COMP30120

Naïve Bayes Classifier

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Overview

- Probability-based Learning
- Bayes Theorem
- Naïve Bayes Classifier
- Examples & Exercises

- Text Classification with Naïve Bayes
- Numeric Features
- Naïve Bayes in Weka

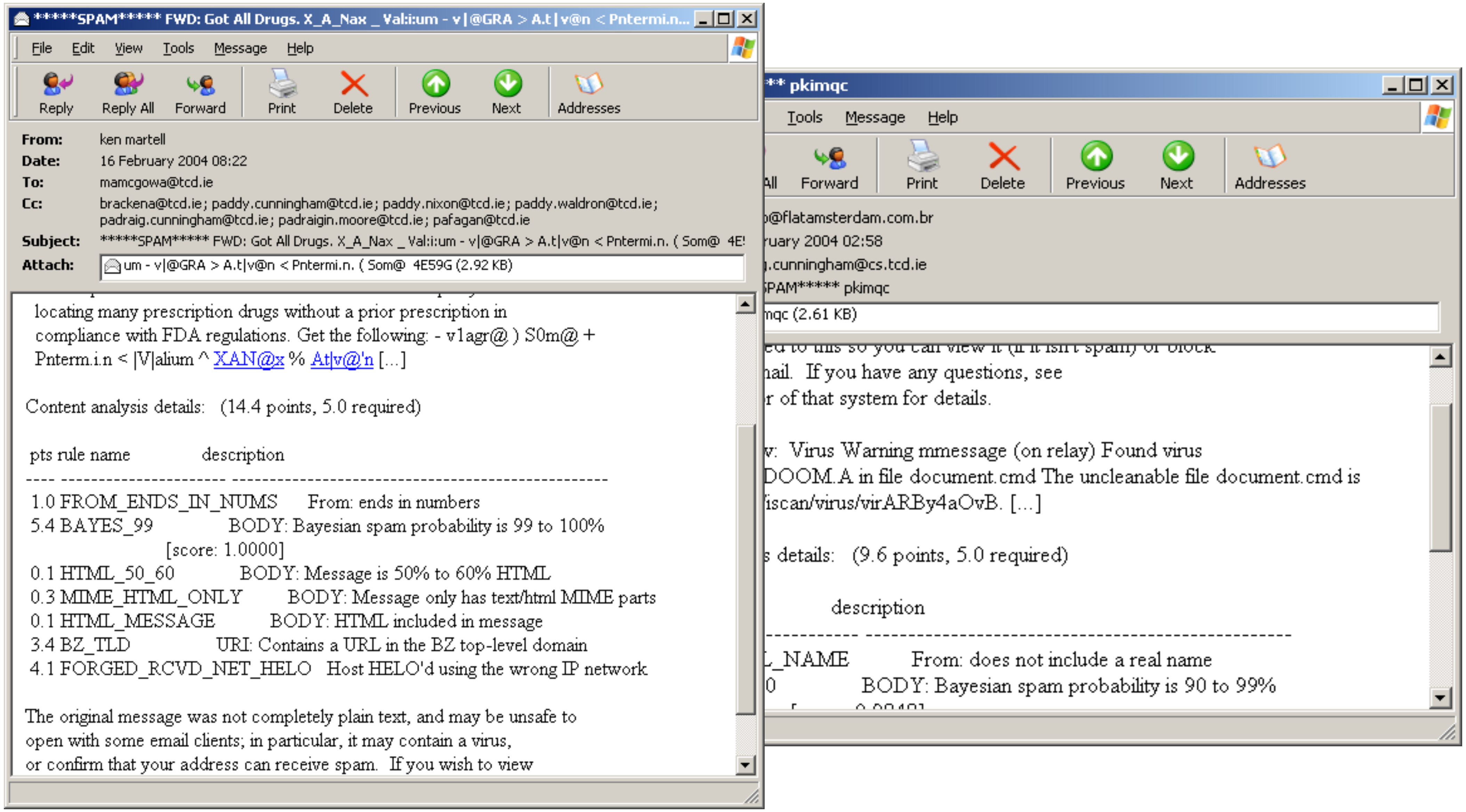
Probability-based Learning

- **Key Idea:** Use estimates of likelihoods to determine the most likely prediction which should be made (e.g. “the email X is more likely to be spam than non-spam”).
- Revise these predictions based on the data we collect.
- Most common probabilistic approach for classification is Naïve Bayes, an eager learning approach based on Bayes Theorem.

