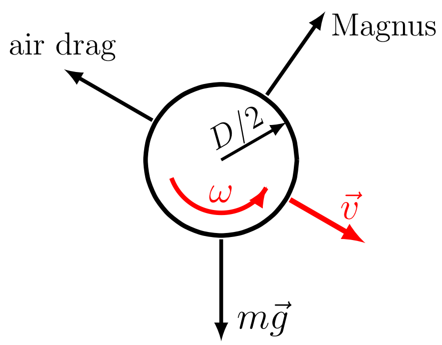
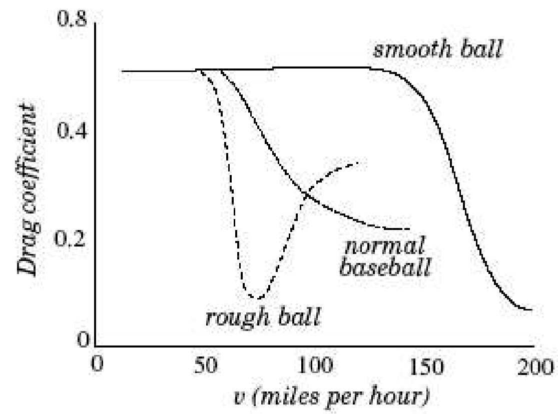
# Chapter 2. Projectiles and Charged Particles

## 2.1 An Example of Projectile Motion — Baseball

There are three forces on a moving and spinning baseball: gravity, air resistance (drag), and Magnus (left) force.




### Air Resistance — Air Drag

An empirical formula of the air drug on a projectile, in general, from wind tunnel experiments is

$$\vec{F}_D \;=\; -3\pi D\mu\,\vec{v} - \frac{1}{2}\pi (D/2)^2 \rho\, C_D v\,\vec{v} \;=\; -(f_{lin} + f_{quad})\,\hat{v} \tag{1}$$

where $\hat{v} = \vec{v}/|\vec{v}|$, $\mu$ is the viscosity of the air, $\rho$ is the air density, and $C_D$ is the drag coefficient. The 1st term $f_{lin}$ in the air drag is linearly proportional to the speed $v$ when $v$ is much smaller than the speed of sound ($\sim$ 770 mph) and it is due to the viscous drag of the medium. The 2nd term $f_{quad}$ is due to the collision between a moving object and the particles in the medium, which is proportional to $v^2$ when $v$ is not too big.

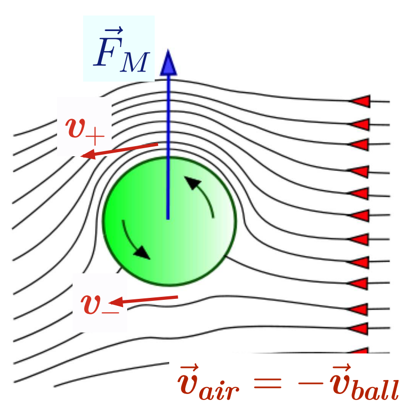
**Cases of Relative Importance of Linear and Quadric Drags**

| | $D$ | $v$ | $f_{quad}/f_{lin} \sim Dv$ | Important Air Drag |
|---|---|---|---|---|
| baseball | 7 cm | 40 m/s | $\sim 10^3$ | quadratic drag |
| raindrop | 1 mm | 0.6 m/s | $\sim 1$ | both linear & quadratic drag |
| Millikan oil drop | 1.5 $\mu$m | $5\times 10^{-5}$ m/s | $\sim 10^{-7}$ | linear drag |

### Magnus Force—Left

The Magnus force is a dynamic left force due to a pressure difference between two sides of a spinning and moving ball. Consider a situation that the angular velocity of a ball is perpendicular to its linear velocity, $\vec{\omega} \perp \vec{v}$. The Bernoulli's equation for the air flow above and below the baseball is

$$P_+ + \frac{1}{2}\rho\, v_+^2 = P_- + \frac{1}{2}\rho\, v_-^2 \tag{2}$$



where $\rho$ is the density of air, $P_\pm$ and $v_\pm$ are the air pressure and the air speed with respect to the ball above (+) and below (−) the ball, respectively,

$$v_\pm = v \pm \alpha(D/2)\omega \qquad \text{and} \qquad F_M = \pi(D/2)^2(P_- - P_+) = \rho\alpha D\omega v$$

where $\alpha < 1$ is a drag coefficient in boundary layer. In general, the direction of Magnus force is in the direction of $\vec{\omega} \times \vec{v}$ and

$$\vec{F}_M = \gamma(\vec{\omega} \times \vec{v}) \qquad \text{where} \quad \gamma = \pi\alpha\rho D^3/4 \tag{3}$$

