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EXCELENCIA MARÍA DE MAEZTU

Functional renormalization group for cold atom mixtures

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Cold atom mixtures

- **Mixtures of atoms** of different species or in different internal states have attracted significant attention in recent years.
  - Bose-Fermi mixtures
  - Bose-Bose mixtures
  - SU(N) Fermi gases
- Theoretically, mixtures have started to become well described by a variety of approaches.
- Recent experiments have been able to produce and control cold atom mixtures in different configurations and reproduce novel physics.

Outline

We present our recent work on the study of **cold atom mixtures** with the **Functional renormalization group (FRG)**.

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1. Functional renormalization group
2. Repulsive Bose-Bose mixtures
   F. Isaule, I. Morera, A. Polls and B. Juliá-Díaz, PRA **103**, 013318 (2020).
3. Bose Polarons
   F. Isaule, I. Morera, P. Massignan, B. Juliá-Díaz, arXiv:2105.10801 (2021).
4. Conclusions and future work

Microscopic action

- We work in a **field theory** formulation of the many-body problem. We work in terms of a **microscopic action** *S*.

- <u>Example</u>: a weakly interacting Bose gas

$$\mathcal{S} = \int_x \left[\varphi^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu\right)\varphi + \frac{g}{2}(\varphi^\dagger\varphi)^2\right]$$

g: repulsive contact potential

- It defines the grand-canonical partition function

$$\mathcal{Z}[\varphi] = \int D(\varphi, \varphi^\dagger) e^{-\mathcal{S}[\varphi]} \quad \rightarrow \quad \begin{aligned} \Omega &= -T \ln \mathcal{Z} \\ d\Omega &= -PdV - SdT - Nd\mu \end{aligned}$$

Ω: grand-canonical potential

Effective action

- To obtain *Z* we need to integrate the **different paths**

$$\mathcal{Z}[\varphi] = \int D(\varphi, \varphi^\dagger) e^{-\mathcal{S}[\varphi]}$$

- Approximations: mean-field, Gaussian

L. Salasnich and F. Toigo, Phys. Rep. **640**, 1 (2016)

- An alternative is to work in terms of an **effective action** Γ that **already contains the effect of fluctuations**

$$\Gamma[\phi] = -\ln \mathcal{Z}_J[\phi] + \int_x J \cdot \phi \quad \rightarrow \quad \Omega = -T\,\Gamma[\phi_0]$$

$\phi(x) = \langle \varphi(x) \rangle$

J: source fields

- There are different ways to compute Γ. For example, in a perturbative expansion:

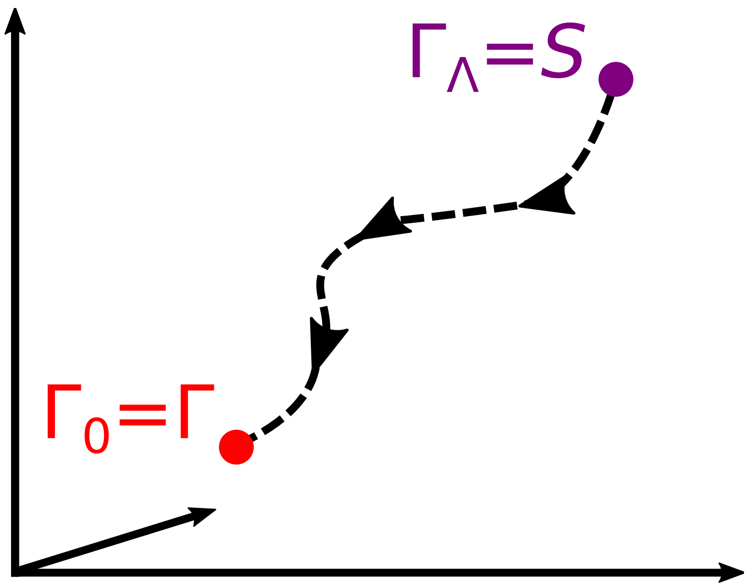
$$\Gamma[\phi] = \mathcal{S}[\phi] + \frac{1}{2}\mathrm{tr}\ln\left(\mathcal{S}^{(2)}[\phi]\right) + \ldots$$

Functional renormalization group (FRG)

- The FRG is a modern **non perturbative** formulation of Wilson's RG.
- A regulator function R_k is added to the theory. It **suppresses all fluctuations for $q<k$**.
- We work in terms of a k-dependent effective action $\boldsymbol{\Gamma_k}$. We follow its flow with a RG equation

$$\boxed{\partial_k \Gamma_k = \frac{1}{2}\mathrm{tr}\left[\partial_k R_k (\Gamma^{(2)} + R_k)^{-1}\right]}$$

C. Wetterich, Phys. Lett. B **301**, 90 (1993)



Functional renormalization group (FRG)

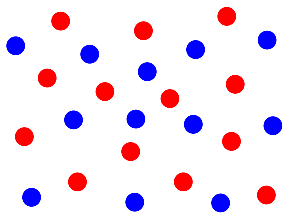
- The FRG is used in a variety of fields: high-energy physics, condensed matter, statistical physics, etc.
- It is particularly useful to study **strongly correlated systems** and critical phenomena.

Recent comprehensive review: N. Dupuis *et al.*, Phys. Rep. **910**, 1 (2021)

- It has been used in different cold atom problems:
  - One-component Bose gases
    S. Floerchinger and C. Wetterich, PRA **77**, 053603 (2008). PRA **79**, 013601 (2009).
  - Fermi gases: BCS-BEC crossover
    S. Floerchinger *et al.*, PRA **81**, 063619 (2010). I. Boettcher *et al.*, PRA **89**, 053630 (2014).
  - Bose gases in optical lattices
    A. Rançon and N. Dupuis, PRA **85**, 063607 (2012), PRA **86**, 043624 (2012).
  - Few bosons: Efimov physics
    S. Floerchinger *et al.*, Few-Body Syst. **51**, 153 (2011). R. Schmidt and S. Moroz, PRA **81**, 052709 (2010).
  - Fermi polaron
    R. Schmidt and T. Enss, PRA **83**, 063620 (2011). von Milczewski *et al.*, arXiv:2104.14017 (2021).

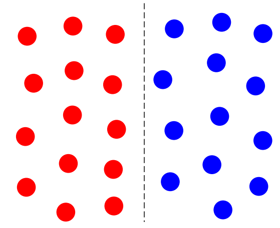
Repulsive Bose-Bose mixtures

Bose-Bose mixtures



- Gases with **two species of bosons** have attracted significant attention in recent years.
- The interplay between the two component of the gas leads to rich physics:
  - Spin drag
  - Phase separation (repulsive interspecies potential)
  - Self-bound droplets (attractive interspecies potential)

    D. Petrov, PRL **115**, 155302 (2015)



- We study **balanced and repulsive Bose-Bose mixtures** with the FRG in **two and three dimensions at zero temperature**.

