Prova 3 cálculo II
Aluno: Felipe Derkian de Sousa Freitas
Matrícula: 1201424418
Boa vista, 08 de Dezembro de 2020

(1) Usando a definição de limite, mostre que

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2+y^2}} = 0.$$

coordenadas polares
$x = K\cos\theta$
$y = K\,\text{sen}\,\theta$

$$= \lim_{K\to 0} \frac{2K\cos\theta\, K\,\text{sen}\,\theta}{\sqrt{(K\cos\theta)^2+(K\,\text{sen}\,\theta)^2}}$$

aplicando o conjugado na raiz e retirando a constante do lim

$$= 2\cos\theta\,\text{sen}\,\theta \cdot \lim_{K\to 0} \frac{K^2}{\sqrt{\frac{(K\cos\theta)^2+(K\,\text{sen}\,\theta)^2}{1} \cdot \frac{(K\cos\theta)^2-(K\,\text{sen}\,\theta)^2}{(K\cos\theta)^2-(K\,\text{sen}\,\theta)^2}}}$$

$$= 2\cos\theta\cdot\text{sen}\,\theta \cdot \lim_{K\to 0} \frac{K^2}{\sqrt{\frac{K^4\cos 2\theta}{(K\cos\theta)^2-(K\,\text{sen}\,\theta)^2}}}$$

$$= 2\cos\theta\,\text{sen}\,\theta \cdot \lim_{K\to 0} \frac{K^2}{\sqrt{K^2}} \cdot \frac{\sqrt{K^2}}{\sqrt{K^2}}$$

Felipe

$$= 2\cos\theta\,\text{sen}\,\theta \cdot \lim_{K\to 0} \frac{\cancel{K^2}\sqrt{K^2}}{\cancel{K^2}}$$

$$= 2\cos\theta\,\text{sen}\,\theta \cdot \sqrt{0^2} = 2\cos\theta\,\text{sen}\,\theta \cdot 0 = 0$$

Portanto o limite de fato é 0.

(2) mostre que a função $f(x,y) = \begin{cases} \frac{2xy}{2x^2+2y^2}, & \text{se } (x,y) \neq (0,0) \\ 0, & \text{se } (x,y) = (0,0) \end{cases}$

$(x,y) = (0,0)$

$$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 0$$

se f é contínua em (0,0) então pegando (x,y) arbitrário (T,T) temos que:

$$\lim_{(x,y)\to(T,T)} f(x,y) = \lim_{(T,T)\to(0,0)} \frac{2.T.T}{2.T^2+2.T^2} = \frac{2T^2}{4T^2} = \frac{1}{2} \neq f(0,0) = 0$$

logo não é contínua para todos os lados em (0,0).

Felipe

③ Dada a função $f(x,y) = \begin{cases} \dfrac{x^3y}{x^2+y^2}, & \text{se } (x,y) \neq (0,0), \\ 0, & \text{se } (x,y) = (0,0). \end{cases}$

mostre que $\dfrac{\partial^2 f}{\partial y \partial x}(0,0) = (0,0)$ e $\dfrac{\partial^2 f}{\partial x \partial y}(0,0) = 1$.

$$\frac{\partial f}{\partial y}(x,y) = \frac{(x^3y)'_y \cdot (x^2+y^2) - \left[(x^3y) \cdot (x^2+y^2)'_y\right]}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{x^3 \cdot (x^2+y^2) - \left[(x^3y) \cdot 2y\right]}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{x^3 \cdot (x^2+y^2) - 2x^3y^2}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{x^3 \cdot (x^2+y^2-2 \cdot y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{x^3 \cdot (x^2-y^2)}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial f}{\partial y}(x,y) = \frac{x^5 - x^3y^2}{(x^2+y^2)^2}}$$