Including both the air drag and the Magnus force, the equation of motion for a projectile

$$\frac{d\vec{v}}{dt} = m\vec{g} + \vec{F}_D + \vec{F}_M \qquad \Longrightarrow \qquad \left\{ \begin{array}{lcl} \dot{v}_x &=& -f(v)v_x + \gamma(\omega_y v_z - \omega_z v_y) \\ \dot{v}_y &=& -f(v)v_y + \gamma(\omega_z v_x - \omega_x v_z) - g \\ \dot{v}_z &=& -f(v)v_z + \gamma(\omega_x v_y - \omega_y v_x) \end{array} \right. \tag{4}$$

has to be solved numerically, where $\vec{F}_D = -f(v)\vec{v}$.

To simplify the problem, in the following, we consider only the case that the spin of an object and, therefore, the Magnus force is negligible.

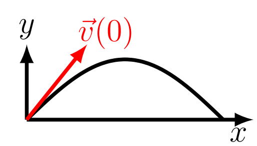
## 2.2 Linear Air Resistance

Without the spin of an object and if the coordinate is chosen in such a way that the initial velocity of the object is in the $x$–$y$ plane, the projectile motion is a two-dimensional motion. In the case of a dominate linear air drag, we only need

$$\vec{F}_D \;=\; -b\,\vec{v} \tag{5}$$

where $b$ is a constant, for the air resistance and the equation of motion can be written as

$$m\,\dot{\vec{v}} = m\,\vec{g} - b\,\vec{v} \qquad \Longrightarrow \qquad \left\{ \begin{array}{l} m\dot{v}_x = -bv_x \\ m\dot{v}_y = -mg - bv_y \end{array} \right. \tag{6}$$




The solution of the $x$-component equation can be obtained as

$$m\frac{dv_x}{dt} = -bv_x \qquad \Longrightarrow \qquad \frac{dv_x}{v_x} = -(b/m)dt \qquad \Longrightarrow \qquad \ln(v_x) - \ln(v_x(0)) = -(b/m)t$$

Therefore

$$\left\{ \begin{array}{lcl} v_x(t) &=& v_x(0)e^{-t/\tau} \\ x(t) &=& x(0) + \displaystyle\int_0^t v_x(t)dt \;=\; x(0) + v_x(0)\tau\left(1 - e^{-t/\tau}\right) \end{array} \right. \tag{7}$$

where $\tau = m/b$ is the damping time scale. The solution of the $y$-component equation can be obtained similarly as

$$\tau\frac{dv_y}{dt} = -v_y - g\tau \qquad \Longrightarrow \qquad \frac{dv_y}{v_y + g\tau} = -\frac{dt}{\tau} \qquad \Longrightarrow \qquad \ln(v_y + g\tau) - \ln(v_y(0) + g\tau) = -\frac{t}{\tau}$$

With the terminal speed being defined as

$$v_{ter} = g\tau = mg/b \tag{8}$$

the solution of the vertical component of motion can be written as

$$\begin{cases} v_y(t) &= -v_{ter} + [v_y(0) + v_{ter}]\, e^{-t/\tau} \overset{t>>\tau}{\geq} -v_{ter} \\ y(t) &= y(0) + \int_0^t v_y(t)dt \;=\; y(0) - v_{ter}t + [v_y(0) + v_{ter}]\,\tau \left(1 - e^{-t/\tau}\right) \end{cases} \tag{9}$$