- **Why use a Naïve Bayes classifier?**
  - Intuitive and easy to implement.
  - Fast to train and to use as a classifier.
  - Suitable for moderate or large data sets with many features.
  - Can deal with missing features.

Application: Spam Filtering

Apache Spamassassin uses Naïve Bayes classification.

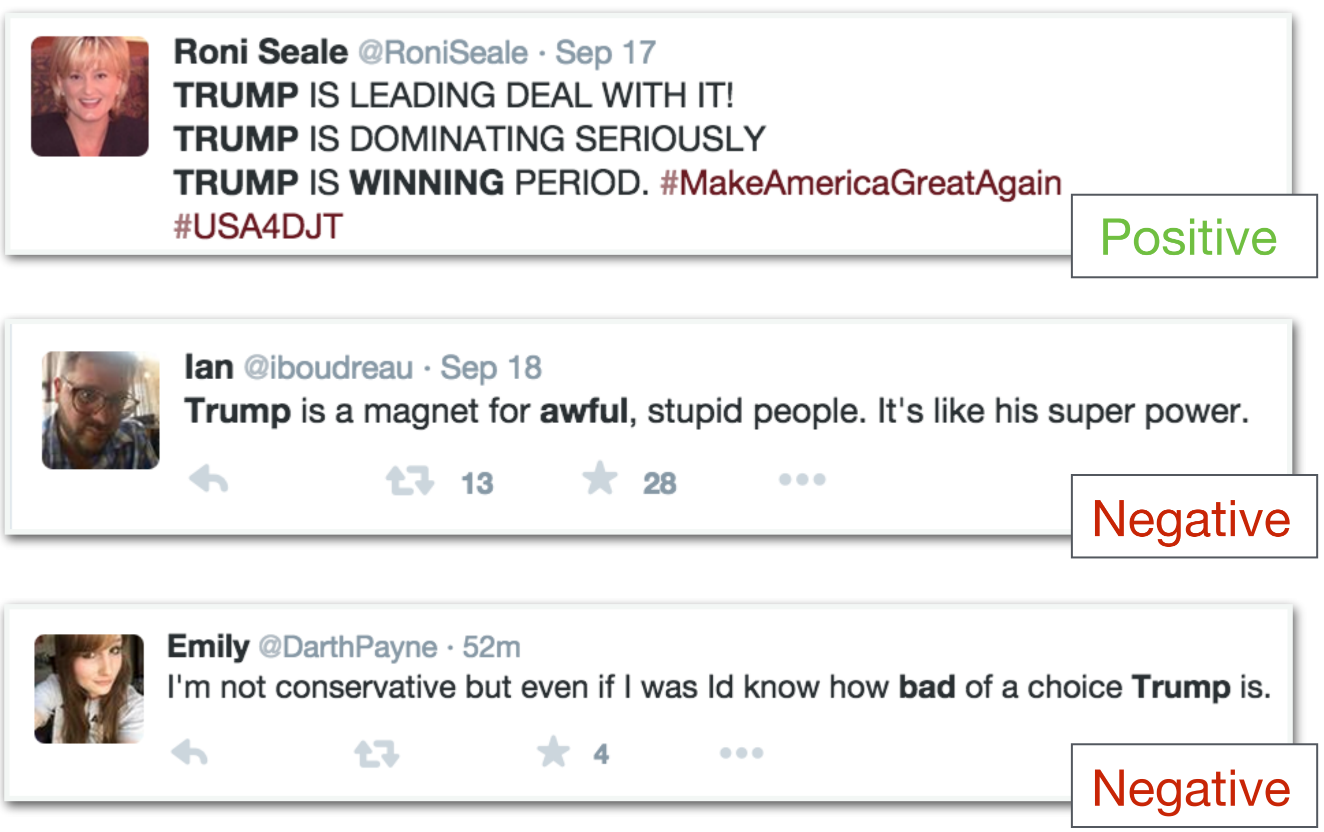


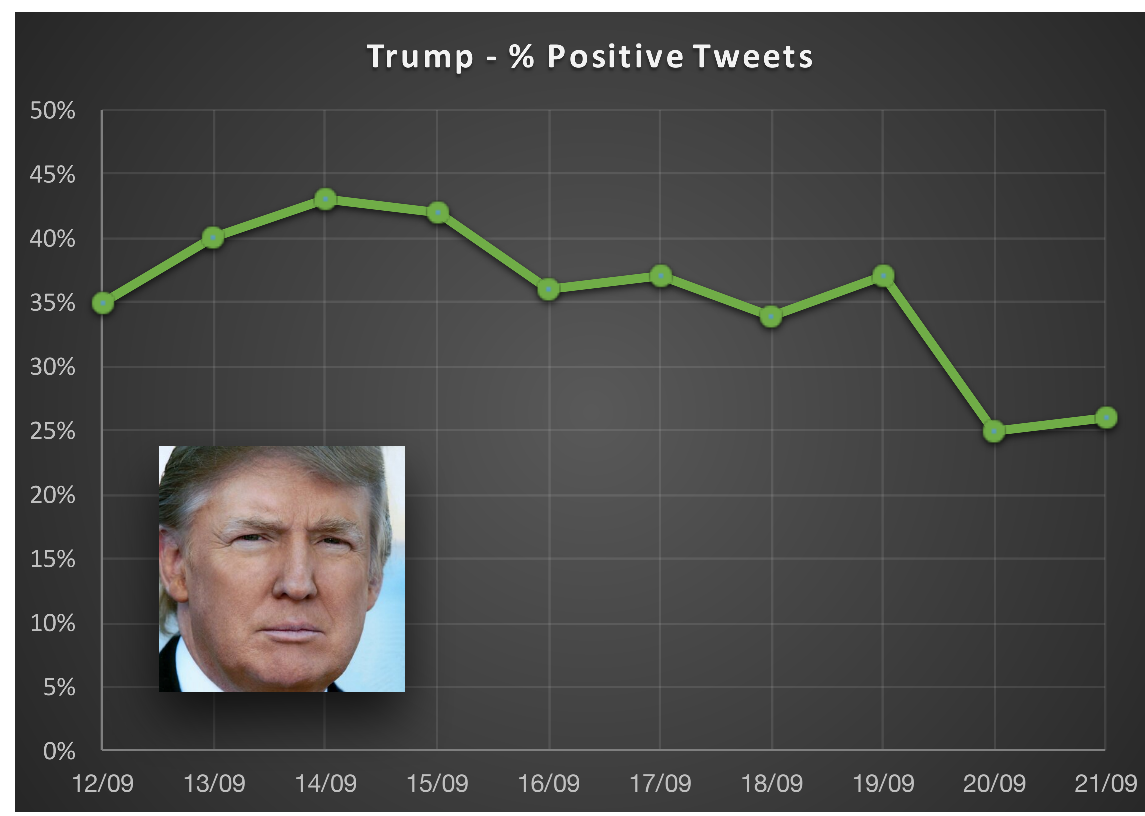
See: http://wiki.apache.org/spamassassin/BayesInSpamAssassin

Application: Sentiment Analysis

Task: Classify sentiment of tweets as “positive” or “negative”.

1. Crowdsource users to label a small subset of tweets as either “positive” or “negative” (i.e. training data).
2. Apply Naïve Bayes classifier to automatically label a much larger set of tweets on an ongoing basis.
3. Plot value of % of positive tweets over time.





Notation

$P(X)$	Probability of event *X* happening.
$P(X\|Y)$	Conditional probability of event *X* happening, given that event *Y* has happened.

What is the probability of a given hypothesis *h* being true ("the event"), given the observed training data *D* ("the evidence")?

Let h denote the hypothesis, D denote the data.

Prior probability of data

$P(D)$: Probability of the data D.

Prior probability of hypothesis - "initial beliefs"

$P(h)$: Probability of the hypothesis h.

Posterior probability

$P(h|D)$: Probability of the hypothesis h given the data D.

