

Part A: Calculator NOT Allowed

$$1. \lim_{x \rightarrow \infty} \frac{-2x^2 + 7x - 3}{1x^2 + 2x - 3} = \lim_{x \rightarrow \infty} \frac{\frac{-2x^2}{x^2} + \frac{7x}{x^2} - \frac{3}{x^2}}{\frac{1x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}} = \lim_{x \rightarrow \infty} \frac{-2 + \frac{7}{x} - \frac{3}{x^2}}{1 + \frac{2}{x} - \frac{3}{x^2}} = -2 \quad \boxed{\text{B}}$$

$$2. \int x^{-2} dx = \frac{x^{-1}}{-1} + C = -x^{-1} + C \quad \boxed{\text{D}}$$

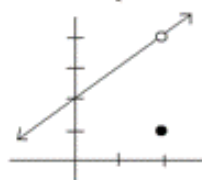
$$3. \text{Product Rule } \rightarrow f'(x) = (x-1)[3(x^2+2)^2(2x)] + (x^2+2)^3(1) \\ = (x^2+2)^2 \{6x(x-1) + (x^2+2)\} = (x^2+2)^2 (7x^2 - 6x + 2) \quad \boxed{\text{D}}$$

$$4. \int \sin 2x dx + \int \cos 2x dx = -\frac{1}{2} \cos 2x + \frac{1}{2} \sin 2x + C \quad \boxed{\text{B}}$$

$$5. \lim_{x \rightarrow 0} \frac{\cancel{x^2}(5x^2+8)}{\cancel{x^2}(3x^2-16)} = -\frac{1}{2} \quad \text{Therefore, } \boxed{\text{A}} \quad \text{Or use L'Hopital's Rule Twice}$$

$$6. f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 1 & x = 2 \end{cases}$$

I) f has a limit at $x = 2 \Rightarrow \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} f(x) = 4$
 II) Continuous at 2? NO b/c $f(2) = 1 \neq 4$
 III) Differentiable at 2? Since not continuous at 2, NO.



So, only I is true $\boxed{\text{A}}$ |

$$7. \int_0^1 v(t) dt = s(1) - s(0) \quad (\text{FTC}) \quad s(1) = s(0) + \int_0^1 v(t) dt = 2 + (t^3 + 3t^2) \Big|_0^1 = 2 + 4 = 6 \quad \boxed{\text{B}}$$

$$8. f(x) = \cos(3x) \rightarrow f'(x) = -3\sin(3x) \quad \text{Therefore, } f'\left(\frac{\pi}{9}\right) = -3\sin\left(\frac{\pi}{3}\right) = -3\left(\frac{\sqrt{3}}{2}\right) = \frac{-3\sqrt{3}}{2} \quad \boxed{\text{E}}$$

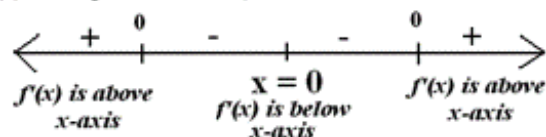
$$9. g'(x) = f(x) \quad \text{So } g \text{ has a relative maximum when } g'(x) \text{ or } f(x) \text{ changes from } + \text{ to } -. \\ \text{This happens at } x = 1. \quad \boxed{\text{D}}$$

10. Because $f(x)$ is decreasing, Right Riemann Sum $< \int_1^3 f(x)dx < \text{Left Riemann Sum}$

Since decreasing and concave down, Right Riemann Sum $<$ both Midpoint Riemann Sum and Trapezoidal Sum, therefore answer is **C**.



$f'(x)$ chart (first derivative)



11. Therefore, Choice **B**.

12. $f'(x) = e^{2/x}(-2x^{-2}) = \frac{-2e^{2/x}}{x^2}$ **D**

13. Since $f(x) = x^2 + 2x \rightarrow f(\ln x) = (\ln x)^2 + 2 \ln x \Rightarrow \frac{d}{dx}(f(\ln x)) = 2(\ln x)\left(\frac{1}{x}\right) + 2\left(\frac{1}{x}\right) = \frac{2 \ln x + 2}{x}$ **A**

14. Nothing about $f'(x)$ is given, eliminating A,B and C. **E**

Only 29% of the students answered this question correctly; common wrong answer is D.