For $\tau >> 1$ and $t << \tau$, the air drag is negligible,

$$e^{-t/\tau} \sim 1 - t/\tau + (t/\tau)^2/2$$

The motion is then reduced to the projectile motion without the air resistance,

$$\begin{cases} v_x(t) &= v_x(0)e^{-t/\tau} \;\simeq\; v_x(0) \\ v_y(t) &= -v_{ter} + [v_y(0) + v_{ter}]\, e^{-t/\tau} \;\simeq\; -v_{ter} + [v_y(0) + v_{ter}]\,(1 - t/\tau) \\ &\simeq v_y(0) - [v_y(0) + g\tau]\,(t/\tau) \;\simeq\; v_y(0) - gt \\ x(t) &= x(0) + v_x(0)\tau \left(1 - e^{-t/\tau}\right) \;\simeq\; x(0) + v_x(0)t \\ y(t) &= y(0) - v_{ter}t + [v_y(0) + v_{ter}]\,\tau \left(1 - e^{-t/\tau}\right) \\ &\simeq y(0) - v_{ter}t + [v_y(0) + v_{ter}]\left[t - (t^2/\tau)/2\right] \\ &\simeq y(0) + v_y(0)t - gt^2/2 \end{cases}$$

### 2.4 Quadratic Air Resistance

When the initial velocity is in the $x$-$y$ plane, the motion of an object with the gravity and the air drag is a two-dimensional motion. In the case of a dominate quadratic air drag, the air resistance is

$$\vec{F}_D \;=\; -c\left(v_x^2 + v_y^2\right)^{1/2} \vec{v} \tag{10}$$

where $\vec{F}_D$ is a quadratic force along the opposite direction of $\vec{v}$ and $c$ is a constant. The equation of motion is

$$m\,\dot{\vec{v}} = m\,\vec{g} + \vec{F}_D \qquad \Longrightarrow \qquad \begin{cases} m\dot{v}_x = -c\left(v_x^2 + v_y^2\right)^{1/2} v_x \\ m\dot{v}_y = -c\left(v_x^2 + v_y^2\right)^{1/2} v_y - mg \end{cases} \tag{11}$$

This is a coupled two-dimensional nonlinear ODE and we don't know how to solve it analytically. It can, however, be solved numerically. A special case that can be solved analytically is when the initial velocity is along the vertical direction. When $\vec{v} = v_y\vec{e}_y$, the net force $m\,\vec{g} + \vec{F}_D$ is along the vertical direction and the object moves only along the vertical direction. The equation of motion is then reduced to a one-dimensional ODE problem that is always solvable analytically. In textbook, another case of one-dimensional motion when an object moves along the horizontal direction is also considered. With the presence of the gravity, however, an object cannot be restricted to move only horizontally and one-dimensional horizontal motion is not possible in reality.

**One-Dimensional Vertical Motion When** $\vec{v}(0) = v_y(0)\vec{e}_y$

The equation of motion of an object moving along the vertical direction with a quadratic air drag is

$$\dot{\vec{v}}_y = -c|v_y|v_y - mg \tag{12}$$

In the following, we consider the cases of an object moving downward ($v_y < 0$) and upward ($v_y > 0$), separately.

**(a) Object moves downward,** $\vec{v} = -v\vec{e}_y$ with $v = |\vec{v}|$

The equation of motion in this case becomes

$$m\frac{dv_y}{dt} = -mg + cv_y^2 \qquad \overset{v_y=-v}{\Longrightarrow} \qquad m\frac{dv}{dt} = mg - cv^2 \tag{13}$$

where the gravity and the air drag are in the opposite direction. When the magnitudes of the forces equal each other, the net force vanishes and the vertical speed becomes a constant, $v = V_{ter}$. This terminal speed is

$$mg - cv_{ter}^2 = 0 \qquad \Longrightarrow \qquad v_{ter} = \sqrt{mg/c} \tag{14}$$

The motion in the vertical direction can be solved from

$$\frac{dv}{dt} = g\left(1 - \frac{c}{mg}v^2\right) \qquad \Longrightarrow \qquad \frac{dv}{1-(v/v_{ter})^2} = g\,dt \tag{15}$$

**When** $v(0) < v_{ter}$

In this case, the net force in Eq. (13) points downward and the speed of the object increases until reaches the terminal speed. The first integral of motion solved from Eq. (15) is

$$g\,t = \int_{v(0)}^{v(t)} \frac{dv}{1-(v/v_{ter})^2} \overset{v\le v_{ter}}{=} v_{ter}\left[\tanh^{-1}\left(\frac{v(t)}{v_{ter}}\right) - \tanh^{-1}\left(\frac{v(0)}{v_{ter}}\right)\right] \tag{16}$$

