Noise Detection and Dependence

Felipe Tala*
University of Kansas \\
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In this experiment, we explore the fundamentals of Johnson noise. In a few words, Johnson noise is present everywhere, and an issue most electronics have to deal with. Companies spend thousands of dollars trying to get rid of Johnson noise between their antennas, MRIs, sensors, and many other electronic components. By utilizing an oscilloscope, a multimeter, a low-level electronics device*, and a high-level electronics device**, we measured the voltage through a non inverting amplifier, caused by the gain of the amplifier and the presence of Johnson noise through it. This paper studies the relationship between Johnson Noise Voltage and resistance, gain, low-pass frequency, bandwidth and temperature. These were observed to have a sigmoid, decaying logarithmic, exponential, and linear relationship with Johnson Noise respectively.

I. INTRODUCTION

Every student who has taken a basic course on electrical circuits knows about Ohm's Law, $V = IR$, V being the voltage change in a resistor, I being the current that goes through it, and R being the component's resistance. They also know that resistors are DC components. Therefore, if $I = 0$, there will be no voltage drop at the resistor. Well, you have been lied to all these years.

In 1926, experimental physicist John Johnson working in the physics division at Bell Labs was researching noise in electronic circuits. He discovered that there was an irreducible low level of noise in resistors whose power was proportional to temperature. Harry Nyquist, a theorist in that division, got interested in the phenomenon and developed an elegant explanation based on fundamental physics.[1]

The concept of Johsnon noise can be described as the thermal agitation of electrons in a resistor which give rise to random fluctuations in the voltage across its terminals. Johnson Noise is the mean-square electromotive force in conductors due to thermal agitation of the electromagnetic modes which are coupled to the thermal environment by the charge carriers.[2]

It was this breakthrough and understanding of Noise that let companies in the US spend thousands of dollars nowadays to reduce their noise margins and achieve the most effective, fast responding electronics devices. Engineers and Physicists have to deal with noise in transistors for phones, computers and smart-watches, in radars, and in transmission lines, just to name a few areas. It would be hard to imagine how much would we have advanced as a species if Johnson-Nyquist noise had never been discovered.

Yet, as much as professionals have to deal with noise in different areas, most of them have never measured the irreducible value of Johnson noise. This paper explores Johnson Noise in greater detail.

* Also at Physics Department, University of Kansas; felipe.tala@ku.edu

II. METHOD

A. Setup

To make Johnson noise measurements, we utilized an oscilloscope, a digital multimeter (DMM), a low-level electronics device (LLE), composed of a pre-amplifier, a temperature module, and a lower half of panel; and a high-level electronics device (HLE), composed of two filters, a gain section, a multiplier, and an output section, and a dewar. The following figures show us the complete apparatus setup, the low-level electronics interior setup, and the high-level electronics parts respectively.

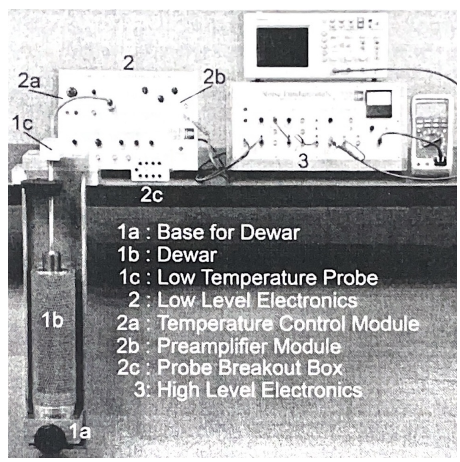

FIG. 1. Complete experiment Setup

Figure 1 displays the full setup of the devices used in the experiment, including the HLE, LLE, oscilloscope and DMM.

In Fig. 2 we can observe that the LLE is composed of two potetiometers, labelled R_{in} and R_f, and a variable capacitor labelled C_f. These are interconnected by two operational amplifiers (op-amps). The LLE circuit can be simplified to a version of a pre-amplifier. We can also observe that the temperature transducer and heater is composed by a voltage follower op-amp, a current source and a monitor.

