Aula 05 – Fourier II

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Processamentos de sinais

Aviso

Trabalho T2
será no dia 27/09/2019

Transformada Rápida de Fourier

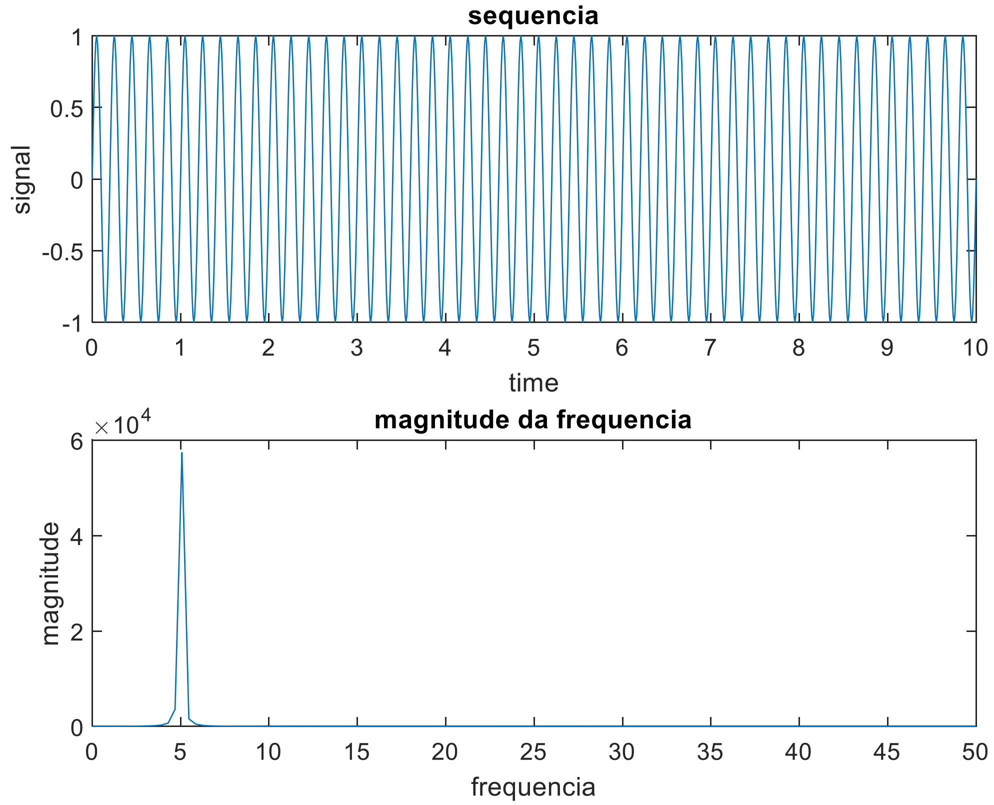
- As transformadas rápidas de Fourier, mais conhecidas por FFT (*Fast Fourier Transforms*) nada mais são do que algoritmos para computador desenvolvidos especialmente para realizar a transformada discreta de Fourier de forma rápida e eficiente.

- FFT é o método computacional mais eficiente para implementação da DFT (*Discrete Fourier Transform* – Transformada Discreta de Fourier).

Magnitude

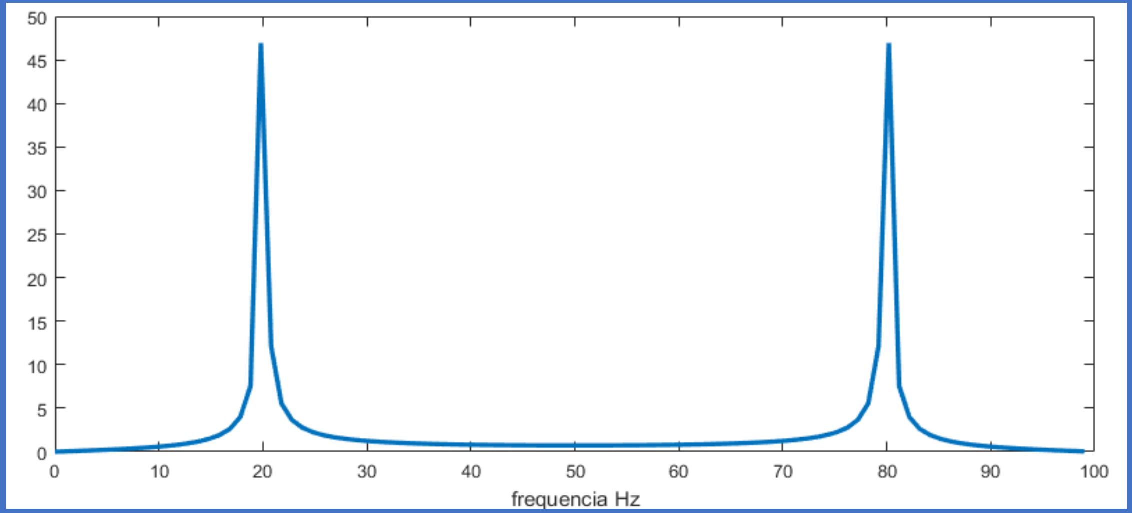
$$|X(e^{-j\omega})| = |X(e^{j\omega})| \quad \text{(even symmetry)}$$

```
close all
clear
clc
fs = 100
t = 0:1/fs:10-1/fs;
x = sin(5*2*pi*t);
y = fft(x);
freq = (0:numel(y)-1)*fs/numel(y);
% freq = freq(1:floor(numel(y)/2))
% y = y(1:floor(numel(y)/2))
subplot(2,1,1); plot(t,x);
title('sequencia');xlabel('time');
ylabel('signal')
subplot(2,1,2);plot(freq,(2*abs(y)).^2);
title('magnitude da frequencia');
xlabel('frequencia');ylabel('magnitude')
```