Bayes Classification

"The probability that an event has happened given a set of evidence for it is equal to the probability of the evidence being caused by the event by the probability of the event itself." (Kelleher et al, 2015)

- Bayes Theorem: Rule states that for each possible hypothesis *h*

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- For classification, each *h* corresponds to a possible class label.

  Q. What is the probability of a given example taking this class?

- If we knew $P(h|D)$ we could classify the data perfectly.
- Since we generally do not know $P(h|D)$, we try to estimate it from the data using Bayes Rule.

Bayes Classification

- We usually want the most likely hypothesis for our data.
- Formally, we are looking for the Maximum Aposteriori Hypothesis (MAP):

$$h_{MAP} = \arg\max_{h \in H} P(h|D)$$

- Example: Two competing hypotheses h_0 and h_1 for data set X

$$P(h_0|X) > P(h_1|X) \implies \text{choose } h_0$$

$$P(h_0|X) < P(h_1|X) \implies \text{choose } h_1$$

$$P(h_0|X) = P(h_1|X) \implies \text{choose either}$$

- In classification, we want to find the most likely class label for a given example among all possible class labels.

Example: Bayes Classification

- **Task:** Classify sentiment of tweets as “positive” or “negative”.

  $P(h_0)$ Probability of any tweet being classed “positive”.

  $P(h_1)$ Probability of any tweet being classed “negative”.

- Want to test hypothesis *h_0* - is a particular tweet *t* positive?

  $P(h_0|t)$ Probability of a positive class prediction for the tweet *t.* This is our target result.

  $P(t|h_0)$ Probability of the tweet *t*, given that it is positive. Calculated based on the data.

- We could rewrite the task with Bayes Theorem as follows:

$$P(h_0|t) = \frac{P(t|h_0)P(h_0)}{P(t)} = \frac{P(\text{tweet}|\text{positive})P(\text{positive})}{P(\text{tweet})}$$

Example: Bayes Classification

- Let's say that we know a-priori 60% of all tweets are positive and 40% of tweets are negative. $\Rightarrow P(positive) = 0.6$
- In addition, the probability of a tweet *t* is constant, so we can remove the denominator from the calculation:

$$P(positive|tweet) = \frac{P(\text{tweet}|\text{positive})P(\text{positive})}{P(\text{tweet})}$$

$$\Rightarrow \quad P(positive|tweet) = P(\text{tweet}|\text{positive}) \times 0.6$$

- But we still need some way of calculating the probability of a particular tweet (as described by its features), given the assumption that it has the class label "positive".

Definition: Bayes Classifier

Classifier Inputs:

A set of labels $V = \{v_1, v_2, \dots\}$

A set of examples $X = \{x_1, x_2, \dots\}$, each represented by features $\{f_1, f_2, \dots, f_n\}$

Classifier Objective:

Find the most probable class label v for x according to:

$$v_{MAP} = \arg\max_{v_j \in V} P(v_j | f_1, f_2 \dots f_n)$$

$$\begin{aligned} v_{MAP} &= \arg\max_{v_j \in V} \frac{P(f_1, f_2 \dots f_n | v_j) P(v_j)}{P(f_1, f_2 \dots f_n)} \\ &= \arg\max_{v_j \in V} P(f_1, f_2 \dots f_n | v_j) P(v_j) \end{aligned}$$

Problem: Difficult to estimate $P(f_1, f_2 \dots f_n | v_j)$

Naïve Bayes Classifier

- Key Idea: Apply Bayes Theorem with the "naïve" assumption that all features in the data are *conditionally independent*:

$$P(f_1, f_2 \dots f_n | v_j) = \prod_i P(f_i | v_j)$$

i.e. the value of a particular feature is unrelated to the presence or absence of any other feature, given class label v_j

- Based on this assumption, the objective of the Naïve Bayes classifier becomes:

Find the most probable class label v for x according to:

$$v_{NB} = \arg\max_{v_j \in V} P(v_j) \prod_i P(f_i | v_j)$$

i.e. (Class Probability) x (Product of Class-Feature Probabilities)

Example: Canoeing

Q. “Will we go canoeing today?”

Binary classification task (Canoeing = {Yes,No}), with examples described by 5 different features:

