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Bound impurities in a one-dimensional Bose lattice gas

arXiv:2402.03070

Felipe Isaule

Instituto de Física, PUC

Collaborators:

A. Rojo-Francas, B. Juliá-Díaz

Universitat de Barcelona

Special thanks to J. Martorell

Universitat Politècnica de Catalunya

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felipeisaule.github.io



felipe.isaule@uc.cl



[@felipeisaule](https://twitter.com/felipeisaule)

Outline

We study **two bosonic mobile impurities** interacting with a **bosonic bath in one-dimensional optical lattices**.

1. Impurities in ultracold atom gases
2. Model
3. Ground-state properties
4. Quench dynamics
5. Conclusions

Outline

1. Impurities in ultracold atom gases

2. Model

3. Ground-state properties

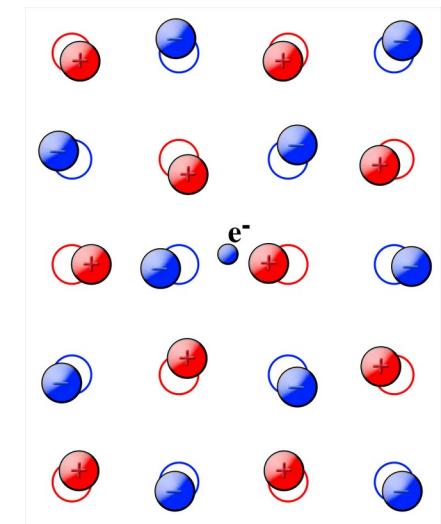
4. Quench dynamics

5. Conclusions

Impurities and quantum mixtures

- The study of **impurities in quantum mediums** has a long history.
- Impurities are usually understood as **dressed quasiparticles** referred to as **polarons**.
- The **experimental** progress realising **ultracold atomic mixtures** offer a unique setting to probe impurities.

C. Baroni, G. Lamporesi, and M. Zaccanti, arxiv:2405.14562 (2024).



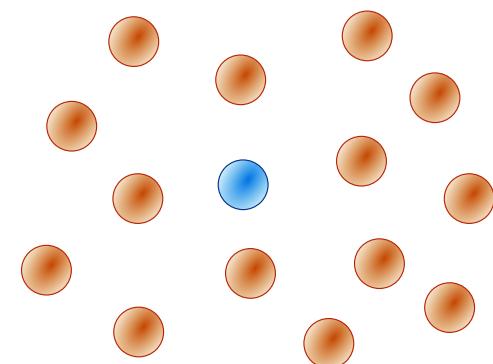
Electrons in a ionic crystal.
L. Landau and S. Pekar, Zh. Eksp. Teor. Fiz. **18**, 419 (1948).

- Impurities are realised with **highly-imbalanced mixtures**.

Bose polarons and bipolarons

- **Bose polarons**, impurities immersed in a **BEC** gas, were achieved in landmark experiments in 2016.
N. B. Jørgensen et al., PRL **117**, 055302 (2016). M.-G. Hu et al., PRL **117**, 055301 (2016).
- The study of **two impurities** immersed in a BEC has also received increasing theoretical attention, as they can form bound **bipolarons**.

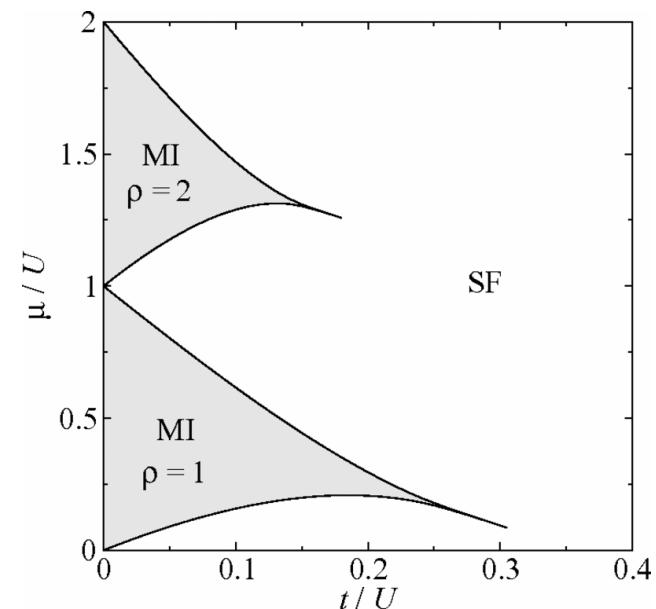
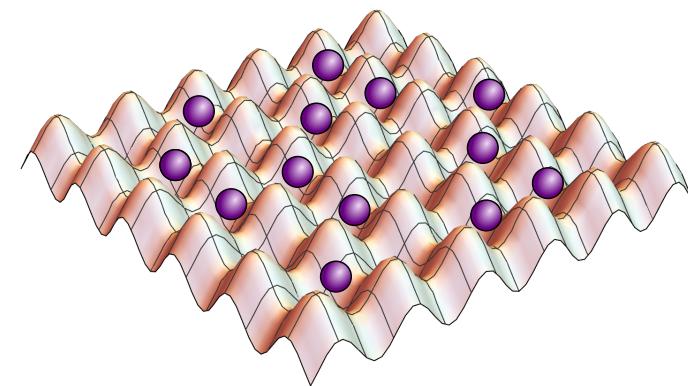
A. Camacho-Guardian *et al.*, PRL **121**, 013401 (2018).



Ultracold atoms in optical lattices

- Ultracold atoms in **optical lattices** offer another interesting setting to study many-body physics.

Gross and I. Bloch, Science 357, 995 (2017).



Phase diagram of the 1D BH model.
S. Ejima et al., PRA 85, 053644 (2012).

- **Bosons in tight optical lattices are described by the Bose-Hubbard model**

$$\hat{H} = -t \sum_{\langle i,j \rangle} \left(\hat{a}_i^\dagger \hat{a}_j + \text{h.c.} \right) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) .$$

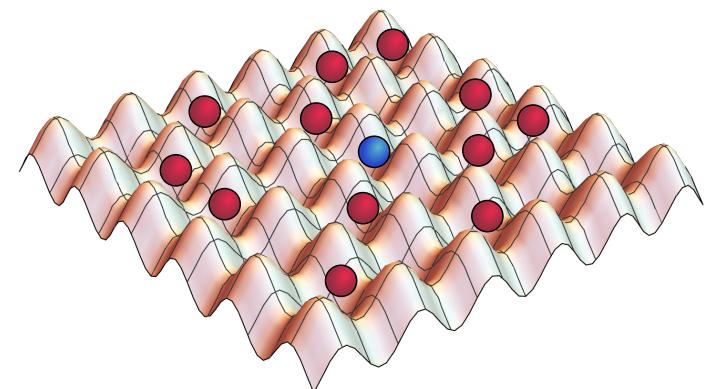
t : tunnelling
 $U > 0$: boson-boson interaction

- The Bose-Hubbard model shows a **superfluid-to-Mott insulator phase transition**.

