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Ultracold polarons and impurities

From lattice polarons to effective field theories

Felipe Isaule

ANID Postdoctoral Fellow
Instituto de Física, PUC, Chile

Quantum Systems Unit, OIST
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Ultracold polarons and impurities

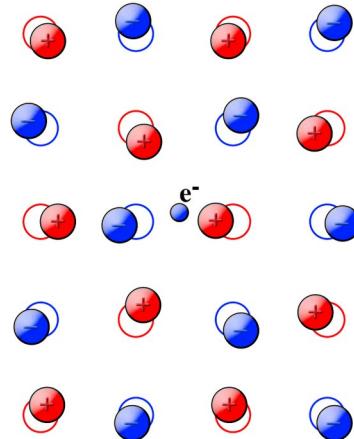
1. Quantum **polarons and impurities** in ultracold atomic systems.
2. One-dimensional **harmonically confined lattice polarons and counterflows**.
3. Polaron physics with the **functional renormalisation group**.
4. Summary and **future work**.

Ultracold polarons and impurities

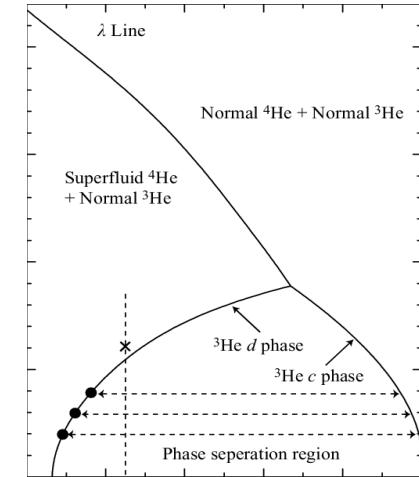
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Polarons and impurities

- The study of **impurities** immersed in **quantum mediums** has a **long history** and is **relevant in many fields** of physics.

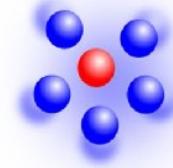


Electrons in an ionic crystal
L. Landau and S. Pekar, Zh. Eksp. Teor. Fiz. **18**, 419 (1948).

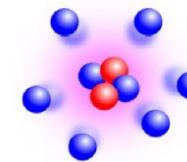


^3He impurities in ^4He
G. Baym and C. Pethick, "Landau Fermi-Liquid Theory: Concepts and Application" (1991).

Polaronic proton



Polaronic alpha particle



Impurities in nuclear systems
Tajima et al., AAPPS Bulletin, **34**, 9 (2024).

- Such **impurities** often form dressed **quasiparticles** referred to as **polarons**.

Polarons in ultracold atomic gases

- The study of polarons has been revitalised thanks to their experimental realisation in ultracold atomic mixtures with a high population imbalance.

P. Massignan et al., Rep. Prog. Phys. **77**, 034401 (2014). F. Grusdt et al. Rep. Prog. Phys. **88**, 066401 (2025).

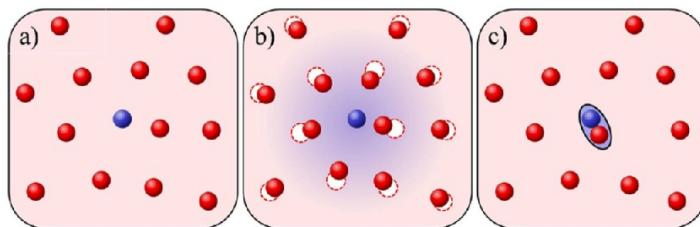


Illustration of a Fermi Polaron
A. Schirotzek et al., PRL **102**, 230402 (2009).

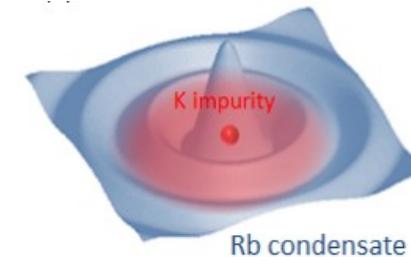


Illustration of a Bose Polaron
M.-G. Hu et al., PRL **117**, 055301 (2016).

- Several relevant properties can be examined: quasiparticle or **polaron energies**, **residue** and **orthogonality catastrophe**, coherence, effective masses, etc.
- They could also be used to **probe** and **manipulate** quantum **many-body systems**: transport properties, **phase transitions**, **mediated interactions**, etc.

Polarons in ultracold atomic gases

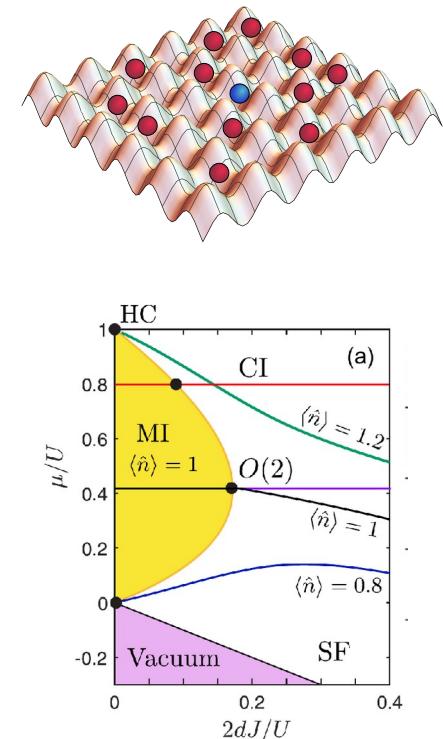
- Different configurations can be studied:
 - **Statistics:** Fermi and Bose polarons.
 - **Number of impurities:** Bipolarons and multipolarons.
 - **Confinement:** Optical lattices, harmonic traps, dimensionality.
 - **Stationary properties or dynamics.**
- Well-posed problem:
 - Variational approaches, Quantum Monte-Carlo (QMC), Exact Diagonalisation (ED), Tensor Networks, Effective Field Theories (EFT), amongst others.
- **Polarons offer a rich combination of few- and many-body physics**, and they are also a **starting point to study more complicated configurations.**

Ultracold polarons and impurities

1. Quantum **polarons and impurities** in ultracold atomic systems.
2. One-dimensional **harmonically confined lattice polarons and counterflows**.
3. Polaron physics with the **functional renormalisation group**.
4. Summary and **future work**.

Lattice polarons

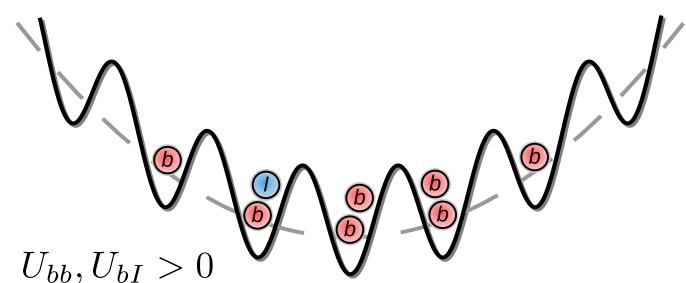
- A rich platform for studying ultracold atomic impurities is tight optical lattices.
- Such impurities are now referred to as **lattice polarons**.
- The study of **lattice polarons** in **bosonic media**s has gained significant attention in the past few years.
- One relevant question is how the **polaron properties** change across the **superfluid-to-Mott insulator (SF-MI) transition**.
- We have tried to address this question in **one-dimensional lattices** with **numerical techniques**.



V. E. Colussi, F. Caleffi, C. Menotti, and A. Recati, Phys. Rev. Lett **130**, 17, 3002 (2023).

Harmonically confined lattice polarons

- We have recently studied a **single impurity** interacting with a **bosonic bath** in a **one-dimensional harmonically confined optical lattice**.