Example	Rain Recently (RR)	Rain Today (RT)	Temp (T)	Wind (W)	Sunshine (S)	Canoeing (C)
1	Moderate	Moderate	Warm	Light	Some	Yes
2	Light	Moderate	Warm	Moderate	None	No
3	Moderate	Moderate	Cold	Gale	None	No
4	Moderate	Moderate	Warm	Light	None	Yes
5	Moderate	Light	Cold	Light	Some	No
6	Heavy	Light	Cold	Moderate	Some	Yes
7	Light	Light	Cold	Moderate	Some	No
8	Moderate	Moderate	Cold	Gale	Some	No
9	Heavy	Heavy	Warm	Moderate	None	Yes
10	Light	Light	Cold	Light	Some	No

➡ How can we use a Naïve Bayes Classifier for this problem?

Example: Canoeing

Test new input example for hypothesis 1: *Canoeing=Yes*

Example	Rain Recently (RR)	Rain Today (RT)	Temp (T)	Wind (W)	Sunshine (S)	Canoeing (C)
1	Moderate	Moderate	Warm	Light	Some	Yes
2	Light	Moderate	Warm	Moderate	None	No
3	Moderate	Moderate	Cold	Gale	None	No
4	Moderate	Moderate	Warm	Light	None	Yes
5	Moderate	Light	Cold	Light	Some	No
6	Heavy	Light	Cold	Moderate	Some	Yes
7	Light	Light	Cold	Moderate	Some	No
8	Moderate	Moderate	Cold	Gale	Some	No
9	Heavy	Heavy	Warm	Moderate	None	Yes
10	Light	Light	Cold	Light	Some	No

Example	Rain Recently (RR)	Rain Today (RT)	Temp (T)	Wind (W)	Sunshine (S)	Canoeing (C)
X0	Moderate	Moderate	Cold	Light	Some	???

Apply NB for *Canoeing=Yes* by calculating product of probabilities for input's feature values and class probability:

(Product of Class-Feature Probabilities) x (Class Probability)

$$P(C) = (2/4 \times 2/4 \times 1/4 \times 2/4 \times 2/4) \times 4/10$$

$$P(C) = 0.00625$$

Class Probability

$P(C) = 4/10$

Feature: Rain Recently

$P(L\_RR|C) = 0/4$

$P(M\_RR|C) = 2/4$

$P(H\_RR|C) = 2/4$

Feature: Rain Today

$P(L\_RT|C) = 1/4$

$P(M\_RT|C) = 2/4$

$P(H\_RT|C) = 1/4$

Feature: Temp

$P(C\_T|C) = 1/4$

$P(W\_T|C) = 3/4$

Feature: Wind

$P(L\_W|C) = 2/4$

$P(M\_W|C) = 2/4$

$P(G\_W|C) = 0/4$

Feature: Sunshine

$P(S\_S|C) = 2/4$

$P(N\_S|C) = 2/4$

Example: Canoeing

Test new input example for hypothesis 2: *Canoeing=No*

Example	Rain Recently (RR)	Rain Today (RT)	Temp (T)	Wind (W)	Sunshine (S)	Canoeing (C)
1	Moderate	Moderate	Warm	Light	Some	Yes
2	Light	Moderate	Warm	Moderate	None	No
3	Moderate	Moderate	Cold	Gale	None	No
4	Moderate	Moderate	Warm	Light	None	Yes
5	Moderate	Light	Cold	Light	Some	No
6	Heavy	Light	Cold	Moderate	Some	Yes
7	Light	Light	Cold	Moderate	Some	No
8	Moderate	Moderate	Cold	Gale	Some	No
9	Heavy	Heavy	Warm	Moderate	None	Yes
10	Light	Light	Cold	Light	Some	No

Example	Rain Recently (RR)	Rain Today (RT)	Temp (T)	Wind (W)	Sunshine (S)	Canoeing (C)
X0	Moderate	Moderate	Cold	Light	Some	???

