Apa - 28/03/2025 Phillipi Jesus Cardoso

a) $n - 100 \le c(n - 200)$

$\frac{n-100}{n-200} \le c$

$\lim_{n \to \infty} \frac{n-100}{n-100} = \lim_{n \to \infty} \frac{1 - \frac{100}{n}}{1 - \frac{200}{n}} = \frac{1}{1} = 1 \le c \quad 300 \le n$

$f(n-100) = O(g(n-200))$ com $c = 2$ e $n_0 = 300$

Para $c = 2$

$\frac{n-100}{n-200} \le 2$

$n - 100 \le 2(n-200)$

$n - 100 \le 2n - 400$

$-100 + 400 \le 2n - n$

b) $n^{\frac{1}{2}} \le c n^{\frac{2}{3}}$

$\frac{n^{\frac{1}{2}}}{n^{\frac{2}{3}}} \le c$

$n^{\frac{1}{2} - \frac{2}{3}} \le c$

$n^{\frac{3-4}{6}} \le c$

$n^{-\frac{1}{6}} \le c$

$\sqrt[6]{\frac{1}{n}}$

Para $c = 1$ e $n_0 = 1$ $n^{\frac{1}{3}}$ é $O(n^{\frac{1}{3}}$

$n^{\frac{1}{2}}$ é $O(n^{\frac{2}{3}})$

c) $100n + \log n \le c((n + (\log n)^2)$

$\log n$ cresce menos que $100n$ logo $100n$ é o termo dominante, já n também cresce mais que $(\log n)^2$

$100n \log n \le c((n + (\log n)^2))$

$\frac{100n \log n}{n} \le c\, n + \frac{(\log n)^2}{n}$

$\lim_{n \to \infty} \left(100 + \frac{\log n}{n}\right) \le c\left(1 + \frac{(\log n)^2}{n}\right) \lim_{n \to \infty}$

$100 + 0 \le c\ 1 + 0$

$100 \le c$

d) $n \log n \leq C\, 10n \log 10n$

$\frac{n \log n}{n} \leq \frac{C\, 10n \log n + 10n \log(10)}{n}$

$\log n \leq C(10 \log n + 10 \log 10)$

$\frac{\log n}{\log n} \leq C\, 10 \frac{\log n}{\log n}$ $\qquad C = \frac{1}{10}$

$1 \leq 10c$

$\frac{1}{10} \leq C$

$n \log n \leq \frac{1}{10} 10n \log n + 10n \log(10)$

$n \log n \leq n \log n + n \log(10)$

$n \log n - n \log n \leq n \log(10)$

$0 \leq n \log(10)$

Verdade pra $n_0 \geq 1$

e) $\log 2n \leq c \log 3n$

$\log 2 + \log n \leq c \log 3 + \log n$

$\log 2 \leq c \log 3$

$\frac{\log 2}{\log 3} \leq C$ $\qquad C = \frac{\log 2}{\log 3}$

$\log 2 + \log n \leq$

log 2 e log 3 = constantes, sendo assim o g(n) vai sempre ser maior,,

$f(n) = \Theta(g(n))$

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f) $10 \log n \leq c \log n^2$

$10 \log n \leq c\, 2 \log n$

$\frac{10 \log n}{2 \log n} \leq c$

$5 \leq c$

Para $c = 5$ e $n_0 = 1$

$f(n) = O(g(n))$

$10 \log n \leq 5 \cdot 2 \log n$

$10 \log n \leq 10 \log n$

g) $n^{1,01} \leq c\, n \log^2 n$

$\frac{n^{1,01}}{n} \leq \frac{c\, n \log^2 n}{n}$

$n^{0,01} \leq c \log^2 n$

f cresce mais que g, não é $O(g)$

h) $\frac{n^2}{\log n} \leq c\, n (\log n)^2$

$\lim_{n \to \infty} \frac{\frac{n^2}{\log}}{(n \log n)^2} = \lim_{n \to \infty} \frac{n}{\log n^3}$

CRESCIMENTO LINEAR

Logaritmo

$f(n)$ não é $O(g(n))$ pq cresce mais que f.

i) $n^{0,1} \leq c$

k) $\sqrt{n} \leq c (\log n)^3$

$n^{\frac{1}{2}} \leq c (\log n)^3$

$n^{\frac{1}{2}}$ cresce mais rápido que $\log n$.

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l) $n^{\frac{1}{2}} \le C \; 5^{\log n}$ $\qquad f(n) = O(g(n))$

$n^{\frac{1}{2}} \le C \; n^{\frac{\log 5}{\log 2}}$

$n^{\frac{1}{2}} \le C \; n^{2{,}32}$ $\qquad$ g cresce mais que f.

m) $n 2^{n} \le C \, 3^{n}$

$\frac{n 2^{n}}{2^{n}} \le C \frac{3^{n}}{2^{n}}$ $\qquad$ Para $n_0 = 1$ $\quad C = 1$

$n \le C \left(\frac{3}{2}\right)^{n}$ $\qquad 1 \le 1 \cdot \frac{3}{2}$

$1 \le \frac{3}{2}$ $\qquad f(n) = O(g(n))$

n) $2^{n} \le C \, 2^{n+1}$

$\frac{2^{n}}{2^{n+1}} \le C$ $\qquad$ Para $C = \frac{1}{2}$ $\quad n_0 = 1$

$2^{n-n-1} \le C$ $\qquad 2 \le \frac{1}{2} 2^{2}$

$\frac{1}{2} \le C$ $\qquad f(n) = O(g(n))$

$2 \le \frac{4}{2}$

o) $n! \le C 2^{n}$

p) $(\log n) \le C \; 2 (\log n)^{2}$

$\left(\frac{\log_2 n}{\log_2 10}\right)^{\frac{\log_2 n}{\log_2 10}} \le C \; n^{\log_2 n}$

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q1 $\sum_{i=1}^{n} i^k \leq c\, n^{k+1}$ for $k=2$

$i^2 \leq n^3$

$\frac{n(n+1)(2n+1)}{6} \leq c n^3$

$2n^3 + 3n^2 + n \leq c6n^3$

$2 + \frac{3}{n} + \frac{1}{n^2} \leq c6$