- We have investigated this system using the **density matrix renormalisation group (DMRG)** for **large lattices** with a **large number of particles**.
S. R. White, Phys. Rev. Lett. **69**, 2863 (1992). U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005).
 - The latter enables us to examine **sharp phase transitions** instead of the smooth crossovers observed for a few particles with ED.

S. R. White, Phys. Rev. Lett. **69**, 2863 (1992). U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005).

Harmonically confined Bose-Hubbard model

- But firstly, a reminder on **bosons** in a **harmonically confined optical lattice**.

$$\hat{H} = -t \sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.} \right) + V_{\text{ho}} \sum_i i^2 \hat{n}_i + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1).$$

Tunnelling Harmonic trap Boson-boson repulsion

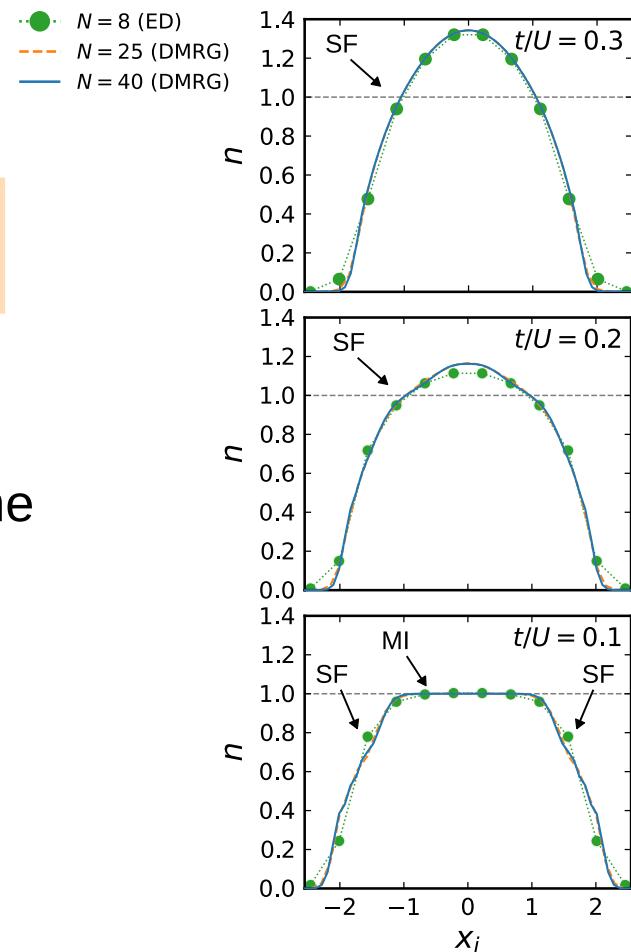
- The system shows **SF and MI domains**.
- For **large enough systems**, the properties become **invariant for equal characteristic density**

G. G. Batrouni *et al.*, Phys. Rev. A **78**, 023627 (2008)

$$\tilde{\rho} = dN/\xi, \quad \xi = d\sqrt{t/V_{\text{ho}}}.$$

- The profiles scale with the **rescaled length**

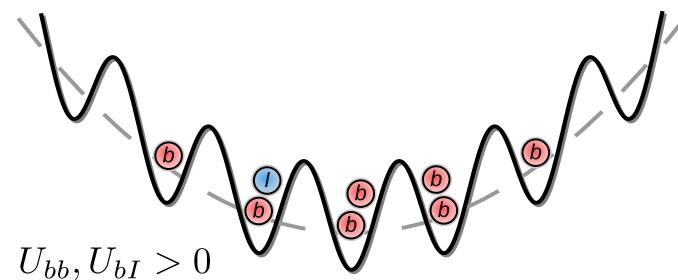
$$x_i = i d/\xi_i.$$



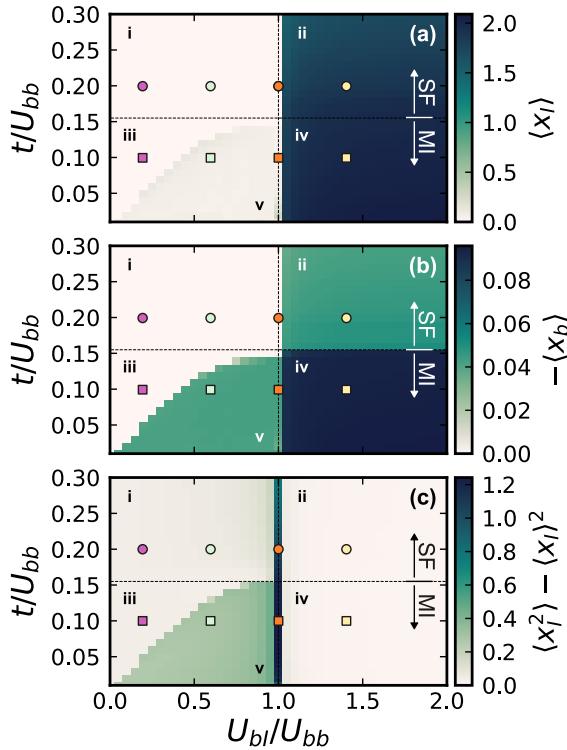
F. Isaule, A. Rojo-Francàs, L. Morales-Molina and B. Juliá-Díaz, Phys. Rev. Lett. **135**, 023404 (2025)

Harmonically confined lattice polarons

- We **considered** a **bath** with a characteristic density that is **completely SF** for $t/U_{bb} > 0.155$ and that has a **MI domain of unity filling** for $t/U_{bb} < 0.155$.
- This choice enables us to nicely **examine the behaviour of the impurity** across the **SF-MI phase transition**.
- We performed **DMRG simulations** for $N_b=40$ bosons. We also obtained **identical results for other choices**, and even a **qualitative agreement** with **ED**.



Harmonically confined lattice polarons: Density profiles



- The system shows **well-defined phases**.

Density profiles:

$$n_\sigma(i) = \langle \hat{n}_{i,\sigma} \rangle$$

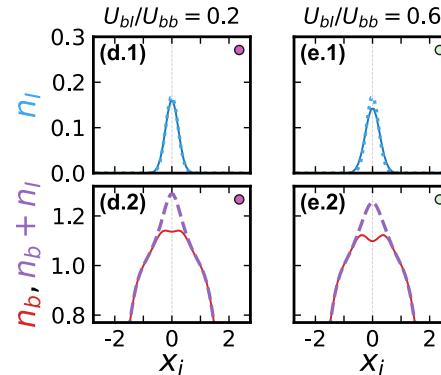
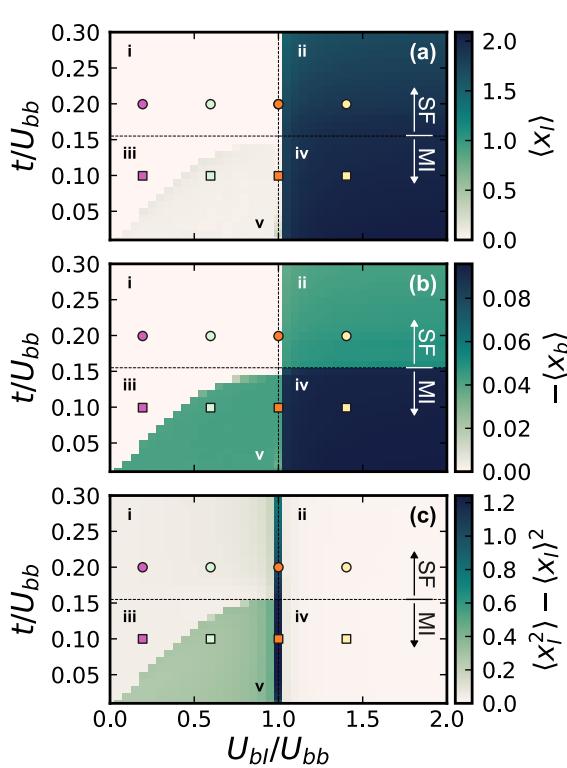
Average position:

$$\langle x_\sigma \rangle = \frac{1}{N_\sigma} \sum_i x_i n_\sigma(i)$$

Impurity cloud:

$$\langle x_I^2 \rangle = \frac{1}{N_\sigma} \sum_i x_i^2 n_I(i)$$

Harmonically confined lattice polarons: Density profiles



Density profiles:

$$n_\sigma(i) = \langle \hat{n}_{i,\sigma} \rangle$$

Average position:

