UNIVERSIDADE FEDERAL DE RORAIMA
DISCIPLINA DE CÁLCULO II
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LISTA 2

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Lista 2 Felipe Kerkhov

5.1 Integração por partes

① $\int x \operatorname{sen}\left(\frac{x}{2}\right) dx$

$u = x \quad dv = \sin\left(\frac{x}{2}\right) dx$

$du = dx \quad v = \int \sin\left(\frac{x}{2}\right) dx$

$uv - \int v\, du$

$= x \cdot -2\cos\left(\frac{x}{2}\right) - \int -2\cos\left(\frac{x}{2}\right) dx$

$v = \int \sin u \cdot 2\,du \quad u = \frac{x}{2}$

$= -2x\cos\left(\frac{x}{2}\right) + 2\int \underbrace{\cos\left(\frac{x}{2}\right) dx}_{①}$

$v = 2\int \sin u\, du \quad du = \frac{1}{2}\, dx$

$= -2x\cos\left(\frac{x}{2}\right) + 2\cdot\left(2\sin\left(\frac{x}{2}\right)\right)$

$v = 2 \cdot -\cos\frac{x}{2} + c \quad dx = 2du$

$= -2x\cos\left(\frac{x}{2}\right) + 4\sin\left(\frac{x}{2}\right) + C$

$v = -2\cos\left(\frac{x}{2}\right) + c$

① $\int \cos\left(\frac{x}{2}\right) dx$

$u = \frac{x}{2}$

$du = \frac{1}{2}\, dx$

$dx = 2\, du$

$= \int \cos(u) \cdot 2\, du$

$= 2\int \cos(u)\, du$

$= 2\sin\left(\frac{x}{2}\right) + c$

Felix

③ $\int T^2 \cos T \, dT$

$uv - \int v \, du$

$u = T^2 \qquad dv = \cos T \, dT$

$du = 2T \, dT \qquad v = \int \cos T \, dT$

$v = \sin T + c$

$= T^2 \cdot \sin T - \int \sin T \cdot 2T \, dT$

$= T^2 \sin T - 2 \int \sin(T) \cdot T \, dT$ ①

$= T^2 \sin T - 2 \cdot \left[-T \cos T + \int \cos T \, dT \right]$ ②

$= T^2 \sin T - 2 \cdot \left[-T \cos T + \sin T \right]$

$= T^2 \sin(T) + 2T \cos(T) - 2 \sin T + C$

felipe

① $\int \sin T \cdot T \, dT$

$u = T \qquad dv = \sin T \, dT$

$du = dT \qquad v = \int \sin T \, dT$

$v = -\cos T + c$

$T \cdot -\cos T - \int -\cos T \, dT$

$-T \cos T + \int \cos T \, dT$

② $\int \cos T \, dT$

$\sin T + c$

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④ $\int x^2 \operatorname{sen} x \, dx$

$uv - \int v \, du$

$u = x^2 \qquad dv = \operatorname{sen} x \, dx$

$du = 2x \, dx \qquad v = \int \operatorname{sen} x \, dx$

$v = -\cos x + c$

$= x^2 \cdot -\cos x - \int -\cos x \cdot 2x \, dx$

$= -x^2 \cos x - \left[-2 \int \cos x \cdot x \, dx \right]$ ①

$= -x^2 \cos x - \left[-2 \cdot \left[x \sin x - \int \sin x \, dx \right] \right]$ ②

$= -x^2 \cos x - \left[-2 \cdot (x \sin x - (-\cos x)) \right]$

$= -x^2 \cos x + 2 \cdot (x \sin(x) + \cos(x)) + C$

felipe

① $\int \cos x \cdot x \, dx$

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = \int \cos x \, dx$

$v = \sin x + c$

$x \sin x - \int \sin x \, dx$

② $\int \sin x \, dx$

$-\cos x + c$

(5) $\int_1^2 x \ln x \, dx$

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{x^2}{2} + c$

$uv - \int v\,du$

$= \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx$

$= \frac{x^2 \ln x}{2} - \frac{1}{2}\int x\,dx$

$= \frac{x^2 \ln x}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + c$

$= \left[\frac{x^2 \cdot \ln x}{2} - \frac{x^2}{4}\right]_1^2$

$f(b) - f(a)$

$= \left(\frac{2^2 \cdot \ln(2)}{2} - \frac{2^2}{4}\right) - \left(\frac{1^2 \cdot \ln(1)}{2} - \frac{1^2}{4}\right)$

