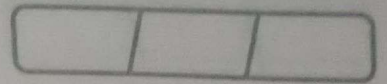


Apa - 28/03/2025

Phillipi Jesus Carlos Pires C=2



a)  $n - 100 \leq C(n - 200)$

$$n - 100 \leq C$$

$$n - 200$$

$$n - 100 \leq 2$$

$$n - 200$$

$$n - 100 \leq 2(n - 100)$$

$$n - 100 \leq 2n - 200$$

$$-100 + 100 \leq 2n - n$$

$$\lim_{n \rightarrow \infty} \frac{n - 100}{n - 200} = \lim_{n \rightarrow \infty} \frac{1 - \frac{100}{n}}{1 - \frac{200}{n}} = \frac{1}{1} = 1 \leq C \quad 200 \leq n$$

$$f(n - 100) = O(g(n - 200)) \text{ com } C=2 \text{ e } n_0 = 300$$

b)  $n^{\frac{1}{2}} \leq Cn^{\frac{2}{3}}$

$$n^{\frac{1}{2}} \leq C$$

$$n^{\frac{1}{2}}$$

$$n^{\frac{1}{2} - \frac{2}{3}} \leq C$$

Para  $C=1$  e  $n_0 = 1n^{\frac{1}{3}}$

$$O(n^{\frac{1}{3}})$$

$$n^{\frac{1}{2}} \text{ é } O(n^{\frac{2}{3}})$$

$$n^{\frac{3}{6} - \frac{4}{6}} \leq C$$

$$n^{-\frac{1}{6}} \leq C$$

$$\sqrt[6]{1}$$

$$n$$

c)  $100n + \log n \leq C(n + (\log n)^2)$

$\log n$  cresce menos que  $100n$  logo  $100n$  é o termo dominante, já  $n$  também cresce mais que  $(\log n)^2$

$$100n + \log n \leq C(n + (\log n)^2)$$

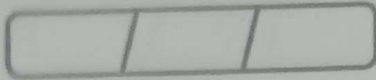
$$100n + \log n \leq C \frac{n + (\log n)^2}{n}$$

$$\lim_{n \rightarrow \infty} \left( 100 + \frac{\log n}{n} \right) \leq C \left( 1 + \frac{(\log n)^2}{n} \right) \lim_{n \rightarrow \infty}$$

$$100 + 0 \leq C \cdot 1 + 0$$

$$100 \leq C$$

FORONI



$$\begin{aligned} d) \quad n \log n &\leq C \cdot 10n \log 10n \\ n \log n &\leq C \cdot 10n \log n + 10n \log(10) \\ \log n &\leq C \cdot 10 \log n + 10 \log(10) \end{aligned}$$

$$\begin{aligned} \frac{\log n}{\log n} &\leq C \cdot 10 \frac{\log n}{\log n} \\ 1 &\leq 10C \\ 1 &\leq C \\ 10 & \end{aligned} \quad C = \frac{1}{10}$$

$$n \log n \leq \frac{1}{10} 10n \log n + 10n \log(10)$$

$$n \log n \leq n \log n + n \log(10)$$

$$\begin{aligned} n \log n - n \log n &\leq n \log(10) \\ 0 &\leq n \log(10) \end{aligned}$$

Verdade para  $n \geq 1$

$$\begin{aligned} e) \quad \log 2n &\leq C \log 3n \\ \log 2 + \log n &\leq C \log 3 + \log n \\ \log 2 &\leq C \log 3 \\ \log 2 &\leq C \\ \log 3 & \end{aligned}$$

$$C = \frac{\log 2}{\log 3}$$

$$\log 2 + \log n \leq$$

$\log 2$  e  $\log 3$  = constantes, sendo assim o  $g(n)$  não sempre ser maior,

$$f(n) = \Theta(g(n))$$

$$f) \quad 10 \log n \leq c \log n^2$$

$$10 \log n \leq c \cdot 2 \log n$$

$$\frac{10 \log n}{2 \log n} \leq c$$

$$5 \leq c$$

Para  $c=5$  ou  $n_0=1$

$$f(n) = O(g(n))$$

$$10 \log n \leq 5 \cdot 2 \log n$$

$$10 \log n \leq 10 \log n$$

$$g) \quad n^{1,01} \leq c n \log^2 n$$

$$\frac{n^{1,01}}{n} \leq c \frac{n \log^2 n}{n}$$

$$n^{0,01} \leq c \log^2 n$$

$f$  cresce mais que  $g$ .

$n$  não é  $O(g)$

$$h) \quad \frac{n^2}{\log n} \leq c n (\log n)^2$$

$$\lim_{n \rightarrow \infty} \frac{\frac{n^2}{\log n}}{n (\log n)^2} = \lim_{n \rightarrow \infty} \frac{n}{\log n^3}$$

Logaritmo

$f(n)$  não é  $O(g(n))$  pq cresce mais que  $g$ .

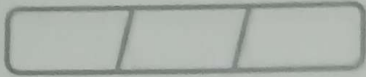
$$i) \quad n^{0,2} \leq c$$

$$k) \quad \sqrt{n} \leq c (\log n)^3$$

$$n^{\frac{1}{2}} \leq c (\log n)^3$$

$n^{\frac{1}{2}}$  cresce mais rápido que  $\log n$ .





$$l) n^{\frac{1}{2}} \leq C \cdot 5 \lg n$$

$$n^{\frac{1}{2}} \leq C \cdot n^{\frac{1}{2}}$$

$$n^{\frac{1}{2}} \leq C \cdot n^{\frac{1}{2}}$$

$$f(n) = O(g(n))$$

g cresce mais que f.

$$m) n^{2^n} \leq C \cdot 3^n$$

$$\frac{n^{2^n}}{2^n} \leq C \cdot \frac{3^n}{2^n}$$

$$n \leq C \left(\frac{3}{2}\right)^n$$

$$\text{Para } n_0 = 1 \quad C = 1$$

$$1 \leq \frac{1 \cdot 3}{2}$$

$$\frac{1 \leq 3}{2}$$

$$f(n) = O(g(n))$$

$$n) 2^n \leq C \cdot 2^{n+1}$$

$$\frac{2^n}{2^{n+1}} \leq C$$

$$2^{n-n-1} \leq C$$

$$\frac{1}{2} \leq C$$

$$\text{Para } C = \frac{1}{2}$$

$$n_0 = 1$$

$$\frac{2 \leq 1 \cdot 2^2}{2}$$

$$\frac{2 \leq 4}{2}$$

$$f(n) = O(g(n))$$

$$o) n! \leq C \cdot 2^n$$

$$p) (\lg n) \leq C \cdot 2 (\lg n)^2$$

$$\left( \frac{\lg_2 n}{\lg_2 10} \right) \frac{\lg_2 n}{\lg_2 10} \leq C \cdot n \cdot \lg_2 n$$

Q 1

$$\sum_{i=1}^n i^k \leq C n^{k+1} \quad \text{for } k=2$$

$$\frac{i^2}{6} \leq n^3$$

$$\frac{n(n+1)(2n+1)}{6} \leq C n^3$$

$$2n^3 + 3n^2 + n \leq C 6n^3$$

$$\frac{2}{n} + \frac{3}{n^2} + \frac{1}{n^3} \leq C 6$$