For $v(0) = 0$, Eq. (16) yields

$$g\,t = v_{ter}\tanh^{-1}\left(\frac{v(t)}{v_{ter}}\right) \qquad \Longrightarrow \qquad v(t) = v_{ter}\tanh\left(\frac{g\,t}{v_{ter}}\right) \tag{17}$$

and the vertical position of the object, the 2nd integral of motion, is

$$y(t) = y(0) - v_{ter}\int_0^t \tanh\left(\frac{g\,t}{v_{ter}}\right)dt = y(0) - \frac{v_{ter}^2}{g}\ln\left[\cosh\left(\frac{g\,t}{v_{ter}}\right)\right] \tag{18}$$

**When $v(0) > v_{ter}$**

The net force in Eq. (13) points upward, the opposite direction to the velocity, and the speed of the object decreases until reaches the terminal speed. Afterward, the object drops with the terminal speed. The first integral of motion solved from Eq. (15) can be calculated as

$$\begin{aligned} g\,t &= \int_{v(0)}^{v(t)} \frac{dv}{1-(v/v_{ter})^2} = \frac{v_{ter}}{2}\int_{\xi(0)}^{\xi(t)} \left(\frac{1}{\xi+1} - \frac{1}{\xi-1}\right) d\xi \overset{\xi\geq 1}{=} \frac{v_{ter}}{2} \ln\left(\frac{\xi+1}{\xi-1}\right)\Bigg|_{\xi(0)}^{\xi(t)} \\ &= \frac{v_{ter}}{2}\left[\ln\left(\frac{v(t)+v_{ter}}{v(t)-v_{ter}}\right) - \ln\left(\frac{v(0)+v_{ter}}{v(0)-v_{ter}}\right)\right] \qquad \text{with } \xi = v/v_{tar} \end{aligned}$$

which yields

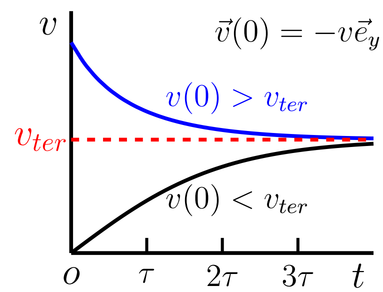
$$\left(\frac{v(t)-v_{ter}}{v(t)+v_{ter}}\right) = \alpha e^{-(2g/v_{tar})t}$$

or

$$v(t) = v_{ter}\left(\frac{1+\alpha\, e^{-t/\tau}}{1-\alpha\, e^{-t/\tau}}\right) \xrightarrow{t>>\tau} v_{ter} \tag{19}$$

where $\alpha = (v(0)-v_{ter})/(v(0)+v_{ter})$ and

$$\tau = v_{tar}/2g = \sqrt{m/4gc} \tag{20}$$




**(b) Object moves upward, $\vec{v} = v\vec{e}_y$ with $v = |\vec{v}|$**

The equation of motion in this case becomes

$$m\frac{dv_y}{dt} = -mg - cv_y^2 \overset{v_y=v}{\Longrightarrow} \frac{dv}{dt} = -g\left(1+\frac{v^2}{v_{ter}^2}\right) \Longrightarrow \frac{dv}{1+(v/v_{ter})^2} = -g\,dt$$

where $v_{ter} = \sqrt{mg/c}$. In this case, the air drag and the gravity are in the same direction and $v_{ter}$ does not have the meaning of the terminal speed. The first integral of motion is then

$$-g\,t = \int_{v(0)}^{v(t)} \frac{dv}{1+(v/v_{ter})^2} = v_{ter}\left[\arctan\left(\frac{v(t)}{v_{ter}}\right) - \arctan\left(\frac{v(0)}{v_{ter}}\right)\right] \tag{21}$$

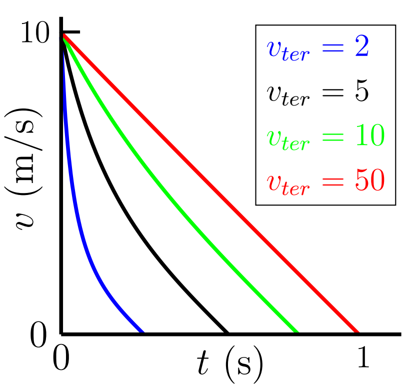
which yields

$$v(t) = v_{ter}\tan\left(\beta - \frac{gt}{v_{ter}}\right)$$

where

$$\beta = \arctan\left(\frac{v(0)}{v_{ter}}\right)$$

Note that the initial vertical velocity $v(0)$ cannot be zero in this case, otherwise the object cannot move upward.