D is not the correct answer because we do not know the sign of $f''(x)$ to the immediate left & right of $x = 1$.

Just because $f''(x) = 0$ doesn't mean there is an inflection point – the sign of $f''(x)$ must change to be an inflection point.

15. $\int \frac{x}{x^2-4} dx \rightarrow \text{Let } u = x^2 - 4, \frac{du}{dx} = 2x \rightarrow \frac{du}{2} = x dx \Rightarrow \int \frac{x}{x^2-4} dx \rightarrow \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln |x^2 - 4| + C$

OR $\int \frac{x}{x^2-4} dx = \boxed{} \ln |x^2 - 4| + C \Rightarrow \text{take deriv and compare: } \frac{\boxed{} 2x}{x^2 - 4} \Rightarrow 2\boxed{} = 1 \text{ so } \boxed{} = \frac{1}{2} \Rightarrow$

$\int \frac{x}{x^2-4} dx = \frac{1}{2} \ln |x^2 - 4| + C$ Answer: **C**

16. Chain Rule and Implicit Differentiation: $\cos(xy)[xy' + y(1)] = 1 \Rightarrow (xy')\cos(xy) + y\cos(xy) = 1$

$\frac{dy}{dx} = \boxed{y' = \frac{1 - y\cos(xy)}{x\cos(xy)}}$ Answer: **D** *Only 37% of students answered it correctly!

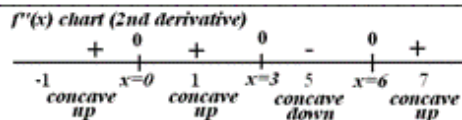
17. By the Second Fundamental Theorem of Calculus, $g'(x) = f(x)$ and so $g''(x) = f'(x)$
 g has a point of inflection where $g'' = f'$ changes sign. This happens when $x = 2$ and $x = 5$ **C**

18. slope of the tangent line is $y' = 2x + 3 = m$
 $x + y = k \rightarrow y = -x + k \Rightarrow m = -1$
 Solve $2x + 3 = -1 \rightarrow 2x = -4 \rightarrow x = -2$, $\rightarrow f(-2) = (-2)^2 + 3(-2) + 1 = -1 = y$
 $\Rightarrow x + y = k \Rightarrow -2 + (-1) = \boxed{-3}$ **A**

19. For horizontal asymptotes, both $\lim_{x \rightarrow \pm\infty} f(x)$ must be considered.

$$y = \frac{5+2^x}{1-2^x} = \frac{\frac{5}{2^x} + 1}{\frac{1}{2^x} - 1} \Rightarrow \lim_{x \rightarrow +\infty} \left(\frac{\frac{5}{2^x} + 1}{\frac{1}{2^x} - 1} \right) = \frac{1}{-1} = -1 \text{ and } \lim_{x \rightarrow -\infty} \left(\frac{5+2^x}{1-2^x} \right) = \lim_{x \rightarrow -\infty} \frac{5 + \frac{1}{2^x}}{1 - \frac{1}{2^x}} = \frac{5+0}{1-0} = 5 = 5$$

OR L'Hopital: $\lim_{x \rightarrow +\infty} \frac{2^x \ln 2}{-2^x \ln 2} = -1$ and $\lim_{x \rightarrow -\infty} \left(\frac{5+2^x}{1-2^x} \right) = 5$ as shown above. So answer: **E**



20. Points of inflection at $x = 3$ and $x = 6$, so Answer: **D**

21. Velocity increasing $\leftrightarrow v' = x'' > 0$ when $x(t)$ is concave up on $0 < t < 2 \rightarrow$ Answer: **A**

22. p changes in direct proportion to the product of p and the difference of N and p where P is the number of people who have heard the rumor and $N - P$ is the number of people who have not heard the rumor.