$= \left(\frac{4 \cdot \ln(2)}{2} - \frac{4}{4}\right) - \left(\frac{1 \cdot \ln(1)}{2} - \frac{1}{4}\right)$

$= (2 \cdot \ln(2) - 1) - \left(\cancel{\frac{\ln(1)}{2}} - \frac{1}{4}\right)$ Selçin

$= 2\ln(2) - 1 + \frac{1}{4}$

$= 2\ln(2) - \frac{3}{4}$

(6) $\int_1^e x^3 \ln x \, dx$

$u = \ln x \qquad dv = x^3 dx$

$du = \frac{1}{x} dx \qquad v = \int x^3 dx$

$v = \frac{x^4}{4} + c$

$uv - \int v \, du$

$= \ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{\cancel{x}} dx$

(1) $\int x^3 dx$

$\frac{x^4}{4} + c$

$= \frac{x^4 \ln x}{4} - \frac{1}{4} \int x^3 dx$ (1)

$= \frac{x^4 \ln x}{4} - \frac{1}{4} \cdot \left[\frac{x^4}{4}\right]$

$= \left[\frac{x^4 \ln x}{4} - \frac{x^4}{16}\right]_1^e$

Leleji

$= \left(\frac{e^4 \ln(e)}{4} - \frac{e^4}{16}\right) - \left(\cancel{\frac{1^4 \cdot \ln(1)}{4}} - \frac{1^4}{16}\right)$

$= \frac{e^4 \cdot 1}{4} - \frac{e^4}{16} - \left(-\frac{1}{16}\right)$

$= \left(\frac{e^4}{4} - \frac{e^4}{16}\right) + \frac{1}{16}$

$= \frac{4e^4 - e^4}{16} + \frac{1}{16}$

$= \frac{3e^4}{16} + \frac{1}{16}$ ✱

⑦ $\int tg^{-1}(y)\,dy$

$u = \operatorname{arctg}(y)$ $\quad dv = 1\,dy$

$du = \frac{1}{y^2+1}\,dy$ $\quad v = y + C$

$u.v - \int v\,du$

$= \operatorname{arctg}(y).y - \int \frac{y}{y^2+1}\,dy$ ①

$= y\operatorname{arctg}(y) - \frac{1}{2}.\ln(y^2+1) + C$ *

Felipe

① $\int \frac{y}{\underbrace{y^2+1}_{u}}\,dy$ $\quad u = y^2+1$

$du = 2y\,dy$

$= \int \frac{1}{u}.\frac{du}{2}$ $\quad y\,dy = \frac{du}{2}$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}.\ln(u) + C$

$= \frac{1}{2}\ln(y^2+1) + C$

⑧ $\int \sin^{-1} y\,dy$

$u = \arcsin(y)$ $\quad dv = 1\,dy$

$du = \frac{1}{\sqrt{1-y^2}}\,dy$ $\quad v = \int 1\,dy$

$v = y + C$

$u.v - \int v\,du$

$= \arcsin(y).y - \int y.\frac{1}{\sqrt{1-y^2}}\,dy$ ①

$= \arcsin(y).y - \left(-\sqrt{1-y^2}\right)$

$= y\arcsin(y) + \sqrt{1-y^2} + C$ *

Felipe

① $\int \frac{y}{\sqrt{1-y^2}}\,dy$ $\quad u = 1-y^2$

$du = -2y\,dy$

$= \int \frac{1}{\sqrt{u}}.\frac{du}{-2}$ $\quad \frac{du}{-2} = y\,dy$

$= -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$= -\frac{1}{2}\int u^{-1/2}\,du$