Bose-Bose mixtures

- A Bose-Bose mixture is described by

$$\mathcal{S} = \int_x \left[\sum_{a=A,B} \psi_a^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_a} - \mu_a \right) \psi_a + \sum_{a,b=A,B} \frac{g_{ab}}{2} |\psi_a|^2 |\psi_b|^2 \right]$$

$$g_{AB} = g_{BA}$$

- Interaction is connected to physical scattering via the *T*-matrix

$$T_{ab} = \begin{cases} \frac{2\pi/m_r}{\ln(-2/m_r|\mu_a+\mu_b|a_{ab}^2)-2\gamma_E} & : d=2 \\ \frac{2\pi a_{ab}}{m_r} & : d=3 \end{cases}$$

a_{ab}: s-wave scattering length
m_r: reduced mass

- We consider the balanced mixture:

$$m = m_A = m_B, \qquad \mu = \mu_A = \mu_B, \qquad g = g_{AA} = g_{BB}$$

$$a = a_{AA} = a_{BB}$$

FRG for repulsive Bose-Bose mixtures

We propose an ansatz for Γ based on a **derivative expansion**:

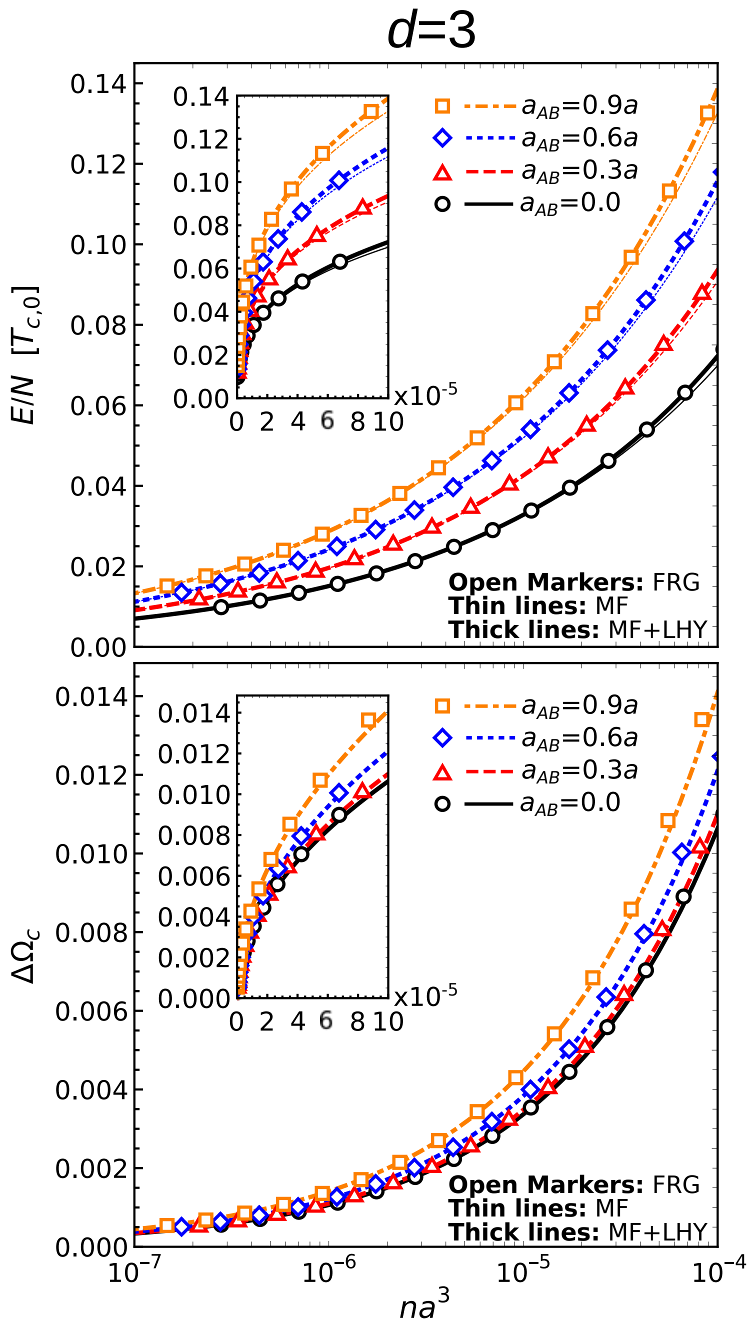
$$\Gamma_k[\phi] = \int_x \left[\sum_{a=A,B} \psi_a^\dagger \left(S\partial_\tau - \frac{Z}{2m}\nabla^2 - V\partial_\tau^2 \right) \psi_a + U(\rho_A, \rho_B) \right]$$

$$U(\rho_A, \rho_B) = -P + \frac{\lambda}{2}(\rho_A - \rho_0)^2 + \frac{\lambda}{2}(\rho_B - \rho_0)^2 + \lambda_{AB}(\rho_A - \rho_0)(\rho_B - \rho_0)$$