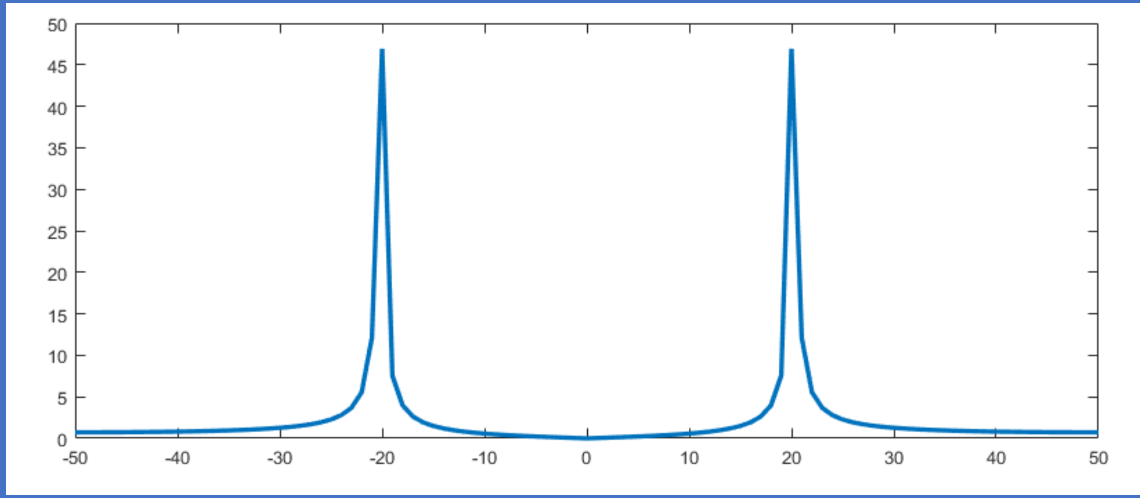


Função fftshift

```
fs = 1e2;
t = 0:1/fs:1;
st = sin(2*pi*20*t);
x = fft(st);
n = numel(x)
freq = linspace(0,n,n).*fs/n
plot(freq,abs(x),'LineWidth',2.5);
```




```
fs = 1e2;
t = 0:1/fs:1;
st = sin(2*pi*20*t);
x = fft(st);
x = fftshift(x);
n = numel(x)
freq = linspace(-n/2,n/2,n).*fs/n;
plot(freq,abs(x),'LineWidth',2.5);
```

