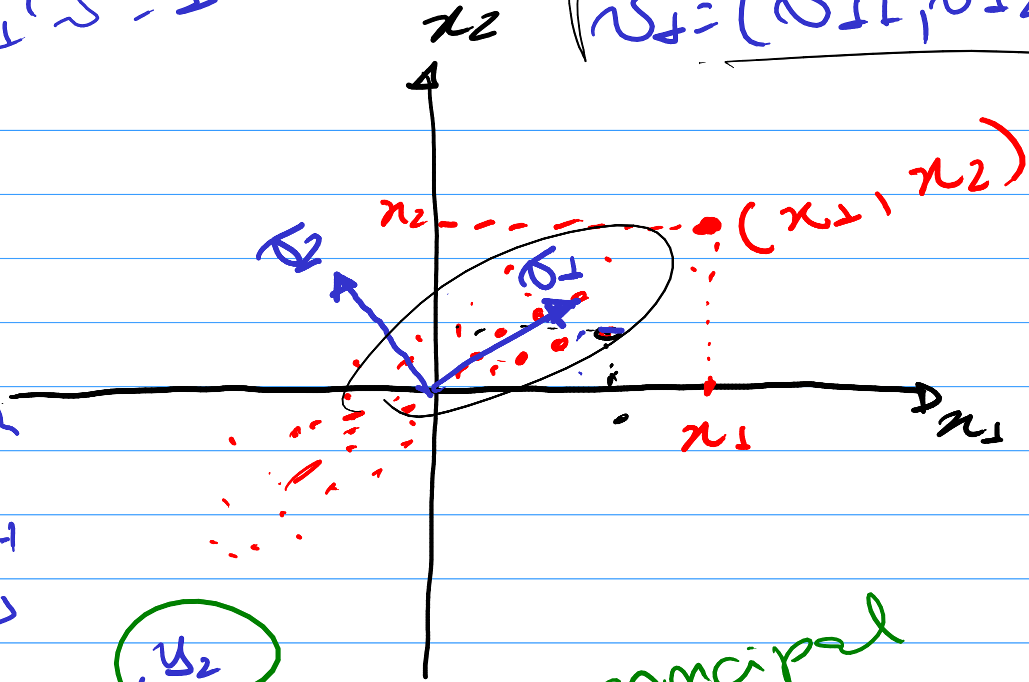
$v_1^T v_2 = 0$

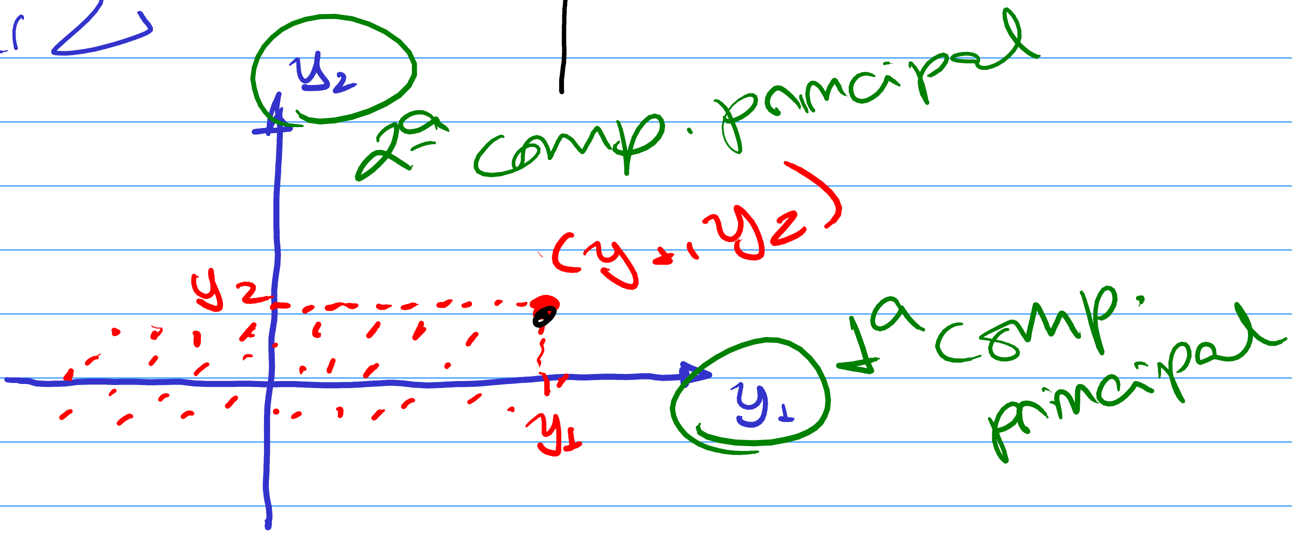
$v_1^T v = 1$

$v_1 = (v_{11}, v_{12})$

$\underbrace{v_1, v_2}$ direção das componentes

autovetores Σ





$$(x_1, x_2) \xrightarrow{?} (y_1, y_2)$$

$$y_1 = v_{11} \cdot x_1 + v_{12} \cdot x_2 = v_1^T x$$

$$y_2 = v_{21} \cdot x_1 + v_{22} \cdot x_2 = v_2^T x$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$y = U^T x$$