Apply NB for *Canoeing=No* by calculating product of probabilities for input's feature values and class probability:

$$P(NC) = (3/6 \times 3/6 \times 5/6 \times 2/6 \times 4/6) \times 6/10$$

$$P(NC) = 0.028$$

Class Probability

$P(NC) = 6/10$

Feature: Rain Recently

$P(L\_RR|NC) = 3/6$

$P(M\_RR|NC) = 3/6$

$P(H\_RR|NC) = 0/6$

Feature: Rain Today

$P(L\_RT|NC) = 3/6$

$P(M\_RT|NC) = 3/6$

$P(H\_RT|NC) = 0/6$

Feature: Temp

$P(C\_T|NC) = 5/6$

$P(W\_T|NC) = 1/6$

Feature: Wind

$P(L\_W|NC) = 2/6$

$P(M\_W|NC) = 2/6$

$P(G\_W|NC) = 2/6$

Feature: Sunshine

$P(S\_S|NC) = 4/6$

$P(N\_S|NC) = 2/6$

Example: Canoeing

- Calculate probabilities for two hypotheses (class labels):

  Yes `P(C)  = (2/4 x 2/4 x 1/4 x 2/4 x 2/4) x 4/10 = 0.00625`

  No `P(NC) = (3/6 x 3/6 x 5/6 x 2/6 x 4/6) x 6/10 = 0.028`

- Normalise probabilities to sum to 1:

$$P(C)' = \frac{P(C)}{P(C) + P(NC)} = 0.18$$

$$\boxed{P(NC)' = \frac{P(NC)}{P(C) + P(NC)} = 0.82}$$

Output: *Canoeing=No*

- **Exercise:** Classify new examples using Naïve Bayes…

Example	Rain Recently (RR)	Rain Today (RT)	Temp (T)	Wind (W)	Sunshine (S)	Canoeing (C)
X1	Heavy	Moderate	Warm	Light	Some	???

Example	Rain Recently (RR)	Rain Today (RT)	Temp (T)	Wind (W)	Sunshine (S)	Canoeing (C)
X2	Light	Moderate	Warm	Light	Some	???

Text Classification

- **Naïve Bayes for text:** Each word in the vocabulary of a collection of documents is a feature; assume independence between word occurrences.
- Input: Examples *X* (set of documents), *V* (class labels)

LEARN_NB_TEXT(X, V):

- $Vocabulary \leftarrow$ set of all unique words in X
- FOR EACH $v_j \in V$
  - $Docs_j \leftarrow$ subset of documents from X with class label v_j
  - $P(v_j) \leftarrow \frac{|Docs_j|}{|X|}$
  - $Text_j \leftarrow$ concatenation of all text from $Docs_j$
  - $n \leftarrow$ total number of word positions in $Text_j$
  - FOR EACH word $w_k \in Vocabulary$
    - $n_k \leftarrow$ number of occurrences of word w_k in $Text_j$
    - $P(w_k|v_j) \leftarrow \frac{n_k}{n}$

Text Classification

- Once we have computed word probabilities for each class, we can use these to predict the class of a new input document *Doc.*
- Words not present in *Vocabulary* are not considered.

CLASSIFY_NB_TEXT(Doc):

- $Positions \leftarrow$ all word positions in Doc with words from $Vocabulary$
- Return class label v_{NB} such that:

$$v_{NB} = \arg\max_{v_j \in V} P(v_j) \prod_{i \in Positions} P(w_i | v_j)$$

- But… words are not independent of one another (e.g. United + States, Barack + Obama, Enda + Kenny).
- Often the conditional independence assumption is violated. Despite this, in practice Naïve Bayes classifiers perform well.

Numeric Features

- How to classify when features take numeric values?

Example	Rain Recently (RR)	Rain Today (RT)	Temp (T)	Wind (W)	Sunshine (S)	Canoeing (C)
X0	Moderate	Moderate	$\boxed{9}$	Light	Some	???