J. K. Freericks and H. Monien, EPL 26, 545 (1994).

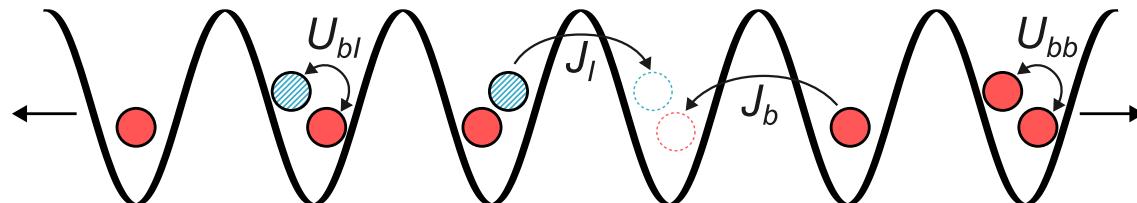
Lattice polarons

- The study of **impurities in optical lattices** has received renewed interest.
- These are often called **lattice polarons**, even though optical lattices do not support phonon excitations.
- Several recent works have studied **Bose lattice polarons**.
V. Colussi, C. Menotti C and A. Recati, PRL **130**, 173002 (2023).
M. Santiago-García, S. Castillo-López and A. Camacho-Guardian, NJP **26**, 063015 (2024).
F. Caleffi, M. Capone,I. DeVega and A. Recati, NJP **23**, 033018 (2021).
- Lattice polarons in **fermionic mediums** have also been studied.
I. Amelio and N. Goldman, Scipost Physics **16**, 056 (2024).



Lattice bipolarons

- Bipolaron-like physics in optical lattices with bosonic baths has also attracted significant attention.
M. Pasek and G. Orso, PRB **100**, 245419 (2019).
S. Ding, G. A. Domínguez-Castro, A. Julku, A. Camacho-Guardian and G. M. Bruun, SciPost Phys. **14**, 143 (2023).
- Systems with purely **repulsive interactions** can induce the formation of a **bound state between two impurities**.
K. Keiler, S. I. Mistakidis and P. Schmelche, NJP **22**, 083003 (2020),
- We study **stationary** and **quench-induced dynamics** properties of **two mobile impurities** interacting with a **bosonic bath** and immersed in a **one-dimensional optical lattice**.



Outline

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Model

- We consider a **two-component Bose-Hubbard Hamiltonian**

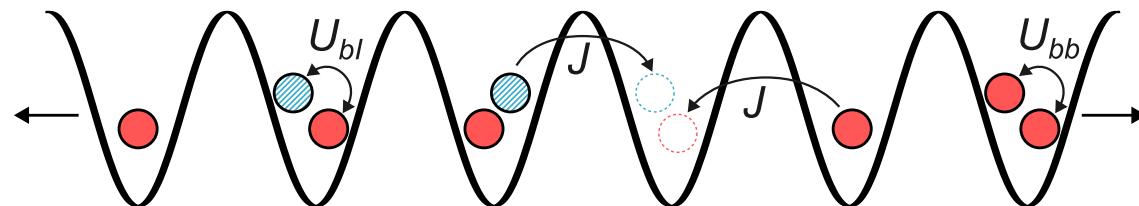
$$\hat{H} = \underbrace{-J \sum_{\sigma=b,I} \sum_i (\hat{a}_{i,\sigma}^\dagger \hat{a}_{i+1,\sigma} + \text{h.c.})}_{\text{Tunnelling}} + \underbrace{\frac{U_{bb}}{2} \sum_i \hat{n}_{i,b}(\hat{n}_{i,b} - 1)}_{\text{Boson-boson repulsion}} + \underbrace{U_{bI} \sum_i \hat{n}_{i,b} \hat{n}_{i,I}}_{\text{Boson-impurity repulsion}}$$

b: Bath
I: Impurities

- We study stationary properties with the **exact diagonalisation** (ED) method for a fixed number N_σ of particles of each species.

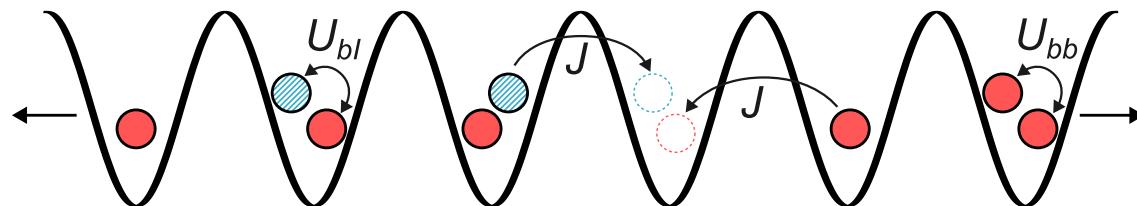
D. Raventós, T. Graß, M. Lewenstein and B. Juliá-Díaz, JPB **50**, 113001 (2017).

- We consider $M=6,7,8,9$ sites, $N_b=M$ bosons in the bath (**unity filling**) and $N_I=2$ impurities.



Model

- We consider **periodic boundary conditions** (a ring), which can be achieved in experiment.
L. Amico, A. Osterloh and F. Cataliotti, PRL **95**, 063201 (2005).
- It has also become possible to realise systems with a **few atoms**.
D. Blume, RPP **75**, 046401 (2012). Sowiński and M. A. García-March, RPP **82**, 104401 (2019).
- The proposed configuration can be produced with **highly-imbalanced atomic mixtures**.
N. B. Jørgensen et al., PRL **117**, 055302 (2016).



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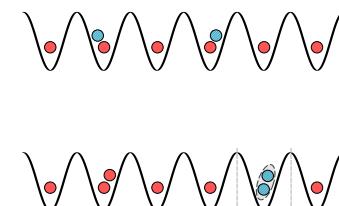
5. Conclusions