When $v(t) = 0$, *i.e.*

$$\tan(\beta - gt/v_{ter}) = 0 \qquad \Longrightarrow \qquad t = \beta v_{ter}/g$$

the object reaches its maximal height. When the air drag is negligible, *i.e.*

$$c \to 0 \qquad \Longrightarrow \qquad v_{ter} = \sqrt{mg/c} \longrightarrow \infty$$

the motion is reduced the projectile without the air resistance,

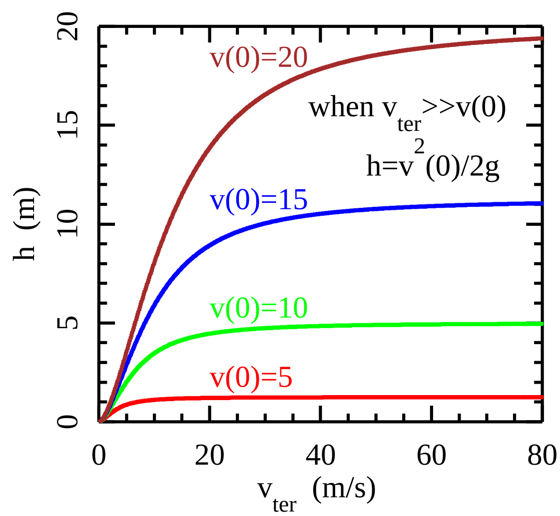
$$v(t) \;=\; \lim_{v_{ter}\to\infty} \left[v_{ter}\tan(\beta - gt/v_{ter})\right] \;=\; v(0) - gt$$

The vertical position of the object can be calculated from $v(t)$ as

$$\begin{aligned} y(t) &= y(0) + v_{ter}\int_0^t \tan\left(\beta - \frac{g\,t}{v_{ter}}\right) dt \\ &= y(0) + \frac{v_{ter}^2}{g}\ln\left[\frac{\cos(\beta - g\,t/v_{ter})}{\cos(\beta)}\right] \end{aligned}$$

and the maximal height is at $\beta - g\,t/v_{ter} = 0$,

$$h \;=\; y_{max} - y(0) \qquad = \qquad -\frac{v_{ter}^2}{g}\ln\left[\cos(\beta)\right] \qquad \xrightarrow{v_{ter}>>v(0)} \qquad v^2(0)/2g$$




## 2.5 Motion of a Charged Particle in a Uniform Magnetic Field

For a charged particle inside a uniform magnetic field, the equation of motion is

$$m\dot{\vec{v}} = q\vec{v} \times \vec{B} \tag{22}$$

Since the $\vec{B}$-field is a uniform and constant field, the coordinate can be chosen to align $\vec{B}$ with $\vec{e}_z$ and therefore

$$\vec{v} \times \vec{B} \;=\; (v_x\vec{e}_x + v_y\vec{e}_y + v_z\vec{e}_z) \times B\vec{e}_z \;=\; v_yB\,\vec{e}_x - v_xB\,\vec{e}_y$$

The equation of motion then becomes

$$m\frac{dv_x}{dt} = qB\,v_y\,, \qquad m\frac{dv_y}{dt} = -qB\,v_x\,, \qquad m\frac{dv_z}{dt} = 0 \tag{23}$$

The motion in the direction of $\vec{B}$ is with a constant velocity, $v_z = constant$. In the $x$-$y$ plane, the equation of motion can be decoupled as

$$\left\{\begin{aligned} \frac{d^2v_x}{dt^2} &= \left(\frac{qB}{m}\right)\frac{dv_y}{dt} = -\left(\frac{qB}{m}\right)^2 v_x \\ \frac{d^2v_y}{dt^2} &= -\left(\frac{qB}{m}\right)\frac{dv_x}{dt} = -\left(\frac{qB}{m}\right)^2 v_y \end{aligned}\right. \qquad \Longrightarrow \qquad \begin{gathered} \frac{d^2u}{dt^2} = -\omega^2 u \\ \text{with } \omega = qB/m \end{gathered} \tag{24}$$

where $\omega$ is called cyclotron frequency. Both $v_x$ and $v_y$ are in an identical harmonic oscillation, called cyclotron oscillation, that can be solved as

$$\left\{\begin{array}{rcl} v_x(t) &=& v_x(0)\cos(\omega t) + \dfrac{\dot{v}_x(0)}{\omega}\sin(\omega t) \\ v_y(t) &=& v_y(0)\cos(\omega t) + \dfrac{\dot{v}_y(0)}{\omega}\sin(\omega t) \end{array}\right. \tag{25}$$