$= -\frac{1}{2}\,\frac{u^{-1/2+1}}{-1/2+1}$

$= -\frac{1}{2}.\frac{2}{1}\,u^{1/2}$

$= -\sqrt{u} = -\sqrt{1-y^2} + C$ *

⑨ $\int x \sec^2 x \, dx$

$u = x \qquad dv = \sec^2 x \, dx$

$du = dx \qquad v = \int \sec^2 x \, dx$

$v = tg(x) + c$

$uv - \int v \, du$

$= x \cdot tg(x) - \int tg(x) \, dx$ ①

$= x \, tg(x) + \ln|\cos(x)| + c$

Felipe

① $\int tg(x) \, dx$

$tg(x) = \dfrac{\operatorname{sen}(x)}{\cos(x)}$

$= \int \dfrac{\operatorname{sen}(x)}{\cos(x)} \, dx$

$u = \cos(x)$

$du = -\operatorname{sen}(x) \, dx$

$= \int -\frac{1}{u} \, du$

$= -\ln|u| + C$

$= -\ln|\cos(x)| + C$

⑩ $\int 4x \sec^2(2x)\,dx$

$u = x$
$du = dx$

$dv = \sec^2(2x)\,dx$
$v = \int \sec^2(2x)\,dx$

$= 4\int x \sec^2(2x)\,dx$

$= 4.\left[x.\frac{tg(2x)}{2} - \int \frac{tg(2x)}{2}\,dx\right]$

$= 4.\left[\frac{x\,tg(2x)}{2} - \frac{1}{2}\int tg(2x)\,dx\right]$ ①

$= 4\left[\frac{x\,tg(2x)}{2} - \frac{1}{2}\left(-\frac{1}{2}\ln(\cos(2x))\right)\right]$

$= 4\left[\frac{x\,tg(2x)}{2} + \frac{1}{4}\ln|\cos(2x)|\right]$

$= 2x\,tg(2x) + \ln|\cos(2x)| + C$

$u = 2x$
$du = 2\,dx$
$dx = \frac{1}{2}du$

$v = \int \sec^2(u).\frac{1}{2}du$

$v = \frac{1}{2}\int \sec^2 u\,du$

$v = \frac{1}{2}tg(u)$

$v = \frac{tg(2x)}{2} + C$

Jelyn

① $\int tg(2x)\,dx$ $\quad tg(x) = \frac{\sin(x)}{\cos(x)}$

$= \int \frac{\sin(2x)}{\cos(2x)}\,dx$

$= \int -\frac{1}{2u}\,du$

$u = \cos(2x)$
$du = -\sin(2x).2dx$
$-\frac{du}{2} = \sin(2x)dx$

$= -\frac{1}{2}\int \frac{1}{u}\,du$

$= -\frac{1}{2}\ln(u)$

$= -\frac{1}{2}\ln(\cos(2x)) + C$

(11) $\int x^3 e^x dx$

$u = x^3$ $\quad$ $dv = e^x dx$

$du = 3x^2 dx$ $\quad$ $v = \int e^x dx$

$v = e^x$

$= x^3 e^x - \int e^x \cdot 3x^2 dx$

$= x^3 e^x - 3\int e^x \cdot x^2 dx$ (1)

$= x^3 e^x - 3 \cdot [x^2 e^x - 2\int e^x \cdot x dx]$ (2)

$= x^3 e^x - 3 \cdot [x^2 e^x - 2 \cdot [x e^x - \int e^x dx]]$

$= x^3 e^x - 3 \cdot [x^2 e^x - 2 \cdot [x e^x - e^x]]$

$= x^3 e^x - 3 \cdot [x^2 e^x - 2x e^x - 2e^x]$

$= x^3 e^x - 3x^2 e^x + 6x e^x + 6e^x + C$

(1) $\int e^x \cdot x^2 dx$ $\quad$ $u = x^2$ $\quad$ $v = e^x$

$du = 2x dx$

$x^2 e^x - \int e^x \cdot 2x dx$

(2) $\int e^x \cdot x dx$ $\quad$ $u = x$ $\quad$ $v = e^x$

$du = dx$

$x e^x - \int e^x dx$

Felipe

(13) $\int (x^2-5x)e^x\,dx$

$u = x^2-5x \qquad v = e^x$

$du = 2x-5\,dx$

$= (x^2-5x).e^x - \int e^x.(2x-5)\,dx$

$= (x^2-5x).e^x - \left[(2x-5).e^x - 2\int e^x\,dx\right]$

$= (x^2-5x)e^x - \left[(2x-5)e^x - 2e^x\right]$

$= (x^2-5x)e^x - e^x(2x-5) + 2e^x + c$

$= x^2e^x - 5xe^x - e^x2x + 5e^x + 2e^x + c$

$= x^2e^x - 5xe^x - e^x2x + 7e^x + c$

$= x^2e^x - 5xe^x - 2xe^x + 7e^x + c$

$= x^2e^x - 7xe^x + 7e^x + c$

Felipe

(1) $\int (2x-5).e^x\,dx$

$(2x-5).e^x - \int e^x.2\,dx$

$u = 2x-5$

$du = 2\,dx$

$v = e^x$

(14) $\int (n^2+n+1)e^n dn$

$u=(n^2+n+1)$ $\quad dv = e^n dn$

$du = 2n\,dn$ $\quad v = e^n + c$

$= (n^2+n+1).e^n - \int e^n . 2n\,dn$

$= (n^2+n+1)e^n - 2\int e^n . n\,dn$ (1)