Static limit

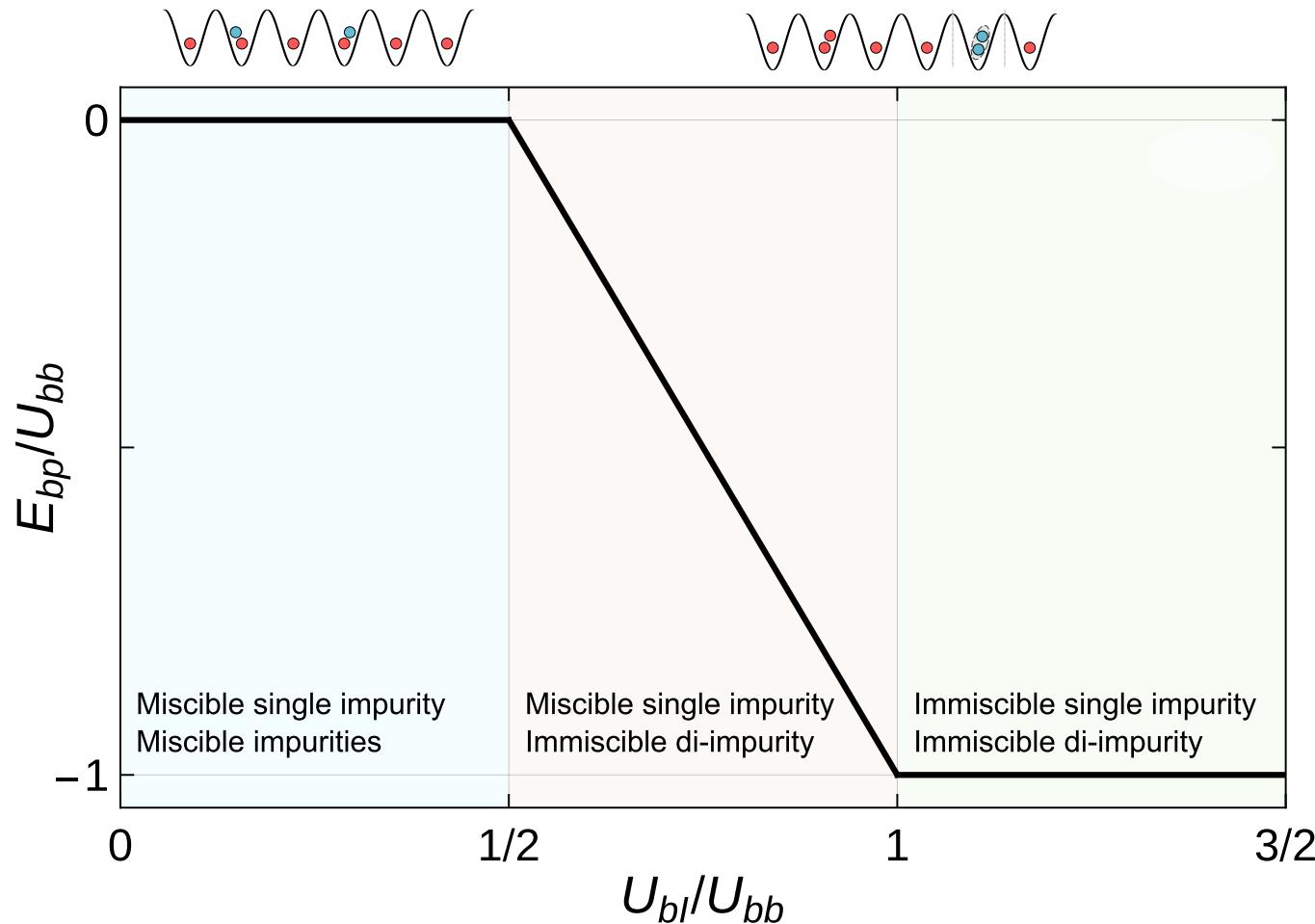
- The ground state in the **static limit** can be examined analytically

$$\hat{H} = \frac{U_{bb}}{2} \sum_i \hat{n}_{i,b} (\hat{n}_{i,b} - 1) + U_{bI} \sum_i \hat{n}_{i,b} \hat{n}_{i,I}.$$

- The bipolaron energy:** $E_{bp} = E(N_I = 2) - 2E(N_I = 1) + E(N_I = 0)$.
A. Camacho-Guardian *et al.*, PRL **121**, 013401 (2018).
- One obtains:

$$E_{bp} = \begin{cases} 0 & : 0 \leq U_{bI} < U_{bb}/2 \\ U_{bb} - 2U_{bI} & : U_{bb}/2 \leq U_{bI} < U_{bb} \\ -U_{bb} & : U_{bb} < U_{bI} \end{cases}$$


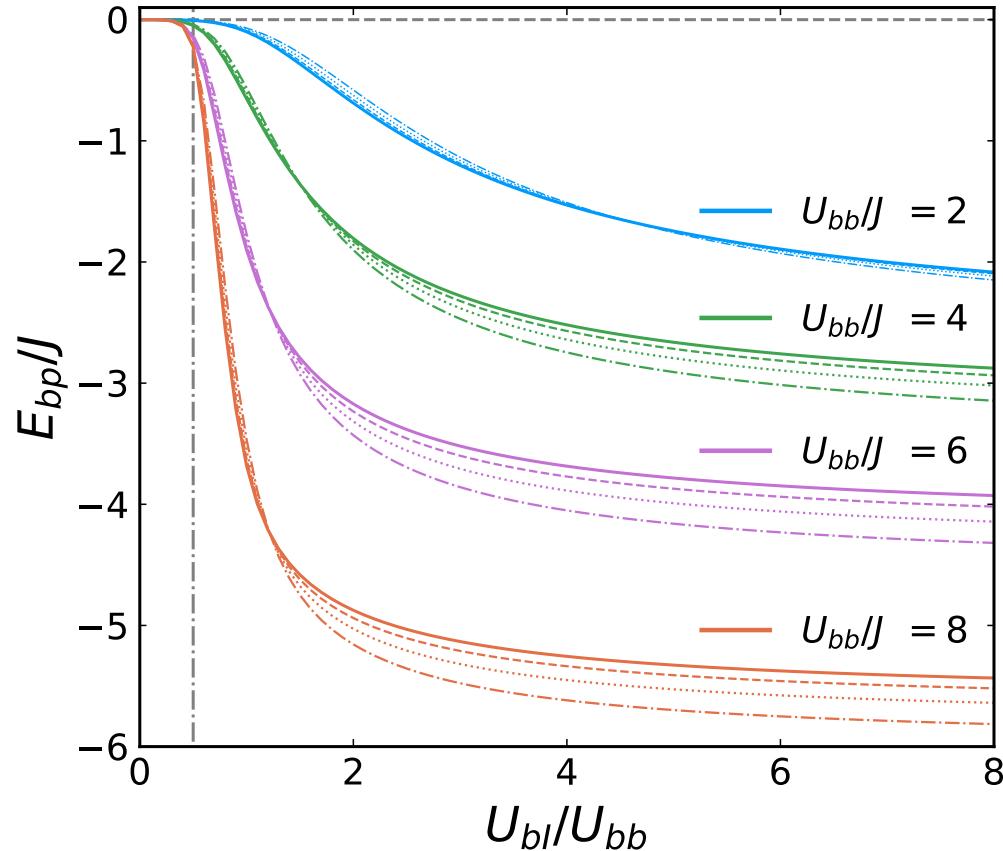
Static limit



- A negative bipolaron energy suggests the formation of **bound states**.