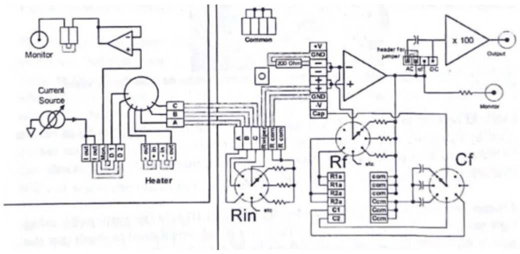

FIG. 2. Temperature transducer and heater, and default condition of the interior of the LLE

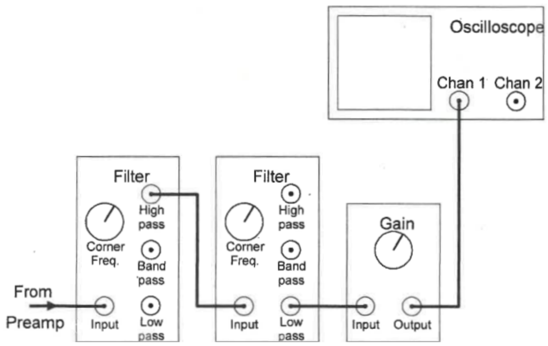

FIG. 3. Cabling diagram of high-level electronics

Figure 3 shows the wiring between the HLE and the oscilloscope for the first measurements of Johnson noise. The low-pass and high-pass filters filter to 0.1 to 100 kHz bandwidth, and the gain part of the HLE scales up the voltage generated from the Johnson noise, making it visible to the naked eye in the oscilloscope.

[3] [4]

B. Measurement Method

To make the first measurements, we made sure that the LLE and HLE were interconnected correctly, and the HLE was connected correctly to the oscilloscope. We continued to set a gain of 300 in the gain device part (Fig. 3). Finally, we connected the HLE gain to a multiplier section that was part of the HLE. 3 sided probe to connect the gain and multiplier to the oscilloscope output channel 1. The HLE output was connected to a digital multimeter. Finally, we connected the LLE to the HLE by connecting the output of the preamplifier to the input of the first HLE filter. Regarding the LLE, we connected its output terminal to the input of the first filter in Fig. 3

We continued to turn on all devices, and set the HLE gain to 300. We changed the values for R_f and R_{in} to observe the change in behavior in the noise output as a function of the preamplifier resistor values. The relationship between the Johnson noise voltage output and the resistance of R_f and R_{in} is given by the following equation:

$$< V_j^2(t) >= 4k_b RT\Delta f \tag{1}$$

In this formula, $V_j^2(t)$ is the Johnson noise measured in volts, k_b is Boltzmann's constant, T is the absolute temperature of the resistor, and Δf is the novel factor (the bandwidth used in the measurements electronics). For this case, R is R_{in}, the input resistance. R_f in the LLE is meant to affect the gain of the circuit, which works together with the HLE gain to allow us to observe a clear Johnson noise signal in the oscilloscope.

We continued to make measurements to find the relationship between the input resistance and the noise voltage. The relationship between the average output voltage and the Johnson noise voltage is given by the following formula:

$$< V_j^2(t) >= \frac{10V_{avg}}{G_1^2 * G_2^2} \tag{2}$$

Where $< V_j^2(t) >$ is the square root of the average Johsnon noise voltage and V_{avg} is the average output voltage from the HLE. G_1 is the first gain given by the LLE, being a constant of 6 for $R_f = 1k\Omega$, and G_2 is the second gain manually set in the HLE. We tracked the Johnson Noise Voltage values by measuring the amplified voltage with a digital multimeter. This voltage measuring method was utilizied for all of this lab's experiments.

For our low-pass frequency measurements, we varied kept all values constant, with $10k$ for R_f, and $1k$ for R_{in}, and continued to vary the low-pass frequency knob observed in the HLE in Fig. 3 to take measurements. This same process was repeated for the bandwidth measurements, with the difference that in the second case both high-pass and low-pass filter knobs observed in Fig. 3 were varied. For our temperature measurements, we utilized around 2 liter of liquid nitrogen. Because liquid nitrogen has an average temperature of 77 Kelvin, it increases the range of temperature for the Noise voltage measurements for R_f. This allows us to study the relationship between temperature and noise voltage. We utilized the liquid nitrogen to fill up the dewar observed in Fig. 1. We continued by measuring the resistance of R_f resistor at a cold resistance value of $10k\Omega$, meaning its resistance when the resistor is immersed in liquid nitrogen, found to be $9.980k\Omega$. We continued to vary the temperature inputted into the HLE through the dewar, and record the output amplified voltage in observed in the HLE. Having the resistance fixed at $10k\Omega$, and the bandwidth fixed at $10kHz$, we could observe a direct relationship between the Johnson Noise Voltage and the temperature, applying II B

For these last temperature measurements, we had to vary the temperature for the heater that warmed up the liquid nitrogen in the dewar. We also varied the bandwidth in the HLE to make measurements of the relationships of both bandwidth and temperature with respect to

Johnson Noise Voltage. The DC voltage measurements were also take with the digital multimeter, and then converted into Johsnon Noise values following II B.