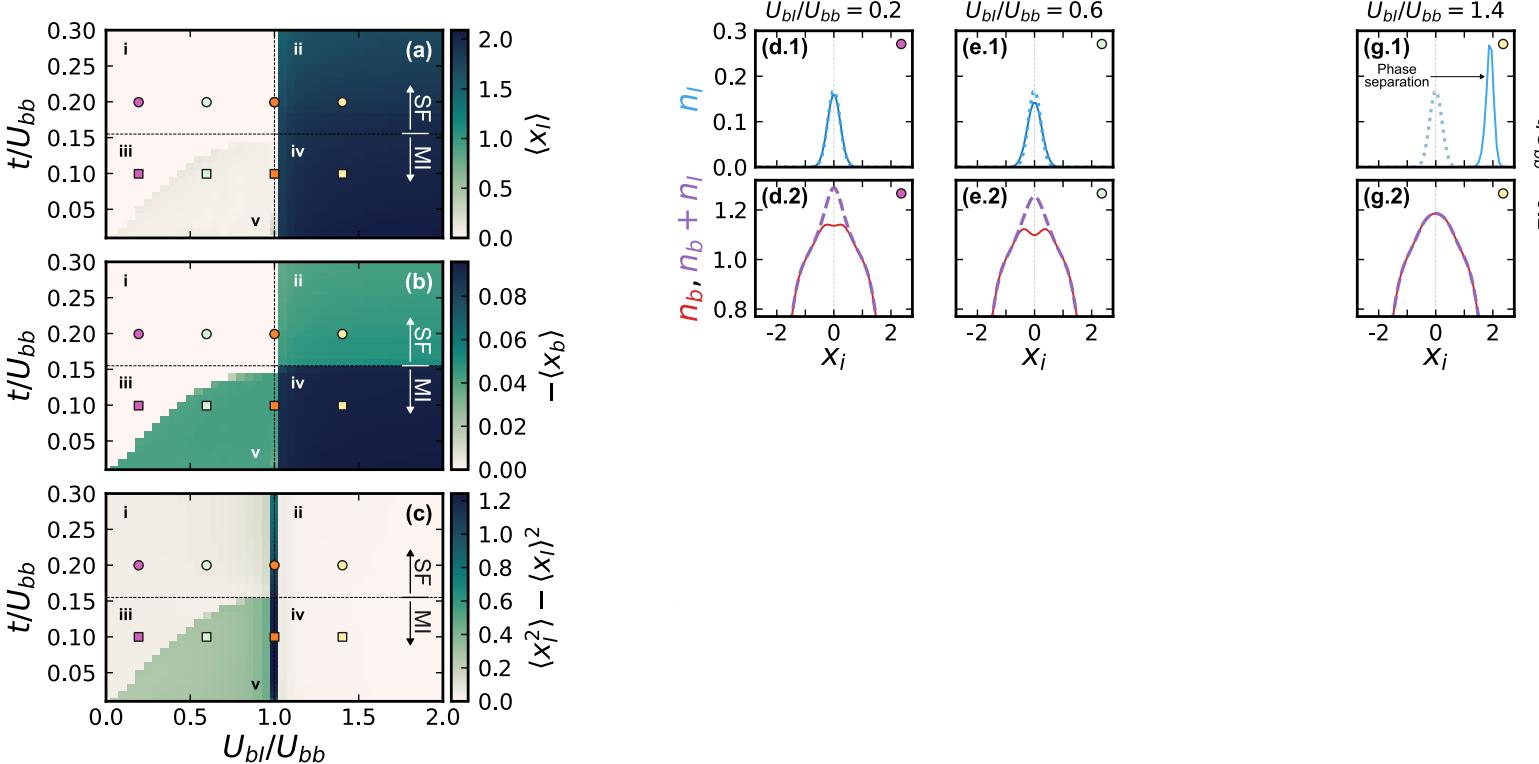
$$\langle x_\sigma \rangle = \frac{1}{N_\sigma} \sum_i x_i n_\sigma(i)$$

Impurity cloud:

$$\langle x_I^2 \rangle = \frac{1}{N_\sigma} \sum_i x_i^2 n_I(i)$$

- Region **i** is a **miscible** phase where the **impurity repels** the **compressible SF bath**.

Harmonically confined lattice polarons: Density profiles



- Region ii is simply a **phase-separated** configuration.

Density profiles:

$$n_\sigma(i) = \langle \hat{n}_{i,\sigma} \rangle$$

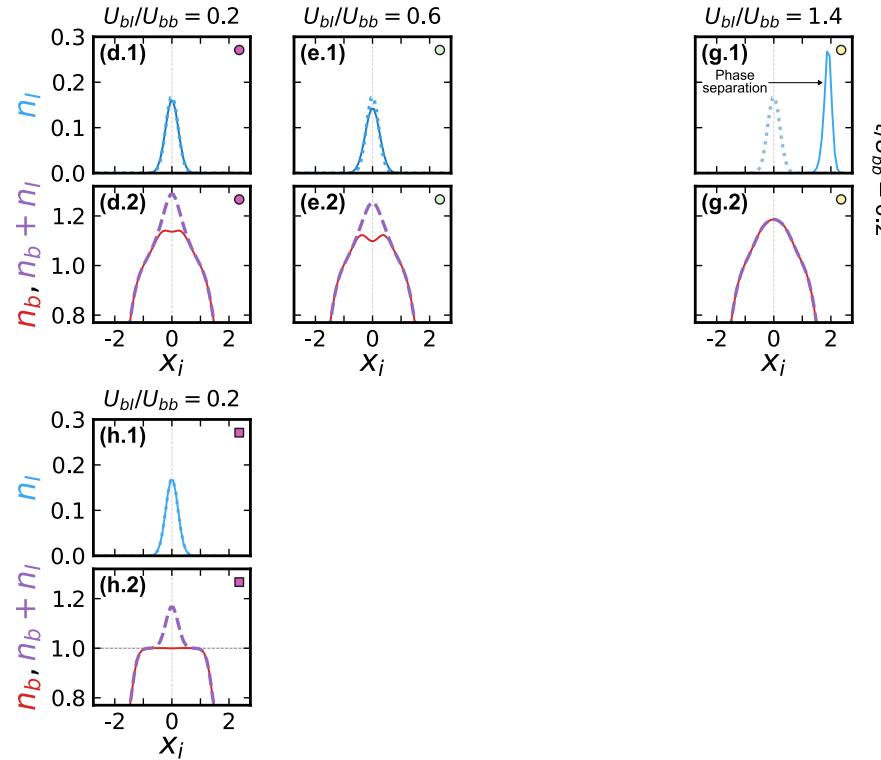
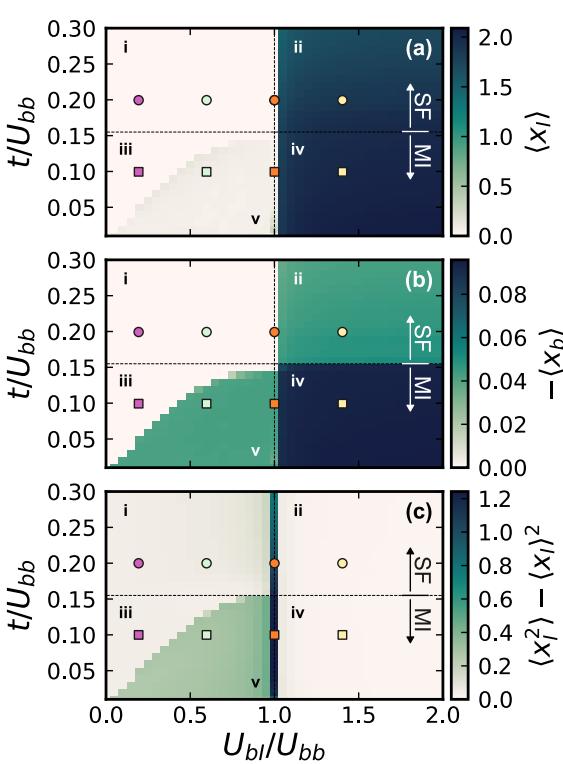
Average position:

$$\langle x_\sigma \rangle = \frac{1}{N_\sigma} \sum_i x_i n_\sigma(i)$$

Impurity cloud:

$$\langle x_I^2 \rangle = \frac{1}{N_\sigma} \sum_i x_i^2 n_I(i)$$

Harmonically confined lattice polarons: Density profiles



Density profiles:

$$n_\sigma(i) = \langle \hat{n}_{i,\sigma} \rangle$$

Average position:

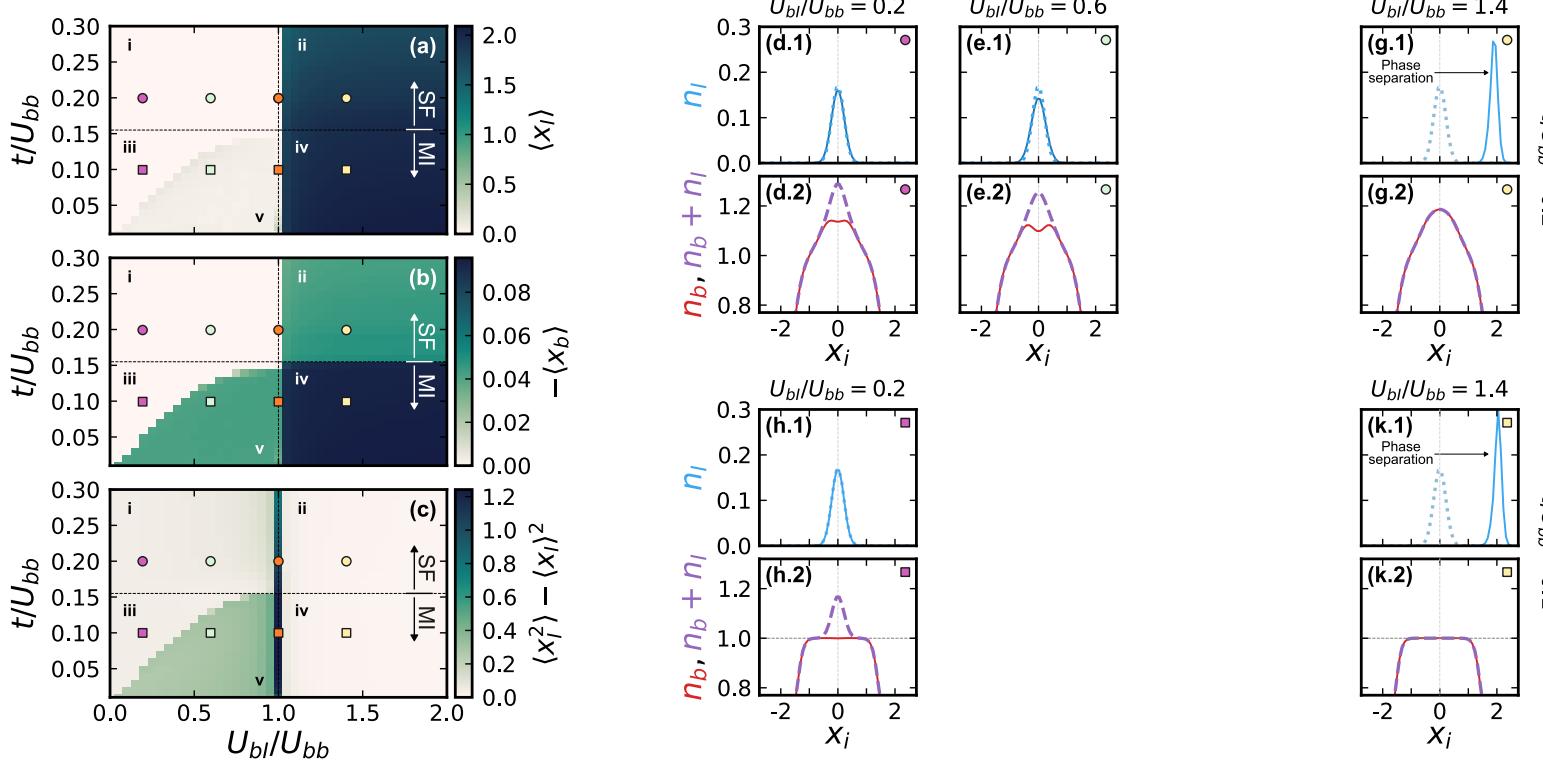
$$\langle x_\sigma \rangle = \frac{1}{N_\sigma} \sum_i x_i n_\sigma(i)$$

Impurity cloud:

$$\langle x_I^2 \rangle = \frac{1}{N_\sigma} \sum_i x_i^2 n_I(i)$$

- Region iii is another **miscible phase**. The **MI bath** remains **undisturbed** due to its **incompressibility**.