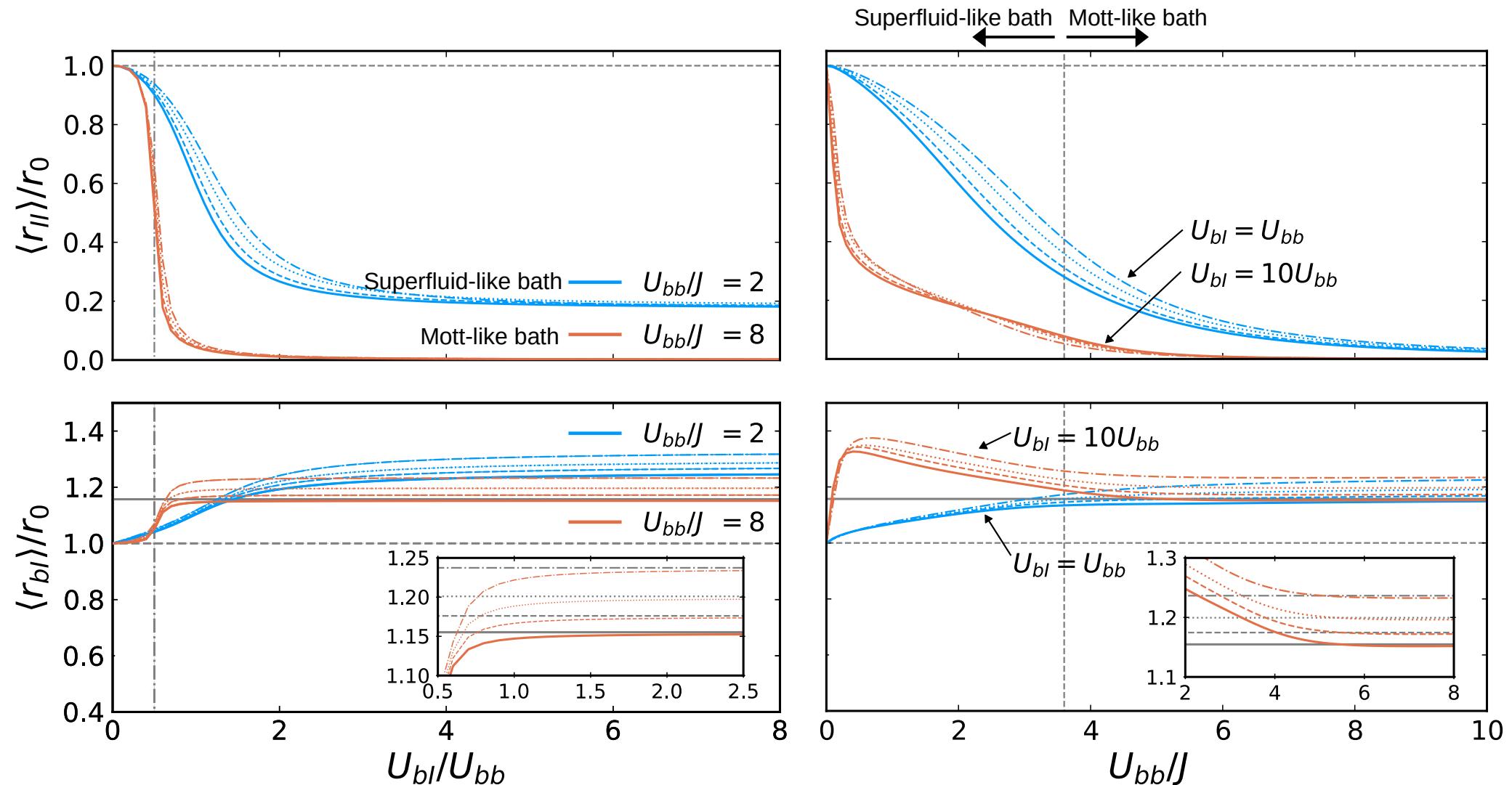
Bipolaron Energy

- Now we examine mobile systems.



- A negative bipolaron energy suggests the formation of **bound states**.

Average distance between particles



- A small $\langle r_{II} \rangle$ signals the formation of a **bound dimer of impurities**.

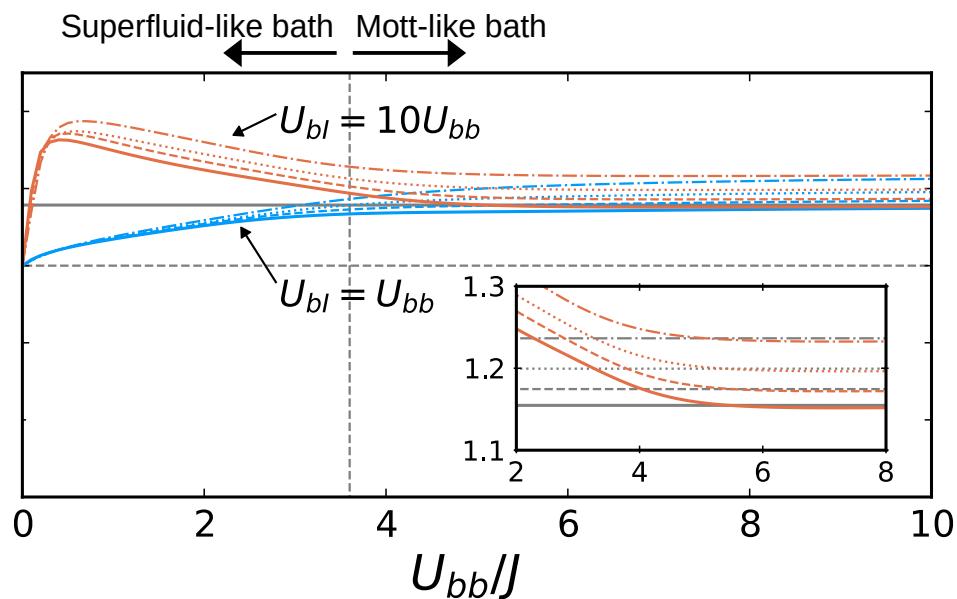
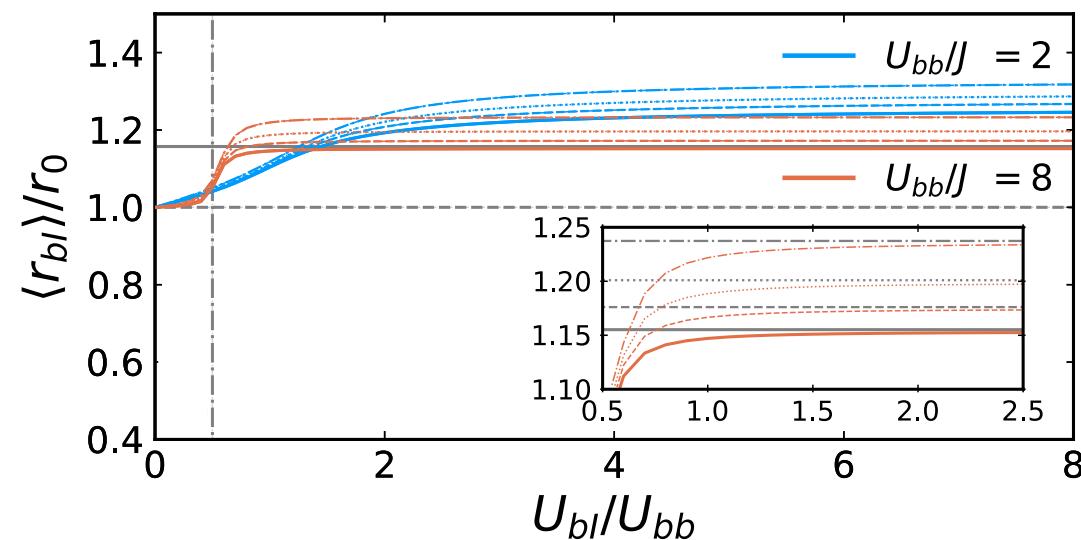
r_0 : Average distance between two free bosons.