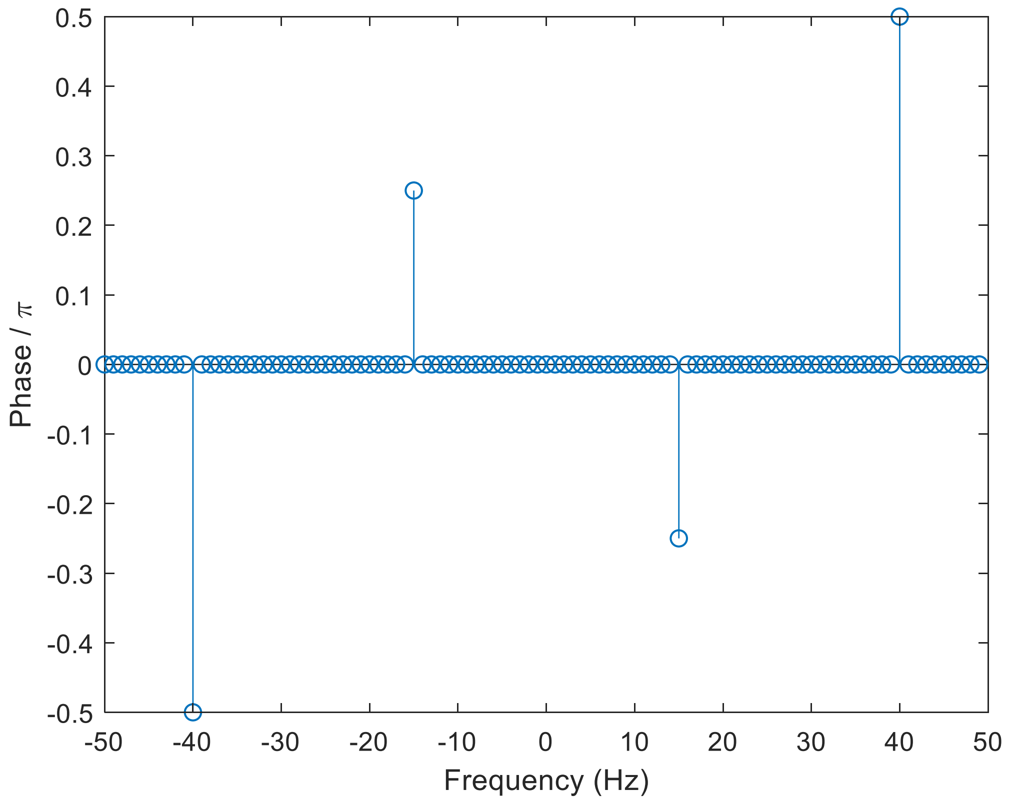


Phase

$$\angle X(e^{-j\omega}) = -\angle X(e^{j\omega}) \quad \text{(odd symmetry)}$$

```
fs = 100;
t = 0:1/fs:1-1/fs;
x = sin(2*pi*15*t - pi/4) -
sin(2*pi*40*t);
% calculo da fft
y = fft(x);
z = fftshift(y);
% criando os vetores
ly = length(y);
f = (-ly/2:ly/2-1)/ly*fs;

%calculando a fase
tol = 1e-6;
z(abs(z) < tol) = 0;
theta = angle(z);
stem(f,theta/pi)
xlabel 'Frequency (Hz)'
ylabel 'Phase / \pi'
```



$$\operatorname{sen}\left(\frac{\pi}{2} - \theta\right) = +\cos\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = +\operatorname{sen}\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = +\cot\theta$$
$$\csc\left(\frac{\pi}{2} - \theta\right) = +\sec\theta$$
$$\sec\left(\frac{\pi}{2} - \theta\right) = +\csc\theta$$
$$\cot\left(\frac{\pi}{2} - \theta\right) = +\tan\theta$$

propriedades da transformada

Periodicidade: A transformada de Fourier de tempo discreto X (ejω) é periódica
em ω com o período 2π.

$$\omega \in [0, 2\pi]$$

$$[-\pi, \pi]$$

$$X(e^{j\omega}) = X(e^{j[\omega+2\pi]})$$

$$\mathrm{Re}[X(e^{-j\omega})] = \mathrm{Re}[X(e^{j\omega})] \quad \text{(even symmetry)}$$

$$\mathrm{Im}[X(e^{-j\omega})] = -\,\mathrm{Im}[X(e^{j\omega})] \quad \text{(odd symmetry)}$$