Since

$$\left\{\begin{array}{rl} \dfrac{dv_x}{dt} =& \omega\, v_y \\ \dfrac{dv_y}{dt} =& -\omega\, v_x \end{array}\right. \qquad \Longrightarrow \qquad \left\{\begin{array}{rl} \dot{v}_x(0) =& \omega\, v_y(0) \\ \dot{v}_y(0) =& -\omega\, v_x(0) \end{array}\right.$$

the solution of the motion on the $x$-$y$ planes is then

$$\left\{\begin{array}{rcccl} v_x(t) &=& v_x(0)\cos(\omega t) + \dfrac{\dot{v}_x(0)}{\omega}\sin(\omega t) &=& v_x(0)\cos(\omega t) + v_y(0)\sin(\omega t) \\ v_y(t) &=& v_y(0)\cos(\omega t) + \dfrac{\dot{v}_y(0)}{\omega}\sin(\omega t) &=& -v_x(0)\sin(\omega t) + v_y(0)\cos(\omega t) \end{array}\right. \tag{26}$$

This is a rotation on the $x$-$y$ plane with the rotational frequency $\omega$,

$$\begin{pmatrix} v_x(t) \\ v_y(t) \end{pmatrix} = \begin{pmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{pmatrix} \begin{pmatrix} v_x(0) \\ v_y(0) \end{pmatrix} = \mathbf{T}_1(\omega t) \begin{pmatrix} v_x(0) \\ v_y(0) \end{pmatrix} \tag{27}$$

where the speed of the rotation $v = \sqrt{v_x^2 + v_y^2}$ is constant, since

$$\begin{aligned} v_x^2(t) + v_y^2(t) &= \begin{pmatrix} v_x(t) & v_y(t) \end{pmatrix} \begin{pmatrix} v_x(t) \\ v_y(t) \end{pmatrix} = \begin{pmatrix} v_x(0) & v_y(0) \end{pmatrix} \mathbf{T}_1^T \mathbf{T}_1 \begin{pmatrix} v_x(0) \\ v_y(0) \end{pmatrix} \\ &= \begin{pmatrix} v_x(0) & v_y(0) \end{pmatrix} \begin{pmatrix} v_x(0) \\ v_y(0) \end{pmatrix} = v_x^2(0) + v_y^2(0) \end{aligned}$$

The trajectory of the charge can be easily integrated from the velocity as

$$\left\{\begin{array}{rcl} x(t) &=& x(0) + \dfrac{v_x(0)}{\omega}\sin(\omega t) - \dfrac{v_y(0)}{\omega}\cos(\omega t) \\ y(t) &=& y(0) + \dfrac{v_x(0)}{\omega}\cos(\omega t) + \dfrac{v_y(0)}{\omega}\sin(\omega t) \\ z(t) &=& z(0) + v_z(0)\, t \end{array}\right. \tag{28}$$

Along the $z$ axis, the charge moves with a constant speed $v_z$. To easily understand the trajectory on the $x$-$y$ plane, we rewrite the trajectory on the $x$-$y$ plane into a matrix form,

$$\begin{aligned}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} &= \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} + \frac{1}{\omega}\begin{pmatrix} \sin(\omega t) & -\cos(\omega t) \\ \cos(\omega t) & \sin(\omega t) \end{pmatrix}\begin{pmatrix} v_x(0) \\ v_y(0) \end{pmatrix} \\ &= \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} + \mathbf{T}_2(\omega t)\begin{pmatrix} v_x(0) \\ v_y(0) \end{pmatrix}\end{aligned}$$

The distance of the charge from it's initial position $(x(0), y(0))$ on the $x$-$y$ plane can then be easily calculated as

$$\begin{aligned}\rho^2 &= [x(t) - x(0)]^2 + [y(t) - y(0)]^2 = \begin{pmatrix} x(t) - x(0) & y(t) - y(0) \end{pmatrix}\begin{pmatrix} x(t) - x(0) \\ y(t) - y(0) \end{pmatrix} \\ &= \begin{pmatrix} v_x(0) & v_y(0) \end{pmatrix}\mathbf{T}_2^T\mathbf{T}_2\begin{pmatrix} v_x(0) \\ v_y(0) \end{pmatrix} = \frac{1}{\omega^2}\begin{pmatrix} v_x(0) & v_y(0) \end{pmatrix}\begin{pmatrix} v_x(0) \\ v_y(0) \end{pmatrix} = \frac{v^2}{\omega^2}\end{aligned}$$