So: $\frac{dp}{dt} = k[p(N - p)]$ Answer: **B**

23. Separation of variables, $\int y dy = \int x^2 dx \Rightarrow \frac{y^2}{2} = \frac{x^3}{3} + C$ $y(3) = -2 \rightarrow \frac{(-2)^2}{2} = \frac{3^3}{3} + C, C = -7$

$$\Rightarrow \frac{y^2}{2} = \frac{x^3}{3} - 7 \rightarrow y^2 = \frac{2x^3}{3} - 14 \Rightarrow y = -\sqrt{\left(\frac{2x^3}{3} - 14\right)}$$

*Square root is negative because $y(3) < 0$ **E**

24. $f(2) = 1 \leftrightarrow (2, 1)$ and $m = f'(2) = 4$

$$y - y_1 = m(x - x_1) \Rightarrow y - 1 = 4(x - 2) \rightarrow y = 4x - 7 \Rightarrow y(1.9) = 4(1.9) - 7 = 7.6 - 7 = .6$$
 B

OR $f(1.9) \approx f(2) + f'(2)(-0.1) = 1 + (4)(-0.1) = 1 - .4 = .6$

$$25. f \text{ is differentiable at } x = 2 \rightarrow f'(x) = \begin{cases} c & , x < 2 \\ 2x - c & , x > 2 \end{cases} \quad \text{So, } c = 2x - c \text{ so } f'(2) \Rightarrow 2c = 2(2) \Rightarrow c = 2$$

f is continuous at $x = 2$ (Diff \rightarrow Cont)

$$\therefore \lim_{x \rightarrow 2^-} (cx + d) = \lim_{x \rightarrow 2^+} (x^2 - cx) \Rightarrow 2c + d = 4 - 2c \Rightarrow d = 4 - 4c = 4 - 4(2) = -4$$

$$\Rightarrow c + d = 2 + (-4) = -2 \quad \boxed{B}$$

$$26. y = \tan^{-1}(4x) \rightarrow \frac{dy}{dx} = \frac{1}{1+(4x)^2} \bullet (4) = \frac{4}{1+16x^2} \quad \left. \frac{dy}{dx} \right|_{x=\frac{1}{4}} = \frac{4}{1+16\left(\frac{1}{4}\right)^2} = \frac{4}{2} = 2 \quad \boxed{A}$$

27. Slope field:

Consider the point $(-1, 1)$. \Rightarrow The slope field indicates $\left. \frac{dy}{dx} \right|_{(-1,1)} = 0 \Rightarrow$ NOT A, B, or D

Consider $(0, 0) \Rightarrow \left. \frac{dy}{dx} \right|_{(0,0)} = 0 \Rightarrow$ NOT E \therefore choice \boxed{C}

$$28. \text{ Since } f(6) = 3, f^{-1}(3) = g(3) = 6 \Rightarrow g'(x) = \frac{1}{f'(g(x))} \Rightarrow g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(6)} = \frac{1}{-2} \quad \boxed{A}$$

ONLY 14% of students answered this correctly.

Part B: Calculator Allowed

76. f increasing $\Rightarrow f' \geq 0$ One could interpret the answer to be $(\mathbb{R}, 1) \cup (1, 3)$ instead, but it is NOT listed as a choice. \therefore [B] $[-2, 3]$

77. $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$, but each do exist. $\therefore \lim_{x \rightarrow 2} f(x)$ DNE \therefore [C]

78. f increasing $\rightarrow f'(x) \geq 0$ Graph: $f'(x) = \sin(x^3 - x) \geq 0$ when curve is above the x -axis on $[-1, 1.691]$ [B]

79. $\int_{-5}^5 f(x) dx = \int_{-5}^2 f(x) dx + \int_2^5 f(x) dx = \int_{-5}^2 f(x) dx - \int_5^2 f(x) dx = -17 - (-4) = -13$ [B]