$$|X(e^{-j\omega})| = |X(e^{j\omega})| \quad \text{(even symmetry)}$$

$$\angle X(e^{-j\omega}) = -\angle X(e^{j\omega}) \quad \text{(odd symmetry)}$$

Espectro de energia

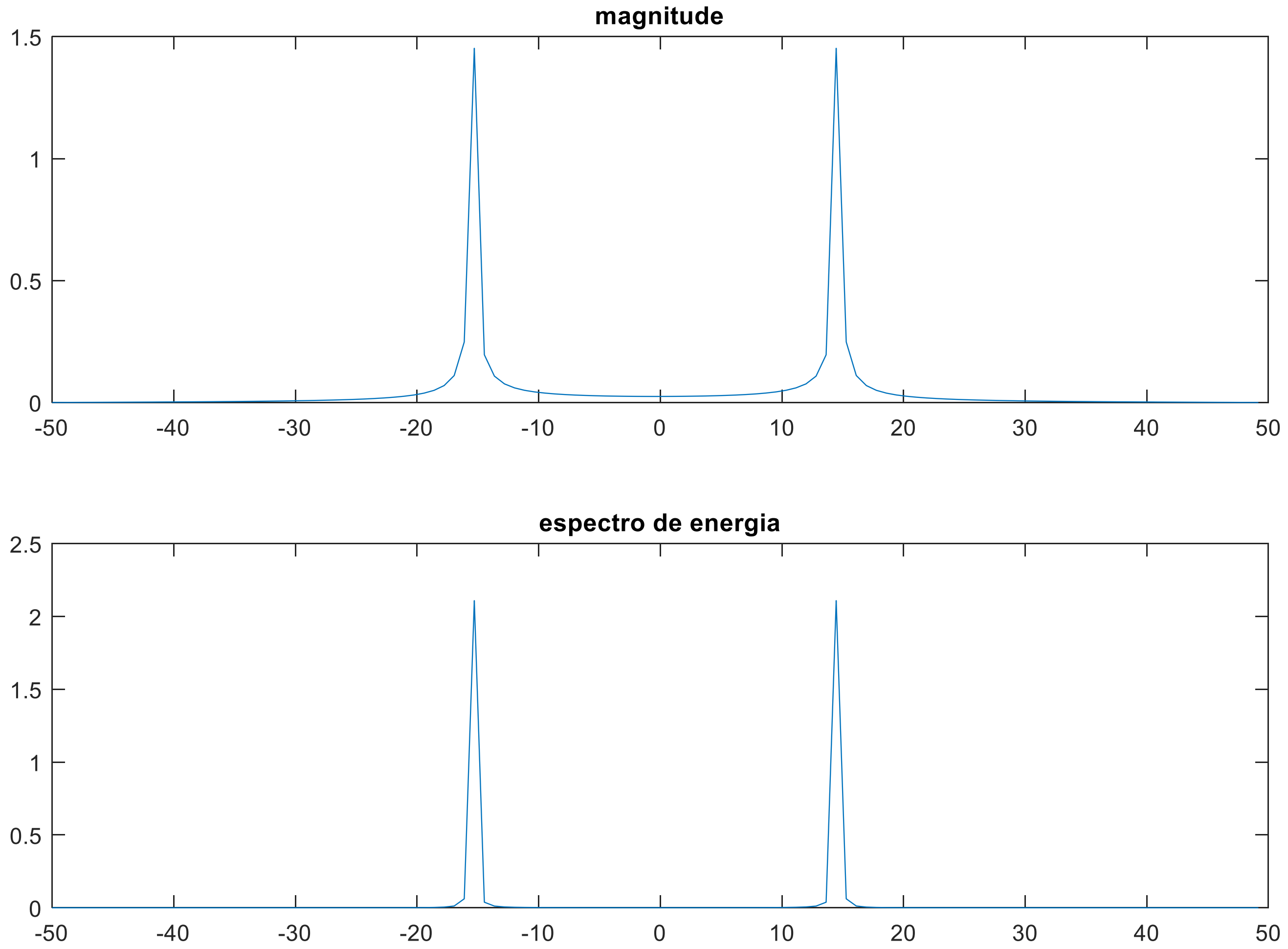
Energia total

$$E_g \approx T_s \sum_{n=1}^{N_0} (x[n])^2$$

$$E_g \approx \frac{f_s}{N} \sum_{n=1}^{N} |X(f_k)|^2$$

Espectro de energia

```
%% espectro de energia
% gerando o sinal
fs = 100;
t = 0:1/fs:1+20/fs;
x = 3*cos(2*pi*15*t);
% calculo da fft
y = fft(x);
z = fftshift(y);
ly = length(y);
f = (-ly/2:ly/2-1)/ly*fs;
% normalizando o espectro na fft
amp=abs(z)/ly;
subplot(2,1,1);plot(f,amp)
%plot espectro de energia
subplot(2,1,2);plot(f,(amp).^2)
% calculando a energia  total
Egt=sum(x.^2)./fs
Egf=sum(amp.^2)*fs/ly
```



Espectro de energia

Auto Correlação

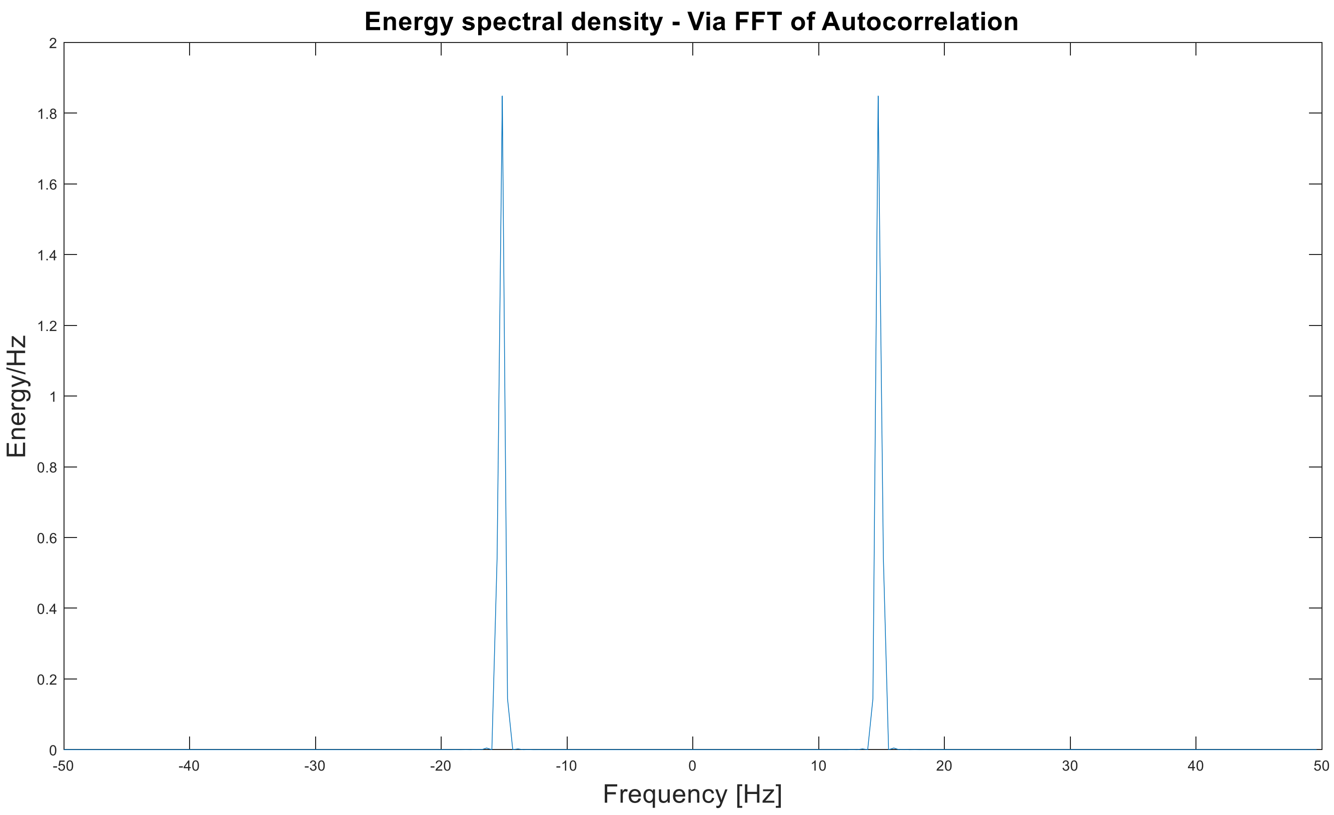
$$\psi_g(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt$$