Average distance between particles

- For **large interactions** U_{bb} and U_{bl} , the average distance between the bath and the impurities converges to:

$$r_s^* = r_0 + r_{F,0}/M.$$

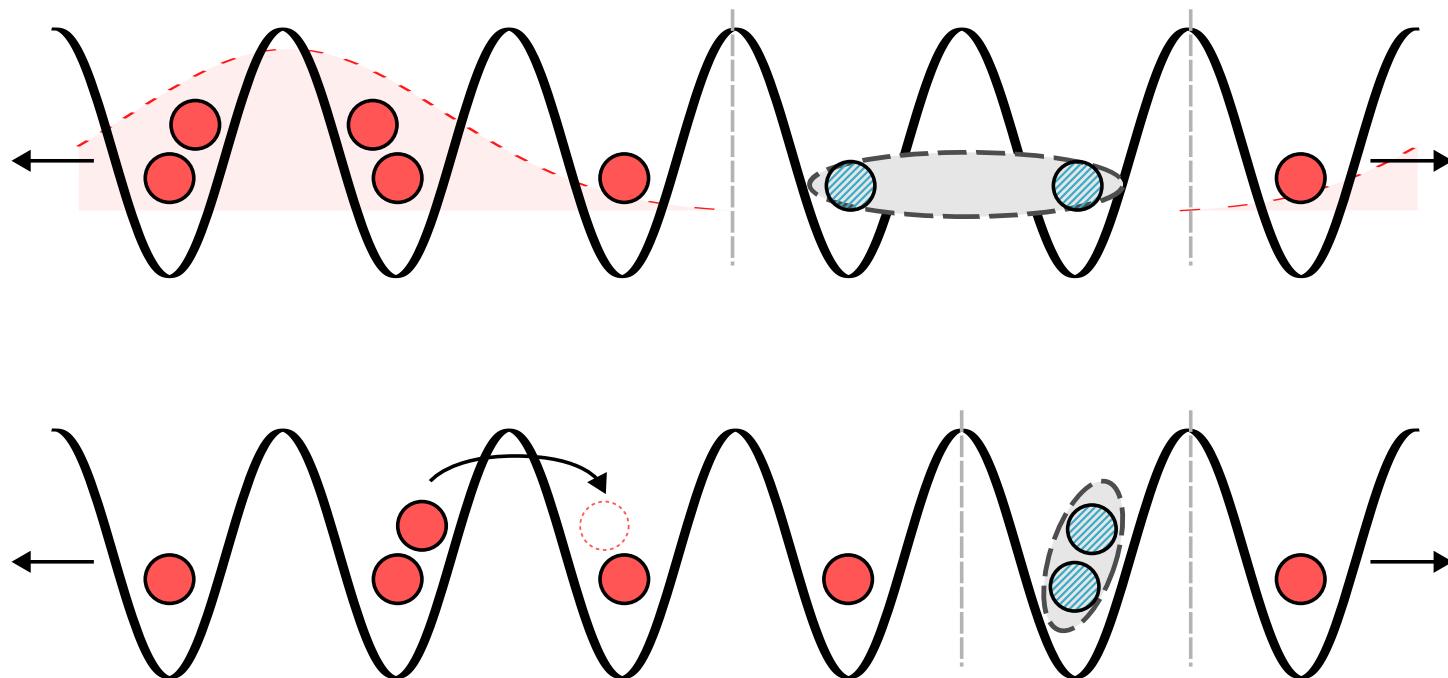
$r_{F,0}$: Average distance between two free fermions of the same spin..



r_0 : Average distance between two free bosons.

Bound impurities

- A large bath-impurity repulsion U_{bi} produces a **phase separation**, inducing the formation of **bound impurities**.
- A Mott-like bath supports the formation of **tightly bound dimers**, while a superfluid-like bath supports **shallow dimers**.