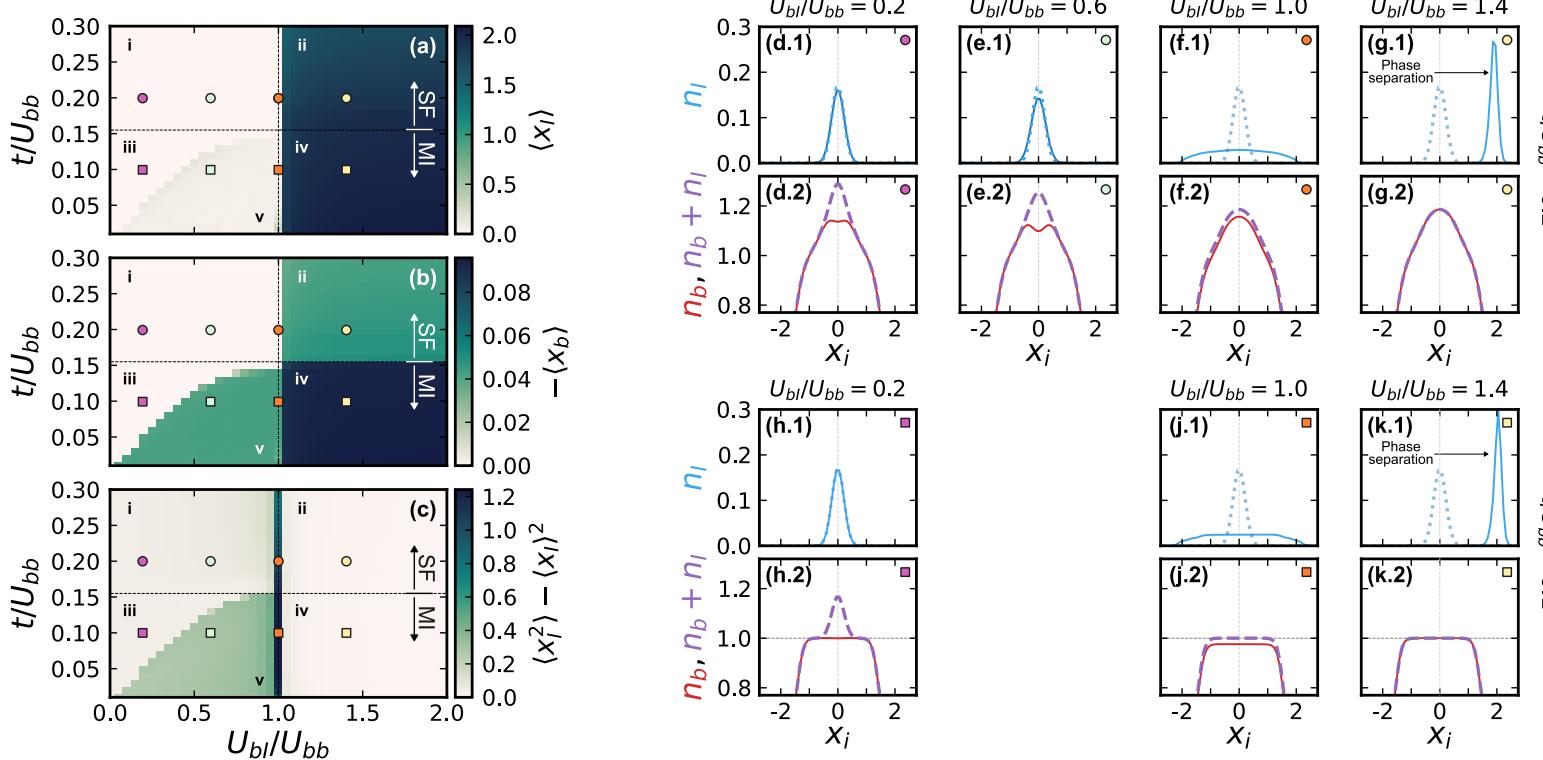
Harmonically confined lattice polarons: Density profiles



- Region **iv** is another **phase-separated** configuration. The MI bath moves further away than an SF bath.

F. Isaule, A. Rojo-Francàs, L. Morales-Molina and B. Juliá-Díaz, Phys. Rev. Lett. **135**, 023404 (2025)

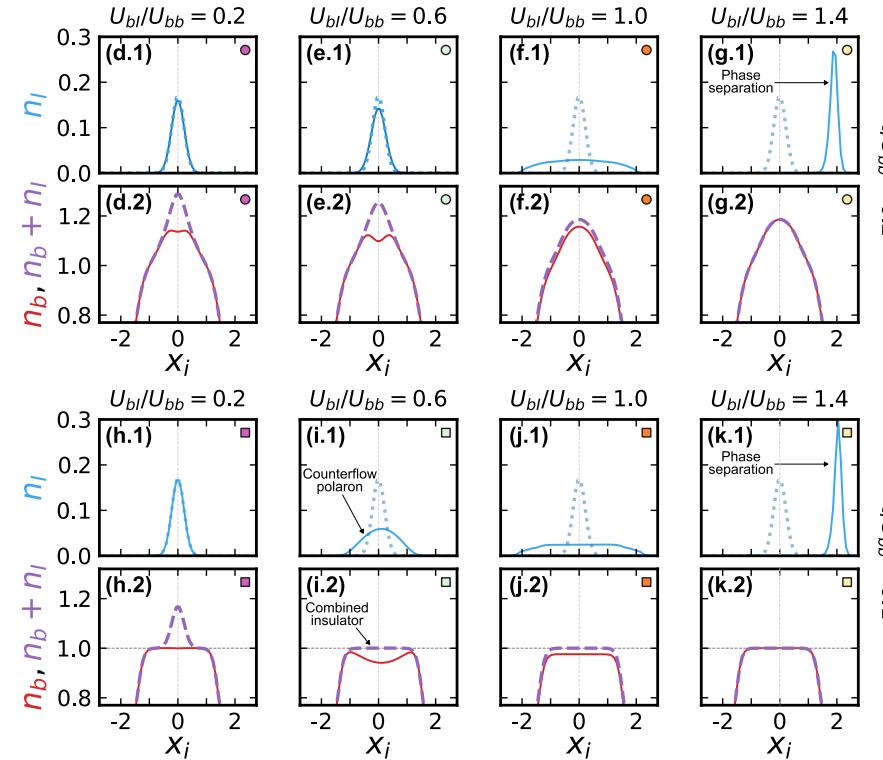
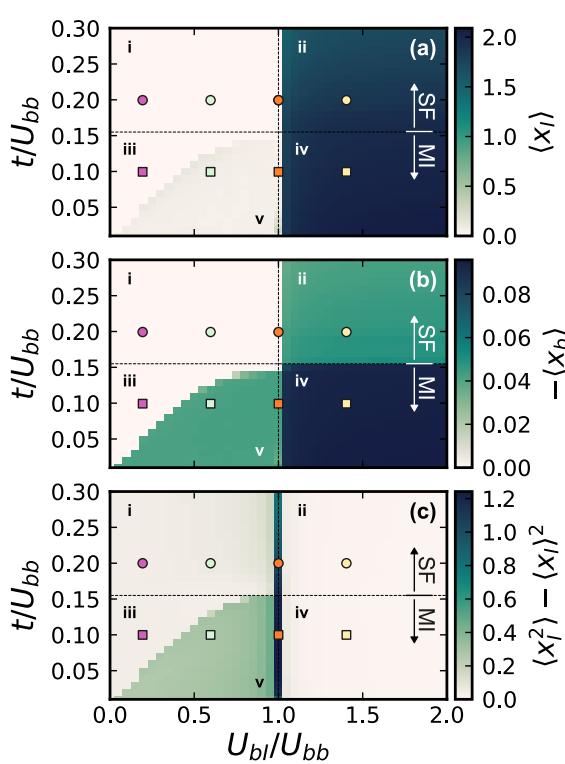
Harmonically confined lattice polarons: Density profiles



- At $U_{bI}=U_{bb}$ the impurity simply behaves as one particle in a **one-component system** with N_b+1 bosons.

F. Isaule, A. Rojo-Francàs, L. Morales-Molina and B. Juliá-Díaz, Phys. Rev. Lett. **135**, 023404 (2025)

Harmonically confined lattice polarons: Density profiles



- Region v is a non-trivial phase where the **impurity** shows an **extended profile** and the **combined profile** of bath and impurity display a domain of **unity filling**.

Harmonically confined lattice polarons: Counterflow

- The new state corresponds to a **counterflow phase**, which **only appears** with **harmonic confinement**.
- It shows **long-range anti-pair order**

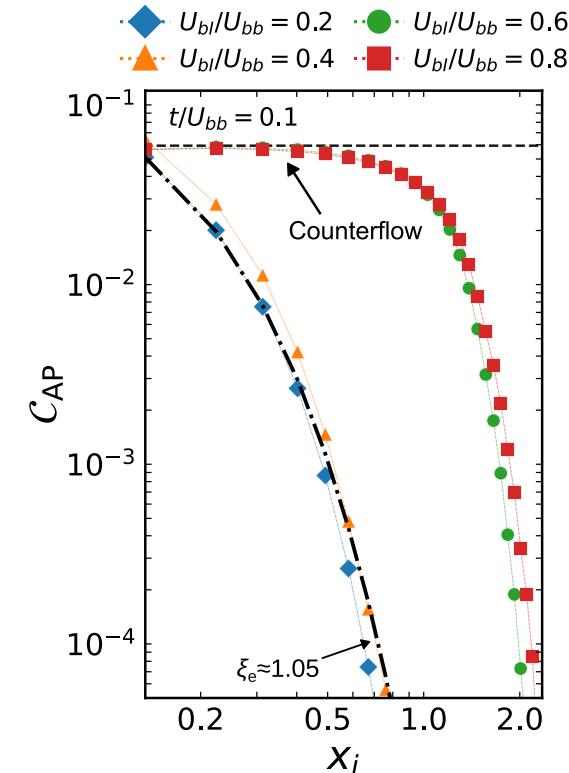
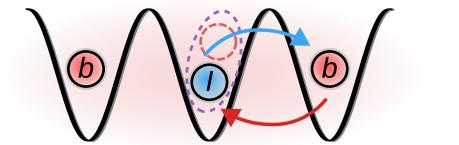
$$\mathcal{C}_{AP} = \langle \hat{a}_{0,I} \hat{a}_{0,b}^\dagger \hat{a}_{i,b}^\dagger \hat{a}_{i,I} \rangle.$$

- **Supercounterflows** have been **studied theoretically** in the past in **two-component bosonic mixtures**.