$$\mathcal{F}\{\psi_g(\tau)\} = \Psi_g(f) \ = |X(f)|^2$$

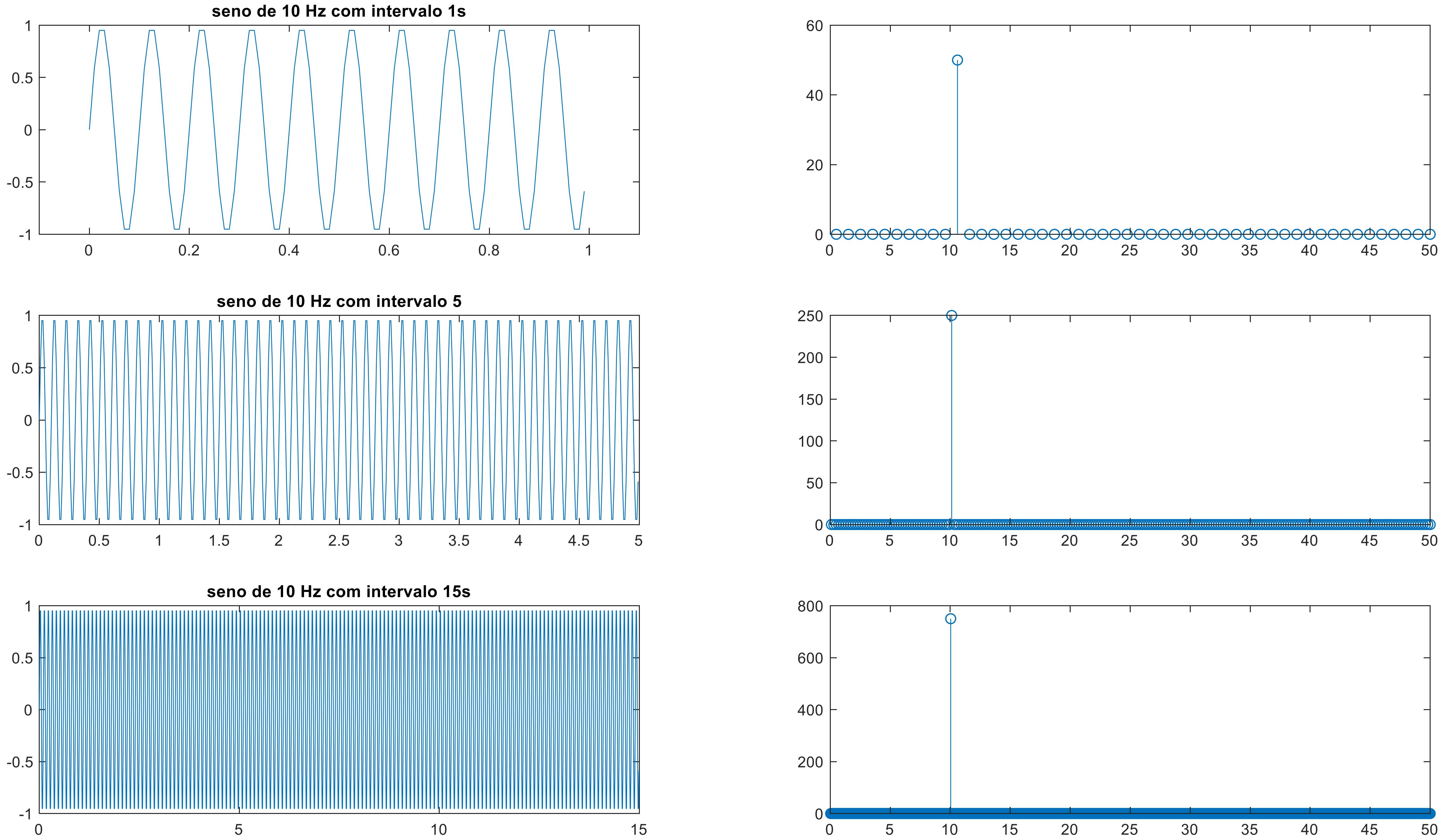
$$\psi_g(\tau_n) = \frac{1}{f_s}\texttt{xcorr(x,x)}$$