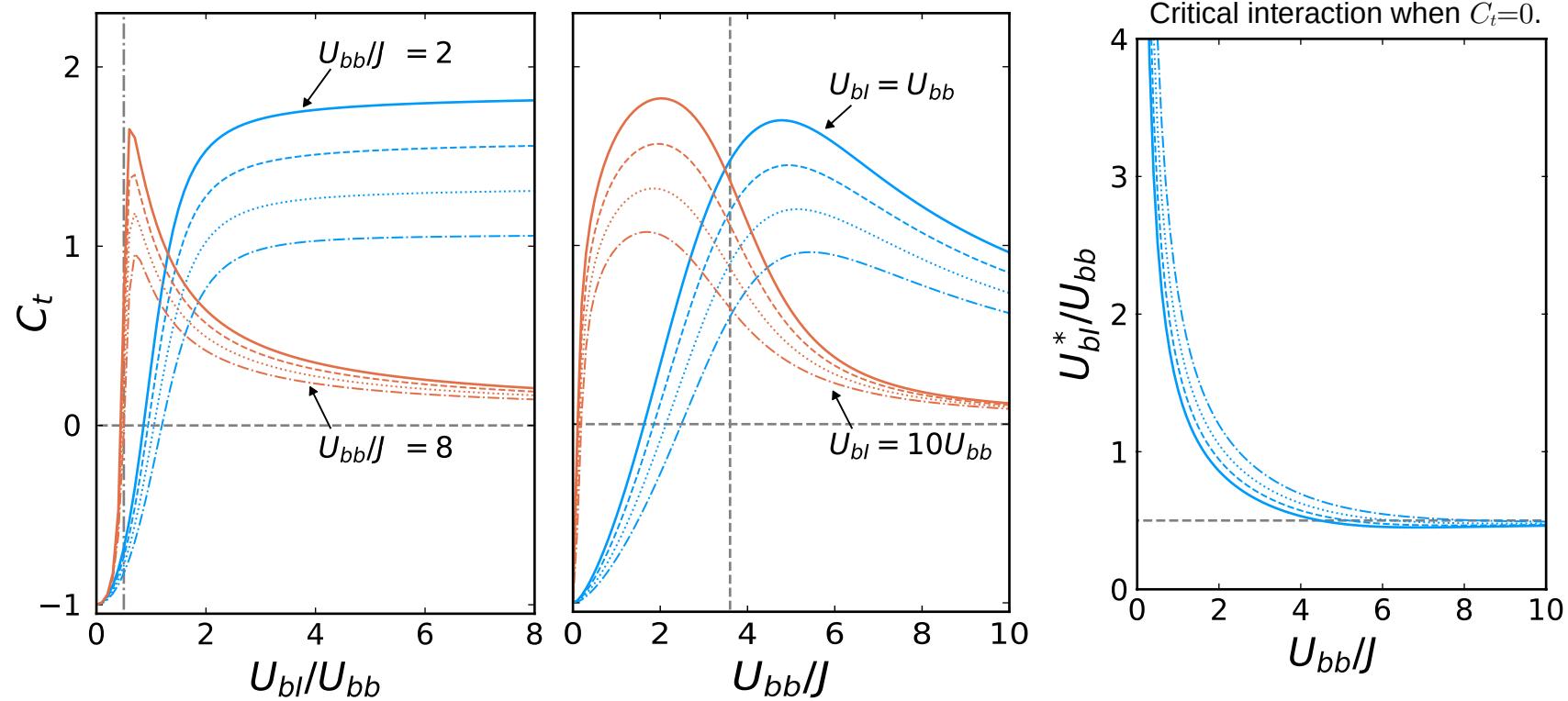


Tunnelling of dimers

- To further characterise the formation of dimers, we examine the **tunnelling correlator**

C. Menotti and S. Stringari, PRA 1, 045604 (2010).

$$C_t = \langle \hat{a}_{i,I}^\dagger \hat{a}_{i,I}^\dagger \hat{a}_{i+1,I} \hat{a}_{i+1,I} \rangle - \langle \hat{a}_{i,I}^\dagger \hat{a}_{i+1,I} \rangle^2.$$



- Dimers form for $U_{bI} > U_{bb}/2$.

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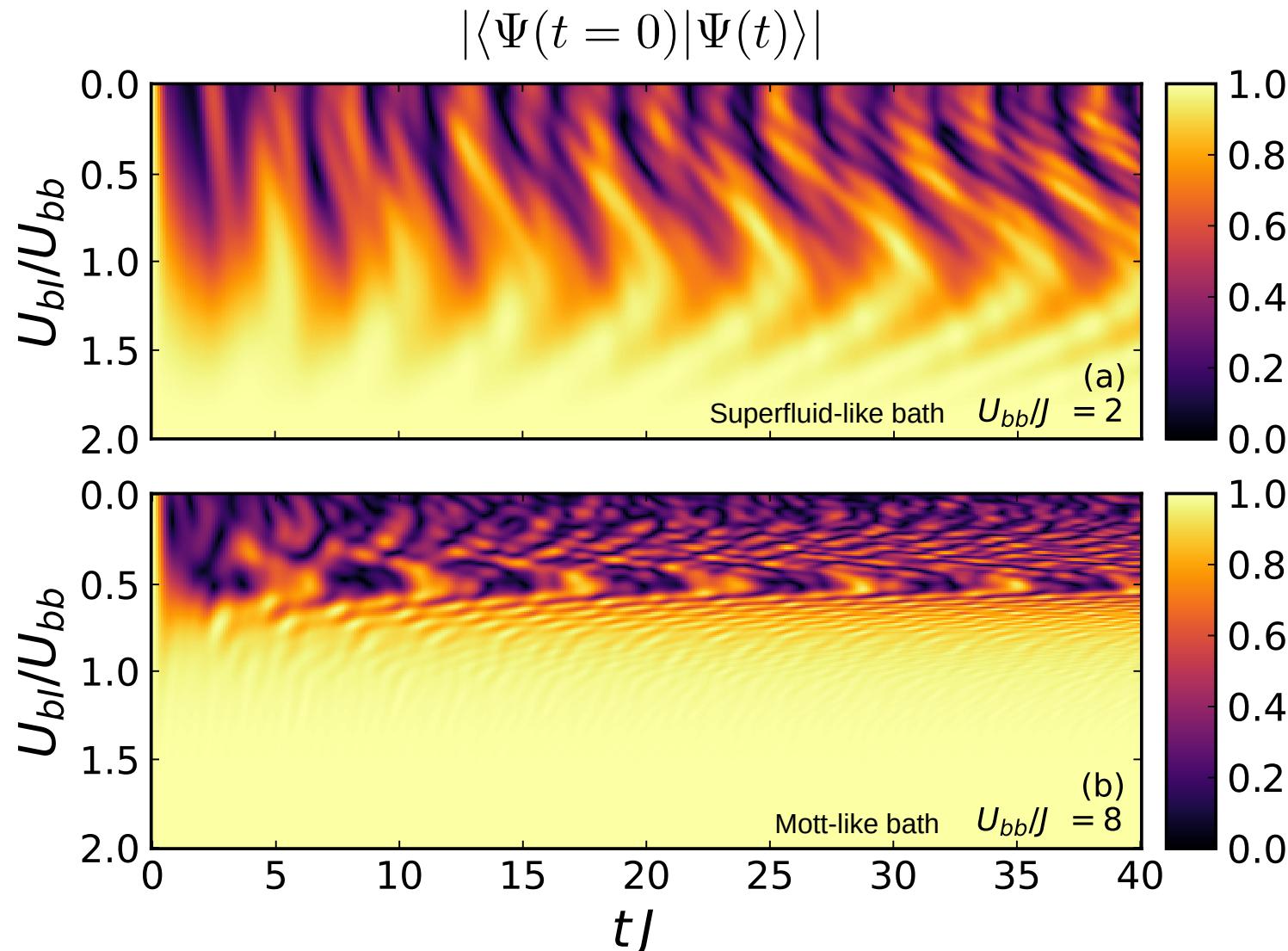
Quench-induced dynamics

- We prepare an **initial state** Ψ_0 from the ground state for chosen interactions U_{bb}/J and U_{bI}/U_{bb} .
- We choose a large U_{bI}/U_{bb} so a **dimer is formed**.
- We perform sudden **quenches** at $t=0$ to a lower value of U_{bb}/J or U_{bI}/U_{bb} .
- We follow the time evolution by numerical exponentiation

$$|\Psi(t)\rangle = e^{i\hat{H}t}|\Psi(t=0)\rangle.$$

- We consider lattices with $M=7$.