$= (n^2+n+1)e^n - 2.[ne^n - \int e^n dn]$

$= (n^2+n+1)e^n - 2.[ne^n - e^n]$

$= (n^2+n+1)e^n - 2ne^n - 2e^n + c$

$= e^n n^2 + e^n n + e^n - 2ne^n - 2e^n + c$

(1) $\int e^n . n\,dn$ $\quad u = n \quad v = e^n$ $\quad du = dn$

$n.e^n - \int e^n dn$

Zelişi

(15) $\int x^5 e^x dx$

$u = x^5$

$du = 5x^4 dx \quad v = e^x$

Zelişi

$= x^5e^x - \int e^x 5x^4 dx$

$= x^5e^x - 5\int e^x . x^4 dx$ (1)

$= x^5e^x - 5.[x^4e^x - 4\int e^x . x^3 dx]$ (2)

$= x^5e^x - 5.[x^4e^x - 4.[x^3e^x - 3\int e^x x^2 dx]]$ (3)

$= x^5e^x - 5[x^4e^x - 4.[x^3e^x - 3.[x^2e^x - 2\int e^x x\,dx]]]$ (4)

$= x^5e^x - 5[x^4e^x - 4.[x^3e^x - 3.[x^2e^x - 2.[xe^x - e^x]]]]$

$= x^5e^x - 5[x^4e^x - 4.[x^3e^x - 3.[x^2e^x - 2xe^x - 2e^x]]]$

$= x^5e^x - 5.[x^4e^x - 4[e^x x^3 - 3x^2e^x + 6xe^x + 6e^x]]$

$= x^5e^x - 5.[xe^x - 4e^x x^3 + 12x^2e^x - 24xe^x - 24e^x]$

$= x^5e^x - 5xe^x + 20e^x x^3 - 60x^2e^x + 120xe^x + 120e^x + c$

(1) $\int e^x x^4 dx$ $\quad u = x^4 \quad du = 4x^3 dx \quad v = e^x$

$x^4e^x - \int e^x 4x^3 dx$

(2) $\int e^x . x^3 dx$ $\quad u = x^3 \quad du = 3x^2 dx \quad v = e^x$

(3) $\int e^x x^2 dx$ $\quad u = x^2 \quad du = 2x\,dx \quad v = e^x$

(4) $\int e^x x\,dx$ $\quad u = x \quad du = dx \quad v = e^x$

$xe^x - \int e^x dx$

$xe^x - e^x + c$

(16) $\int t^2 e^{4t} dt$

$u = t^2$ $\quad dv = e^{4t} dt$

$du = 2t\,dt$ $\quad v = \int e^{4t} dt$

$v = \int e^{u} \cdot \frac{1}{4} \cdot du$ $\quad u = 4t$, $du = 4dt$, $dt = \frac{du}{4}$

$v = \frac{1}{4} \int e^{u} du$

$v = \frac{e^{4t}}{4} + c$

$= t^2 \cdot \frac{e^{4t}}{4} - \int \frac{e^{4t}}{\cancel{4}} \cdot 2t\, dt$

$= t^2 \cdot \frac{e^{4t}}{4} - \frac{1}{2} \int e^{4t} \cdot t\, dt$ ①

$= t^2 \cdot \frac{e^{4t}}{4} - \frac{1}{2} \cdot \left[\frac{te^{4t}}{4} - \frac{1}{4} \int e^{4t} dt\right]$

① $\int e^{4t} \cdot t\, dt$ $\quad u = t$, $du = dt$

$\frac{t \cdot e^{4t}}{4} - \int \frac{e^{4t}}{4} \cdot dt$ $\quad dv = e^{4t} dt$, $v = \int e^{4t} dt$, $v = \frac{e^{4t}}{4} + c$