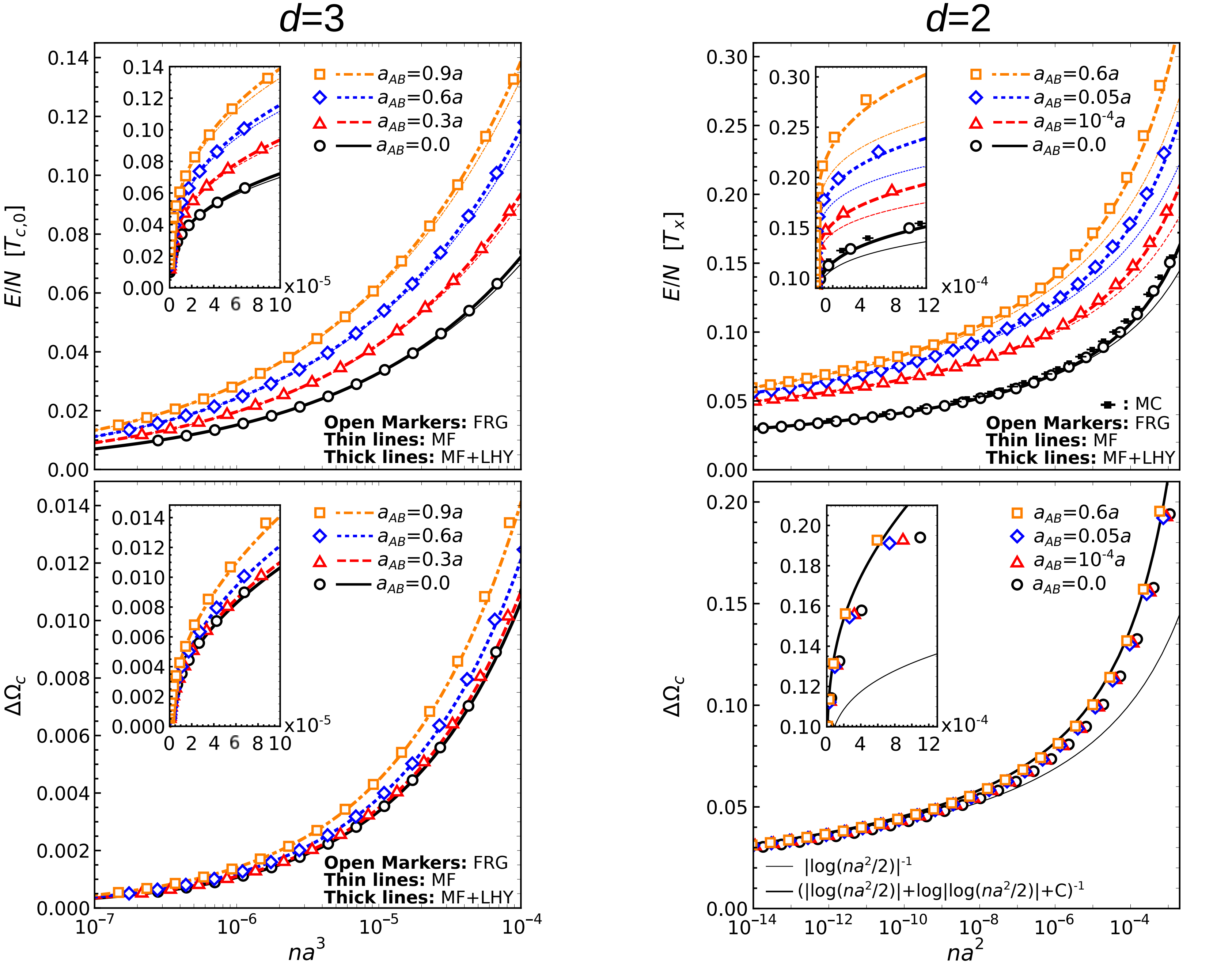
$$\rho_a = \psi_a^\dagger \psi_a$$

- *S, Z, V,* λ, λ_{AB} and ρ_0 flow with *k.*
- $\rho_0 = \langle \rho_A \rangle = \langle \rho_B \rangle$: order parameter.
- <u>Physical inputs:</u> *a*,a_{AB}, μ
- $U(\rho_0)$ gives the grand-canonical potential at *k=0.*

Results: Bose-Bose mixture



Results: Bose-Bose mixture



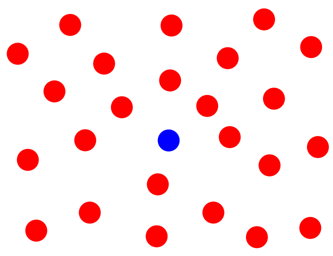
MC: G. Astrakharchik *et al.*, PRA **79**, 051602(R) (2009)

FRG for repulsive Bose-Bose mixtures

- FRG calculations compare favorably with known macroscopic results.
- <u>Future work</u>:
  - Gas at finite temperatures.
  - Imbalanced mixture.
  - Attractive Bose-Bose mixture: liquid phase, dimerization, **strongly-interacting regime**.

Bose Polarons

The Bose polaron



- **Impurity** immersed in a weakly-interacting Bose gas
- Several developments in the past few years:
  - Experimental realization
    N. Jørgensen *et al.*, PRL **117**, 055302 (2016), M. Hu *et al.*, PRL **117**, 055301 (2016)
  - Theoretical description of strong coupling regime
    J. Levinsen et al., PRL **115**, 125302 (2015). N.-E. Guenther *et al.*, PRL **120**, 050405 (2018).
    L. Peña Ardila et al., PRA **99**, 063607 (2019).
- We can study both **repulsive** (positive energies) and **attractive** (negative energies) branches of excitations.
- <u>Attractive branch</u>: **strong-coupling** regime, **three- and more-body correlations**, Efimov physics.
- We study the **repulsive and attractive** branches of Bose polarons in **two and three dimensions at zero temperature**.

The Bose polaron

- We consider a Bose-Bose mixture with **infinite population imbalance**:

$$\mathcal{S} = \int_x \Big[\psi_B^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_B} - \mu_B\right)\psi_B + \psi_I^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_I} - \mu_I\right)\psi_I + \frac{g_{BB}}{2}(\psi_B^\dagger\psi_B)^2 + g_{BI}\psi_B^\dagger\psi_I^\dagger\psi_B\psi_I\Big]$$

μ_B: chemical potential of the medium
μ_I: impurity energy

- We can find the physical **polaron energy** μ_I from the poles of the impurity propagator (spectral function)

$$\det(G_I^{-1}(q=0))\Big|_{\mu_I} = 0$$

Attractive branch

- We can study the regime of strong boson-impurity coupling, including the **unitary limit** in three dimensions

Unitary limit: $a_{BI} \to \infty$

- It is convenient to introduce **dimer fields** via a Hubbard-Stratonovich transformation

$$\phi \sim \psi_B \psi_I$$

- The microscopic action takes the form

$$\mathcal{S} = \int_x \Big[\psi_B^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_B} - \mu_B \right) \psi_B + \psi_I^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_I} - \mu_I \right) \psi_I + \nu_\phi \phi^\dagger \phi + \frac{g_{BB}}{2} (\psi_B^\dagger \psi_B)^2 + h_\Lambda \left(\phi^\dagger \psi_B \psi_I + \phi \psi_B^\dagger \psi_I^\dagger \right) \Big]$$

S. P. Rath and R. Schmidt, PRA **88**, 053632 (2013)

Attractive branch

We propose the following ansatz

$$\Gamma_k = \int_x \Bigg[\psi_B^\dagger \left(S_B\partial_\tau - \frac{Z_B}{2m_B}\nabla^2 - V_B\partial_\tau^2\right)\psi_B + \psi_I^\dagger \left(S_I\partial_\tau - \frac{Z_I}{2m_I}\nabla^2 + U_I(\rho_B)\right)\psi_I$$

$$+\phi^\dagger \left(S_\phi\partial_\tau - \frac{Z_\phi}{2m_\phi}\nabla^2 + U_\phi(\rho_B)\right)\phi + U_B(\rho_B) + H_\phi(\rho_B)\left(\phi^\dagger\psi_B\psi_I + \phi\psi_B^\dagger\psi_I^\dagger\right)\Bigg]$$

where

$$\rho_a = \psi_a^\dagger\psi_a$$

$$U_B(\rho_B) = -P + \frac{\lambda_{BB}}{2}(\rho_B - \rho_0)^2$$

$$U_I(\rho_B) = u_I + \lambda_{BI}(\rho_B - \rho_0) + \frac{\lambda_{BBI}}{2}(\rho_B - \rho_0)^2$$