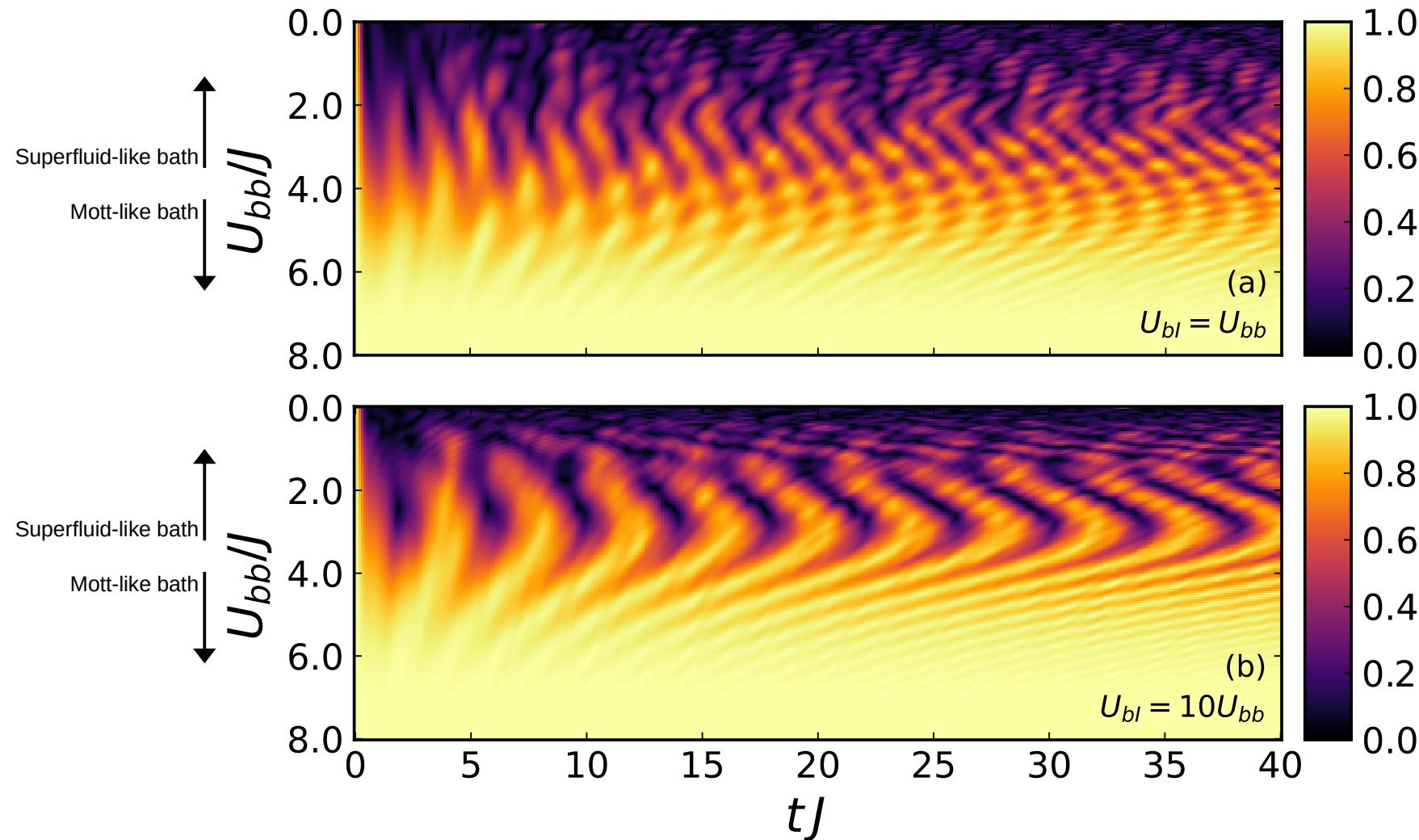
Overlaps: Quench in U_{bI}



- The system shows **collapses** and **revivals** of the dimer states.
- The **periods reach a maximum** around the interaction when $C_t=0$.

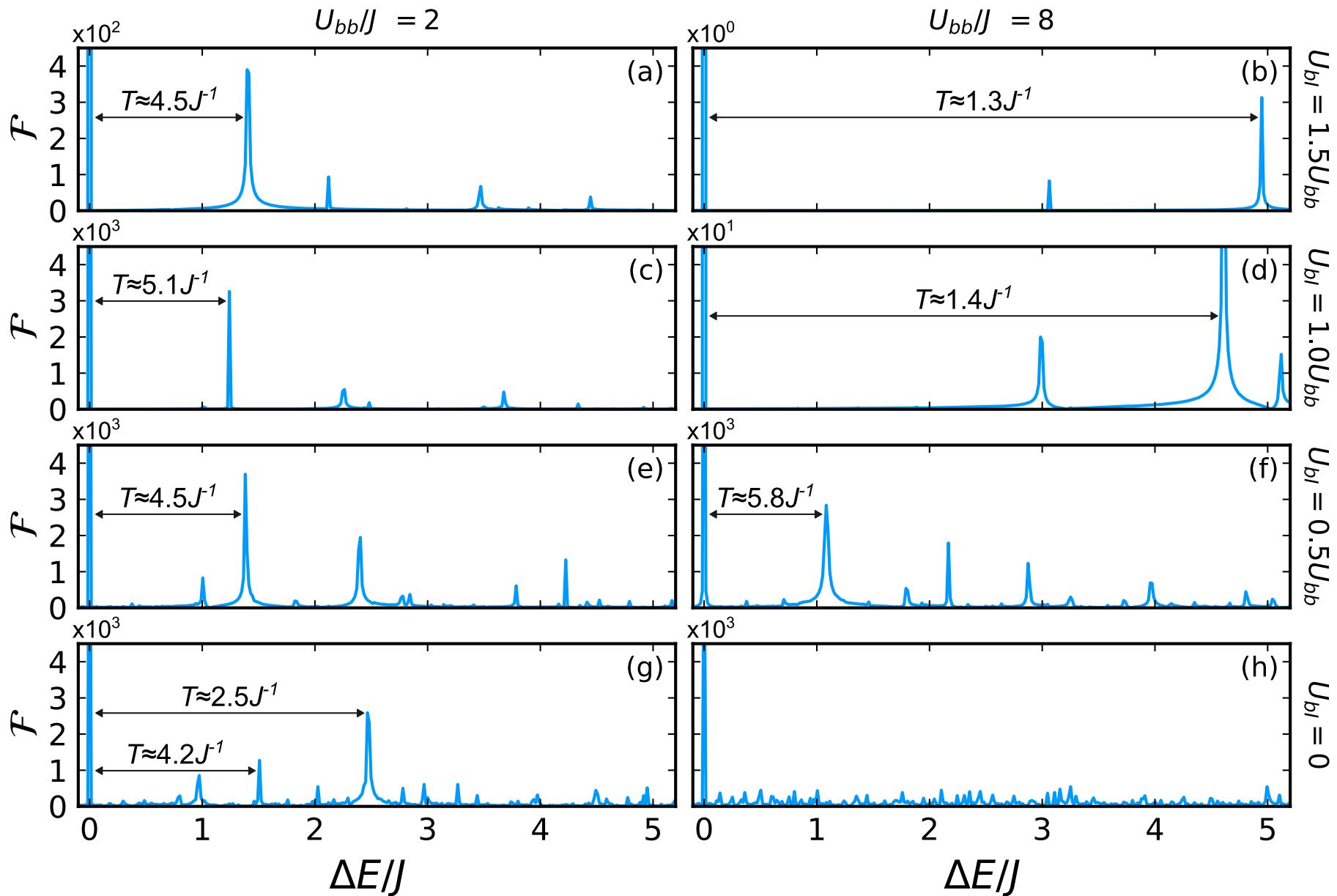
Overlaps: Quench in U_{bb}

$$|\langle \Psi(t=0) | \Psi(t) \rangle|$$



- The system shows **collapses** and **revivals** of the dimer states.
- The periods reach a maximum around the **superfluid-Mott transition region**.

