1)

Give an example of sets a and B for which $a \in B$ but $\mathscr{P}a \notin \mathscr{P}B$.

2)

Show that the following four conditions are equivalent.

(a) $A \subseteq B$, (b) $A - B = \varnothing$,

(c) $A \cup B = B$, (d) $A \cap B = A$.

3)

Is $\mathscr{P}(A - B)$ always equal to $\mathscr{P}A - \mathscr{P}B$? Is it ever equal to $\mathscr{P}A - \mathscr{P}B$?

4)

Let A, B, and C be sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

5)

Assume that $\mathscr{P}A = \mathscr{P}B$. Prove that $A = B$.

6)

Prove that for all sets the following are valid.

(a) $A \subseteq C \;\&\; B \subseteq C \;\Leftrightarrow\; A \cup B \subseteq C$.

(b) $C \subseteq A \;\&\; C \subseteq B \;\Leftrightarrow\; C \subseteq A \cap B$.

7)

(a) Show that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(b) Show that if $A \times B = A \times C$ and $A \neq \varnothing$, then $B = C$.

8)

Assume that f and g are functions with $f \subseteq g$ and dom $g \subseteq$ dom f. Show that $f = g$.

9)

Assume that f and g are functions.

(a) Show that $f \cap g$ is a function.

(b) Show that $f \cup g$ is a function iff $f(x) = g(x)$ for every x in $(\operatorname{dom} f) \cap (\operatorname{dom} g)$.

10)

Show that $(R \circ S) \circ T = R \circ (S \circ T)$ for any sets R, S, and T.

11)

(a) Show that R is symmetric iff $R^{-1} \subseteq R$.

(b) Show that R is transitive iff $R \circ R \subseteq R$.

12)

Show that R is a symmetric and transitive relation iff $R = R^{-1} \circ R$.

13)

Assume that R is a linear ordering on a set A. Show that R^{-1} is also a linear ordering on A.

14)

Assume that $<_A$ and $<_B$ are linear orderings on A and B, respectively. Define the binary relation $<_L$ on the Cartesian product $A \times B$ by:

$$\langle a_1, b_1\rangle <_L \langle a_2, b_2\rangle \quad \text{iff} \quad \text{either } a_1 <_A a_2 \text{ or } (a_1 = a_2 \;\&\; b_1 <_B b_2)$$

Show that $<_L$ is a linear ordering on $A \times B$. (The relation $<_L$ is called *lexicographic* ordering, being the ordering used in making dictionaries.)