$$U_\phi(\rho_B) = u_\phi + \lambda_{B\phi}(\rho_B - \rho_0)$$

$$H_\phi(\rho_B) = h_\phi + h_{B\phi}(\rho_B - \rho_0)$$

$$\langle\rho_B\rangle = \rho_0$$

$$\langle\rho_I\rangle = 0$$

$$\langle\rho_\phi\rangle = 0$$

Attractive branch

We propose the following ansatz

$$\Gamma_k = \int_x \Big[\psi_B^\dagger \left(S_B\partial_\tau - \frac{Z_B}{2m_B}\nabla^2 - V_B\partial_\tau^2\right)\psi_B + \psi_I^\dagger\left(S_I\partial_\tau - \frac{Z_I}{2m_I}\nabla^2 + U_I(\rho_B)\right)\psi_I$$

$$+\phi^\dagger\left(S_\phi\partial_\tau - \frac{Z_\phi}{2m_\phi}\nabla^2 + U_\phi(\rho_B)\right)\phi + U_B(\rho_B) + H_\phi(\rho_B)\left(\phi^\dagger\psi_B\psi_I + \phi\psi_B^\dagger\psi_I^\dagger\right)\Big]$$

where

Boson-boson interaction

$$\rho_a = \psi_a^\dagger\psi_a$$

$$U_B(\rho_B) = -P + \frac{\lambda_{BB}}{2}(\rho_B - \rho_0)^2$$

$$U_I(\rho_B) = u_I + \lambda_{BI}(\rho_B - \rho_0) + \frac{\lambda_{BBI}}{2}(\rho_B - \rho_0)^2$$

$$U_\phi(\rho_B) = u_\phi + \lambda_{B\phi}(\rho_B - \rho_0)$$

$$H_\phi(\rho_B) = h_\phi + h_{B\phi}(\rho_B - \rho_0)$$

$$\langle\rho_B\rangle = \rho_0$$

$$\langle\rho_I\rangle = 0$$

$$\langle\rho_\phi\rangle = 0$$

Attractive branch

We propose the following ansatz

$$\Gamma_k = \int_x \Bigg[\psi_B^\dagger \left(S_B\partial_\tau - \frac{Z_B}{2m_B}\nabla^2 - V_B\partial_\tau^2\right)\psi_B + \psi_I^\dagger\left(S_I\partial_\tau - \frac{Z_I}{2m_I}\nabla^2 + U_I(\rho_B)\right)\psi_I$$

$$+\phi^\dagger\left(S_\phi\partial_\tau - \frac{Z_\phi}{2m_\phi}\nabla^2 + U_\phi(\rho_B)\right)\phi + U_B(\rho_B) + H_\phi(\rho_B)\left(\phi^\dagger\psi_B\psi_I + \phi\psi_B^\dagger\psi_I^\dagger\right)\Bigg]$$

where

Boson-boson interaction

$$\rho_a = \psi_a^\dagger\psi_a$$

$$U_B(\rho_B) = -P + \frac{\lambda_{BB}}{2}(\rho_B-\rho_0)^2$$

$$\langle\rho_B\rangle = \rho_0$$

$$U_I(\rho_B) = u_I + \lambda_{BI}(\rho_B-\rho_0) + \frac{\lambda_{BBI}}{2}(\rho_B-\rho_0)^2$$

$$\langle\rho_I\rangle = 0$$

Boson-impurity interaction

$$U_\phi(\rho_B) = u_\phi + \lambda_{B\phi}(\rho_B-\rho_0)$$

$$\langle\rho_\phi\rangle = 0$$

$$H_\phi(\rho_B) = h_\phi + h_{B\phi}(\rho_B-\rho_0)$$

Attractive branch

We propose the following ansatz

$$\Gamma_k = \int_x \Bigg[\psi_B^\dagger \left(S_B\partial_\tau - \frac{Z_B}{2m_B}\nabla^2 - V_B\partial_\tau^2\right)\psi_B + \psi_I^\dagger\left(S_I\partial_\tau - \frac{Z_I}{2m_I}\nabla^2 + U_I(\rho_B)\right)\psi_I$$

$$+\phi^\dagger\left(S_\phi\partial_\tau - \frac{Z_\phi}{2m_\phi}\nabla^2 + U_\phi(\rho_B)\right)\phi + U_B(\rho_B) + H_\phi(\rho_B)\left(\phi^\dagger\psi_B\psi_I + \phi\psi_B^\dagger\psi_I^\dagger\right)\Bigg]$$

where

Boson-boson interaction

$$\rho_a = \psi_a^\dagger\psi_a$$

$$U_B(\rho_B) = -P + \frac{\lambda_{BB}}{2}(\rho_B-\rho_0)^2$$

$$\langle\rho_B\rangle = \rho_0$$

$$U_I(\rho_B) = u_I + \lambda_{BI}(\rho_B-\rho_0) + \frac{\lambda_{BBI}}{2}(\rho_B-\rho_0)^2$$

$$\langle\rho_I\rangle = 0$$

Boson-impurity interaction

$$U_\phi(\rho_B) = u_\phi + \lambda_{B\phi}(\rho_B-\rho_0)$$

$$\langle\rho_\phi\rangle = 0$$

$$H_\phi(\rho_B) = h_\phi + h_{B\phi}(\rho_B-\rho_0)$$

Three-body coupling

Attractive branch

We propose the following ansatz

$$\Gamma_k = \int_x \Big[\psi_B^\dagger \left(S_B\partial_\tau - \frac{Z_B}{2m_B}\nabla^2 - V_B\partial_\tau^2\right)\psi_B + \psi_I^\dagger\left(S_I\partial_\tau - \frac{Z_I}{2m_I}\nabla^2 + U_I(\rho_B)\right)\psi_I$$

$$+\phi^\dagger\left(S_\phi\partial_\tau - \frac{Z_\phi}{2m_\phi}\nabla^2 + U_\phi(\rho_B)\right)\phi + U_B(\rho_B) + H_\phi(\rho_B)\left(\phi^\dagger\psi_B\psi_I + \phi\psi_B^\dagger\psi_I^\dagger\right)\Big]$$

$$\rho_a = \psi_a^\dagger\psi_a$$

where

$$U_B(\rho_B) = -P + \frac{\lambda_{BB}}{2}(\rho_B - \rho_0)^2$$

$$U_I(\rho_B) = u_I + \lambda_{BI}(\rho_B - \rho_0) + \frac{\lambda_{BBI}}{2}(\rho_B - \rho_0)^2$$