Fourier Analysis: Quench in U_{bI}



- The oscillations are driven by **phase-separated excitations**.

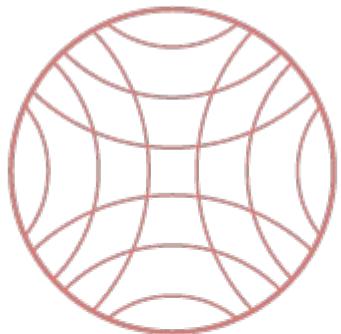
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Conclusions

- We have studied **stationary properties** and **quench-induced dynamics** of two impurities immersed in a 1D Bose lattice gas.
- We have characterised the formation of **bound dimers of impurities**.
- We revealed an intriguing onset of **collapses and revivals** after a quench of the interactions.
- We found that the **oscillations** are driven by **phase-separated excitations**.
- Future work:
 - Consider fermionic baths.
 - Study Rabi (driven) impurities.
 - Employ other theoretical techniques.

Conferences en Chile



QUANTUM OPTICS 2024

December 9 to 13

<https://www.miroptics.cl/quantum-optics-2024-chile/>

Puerto Varas



3rd Workshop on Molecular Quantum Technology - MQT 2024

December 16 to 20

<https://mqt2024.org/>



Thank you!

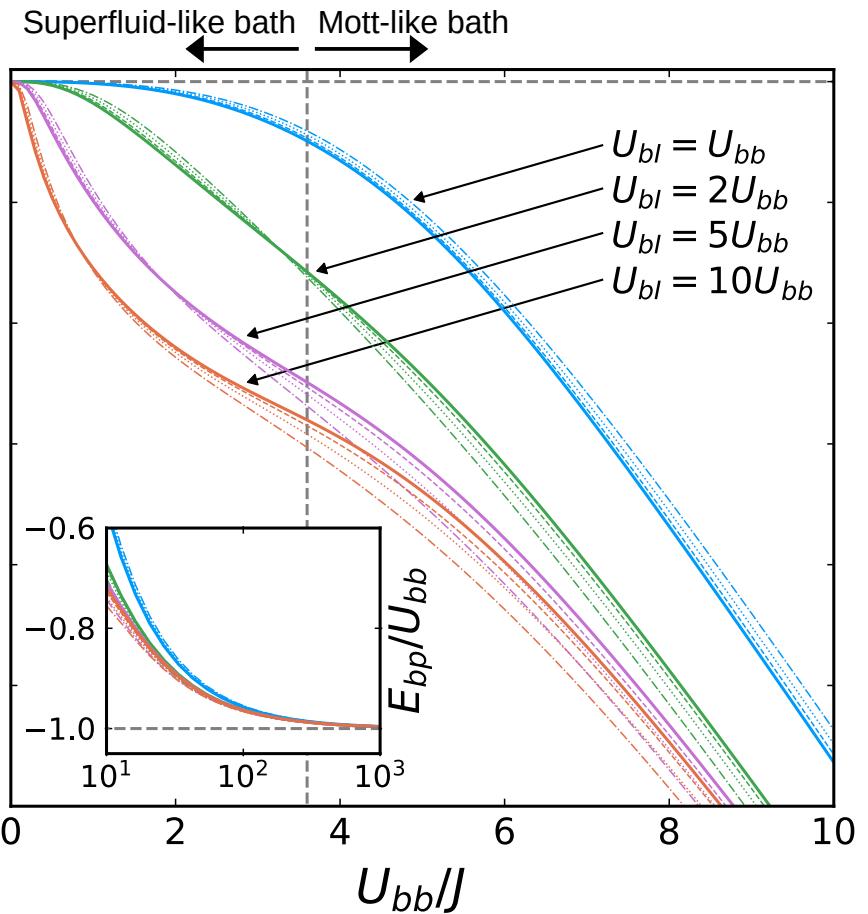
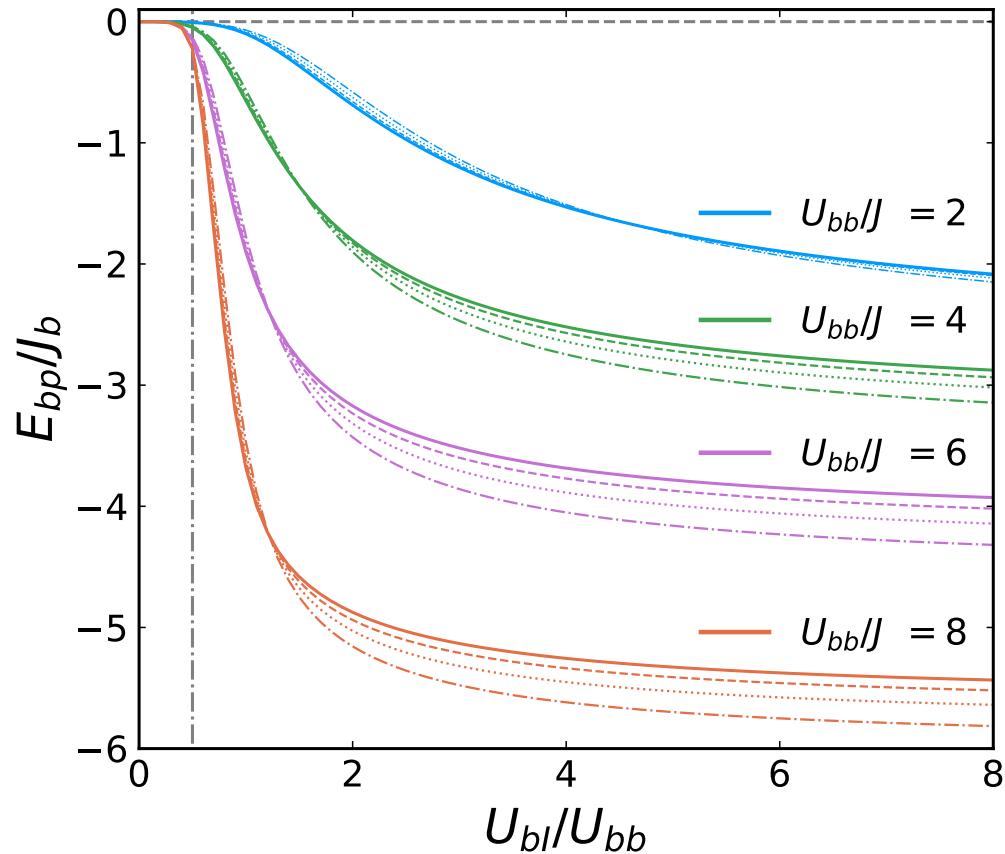


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Bipolaron Energy

- Now we examine mobile systems.



- A negative bipolaron energy suggests the formation of **bound states**.

Average distances: Quench in U_{bI}