$= \frac{t^2 e^{4t}}{4} - \frac{1}{2} \cdot \left[\frac{te^{4t}}{4} - \frac{1}{4} \cdot \frac{1}{4} e^{4t}\right] + c$

$= \frac{t^2 e^{4t}}{4} - \frac{1}{2} \cdot \left[\frac{te^{4t}}{4} - \frac{1}{16} e^{4t}\right]$

$= \frac{t^2 e^{4t}}{4} - \frac{te^{4t}}{8} + \frac{e^{4t}}{32} + c$

Felipe

17 $\displaystyle\int_0^{\pi/2} \theta^2 \operatorname{sen}(2\theta)\, d\theta$

$u = \theta^2$ $\quad dv = \operatorname{sen}(2\theta) d\theta$

$du = 2\theta\, d\theta$ $\quad v = \int \operatorname{sen}(2\theta) d\theta$

$v = \frac{1}{2}\int \sin(u) du$ $\quad u = 2\theta$

$du = 2 d\theta$

$d\theta = \frac{du}{2}$

$v = -\frac{1}{2} \cdot \cos(2\theta) + C$

$= -\frac{\theta^2 \cos(2\theta)}{2} - \int \frac{\cos(2\theta)}{\cancel{2}} \cdot \cancel{2}\theta\, d\theta$

$= -\frac{\theta^2 \cdot \cos(2\theta)}{2} - \int \cos(2\theta) \cdot \theta\, d\theta$ 1

$= -\frac{\theta^2 \cos(2\theta)}{2} - \left[-\frac{\theta \sin(2\theta)}{2} + \frac{1}{2}\int \sin(2\theta) d\theta \right]$ 2

$= -\frac{\theta^2 \cos(2\theta)}{2} - \left[-\frac{\theta \sin(2\theta)}{2} + \frac{1}{2} \cdot \frac{\cos(2\theta)}{2} \right]$

$= -\frac{\theta^2 \cos(2\theta)}{2} - \left[-\frac{\theta \cdot \sin(2\theta)}{2} + \frac{\cos(2\theta)}{4} \right]$

$= -\frac{\theta^2 \cos(2\theta)}{2} + \frac{\theta \sin(2\theta)}{2} - \frac{\cos(2\theta)}{4} + C$ ✗

1 $\int \cos(2\theta)\, \theta\, d\theta$

$u = \theta$

$du = d\theta$

$-\frac{\theta \sin(2\theta)}{2} + \int \frac{\sin(2\theta)}{2} d\theta$ $\quad dv = \cos(2\theta) d\theta$

$v = \int \cos(2\theta) d\theta$

$u = 2\theta$

$du = 2 d\theta$

$v = -\frac{\sin(2\theta)}{2}$

2 $\int \sin(2\theta) d\theta$

$= \frac{1}{2}\int \sin(u) du$ $\quad u = 2\theta$

$du = 2 d\theta$

$\frac{du}{2} = d\theta$

$\frac{\cos(2\theta)}{2}$

Felipe

(18) $$\int_0^{\pi/2} x^3 \cos(2x)\,dx$$

$u = x^3 \quad dv = \cos(2x)dx$

$du = 3x^2 dx \quad v = \int \cos(2x)dx$

$v = \frac{\sin(2x)}{2}$

$$= x^3\frac{\sin(2x)}{2} - \int \frac{\sin(2x)}{2}\cdot 3x^2 dx$$

$$= \frac{x^3 \sin(2x)}{2} - \frac{3}{2}\int \underset{①}{\sin(2x)\cdot x^2 dx}$$

① $\int \sin(2x)\, x^2 dx$

$u = x^2$
$du = 2x\,dx$

$dv = \sin(2x)dx$

$v = \int \sin(2x)dx$

$v = \frac{\cos(2x)}{2} + C$

$$= \frac{x^3\sin(2x)}{2} - \frac{3}{2}\cdot\left[x^2\cdot\frac{\cos(2x)}{2} - \int \frac{\cos(2x)}{\cancel{2}}\cdot \cancel{2}x\,dx\right] \; ②$$

$$= \frac{x^3\sin(2x)}{2} - \frac{3}{2}\cdot\left[\frac{x^2\cdot\cos(2x)}{2} - \left[-x\sin(2x) + \frac{1}{2}\int \underset{③}{\sin(2x)\,dx}\right]\right]$$