A. B. Kuklov and B. V. Svistunov, PRL **90**, 100401 (2003). C. Menotti and S. Stringari, PRA **81**, 045604 (2010).

- They were **realised experimentally** very recently with **binary Mott insulators**.

Y.-G. Zheng et al., Nat. Phys. **21**, 208 (2025).

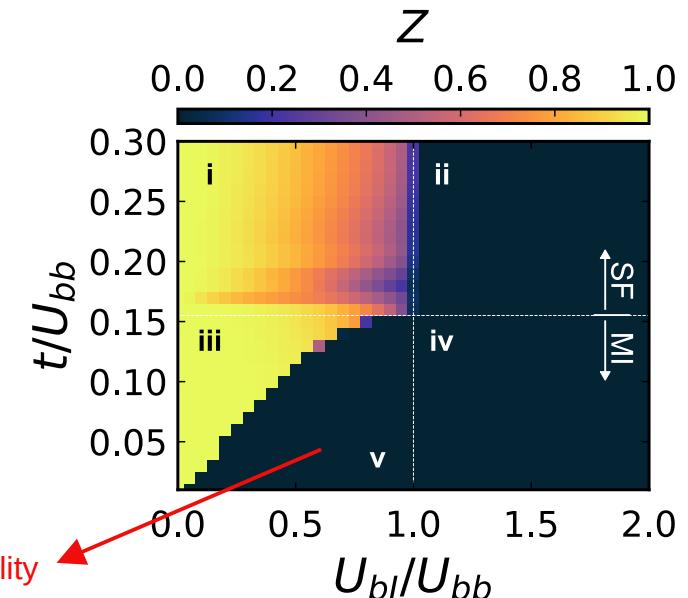


F. Isaule, A. Rojo-Francàs, L. Morales-Molina and B. Juliá-Díaz, Phys. Rev. Lett. **135**, 023404 (2025)

Harmonically confined lattice polarons: Counterflow

- Counterflows are insulators, explaining the combined insulator domain.
- Counterflows require commensurable filling or a harmonic trap.
- We showed that counterflows appear for large population imbalance.
- The impurity forms a correlated state with the whole insulating bath.
- The residue abruptly vanishes at the phase transition.

$$Z(U_{bI}) = |\langle \Psi(U_{bI} = 0) | \Psi(U_{bI}) \rangle|^2.$$



Harmonically confined lattice polarons: Counterflow

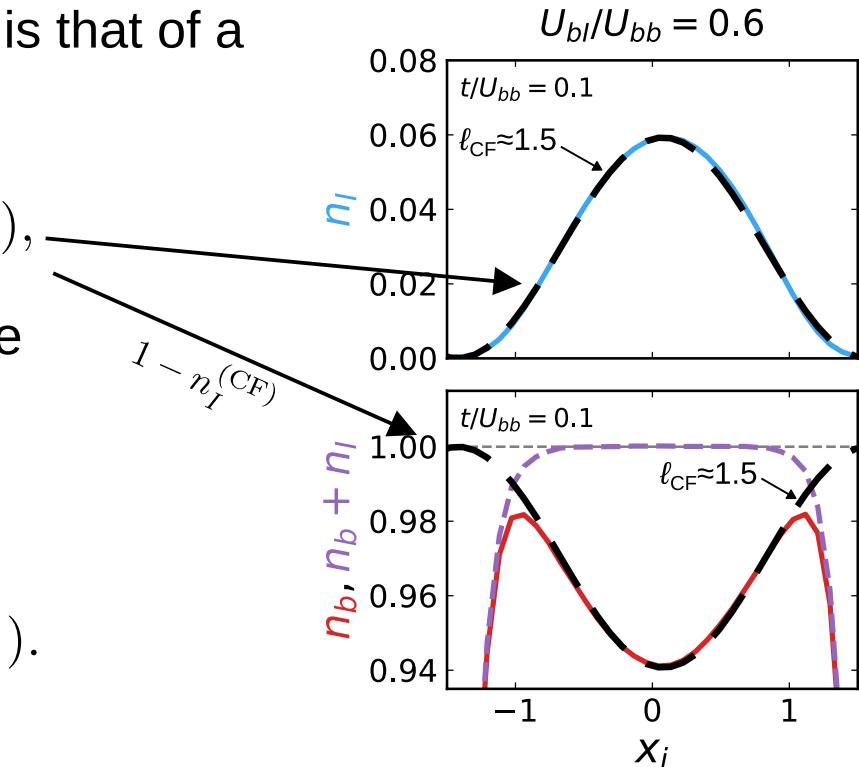
- We have found that the **profile** of the **impurity** is that of a **free particle in a square well**

$$n_I^{(\text{CF})}(x_i) = n_I^{(0)} \cos^2(\pi(x_i - \langle x_I \rangle)/\ell_{\text{CF}}),$$

where only $n_I^{(0)}$ and $\langle x_I \rangle$ are extracted from the calculations.

- The **width** is given by:

$$\sum_i n_I^{(\text{CF})} = 1 \quad \rightarrow \quad \ell_{\text{CF}} = 2/(\xi n_I^{(0)}).$$



Harmonically confined lattice polarons: Counterflow

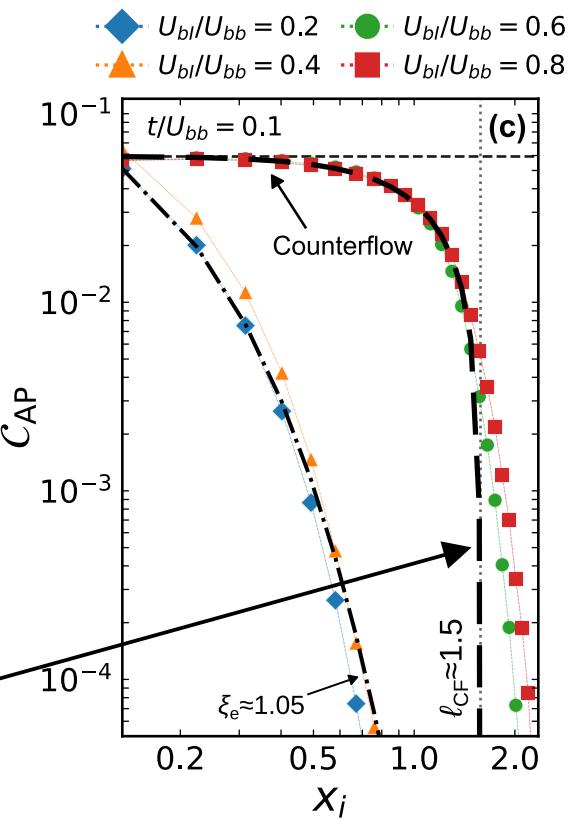
- We have developed a simple **analytical model** to describe the **counterflow** based on **impurity-hole pairs**

$$|\text{MI}\rangle = \prod_i \hat{a}_{i,b}^\dagger |\emptyset\rangle \quad \longrightarrow \quad |\Psi_{\text{CF}}\rangle = \sum_i \alpha_i |i_{\text{IH}}\rangle.$$
$$|i_{\text{IH}}\rangle = \hat{a}_{i,I}^\dagger \hat{a}_{i,b} |\text{MI}\rangle$$

- After some algebra, one gets that $n_I(i) = |\alpha_i|^2$.
- From imposing the **square-well solution**, the **correlator** takes the form

$$C_{(\text{AP})} = \sqrt{n_I^{(0)}} \cos(i\pi(x_i - \langle x_I \rangle)/\ell_{\text{CF}}),$$

which **agrees** with the **numerical solution**.



