

UNIVERSIDADE FEDERAL DE RORAIMA
DISCIPLINA DE ÁLGEBRA LINEAR
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LISTA 19

BOA VISTA, 09 DE DEZEMBRO DE 2020

Lista 19 A2 → Aléio

1) $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ $T(x, y, z) = (2x, 2y, x+y+3z)$

$B = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T(1,0,0) = (2,0,1)$

$T(0,1,0) = (0,2,1)$

$T(0,0,1) = (0,0,3)$

$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ para $\lambda_1 = 2$
 $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2 \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\det(A - \lambda I)$

$= \begin{vmatrix} 2-\lambda & 0 & 0 & 2-\lambda & 0 \\ 0 & 2-\lambda & 0 & 0 & 2-\lambda \\ 1 & 1 & 3-\lambda & 1 & 1 \end{vmatrix}$

$= (2-\lambda) \cdot (2-\lambda) \cdot (3-\lambda)$

$= (2-\lambda)^2 \cdot (3-\lambda) = 0$

logo

$2-\lambda = 0 \rightarrow 3-\lambda = 0$

$\lambda_1 = 2 \quad \lambda_2 = 3$

$\begin{bmatrix} 2x & 0 & 0 \\ 0 & 2y & 0 \\ x & y & 3z \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix}$

$\begin{cases} 2x = 2x & \therefore x = 0 \\ 2y = 2y & \therefore y = 0 \\ x+y+3z = 2z & \therefore z = -x-y \end{cases}$

$\therefore z = 0$

$\vec{v}_1 = (0, 0, 0)$

para $\lambda_2 = 3$

$\begin{cases} 2x = 3x & \therefore x = 0 \\ 2y = 3y & \therefore y = 0 \\ x+y+3z = 3z & \therefore z = 0 \end{cases}$

$\vec{v}_2 = (0, 0, 0)$

$B = \{(0,0,0)\}$

① b) $S: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ definido por $S(x, y, z) = (x, x+y, x-2z)$

$$B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

$$S(1, 0, 0) = (1, 1, 1)$$

$$S(0, 1, 0) = (0, 1, 0)$$

$$S(0, 0, 1) = (0, 0, -2)$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\det(A - \lambda I)$$

$$= \begin{vmatrix} 1-\lambda & 0 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix}$$

$$= (1-\lambda) \cdot (1-\lambda) \cdot (-2-\lambda) = 0$$

$$= (1-\lambda)^2 \cdot (-2-\lambda) = 0$$

luego:

$$(1-\lambda) = 0 \text{ y } (-2-\lambda) = 0$$

$$\boxed{\lambda_1 = 1} \text{ y } \boxed{\lambda_2 = -2}$$

Para $\lambda_1 = 1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x & 0 & 0 \\ x & y & 0 \\ x & 0 & -2z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} x = x & \therefore \boxed{x = x} \\ x + y = y & \therefore \boxed{x = 0} \\ x - 2z = z & \therefore \boxed{x = 3z} \end{cases}$$

$$\vec{v}_1 = (x, 0, \frac{x}{3}) = x(1, 0, \frac{1}{3})$$

Para $\lambda_2 = -2$

$$\begin{cases} x = -2x & \therefore \boxed{x = 0} \\ x + y = -2y & \therefore x = -y \Rightarrow \boxed{y = -x} \\ x - 2z = -2z & \therefore \boxed{x = 0} \end{cases}$$

$$\vec{v}_2 = (0, -x, 0) = x(0, -1, 0)$$

$$B = \{(1, 0, \frac{1}{3}), (0, -1, 0)\}$$

Lista 19 A1 -> Soluções

C) $T: P_2(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ definido por $T(t) = t^2$

$$\Phi = \{t^2, t, 1\}$$

$$T(t^2) = 2t \Rightarrow 2t^2 + 1t + 0.1$$

$$T(t) = 1 \Rightarrow 0t^2 + 1t + 0.1$$

$$T(1) = 0 \Rightarrow 0t^2 + 0t + 0.1$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\det(A - \lambda I)$$

$$= \begin{vmatrix} 2-\lambda & 0 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Selma

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(12) b) A é diagonalizável pois os autovalores encontrados usando o polinômio característico formam uma base e A é diagonal.