② $\int \cos(2x)\, x\,dx$

$u = x$

$du = dx$

$v = \frac{-\sin(2x)}{2}$

$-x\sin(2x) + \int \frac{\sin(2x)}{2}dx$

$$= \frac{x^3\sin(2x)}{2} - \frac{3}{2}\left[\frac{x^2\cdot\cos(2x)}{2} - \left[-x\sin(2x) + \frac{1}{2}\cdot\left[\frac{\cos(2x)}{2}\right]\right]\right]$$

$$= \frac{x^3\sin(2x)}{2} - \frac{3}{2}\cdot\left[\frac{x^2\cos(2x)}{2} - \left[-x\sin(2x) + \frac{\cos(2x)}{4}\right]\right]$$

$$= \frac{x^3\sin(2x)}{2} - \frac{3}{2}\cdot\left[\frac{x^2\cos(2x)}{2} + x\sin(2x) - \frac{\cos(2x)}{4}\right]$$

③ $\int \sin(2x)\,dx$

$\frac{\cos 2x}{2}$

$$= \frac{x^3\sin(2x)}{2} - \frac{3x^2\cos(2x)}{4} - \frac{3\sin(2x)}{2} + \frac{3\cos(2x)}{8} + C$$

Julia

(19) $\int_{2\sqrt{3}}^{2} x \sec^{-1} x \, dx$

$u = \operatorname{arcsec}(x) \quad dv = x\,dx$

$du = \frac{1}{\sqrt{x^2} \cdot \sqrt{x^2-1}} dx \quad v = \frac{x^2}{2} dx$

$= \frac{\operatorname{arcsec} x \cdot x^2}{2} - \frac{1}{2} \int \frac{x^2}{\sqrt{x^2}\sqrt{x^2-1}} dx$ (1)

$= \left[\frac{\operatorname{arcsec} x \cdot x^2}{2} - \frac{1}{2} \cdot \sqrt{x^2-1} \right]_{2\sqrt{3}}^{2}$

$= \left[\frac{\sec^{-1}(2) \cdot 2^2}{2} - \frac{1}{2} \cdot \sqrt{2^2-1} - \left(\frac{\sec^{-1}(2\sqrt{3})(2\sqrt{3})^2}{2} - \frac{\sqrt{(2\sqrt{3})^2-1}}{2} \right) \right]$

$= \left[\sec^{-1}(2) \cdot 2 - \frac{\sqrt{3}}{2} - \left(\sec^{-1}(2\sqrt{3}) \cdot 6 - \frac{\sqrt{11}}{2} \right) \right]$

$= \left[2\sec^{-1}(2) - \frac{\sqrt{3}}{2} - 6\sec^{-1}(2\sqrt{3}) + \frac{\sqrt{11}}{2} \right]$ //

Idur

(1) $\int \frac{x^2}{\sqrt{x^2}\sqrt{x^2-1}} dx$

$= \int \frac{x^{\cancel{2}}}{\cancel{x}\sqrt{x^2-1}} dx$

$= \int \frac{x}{\sqrt{x^2-1}} dx$

$u = x^2 - 1$

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$= \frac{1}{2} \int \frac{1}{\sqrt{u}} du$

$= \frac{1}{2} \int u^{-1/2} du$

$= \frac{1}{2} \cdot \frac{u^{-1/2+1}}{-1/2+1}$

$= \frac{1}{2} \cdot \frac{2}{1} \sqrt{u}$

$= \sqrt{x^2-1} + c$ //

(20) $\int_0^{1/\sqrt{2}} 2x \sin^{-1}(x^2)\, dx$

$= 2\int x \sin^{-1}(x^2)\, dx$

$= 2.\left[\frac{\sin^{-1}(x^2).x^2}{2} - \int \frac{x^2}{2}.\frac{2x}{\sqrt{1-x^4}}\, dx\right]$

$= 2.\left[\frac{\sin^{-1}(x^2).x^2}{2} - \int \frac{x^3}{\sqrt{1-x^4}}\, dx\right]$ ①

$= \cancel{2}.\left[\frac{\sin^{-1}(x^2).x^2}{\cancel{2}} - \left(2\sqrt{1-x^4}\right)\right]$

$= \left[\sin^{-1}(x^2).x^2 - 4\sqrt{1-x^4}\right]_0^{1/\sqrt{2}}$

$= \left[\sin^{-1}\left(\left(\frac{1}{\sqrt{2}}\right)^2\right).\left(\frac{1}{\sqrt{2}}\right) - 4\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^4}\right]$