80. Points of inflection of f occur when f'' changes signs. This can happen when f' changes from increasing to decreasing or decreasing to increasing at relative max/min of the graph of f' or when the graph of f'' passes through the x -axis. When you graph f' , there are 5 relative max/min on $(-2, 2)$. The graph of f'' passes through the x -axis 5 times on $(-2, 2)$. [E]

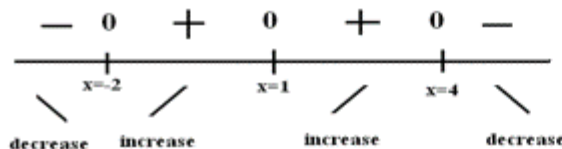
81. Since $G(x)$ is antiderivative for $f(x)$, $\int_2^4 f(x) dx = G(4) - \underbrace{G(2)}_{-7} = G(4) - (-7) \Rightarrow G(4) = -7 + \int_2^4 f(x) dx$ [E]

82. $a(3) = v'(3) = 0.055$ [B] using [nDeriv] on your calculator

83. Graph $f(x) = x^3 - 8x^2 + 18x - 5$ and $g(x) = x + 5$ They intersect at $x=1, 2, 5$

Area = $\int_1^2 (f - g) dx + \int_2^5 (g - f) dx = 11.833$ [B] Or Area = $\int_1^5 |f - g| dx = 11.833$

84. f has a rel max when f' changes from $+$ to $-$. The graph of $f'(x)$ and its chart show that f' changes from $+$ to $-$ at $x=4$. [C]



$$85. \int_{-4}^{-1} f'(x) dx = f(-1) - f(-4) = (-1.5) - (.75) = -2.25 \quad \boxed{\text{B}}$$

86. $v(3) = x'(3) = 0 \leftrightarrow x(t)$ has a horizontal tangent at $t = 3 \therefore$ either C or E.

from the table, $v(1) = x'(1) = 2 \rightarrow x(t)$ is inc. at $t = 1 \therefore$ not E $\boxed{\text{C}}$

$$87. x(3) = x(0) + \int_0^3 v(t) dt = 2 + 4.5115 \approx 6.512 \quad \boxed{\text{D}}$$

$$88. S = 4\pi r^2 \rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(3)(-2) = -48\pi \quad \boxed{\text{C}}$$

89. Think of Rolle's Theorem \rightarrow since $f'(c) \neq 0$ for $-2 < c < 2$, then either f is not continuous $[-2, 2]$ or f is not differentiable on $(-2, 2)$. Answer can't be D because if $f'(k)$ exists, then it is differentiable meeting the conditions of Rolle's Theorem. Hence, the answer is $\boxed{\text{E}}$.

90.

$f'(3) = 2, f''(x) < 0$ on $(2, 4) \rightarrow f'$ must be decreasing on $(2, 4)$

The answer is NOT tables B, D, or E since $f'(c) = \frac{f(4) - f(3)}{4 - 3}$ for $3 \leq c \leq 4$ are not < 2

(i.e. f' is not decreasing from 3 to 4) \uparrow

It is NOT table C because $f'(c) = \frac{f(3) - f(2)}{3 - 2} = 2$ for $2 \leq c \leq 3$, but $f'(3) = 2$ and f' needs to be

dec on $2 < x < 3$ So, answer is $\boxed{\text{A}}$

$$91. f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{3 - (-1)} \int_{-1}^3 \frac{\cos x}{x^2 + x + 2} dx = 0.183 \quad \boxed{\text{C}}$$

92. The population is given as the sum (accumulation) of the population times the population density on the interval from 0 miles to 4 miles:

$$\text{Population} = \int \frac{\text{Pop}}{\text{mi}^2} \cdot \text{mi}^2 = \int_{x=0}^4 \frac{\text{Pop}}{\text{mi}^2} \cdot \text{mi} \cdot dx = \int_0^4 7 \cdot f(x) dx \quad \boxed{\text{B}}$$