*

$$\boxed{\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{x^8 + 7x^6y^2 + 3x^4y^4 - 3x^2y^6}{(x^2+y^2)^4}}$$

$$\frac{\partial^2 f(0,0)}{\partial y \partial x} = \frac{0^8 + 7.0^6.0^2 + 3.0^4.0^4 - 3.0^2.0^6}{(0^2+0^2)^4} = \frac{0}{0}$$

$$\frac{\partial^2 f}{\partial y \partial x}(0,0) = \lim_{T \to 0} \frac{f(0+T,0) - f(0,0)}{T}$$

$$= \lim_{T \to 0} \frac{\dfrac{(0+T)^8 + \cancel{7.(0+T)^6.0^2} + \cancel{3.(0+T)^4.0^4} - \cancel{3.(0+T)^2.(0)^6}}{((0+T)^2+0^2)^4}}{T}$$

$$= \lim_{T \to 0} \frac{\dfrac{T^8}{T^6}}{T} = \frac{T^8}{T^6} \cdot \frac{1}{T} = \frac{T^8}{T^7} = T = 0 \#$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{(x^3y)'_x \cdot (x^2+y^2) - \left[(x^3y)\cdot(x^2+y^2)'_x\right]}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{(3x^2y)\cdot(x^2+y^2) - \left[(x^3y)\cdot 2x\right]}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{3x^4y + 3x^2y^2 - \left[2x^4y\right.}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{x^2\cdot(3x^2y + 3y^2 - 2x^2y)}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial f}{\partial x}(x,y) = \frac{x^2\cdot(x^2y + 3y^2)}{(x^2+y^2)^2}}$$

$$\frac{\partial f}{\partial x\partial y}(x,y) = \frac{(x^4y + 3x^2y^2)'_y\cdot(x^2+y^2)^2 - \left[(x^4y+3x^2y^2)\cdot\left((x^2+y^2)^2\right)'_y\right]}{(x^2+y^2)^4}$$

$$\frac{\partial f}{\partial x\partial y}(x,y) = \frac{(x^4 + 6x^2y)\cdot(x^4 + 2x^2y^2 + y^4) - \left[(x^4y + 3x^2y^2)\cdot(2(x^2+y^2)\cdot 2y)\right]}{(x^2+y^2)^4}$$

$$\frac{\partial f}{\partial x \partial y}(x,y) = \frac{(x^4+6x^2y)\cdot(x^4+2x^2y^2+y^4)-\left[(x^4y+3x^2y^2)\cdot(4x(x^2+y^2)\right]}{(x^2+y^2)^4}$$

$$\frac{\partial f}{\partial x \partial y}(x,y) = \frac{x^8+2x^6y^2+x^4y^4+6x^6y+12x^4y^3+6x^2y^3-\left[(x^4y+3x^2y^2)\cdot(4x^3+4xy^2)\right]}{(x^2+y^2)^4}$$

$$\frac{\partial f}{\partial x \partial y}(x,y) = \frac{x^8+2x^6y^2+x^4y^4+6x^6y+12x^4y^3+6x^2y^3-\left[4y^2x^6+4x^7y+12x^4y^3+12x^5y^2\right]}{(x^2+y^2)^4}$$

$$\frac{\partial f}{\partial x \partial y}(x,y) = \frac{x^8+2x^6y^2+x^4y^4+6x^6y+12x^4y^3+6x^2y^3-4y^2x^6-4x^7y-12x^4y^3-12x^5y^2}{(x^2+y^2)^4}$$

$$\frac{\partial f}{\partial x \partial y}(x,y) = \frac{x^2\cdot(x^6+2x^4y^2+x^2y^4+6x^4y+12x^2y^3+6y^3+4y^2x^4-4x^5y-12x^2y^3-12x^3y^2)}{(x^2+y^2)^4}$$

$$\lim_{t\to 0} \frac{t^2\cdot(t^2+0+0+0+0+0+0+0+0+0+0+0)}{(t^2+0^2)^4}$$

$$\lim_{t\to 0} \frac{t^4}{t^6} = \frac{1}{t^2} = \boxed{1}$$