$= \left[\sin^{-1}\left(\frac{1}{2}\right).\left(\frac{1}{\sqrt{2}}\right) - 4\sqrt{1-\left(\frac{1}{16}\right)}\right]$

$u = \sin^{-1}(x^2)$, $dv = x\, dx$

$du = \frac{1}{\sqrt{1-(x^2)^2}}.2x\,dx$ $\quad v = \int x\, dx$

$v = \frac{x^2}{2} + C$

$du = \frac{2x}{\sqrt{1-x^4}}\, dx$

① $\int \frac{x^3}{\sqrt{1-x^4}}\, dx$

$u = 1-x^4$

$du = -4x^3\, dx$

$\frac{du}{-4} = x^3\, dx$

$= \int \frac{1}{\sqrt{u}}\, du$

$= \int u^{-1/2}\, du$

$= \frac{u^{-1/2+1}}{-1/2+1}$

$= 2\sqrt{u}$

$= 2\sqrt{1-x^4}$

Felipe

(21) $\int e^x \sin x \, dx$

$u = \sin x \quad dv = e^x dx$

$du = \cos x \, dx \quad v = \int e^x dx$

$v = e^x$

$= \sin x \cdot e^x - \int e^x \cdot \cos x \, dx$ (1)

$= \sin x \, e^x - \left[\cos x \, e^x + \int e^x \sin x \, dx\right]$

$= \sin x e^x - \cos x e^x - \int e^x \sin x \, dx$

$\int e^x \sin x \, dx + \int e^x \sin x \, dx = \sin x e^x - \cos x e^x$

$2 \int e^x \sin x \, dx = \sin x e^x - \cos x e^x$

$\int e^x \sin x \, dx = \dfrac{\sin x e^x - \cos x e^x}{2} + C$

(1) $\int e^x \cos x \, dx$

$v = e^x$

$\cos x \cdot e^x + \int e^x \sin x \cdot dx \quad u = \cos x$

$du = -\sin x \, dx$

Telezio

(23) $\int e^{(2x)} \cos(3x)\,dx$

$u = e^{2x} \quad dv = \cos(3x)\,dx$

$du = e^{2x} \cdot 2\,dx \quad v = \frac{\sin(3x)}{3} + C$

$= e^{2x} \cdot \frac{\sin(3x)}{3} - \int \frac{\sin(3x)}{3} \cdot e^{2x} \cdot 2\,dx$

$= \frac{e^{2x} \cdot \sin(3x)}{3} - \frac{2}{3} \int \sin(3x) \cdot e^{2x}\,dx$ (1)

(1) $\int \sin(3x) \cdot e^{2x}\,dx$

$u = e^{2x}$

$du = e^{2x} \cdot 2\,dx$

$dv = \sin(3x)\,dx$

$v = \int \sin(3x)\,dx$

$v = \frac{-\cos 3x}{3} + C$

$-\frac{e^{2x} \cdot \cos(3x)}{3} + \int \frac{\cos(3x)}{3} \cdot e^{2x} \cdot 2\,dx$

$= \frac{e^{2x} \sin(3x)}{3} - \frac{2}{3} \cdot \left\{ \frac{-e^{2x} \cdot \cos(3x)}{3} + \int \frac{\cos(3x)}{3} \cdot e^{2x} \cdot 2\,dx \right]$

$= \frac{e^{2x} \cdot \sin(3x)}{3} - \frac{2}{3} \cdot \left[\frac{-e^{2x} \cos(3x)}{3} + \frac{2}{3} \int \cos 3x \cdot e^{2x}\,dx \right]$

$= \frac{e^{2x} \sin(3x)}{3} + \frac{2e^{2x} \cos(3x)}{9} - \frac{4}{9} \cdot \int \cos 3x \cdot e^{2x}\,dx$

$\int e^{2x} \cos(3x)\,dx + \frac{4}{9} \int \cos 3x\, e^{2x}\,dx = \frac{e^{2x} \sin(3x)}{3} + \frac{2e^{2x} \cos(3x)}{9}$

$\frac{13}{9} \int e^{2x} \cos(3x)\,dx = \frac{e^{2x} \sin(3x)}{3} + \frac{2}{9} e^{2x} \cos(3x)$