III. RESULTS

A. Input Resistance vs Johnson Noise

For our Johsnon noise voltage and input resistance measurements we got the following measurements.

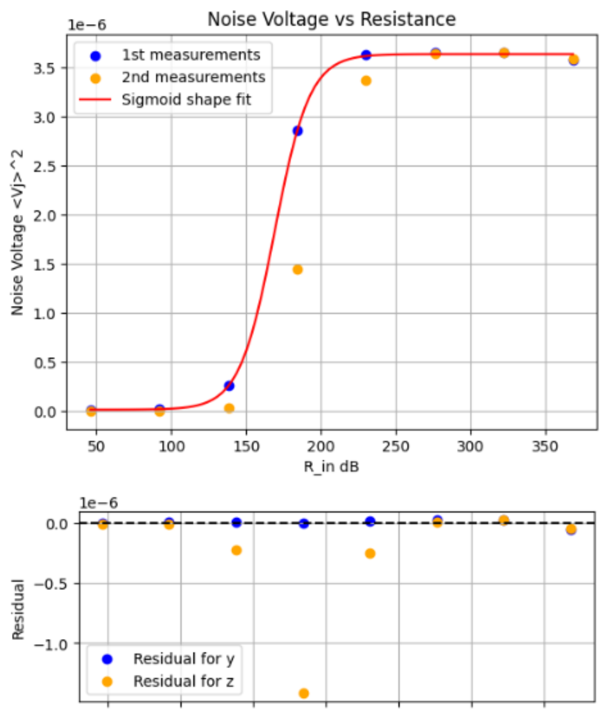

FIG. 4. Noise Voltage vs Resistance

In the figures above, the first figure shows a sigmoid relationship between our input resistance and our output Johnson noise voltage for 2 sets of measurements. The values of R_{in} were varied from $10\Omega to 100 M\Omega$. We made a dB plot to better display the resistance values range. The sigmoid relationship reflects the behavior of a high-pass filter. The second and third plots display the residuals for the first and second set of measurements. For both sets of measurements the room's temperature and humidity were measured with a thermometer. There was a temperature of $69.3°F$ and a humidity of 23%. The low-pass frequency for these measurements was of $f_{low} = 33000Hz$, and the high-pass frequency was of $f_{high} = 1000Hz$.

B. Varying Gain vs Johnson Noise

Four more set of measurements were taken to observe the behavior of Johnson Noise with respect to the change in input Gain. We carried out two set of measurements for $R_{in} = 10\Omega$ and two set of measurements for $R_{in} = 100\Omega$.

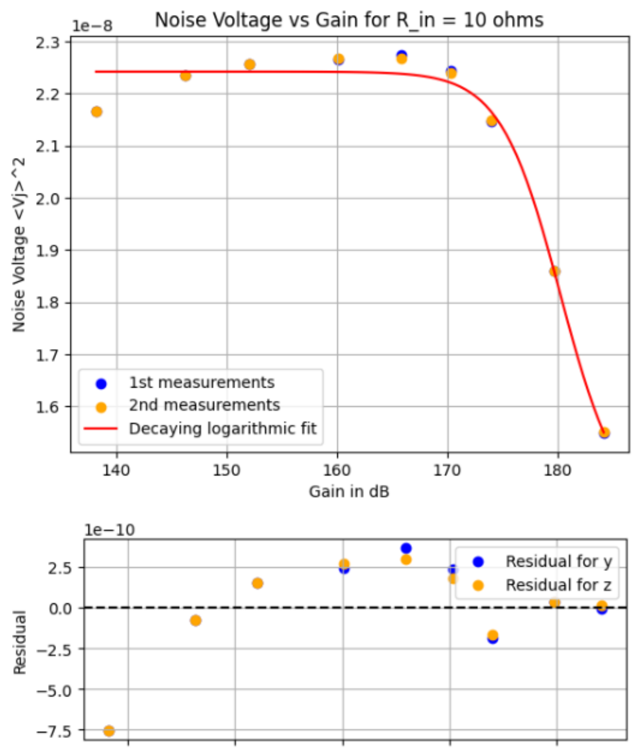

FIG. 5. Noise Voltage vs Gain ($R_{in} = 10\Omega$)

For Fig. 5 & 6, we did a dB fit for the Gain.