$$U_\phi(\rho_B) = u_\phi + \lambda_{B\phi}(\rho_B - \rho_0)$$

$$H_\phi(\rho_B) = h_\phi + h_{B\phi}(\rho_B - \rho_0)$$

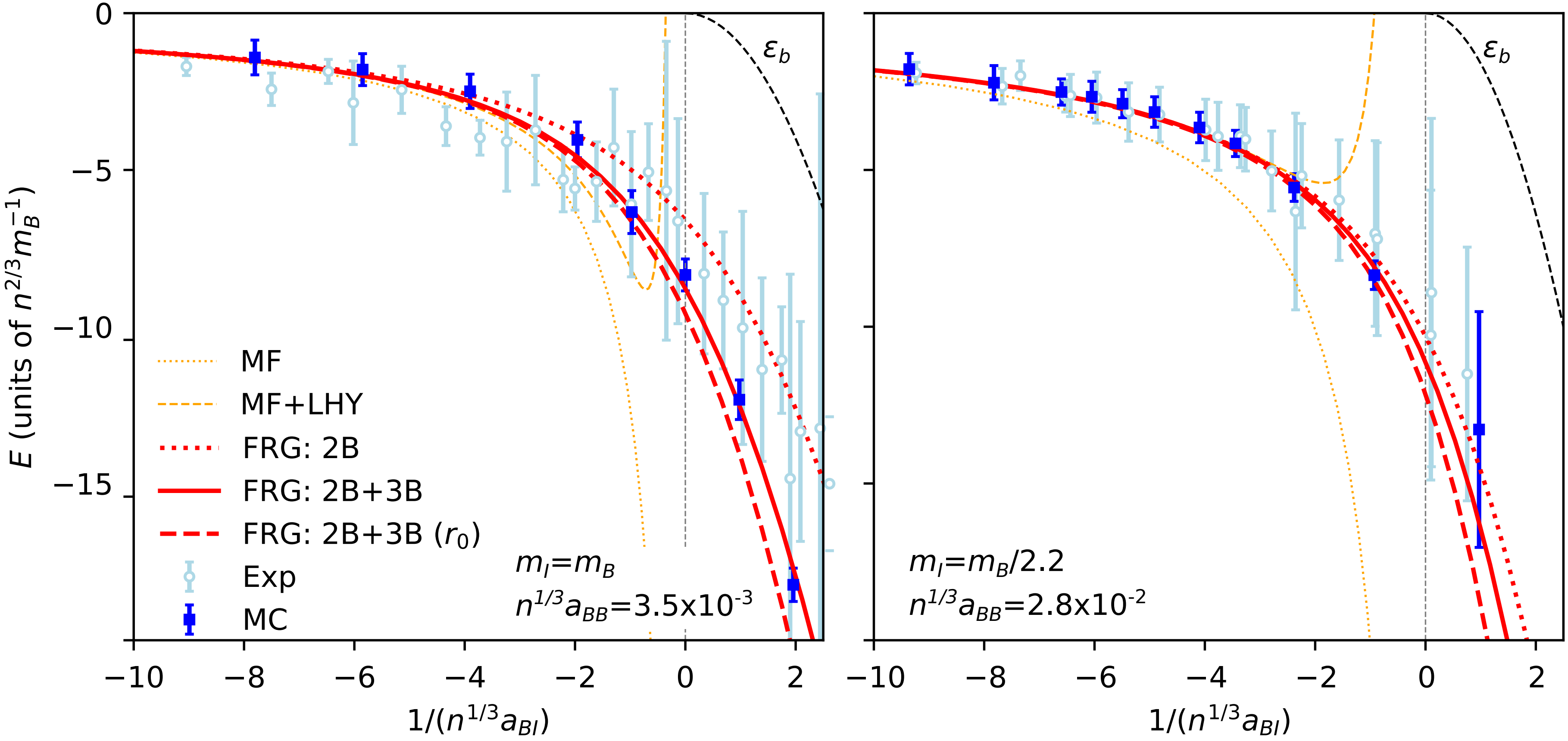
$$\langle\rho_B\rangle = \rho_0$$

$$\langle\rho_I\rangle = 0$$

$$\langle\rho_\phi\rangle = 0$$

<u>Inputs:</u> a_{BB}, a_{BI}, μ_B, μ_I, r_0 (effective range)

Results: Attractive branch (d=3)

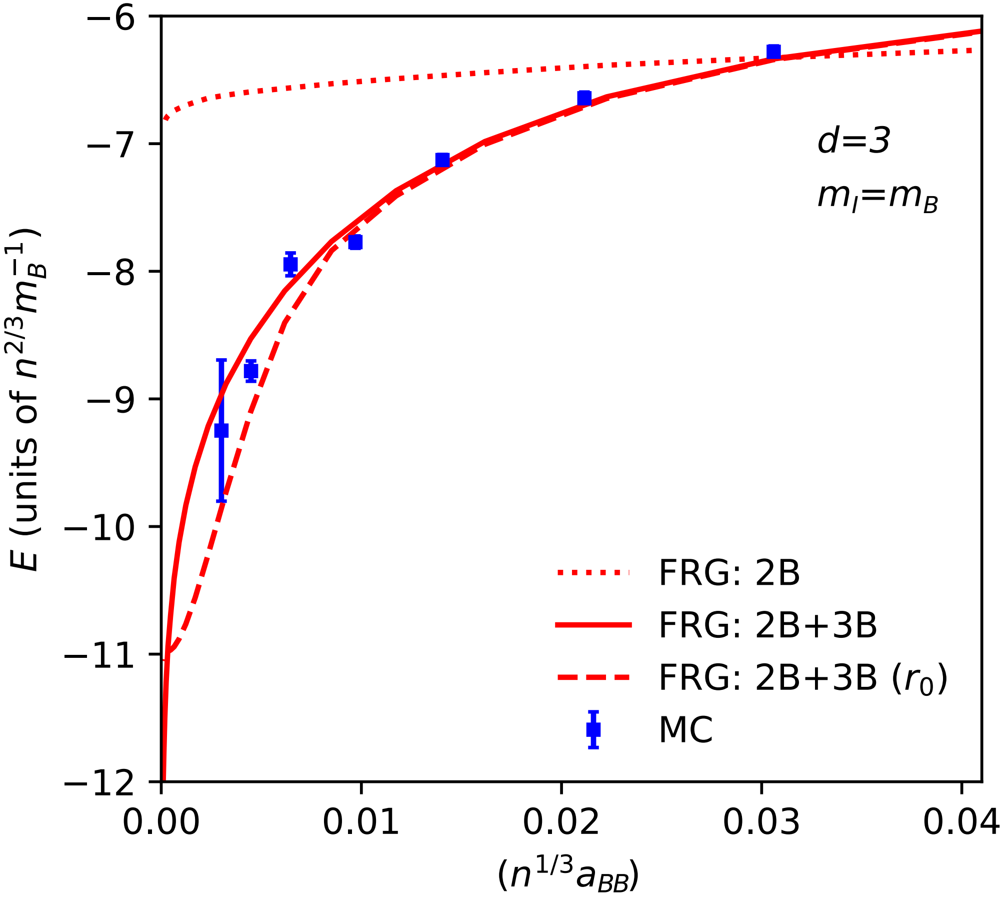


Exp: N.B Jørgensen *et al.*, PRL **117**, 055302 (2016)
M.-G. Hu *et al.*, PRL **117**, 055301 (2016)
MC: L. Peña Ardila *et al.*, PRA **99**, 063607 (2019)

Results: Attractive branch (d=3)