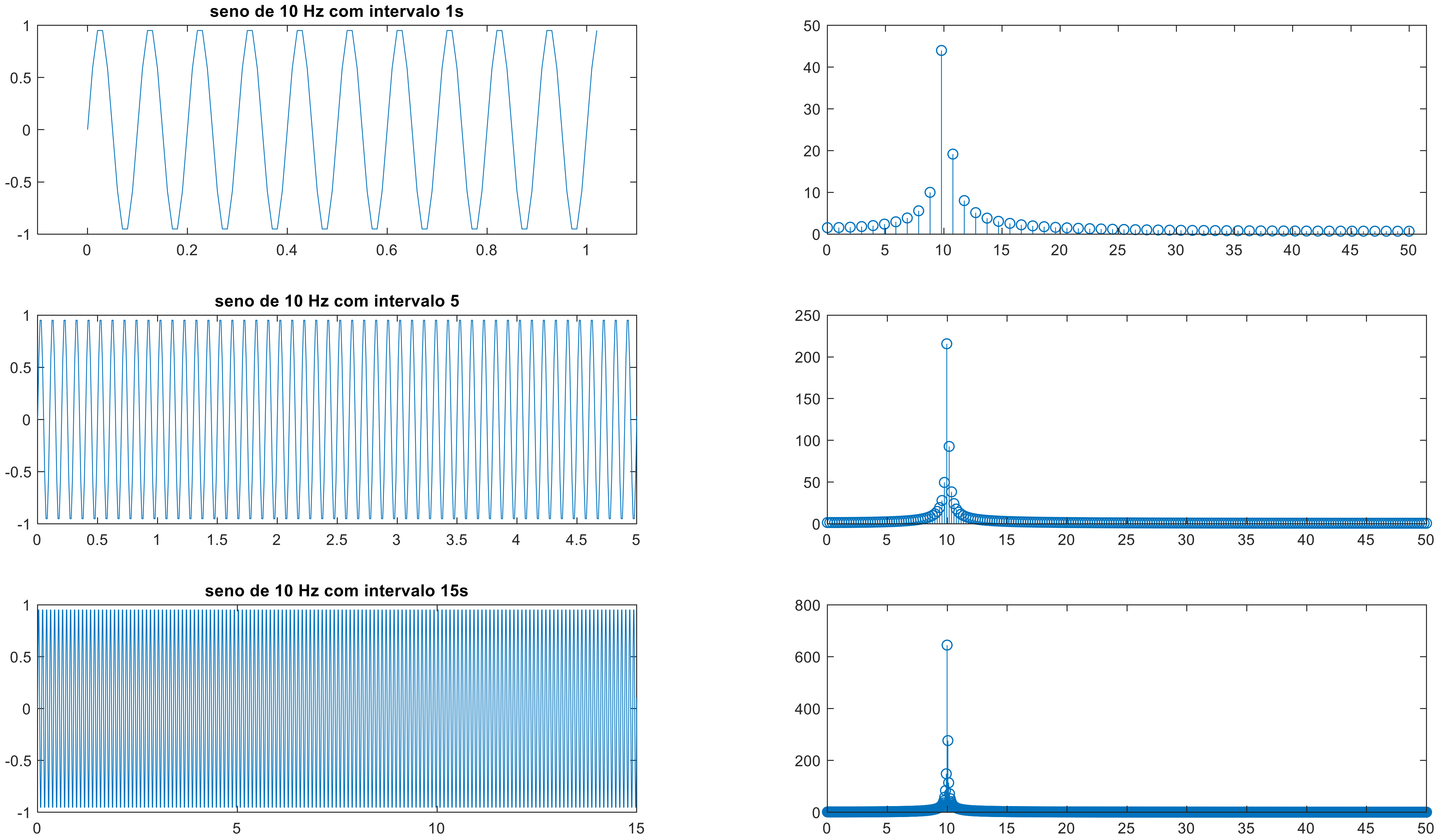
Implementando no matlab

```
%% espectro de energia
%%por autocorrelação
clear
% gerando o sinal
fs = 100;
t = 0:1/fs:1+20/fs;
x = 3*cos(2*pi*15*t);
%calculando a autocorrelação
corr = xcorr(x,x)./fs;
% fazendo fft
y = fft(corr);
z = fftshift(y);
ly = length(y);
f = (-ly/2:ly/2-1)/ly*fs;
amp = abs(z)/ly;
plot(f,amp.^2)
```

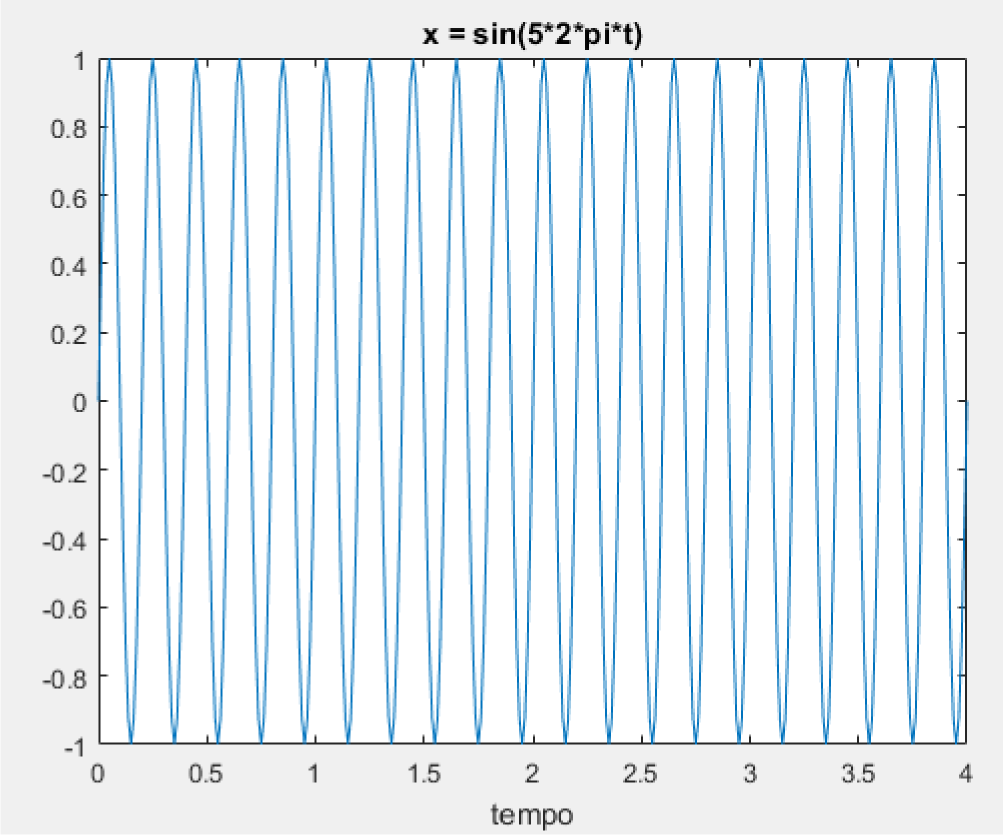


Leakege


seno de 10 Hz com intervalo 1s
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1
60
40
20
0
0
5
10
15
20
25
30
35
40
45
50
seno de 10 Hz com intervalo 5
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40
45
50
seno de 10 Hz com intervalo 15s
1
0.5
0
-0.5
-1
0
5
10
15
800
600
400
200
0
0
5
10
15
20
25
30
35
40
45
50