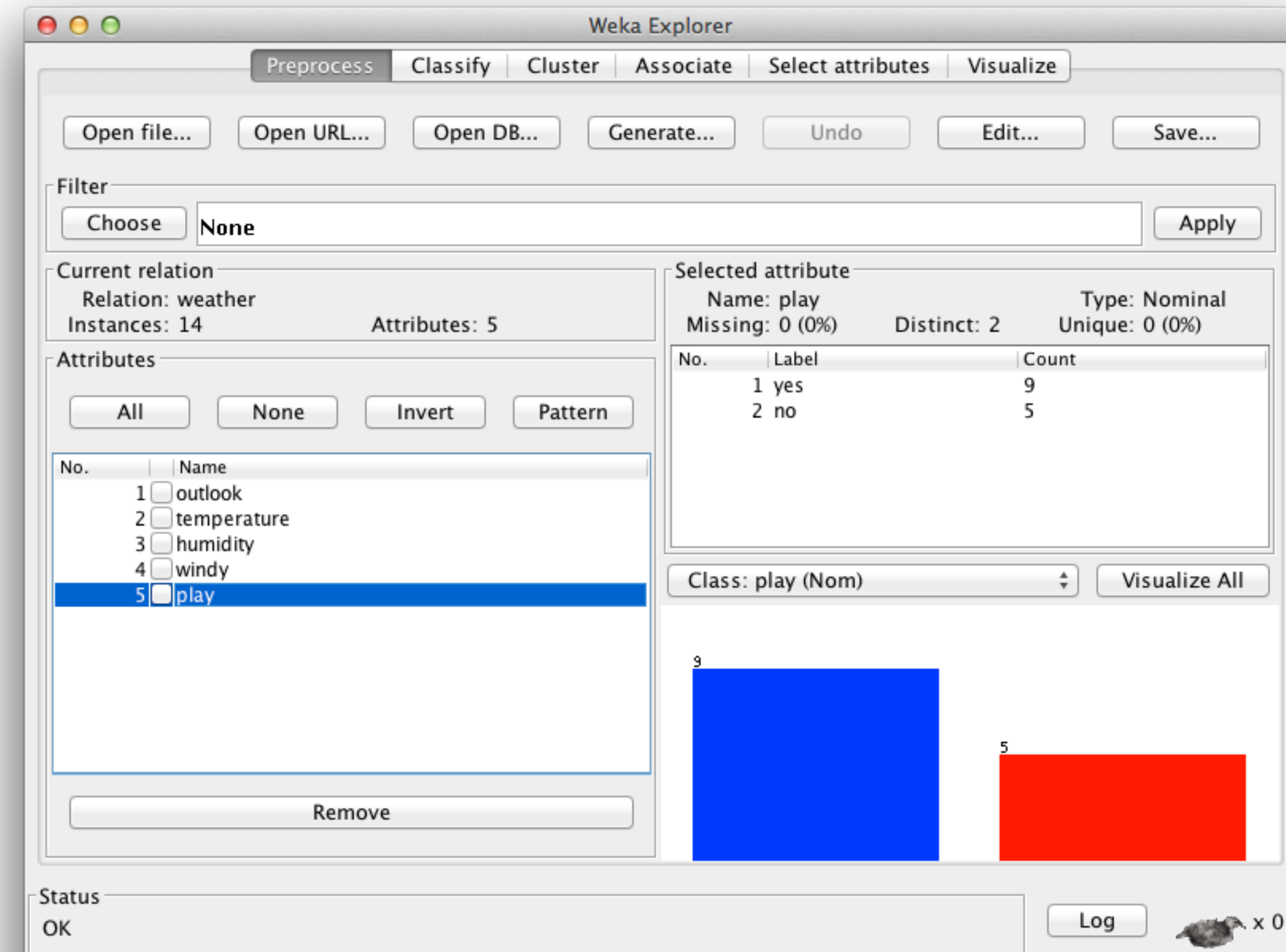
- **Option 1:** Discretise the feature to take fixed number of values. e.g. Temp = {cool, mild, hot}

- **Option 2:** Assume that the feature fits to some distribution. e.g. For a Normal Distribution:

  1. For numeric feature f_i, store mean μ_i and standard deviation σ_i for each class v_j
  2. When classifying, find the probability that the feature value fits the distribution $N(\mu_i, \sigma_i^2)$