$\int e^{2x} \cos(3x)\,dx = \frac{9}{13} \cdot \left(\frac{e^{2x} \sin(3x)}{3} + \frac{2}{9} e^{2x} \cos(3x) \right) + C$

Felipe

(25) $\int e^{\sqrt{3x+9}}\,dx$

$= \int e^{u} \cdot \frac{2}{3}\underbrace{\sqrt{3x+9}}_{u}\,du$

$= \frac{2}{3}\int e^{u} \cdot u\,du$ ①

$= \frac{2}{3} \cdot \left[u e^{u} - e^{u}\right]$

$= \frac{2}{3} \cdot \left[\sqrt{3x+9} \cdot e^{\sqrt{3x+9}} - e^{\sqrt{3x+9}}\right] + c$

felipi

$u = \sqrt{3x+9}$

$du = (3x+9)^{1/2'} \cdot (3x+9)'\,dx$

$du = \frac{1}{2}(3x+9)^{-1/2} \cdot 3\,dx$

$du = \frac{3}{2\sqrt{3x+9}}\,dx$

$dx = \frac{du}{\frac{3}{2\sqrt{3x+9}}}$

$dx = du \cdot \frac{2\sqrt{3x+9}}{3}$

① $\int e^{u} \cdot u\,du$

$f = u$

$df = u\,d_u$

$df = du$

$u \cdot e^{u} - \int e^{u}\,du$

$u e^{u} - e^{u} + c$

$dv = e^{u}\,du$

$v = e^{u} + c$

(26) $\int_0^1 x\sqrt{1-x}\,dx$

$u = 1-x$

$du = -dx$

$-du = dx$

$u - 1 = -x$

$x = 1-u$

$= -\int_0^1 (1-u) \cdot u^{1/2}\,du$

$= -\int_0^1 (u^{1/2} - u^{3/2})\,du$

$= -\left[\int_0^1 u^{1/2}\,du - \int_0^1 u^{3/2}\,du\right]$

$= -\left[\left(\frac{2}{3}u^{3/2}\right)_0^1 - \left(\frac{2}{5}u^{5/2}\right)_0^1\right]$

$= -\left[\frac{2}{3}\cdot\left(u^{3/2}\right)_0^1 - \frac{2}{5}\left(u^{5/2}\right)_0^1\right]$

$= -\left[\frac{2}{3} - \frac{2}{5}\right]$

$= -\frac{2}{3} + \frac{2}{5} = \frac{4}{15}$

falsch

27 $\displaystyle\int_0^{\pi/3} x\,tg^2x\,dx$

$u = x \quad dv = tg^2x\,dx$

$du = dx \quad v = \int tg^2x\,dx$

$v = -x + tgx + c$

$= \cancel{x} = x + tg\,x - \int -x + tg\,x\,dx$

$= tg\,x + \int x + tg\,x\,dx$

$= \left[tg\,x + \frac{x^2}{2} - \ln|\cos(x)| \right]_0^{\pi/3}$

$= tg(\pi/3) + \frac{(\pi/3)^2}{2} - \ln|\cos(\pi/3)|$

① $\int (x + tg\,x)\,dx$

$= \int x\,dx + \int tg\,x\,dx$

$= \frac{x^2}{2} + (-\ln|\cos(x)|$

$= \frac{x^2}{2} - \ln|\cos(x)| + c$

Felipe

(28) $\int \ln(x+x^2)\,dx$

$$= \ln(x+x^2).x - \int x.\frac{(1+2x)}{(x+x^2)}\,dx$$

$$= \ln(x+x^2).x - \int \frac{x+2x^2}{x+x^2}\,dx$$

$$= \ln(x+x^2).x - \int \frac{1+2x}{1+x}\,dx$$

$$= \ln(x+x^2).x - \int \left(2 - \frac{1}{x+1}\right)dx$$

$$= \ln(x^2+x).x - \left[2\int dx - \int \frac{1}{x+1}\,dx\right]$$

$$= \ln(x^2+x).x - \left[2x - \ln|x+1|\right]$$

$$= \ln(x^2+x).x - 2x + \ln(x+1) + c$$

$u = \ln(x+x^2)$

$du = \frac{1}{(x+x^2)}.(1+2x)\,dx$

$du = \frac{(1+2x)}{(x+x^2)}\,dx$

$dv = 1\,dx$

$v = \int 1\,dx$

$v = x + c$

Felipe