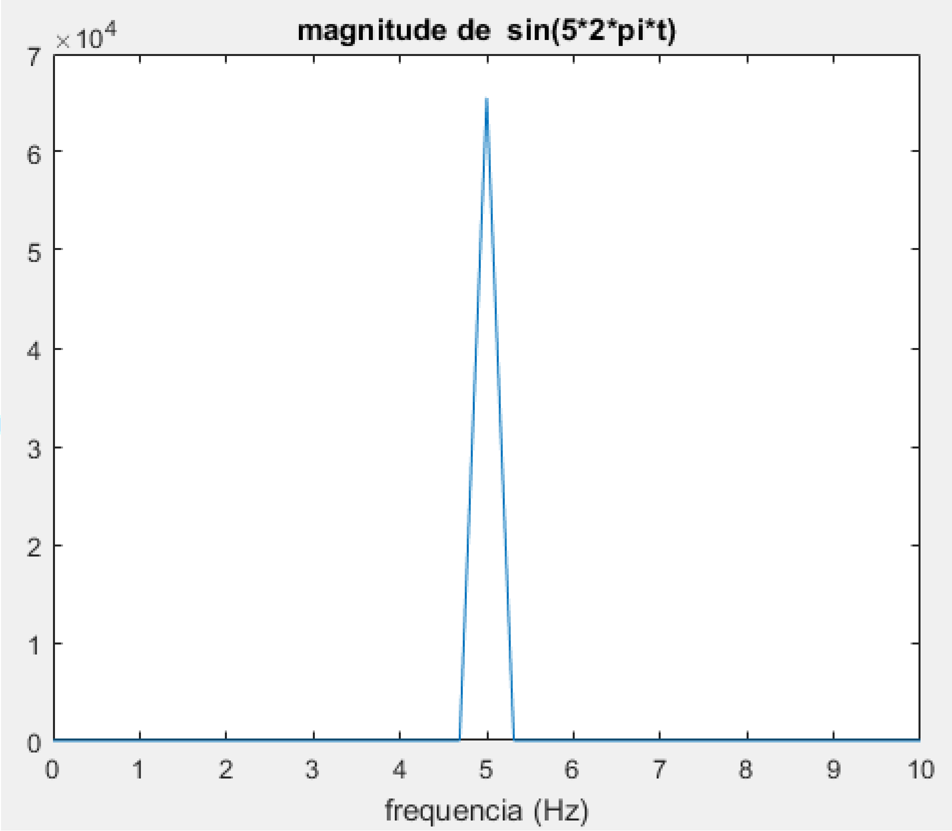

seno de 10 Hz com intervalo 1s
seno de 10 Hz com intervalo 5
seno de 10 Hz com intervalo 15s