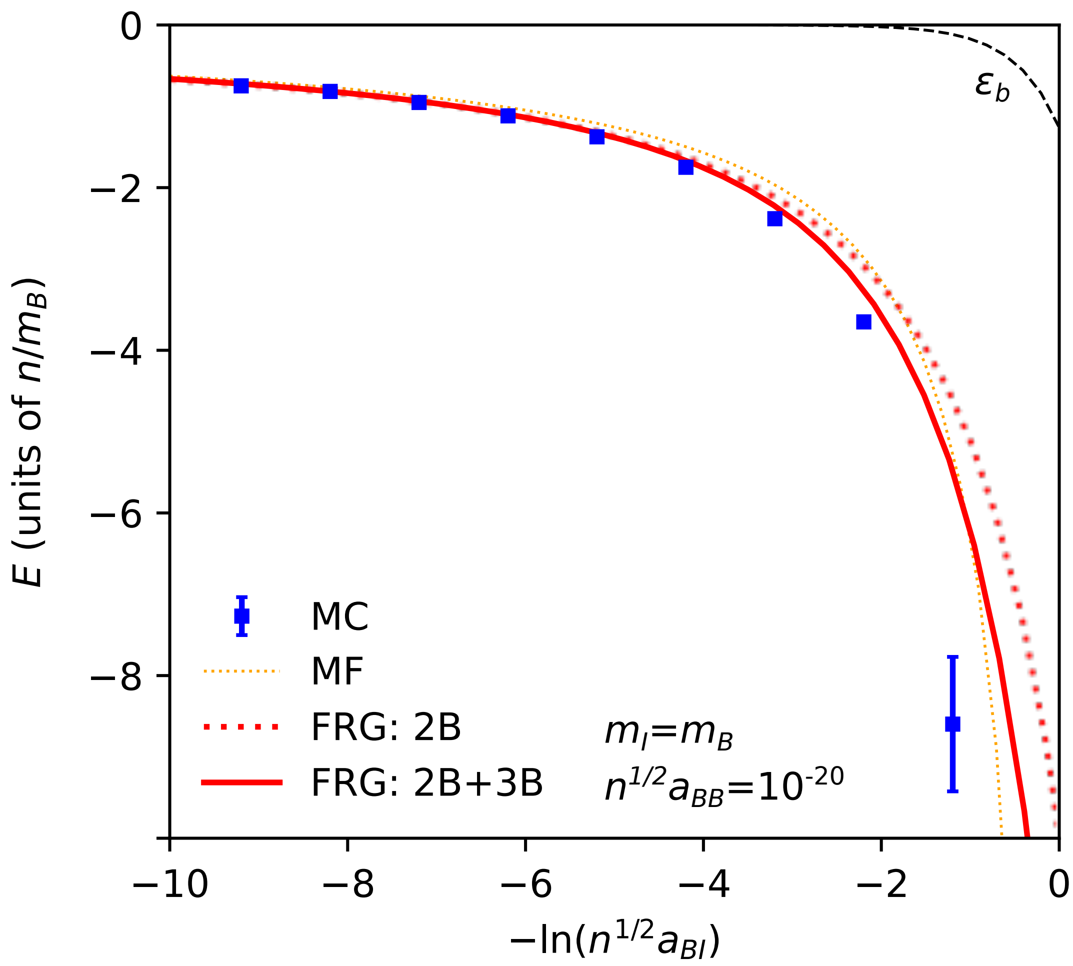
Unitary limit:

$$a_{BI} \to \infty$$



MC: L. Peña Ardila and S. Giorgini, PRA **92**, 033612 (2015)

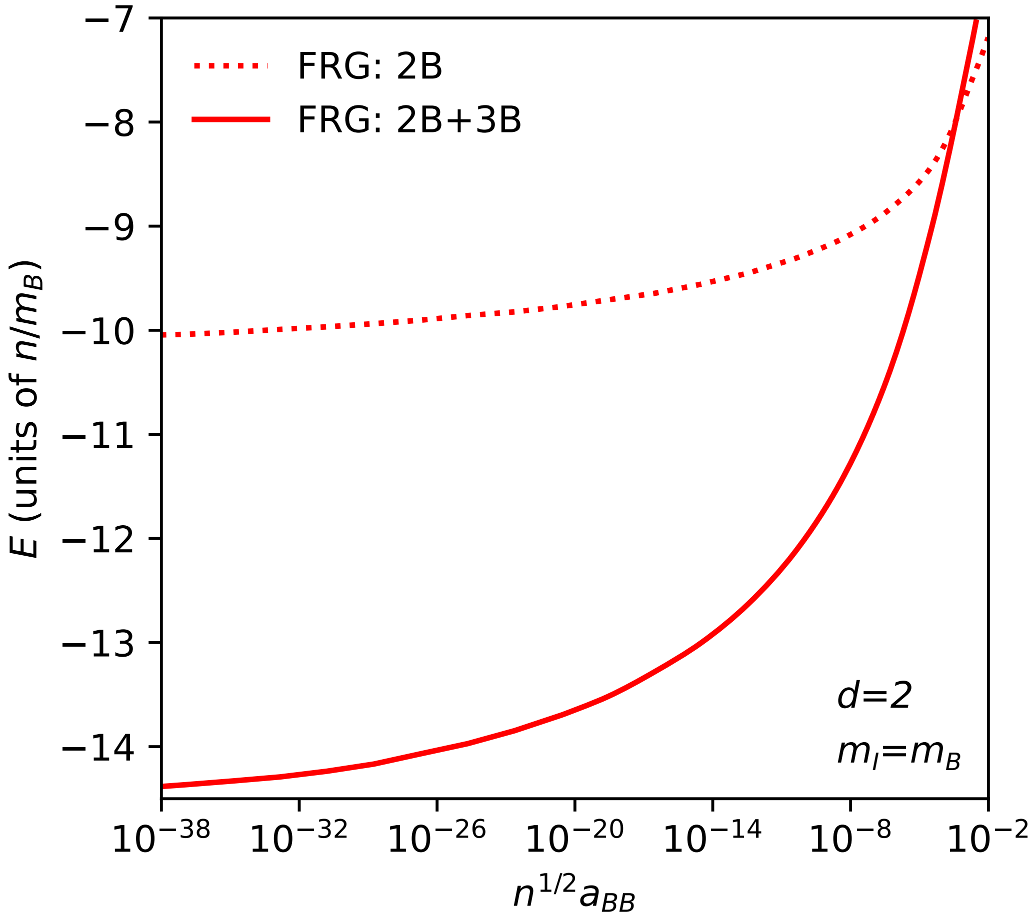
Results: Attractive branch (d=2)



MC: L. Peña Ardila *et al.*, PRR **2**, 023405 (2020)

Results: Attractive branch (d=2)

$$\ln(n^{1/2}a_{BI}) = 0$$



FRG for Bose Polaron

- FRG can describe the **strong-coupling regime** with ease
- The effect of **three and more-body correlations** is conceptually easy to include.
- <u>Future work</u>:
  - Finite temperature
  - Momentum-dependent vertices