Harmonically confined lattice polarons: Outlook

- An **impurity** forms a **correlated counterflow state** with an **insulating bath** in **harmonically confined optical lattices**.
- The **impurity** behaves as a **free particle** in a **square well**.
- Ideas for future work in **harmonically confined lattices**:
 - Study **dynamics**.
 - Consider **multiple impurities**.
 - Examine the **attractive branch** (molecule formation?).
 - Study balanced **binary mixtures** (droplets, supercounterflows, etc).

Ultracold polarons and impurities

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Bose polarons

- A **Bose polaron** is the quasiparticle formed by an **impurity** interacting with a **weakly-interacting Bose gas** (BEC).

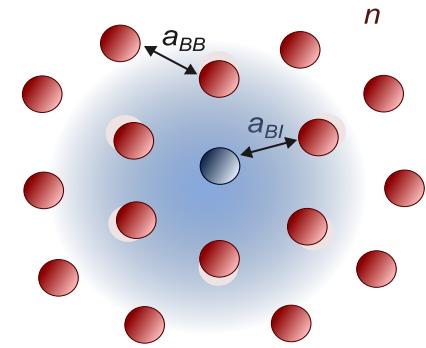
$$\hat{H} = \sum_{k,\sigma=B,I} (\varepsilon_{k,\sigma} - \mu_\sigma) \hat{b}_{k,\sigma}^\dagger \hat{b}_{k,\sigma} + \frac{g_{BB}}{2\mathcal{V}} \sum_{kk'k''} \hat{b}_{k,B}^\dagger \hat{b}_{k',B}^\dagger \hat{b}_{k'-k'',B} \hat{b}_{k'-k'',B}$$

Kinetic energy Chemical potential
Boson-boson repulsion

$$+ \frac{g_{BI}}{\mathcal{V}} \sum_{kk'k''} \hat{b}_{k,B}^\dagger \hat{b}_{k',I}^\dagger \hat{b}_{k'-k'',I} \hat{b}_{k'-k'',B}$$

Bath-impurity interaction

$$\varepsilon_{k,\sigma} = \frac{k^2}{2m_\sigma}$$



- Bose polarons were **achieved experimentally in 2016**.

N. Jørgensen *et al.*, Phys. Rev. Lett. **117**, 055302 (2016), M. Hu *et al.*, Phys. Rev. Lett. **117**, 055301 (2016).

- The branch of **attractive bath-impurity interactions** offers **rich physics**:
 - **Strong coupling regime** (Unitary limit $a_{BI} \rightarrow \infty$), **three- and more-body correlations**, Efimov physics, finite temperatures.

The functional renormalisation group

- **Bose polarons** have been studied with a variety of techniques such as **Quantum MonteCarlo**, variational ansatzë, Frölich model, etc.
- In general, one needs to rely on **non-perturbative methods** to theoretically study the **strongly-interacting regime**.
- One such technique is the **functional renormalisation group (FRG)**.
- The objective of the FRG is to compute an **effective action** Γ (IR) from a microscopic action S (UV).

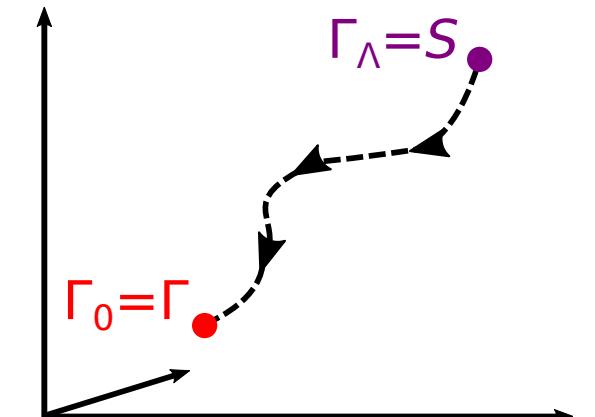
N. Dupuis *et al.*, Phys. Rep. **910**, 1 (2021).

Wetterich equation:

$$\partial_k \Gamma_k = \frac{1}{2} \text{tr} \left[\partial_k R_k (\Gamma^{(2)} + R_k)^{-1} \right]$$

C. Wetterich, Phys. Lett. B **301**, 90 (1993)

Regulator (“cutoff”)



$$\mathcal{Z}[\varphi] = \int D\varphi e^{-S[\varphi]} \rightarrow \Omega_G = -\frac{1}{\beta} \ln \mathcal{Z} = \frac{1}{\beta} \Gamma[\varphi_{cl,0}]$$

The functional renormalisation group

- The FRG is useful to study **strongly correlated systems** and critical phenomena.
- It has been used in different problems in **ultracold atomic** systems:
 - One-component Bose gases
S. Floerchinger and C. Wetterich, Phys. Rev. A **77**, 053603 (2008). Phys. Rev. A **79**, 013601 (2009).
 - Fermi gases: BCS-BEC crossover
S. Floerchinger *et al.*, Phys. Rev. A **81**, 063619 (2010). I. Boettcher *et al.*, Phys. Rev. A **89**, 053630 (2014).
 - Bose gases in optical lattices
A. Rançon and N. Dupuis, Phys. Rev. A **85**, 063607 (2012), Phys. Rev. A **86**, 043624 (2012).
 - Few bosons: Efimov physics
S. Floerchinger *et al.*, Few-Body Syst. **51**, 153 (2011). R. Schmidt and S. Moroz, Phys. Rev. A **81**, 052709 (2010).
 - Fermi polaron
R. Schmidt and T. Enss, Phys. Rev. A **83**, 063620 (2011). von Milczewski *et al.*, Phys. Rev. A **105**, 013317 (2022).

Field-theory description of Bose Polarons

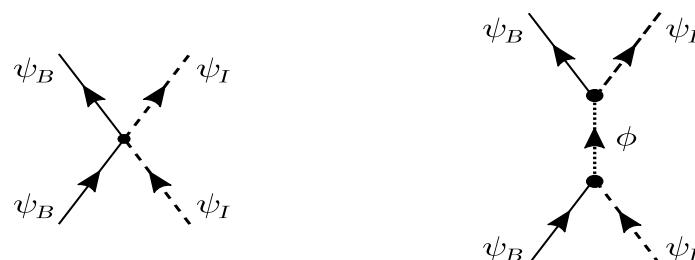
- The microscopic model for the Bose polaron problem reads

S. P. Rath and R. Schmidt, Phys. Rev. A **88**, 053632 (2013).

$$\mathcal{S} = \int_x \left[\sum_{\sigma=B,I} \psi_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_\sigma} - \mu_\sigma \right) \psi_\sigma + \nu_\phi \phi^\dagger \phi + \frac{g_{BB}}{2} (\psi_B^\dagger \psi_B)^2 + h_\Lambda (\phi^\dagger \psi_B \psi_I + \phi \psi_B^\dagger \psi_I^\dagger) \right],$$

Kinetic energy Chemical potential Boson-boson repulsion Bath-impurity interaction

where $\phi \sim \psi_B \psi_I$ are auxiliary dimer fields that mediate the bath-impurity interaction.