Resolução em frequência

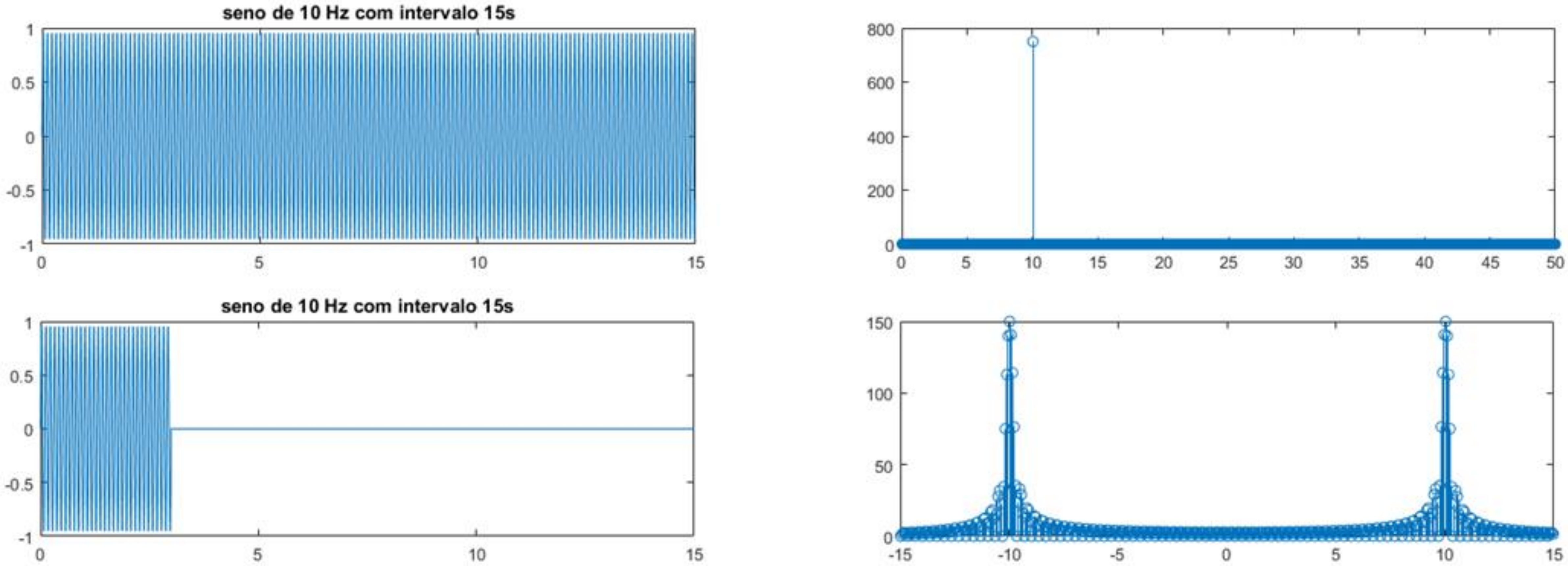


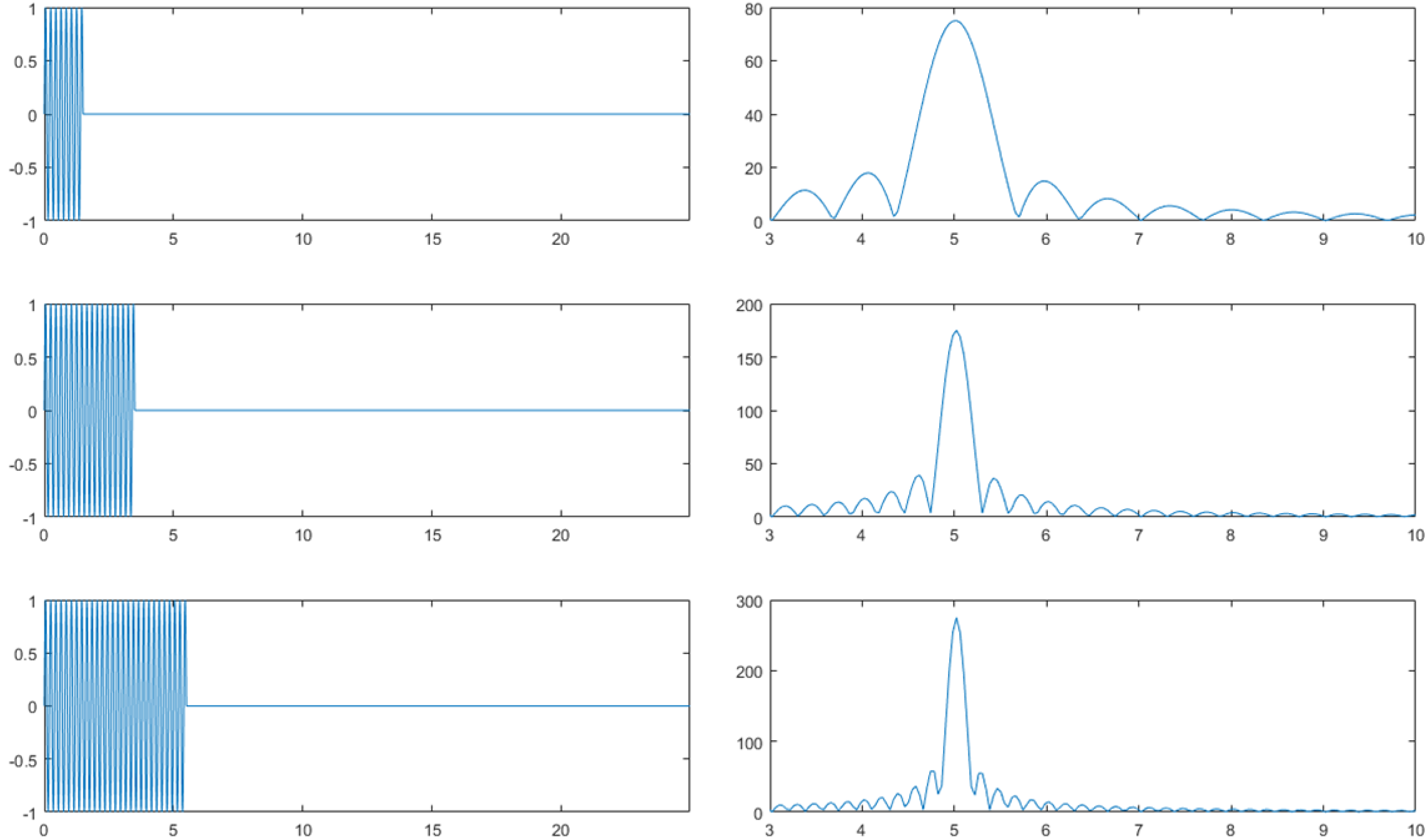
$$\Delta t = \frac{1}{f_s}$$

$$\Delta f = \frac{f_s}{N}(Hz)$$



Zero padding


seno de 10 Hz com intervalo 15s
seno de 10 Hz com intervalo 15s
1
0.5
0
-0.5
-1
0
5
10
15
800
600
400
200
0
0
5
10
15
20
25
30
35
40
45
50
150
100
50
0
-15
-10
-5
0
5
10
15



1
0.5
0
-0.5
-1
0
5
10
15
20
80
60
40
20
0
3
4
5
6
7
8
9
10
200
150
100
50
300

Desafio