Conclusions

- The FRG can provide a successful description of bosonic mixtures.
- Macroscopic properties of **repulsive Bose-Bose mixtures** are well described.
- It gives a good description of the ground state properties of the **Bose polaron** within a derivative expansion. Strong coupling regime is reasonably well described.
- Future work:
  - Bose-Fermi mixtures
  - SU(N) Fermi gases
  - Multiple impurities

Bose Polaron: Repulsive branch

- We employ an ansatz based on that used for repulsive Bose-Bose mixtures

$$\Gamma_k = \int_x \left[\psi_B^\dagger \left(S_B \partial_\tau - \frac{Z_B}{2m_B} \nabla^2 - V_B \partial_\tau^2 \right) \psi_B + \psi_I^\dagger \left(S_I \partial_\tau - \frac{Z_I}{2m_I} \nabla^2 \right) \psi_I + U(\rho_B, \rho_I) \right]$$

$$U(\rho_B, \rho_I) = -P + u_I \rho_I + \frac{\lambda_{BB}}{2} (\rho_B - \rho_0)^2 + \frac{\lambda_{BI}}{2} \rho_I^2$$

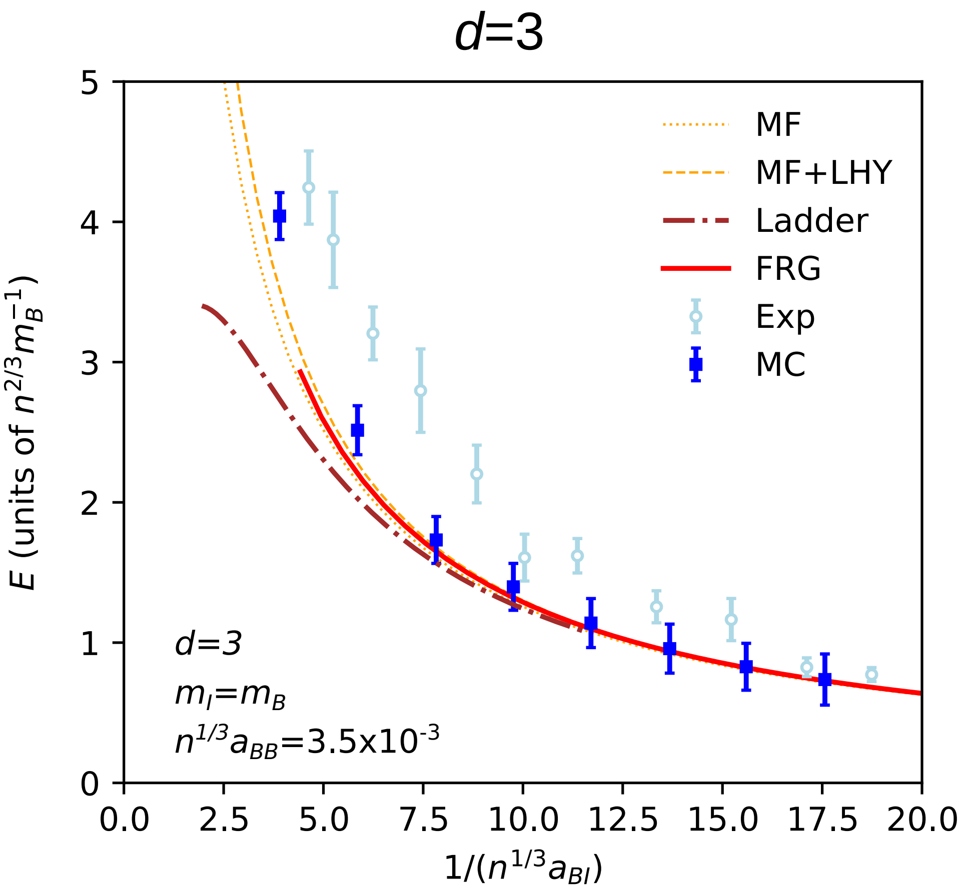
$$\rho_a = \psi_a^\dagger \psi_a$$

- S_B, Z_B, V_B, S_I, Z_I, u_I, λ_{BB}, λ_{BI}, and ρ_0 flow with k.
- The inputs of the calculations are a_{BB}, a_{BI}, μ_B, μ_I.