Hubbard-Stratonovich transformation

F. Isaule, I. Morera, P. Massignan, and B. Juliá-Díaz, Phys. Rev A. **104**, 023317 (2021).

FRG for Bose polarons

- We employ the following **ansatz** (shortened version):

$$\Gamma_k = \int_x \left[\psi_B^\dagger \left(S_B \partial_\tau - \frac{Z_B}{2m_B} \nabla^2 \right) \psi_B + \psi_I^\dagger \left(S_I \partial_\tau - \frac{Z_I}{2m_I} \nabla^2 + m_I^2 \right) \psi_I \right.$$

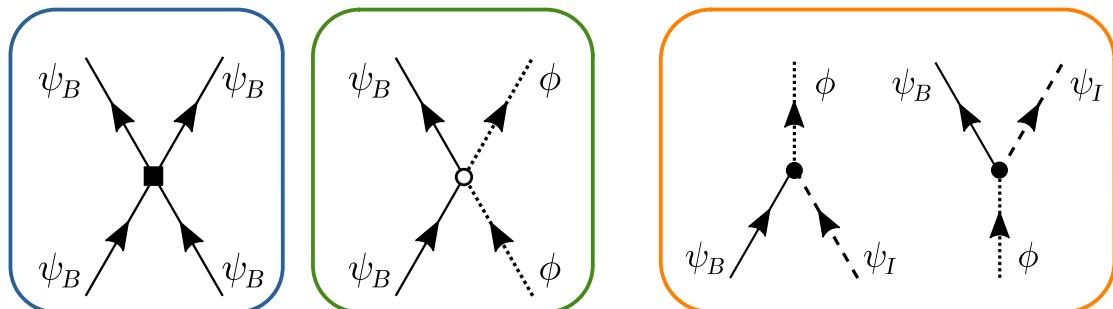
$$+ \phi^\dagger \left(S_\phi \partial_\tau - \frac{Z_\phi}{2m_\phi} \nabla^2 + m_\phi^2 \right) \phi + U_B(|\psi_B|^2) + \lambda_{B\phi} |\psi_B|^2 |\phi|^2 + h \left(\phi^\dagger \psi_B \psi_I + \phi \psi_B^\dagger \psi_I^\dagger \right) \right].$$

Boson-boson
repulsion

Three-body
Bath-impurity
interaction

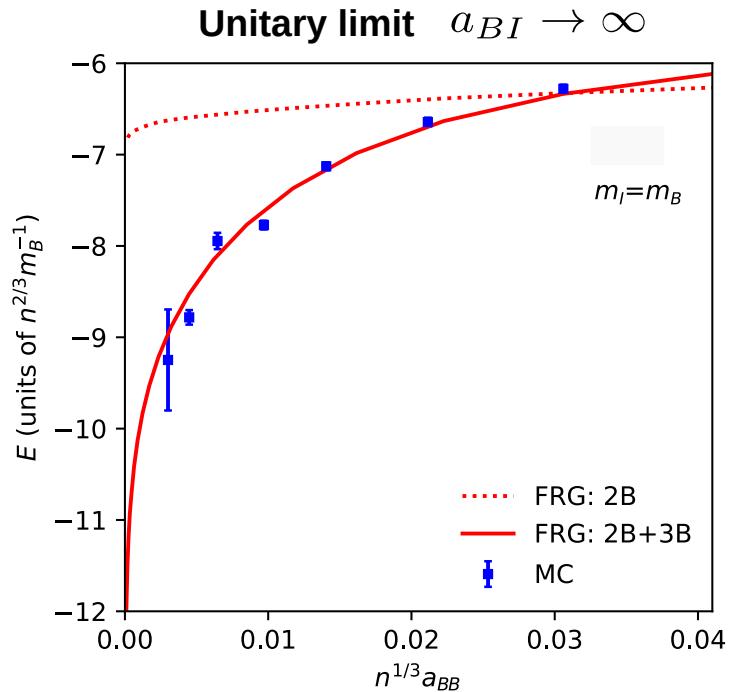
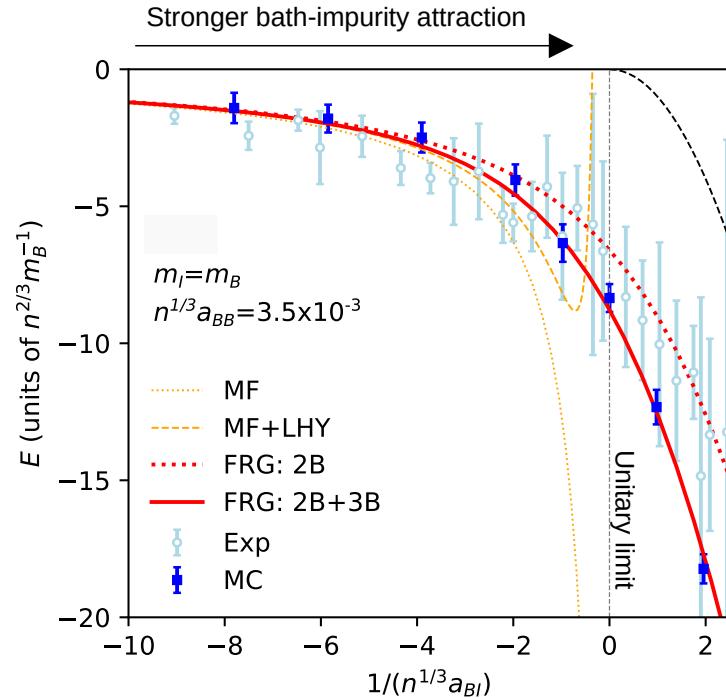
Two-body
Bath-impurity
interaction

- Multi-body correlations** can be added **systematically**.
- We **solved** the RG flow with and without three-body correlations.



F. Isaule, I. Morera, P. Massignan, and B. Juliá-Díaz, Phys. Rev A. **104**, 023317 (2021).

Polaron energy for the three-dimensional Bose Polaron



n : Bath density
 a_{BB} : boson-boson scattering length
 a_{BI} : bath-impurity scattering length

Exp (left): N.B Jørgensen *et al.*, PRL **117**, 055302 (2016).

QMC (left): L. Peña Ardila *et al.*, PRA **99**, 063607 (2019).

QMC (right): L. Peña Ardila and S. Giorgini, PRA **92**, 033612 (2015).

F. Isaule, I. Morera, P. Massignan, and B. Juliá-Díaz, Phys. Rev A. **104**, 023317 (2021).

FRG for Bose polarons

- The **FRG describes** with **high accuracy** the **Bose polaron**, giving a good agreement with Quantum Monte-Carlo.
- It allows us to **quantify** the effect of **three-body correlations** and add them systematically.
- Future work:
 - Polarons across the **BCS-BEC crossover**.
 - Efimov physics in Bose polarons.
 - Bipolarons in BECs.
 - **Bose-Fermi mixtures**.
 - Polarons in **optical lattices**.

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4. Summary and **future work**.

Summary

- We have **numerically** studied **impurities** interacting with **bosonic baths** in **optical lattices** across the **SF-MI transition**.
- We have found the onset of a correlated **counterflow** bath-impurity state in **harmonically confined optical lattices**.
- The **FRG** is a **powerful technique** to study **strongly-interacting polarons**.
- Future work:
 - Further studies of **impurities** and **mixtures** in **harmonically confined optical lattices**.
 - Employ the **FRG** to study **more complicated polaron** configurations.