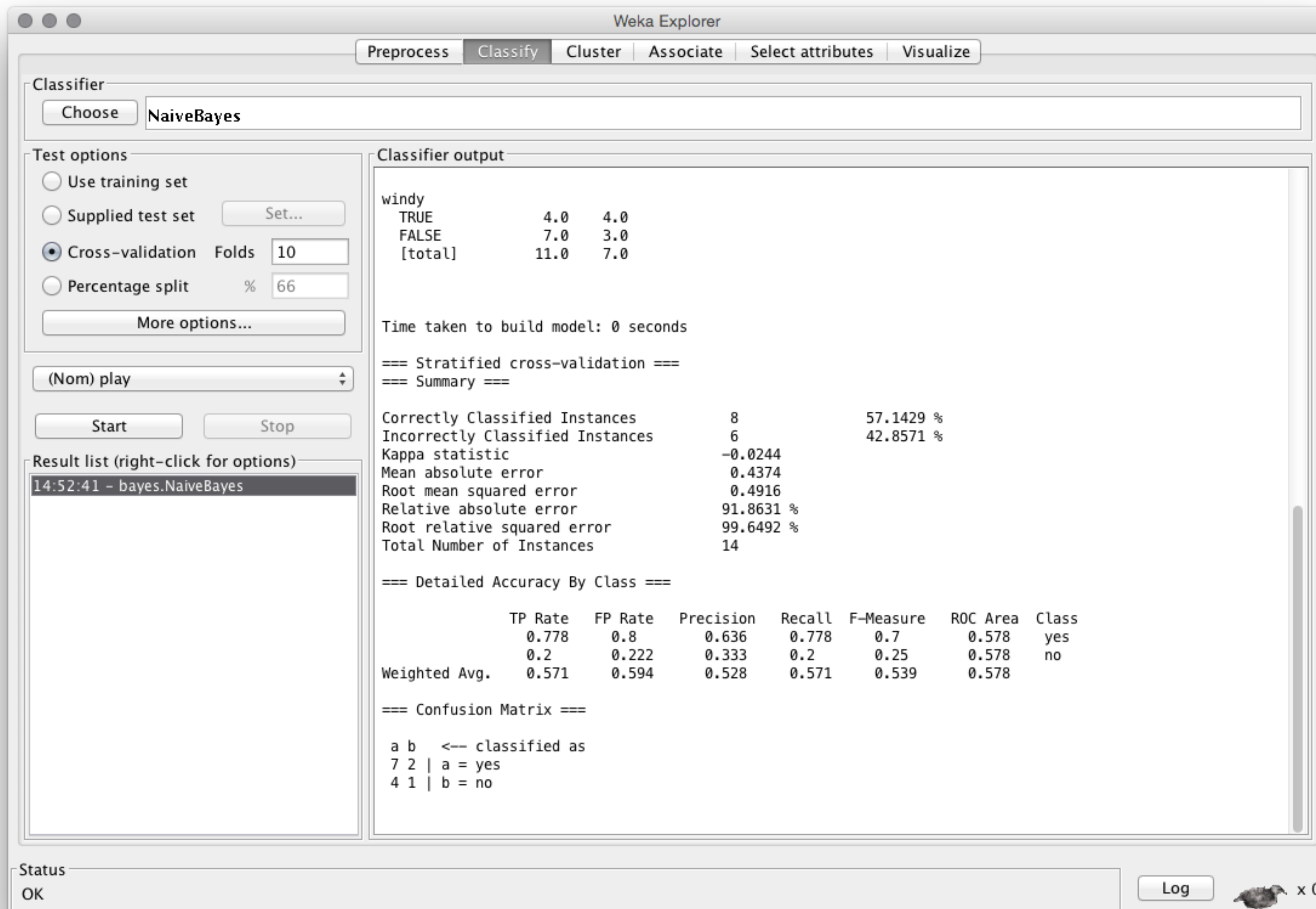
Naïve Bayes in Weka

1. Launch the WEKA application, click on the *Explorer* button.
2. *Open File* - weather.arff



Naïve Bayes in Weka

3. In *Classify* tab, click *Choose* and *Bayes→NaiveBayes*
4. Choose *(Nom) Play* as class label, then click *Start.*



Summary

- Naïve Bayes Classifier
  - Probabilistic approach to classification.
  - Based on key independence assumption. This assumption is often violated, but still works.
- Text Classification with Naïve Bayes
  - Learning: Calculate word probabilities for vocabulary
  - Classifying: Find product of word probabilities in the new document.
- Numeric Features
  - Make feature discrete or assume a distribution.

References

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