$$\langle \rho_B \rangle = \rho_0$$

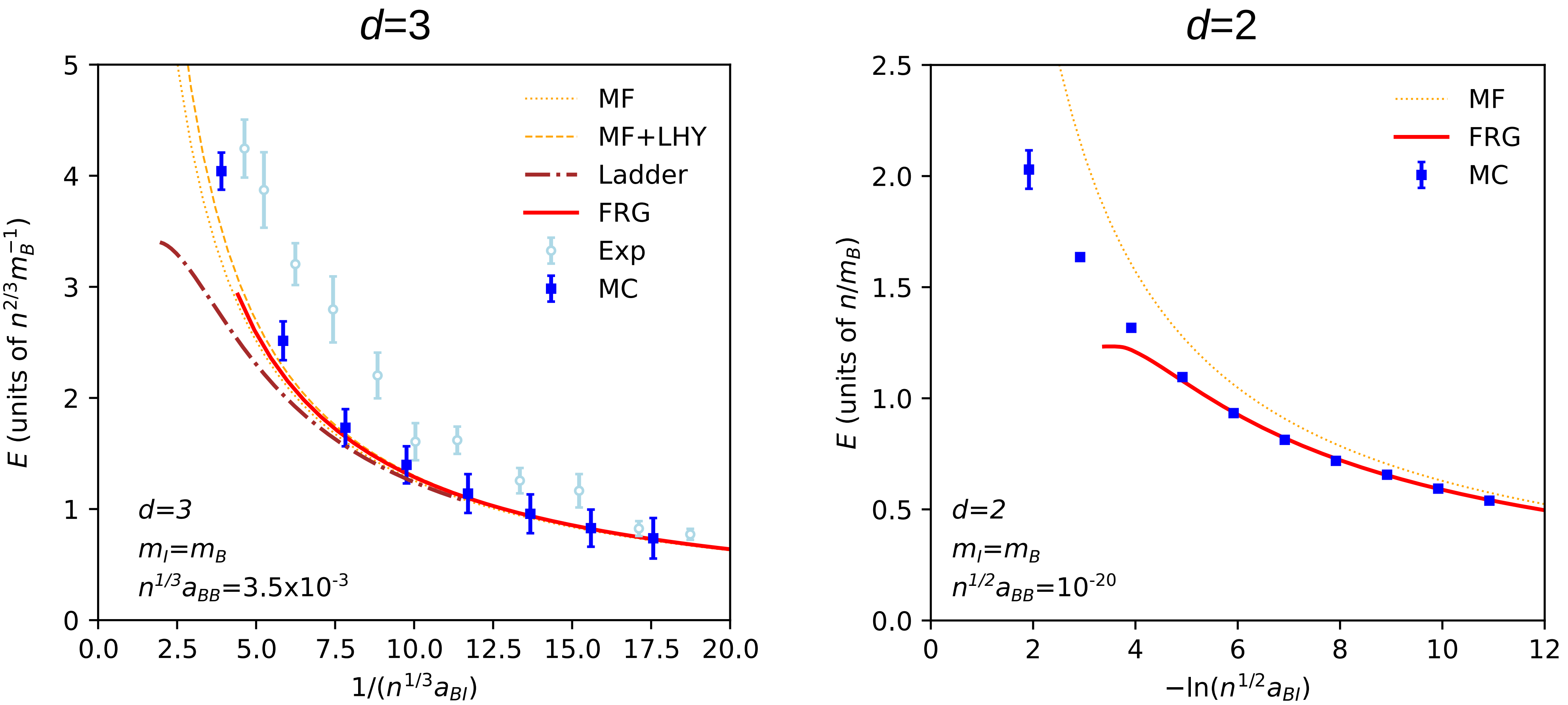
$$\langle \rho_I \rangle = 0$$

<u>Results:</u> Repulsive branch



<u>Ladder:</u> A. Camacho-Guardian *et al.*, PRX **8**, 031042 (2018)
<u>Exp:</u> N.B Jørgensen *et al.*, PRL **117**, 055302 (2016)
<u>MC</u>: L. Peña Ardila *et al.*, PRA **99**, 063607 (2019)

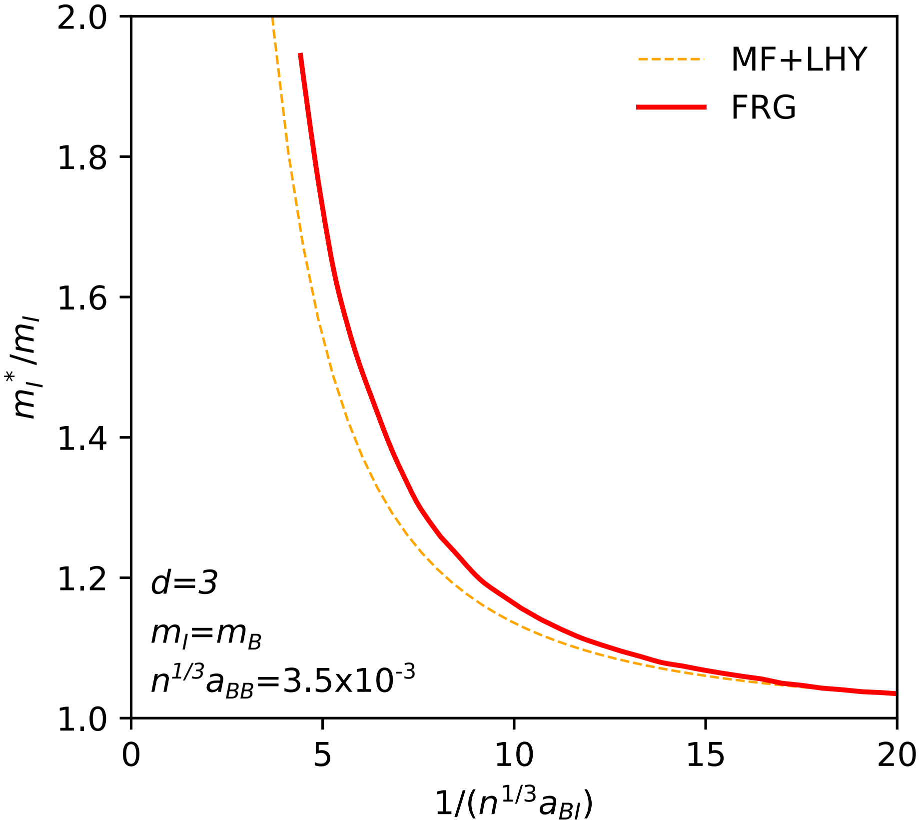
Results: Repulsive branch



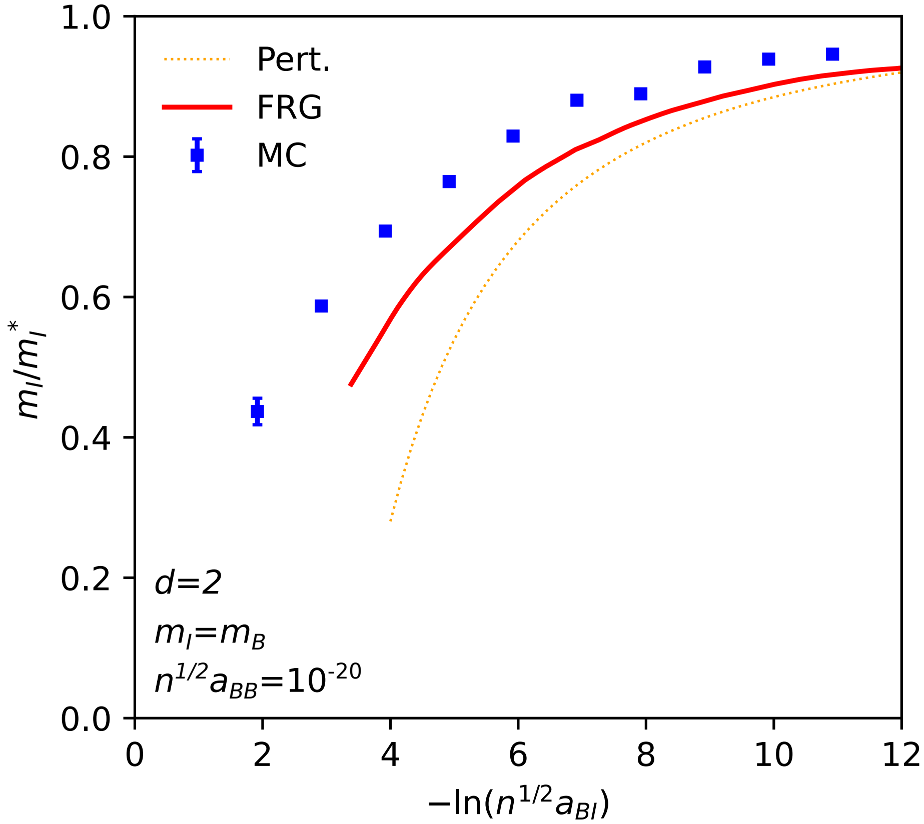
Ladder: A. Camacho-Guardian *et al.*, PRX **8**, 031042 (2018)
Exp: N.B Jørgensen *et al.*, PRL **117**, 055302 (2016)
MC: L. Peña Ardila *et al.*, PRA **99**, 063607 (2019)
MC: L. Peña Ardila *et al.*, PRR **2**, 023405 (2020)

Results: Repulsive branch (effective masses)

d=3



d=2



MC: L. Peña Ardila *et al.*, PRR **2**, 023405 (2020)