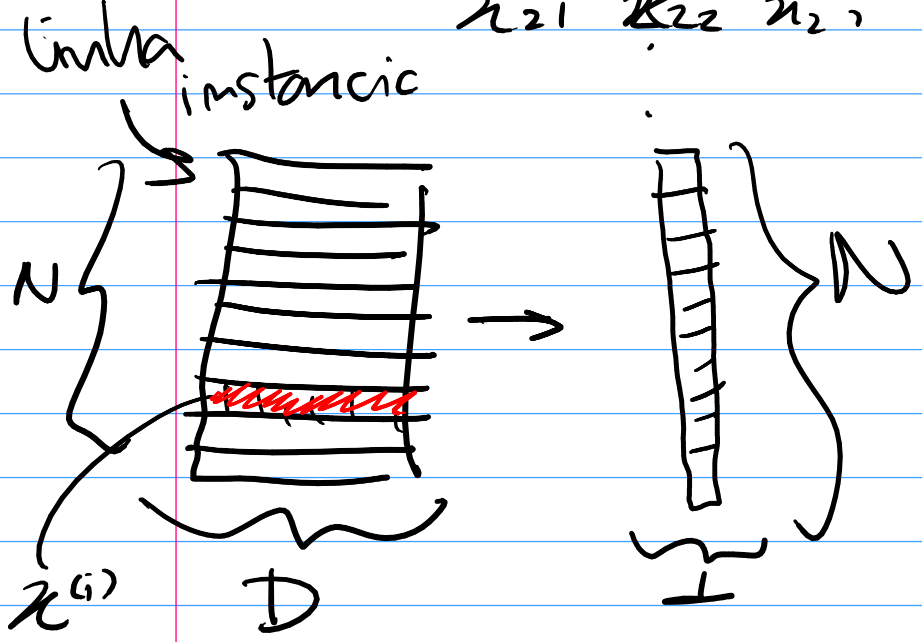
$$U = \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix}$$

v autovetores de Σ

λ autovalores

x_1 x_{11} x_{12} x_{13} ...

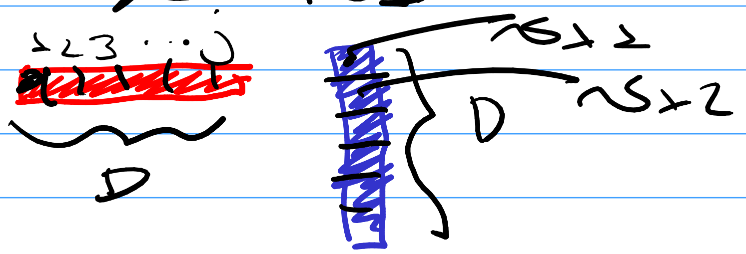
x_{21} x_{22} $x_{2?}$



Só 1ª componente

Para cada instancia $x^{(i)}$

$$y_1^{(i)} = x^{(i)T} w_1$$



$$= x_1^{(i)} \cdot w_{11} + x_2^{(i)} \cdot \boxed{w_{12}}$$

comp.