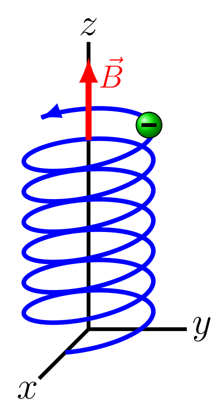
where

$$\mathbf{T}_2^T\mathbf{T}_2 = \frac{1}{\omega^2}$$

The motion on the $x$-$y$ plane is therefore a uniform circular motion with a constant radius $\rho$ and a constant speed $v$, where

$$\rho = \frac{v}{\omega} = \frac{mv}{qB} \qquad \Longleftarrow \qquad m\frac{v^2}{\rho} = qvB \tag{29}$$

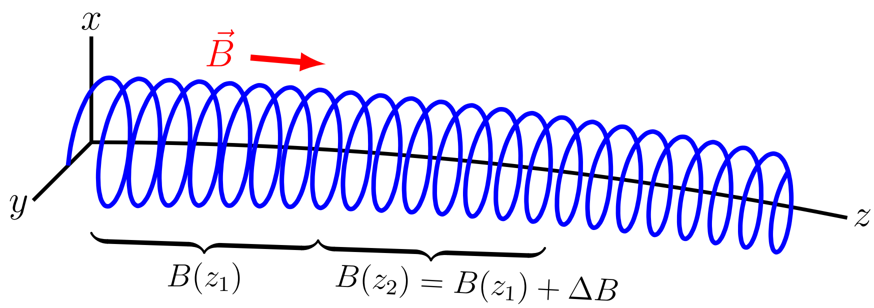
This uniform circular motion on the $x$-$y$ plane plus a linear motion with a constant speed in the $z$ direction forms a helix motion around the magnetic field lines.




**Adiabatic Approximation When Magnetic Field Varys Slowly in Space**

For the case of a charged particle in a non-uniform magnetic field, solving the equation of motion is complicated in general. However, if the change of a $\vec{B}$-field is sufficiently slow compared to the time scale of the cyclotron motion in the $x$-$y$ plane, the result in Eq. (28) for a uniform $B$-field can still be applied piecewisely for the change of a $\vec{B}$-field — adiabatic approximation.

As an example, we consider the strength of the magnetic field changes slowly along the $z$ axis, *i.e.* $\vec{B} = B(z)\,\vec{e}_z$.




The percentage change of the magnetic field in a small interval $\Delta z = v_z \Delta t$ is

$$\frac{\Delta B}{B} = \frac{B(z_2) - B(z_1)}{B} = \frac{1}{B}\frac{\partial B}{\partial z}\Delta z = \frac{1}{B}\frac{\partial B}{\partial z} v_z \Delta t$$

where $B(z_2) = B(z_1)(1 + \Delta B/B)$ and

$$\text{the number of the cyclotron rotations during } \Delta t = \frac{\omega \Delta t}{2\pi}$$

The adiabatic approximation is valid when

$$\frac{\Delta B}{B} << \frac{\omega \Delta t}{2\pi} \qquad \Longrightarrow \qquad \frac{\partial B}{\partial z} << \frac{\omega B}{2\pi v_z} = \frac{qB^2}{2\pi m v_z}$$

*i.e.* there are many cyclotron rotations occurring before there is a noticeable change of the magnetic field. With the adiabatic approximation, the motion of a charged particle in a slowly-varying field can be approximated by the solution with a uniform-field in Eqs. (26) and (28) with the cyclotron frequency and rotation radius changing slowly with the magnetic field as

$$\omega = \frac{qB(z)}{m} \qquad \text{and} \qquad \rho = \frac{mv}{qB(z)}$$

**Homework for Chapter 1**

| Assig. | Problem | Covered Subject |
|---|---|---|
| 2.1 | 2.5, 2.6, 2.7, 2.8, 2.12 | Solve 1D Newton's equation |
| 2.2 | 2.16, 2.19, 2.29, 2.39, 2.41, 2.42 | Projectiles |