$$Var(y_1) = \lambda_1$$

$$Imp(\boxed{x_j}) = |w_{1j}|$$

feat.

$$y_1 = v_{11}\cdot x_1 + v_{12}\cdot x_2 = v_1^T x$$

$$y_2 = v_{21}\cdot x_1 + v_{22}\cdot x_2 = v_2^T x$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$y = U^T x$$

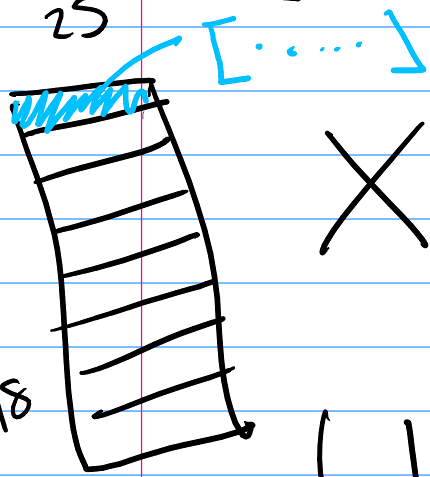
v autovetores de Σ

λ autovalores

$$U = \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix}$$

linha

25



498

$$X \in \mathbb{R}^{498\times 25}$$

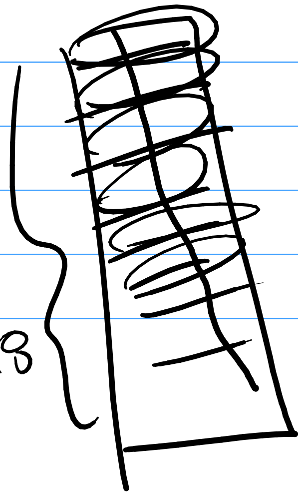
498 pontos
25 dim

$$U \in \mathbb{R}^{25\times 2}$$

$$A \cdot B = C$$

$$M\times \underbrace{N}\ \underbrace{N}\times K \qquad M\times K$$

$$XU \in \mathbb{R}^{498\times 2}$$



498

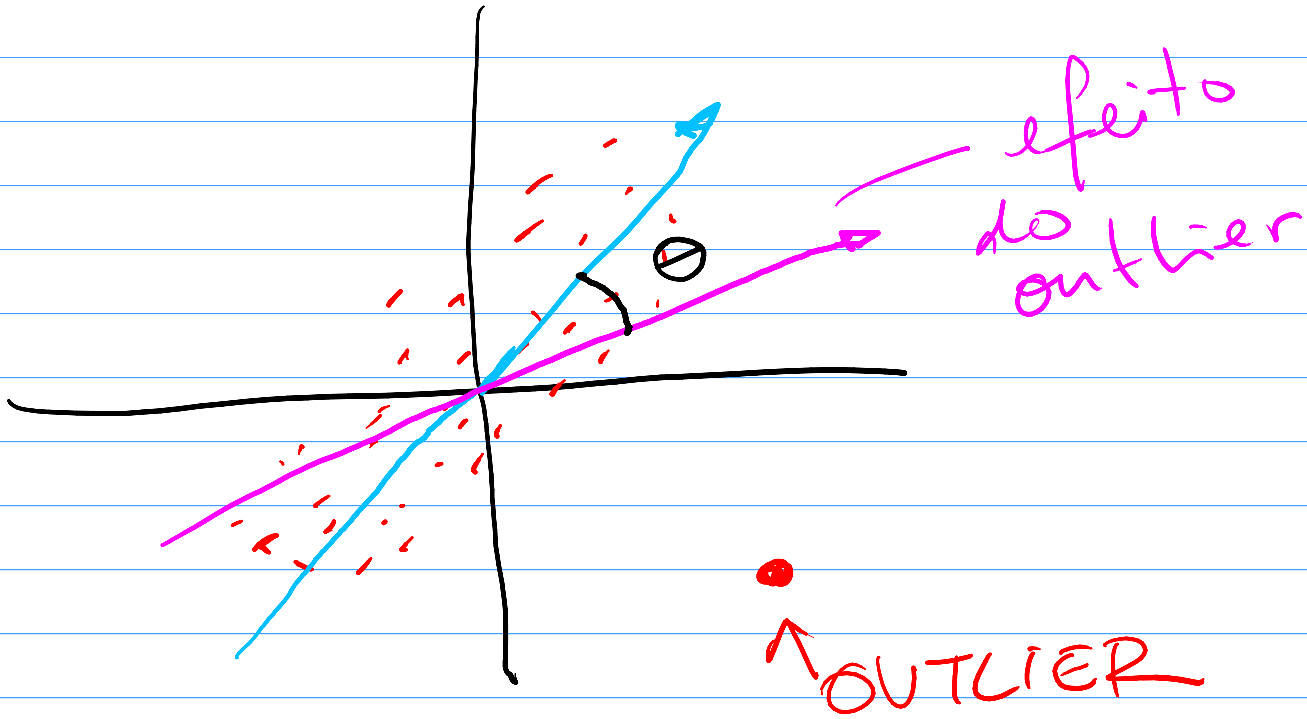
$$y = U^T x$$

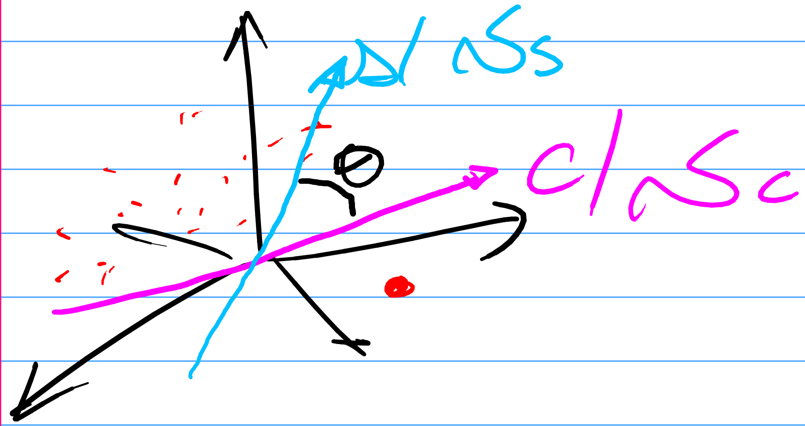
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \longleftarrow v_1 \longrightarrow \\ \longleftarrow v_2 \longrightarrow \end{bmatrix} \begin{array}{c} \text{column} \\ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ 2 \times 1 \end{array}$$

$$= \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

✓ $$= \underset{1 \times \underline{2}}{\begin{bmatrix} x_1 & x_2 \end{bmatrix}} \underset{\underline{2} \times 2}{\begin{bmatrix} v_{11} & v_{12} \\ v_{12} & v_{22} \end{bmatrix}}$$

✗ $$\underset{2 \times 2}{\begin{bmatrix} v_{11} & v_{12} \\ v_{12} & v_{22} \end{bmatrix}} \underset{1 \times 2}{\begin{bmatrix} x_1 & x_2 \end{bmatrix}}$$





$$\cos\theta = \frac{v_s \cdot v_c}{\|v_s\| \cdot \|v_c\|}$$

$$\cos\theta = \frac{a \cdot b}{\|a\|\|b\|}$$

$$\theta = \arccos\left(\frac{a \cdot b}{\|a\| \cdot \|b\|}\right)$$

grand   key

$$\alpha \cdot x_1 + \beta x_2$$

PCA:

$$\alpha y_1 + \beta y_2$$

$$y = \star x_1 + \star x_2 + \star x_3$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} \quad \begin{matrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_4 \end{matrix}$$

$$\Sigma = \begin{bmatrix} \ddots \end{bmatrix}$$