We varied the gain values as follows: $G_2 = 1k, 1.5k, 2k, 3k, 4k, 5k, 6k, 8k$, and $10k$. The second and third plots are the residuals of the first and second measurements respectively. We can observe that the HLE gain has a decaying logarithmic relationship with the noise voltage. As we increase the gain, the Johnson noise voltage decreases. We can see that the 10Ω Gain measurements have a greater voltage difference between the initial value and the peak value voltage compared to those of 100Ω, suggesting the initial values plateau for the noise voltage as we increase R_{in}.

C. Noise Voltage vs Low-Pass Frequency

Finally, we took measurements of the response of the noise voltage as we varied the low-pass frequency filter in the HLE. We carried out one set of measurements, varying the low-pass filter frequency values as follows: $f_{low} = 330, 1000, 3300, 10000, 33000$, and 100000.

In the following figure we can observe an increasing

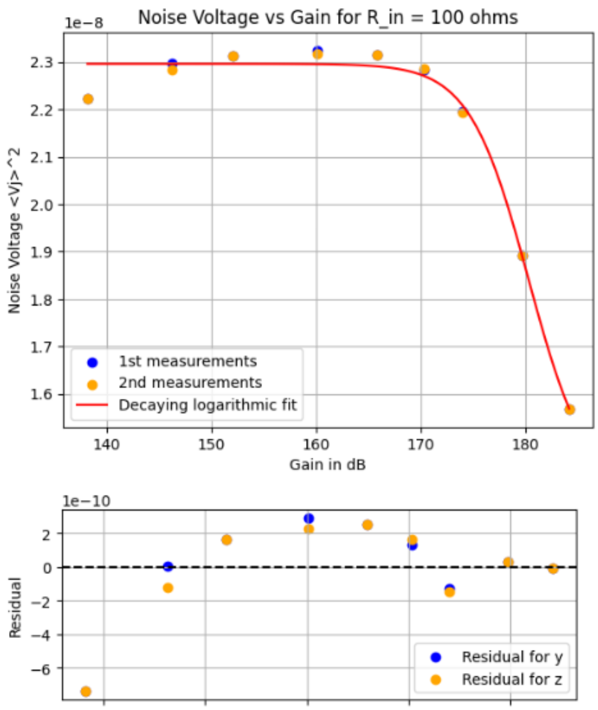

FIG. 6. Noise Voltage vs Gain ($R_{in} = 100\Omega$)

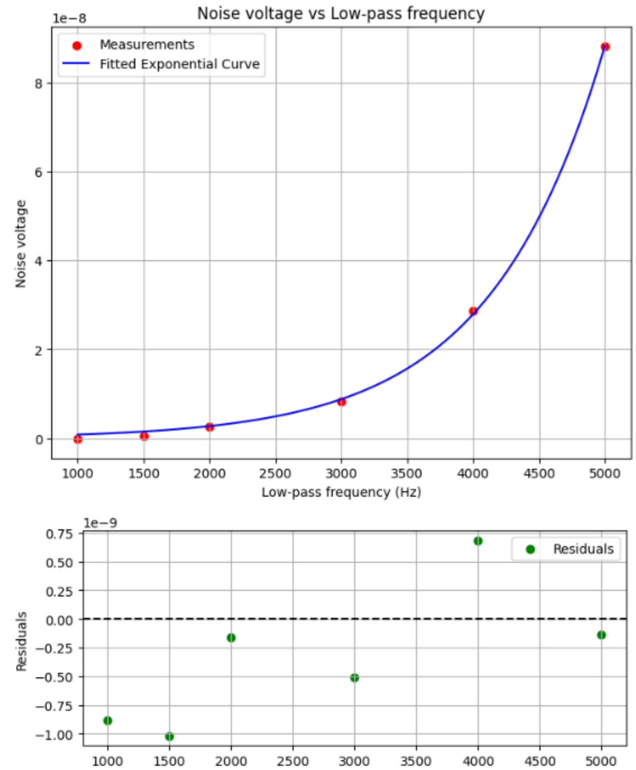

FIG. 7. Noise Voltage vs Low-pass frequency

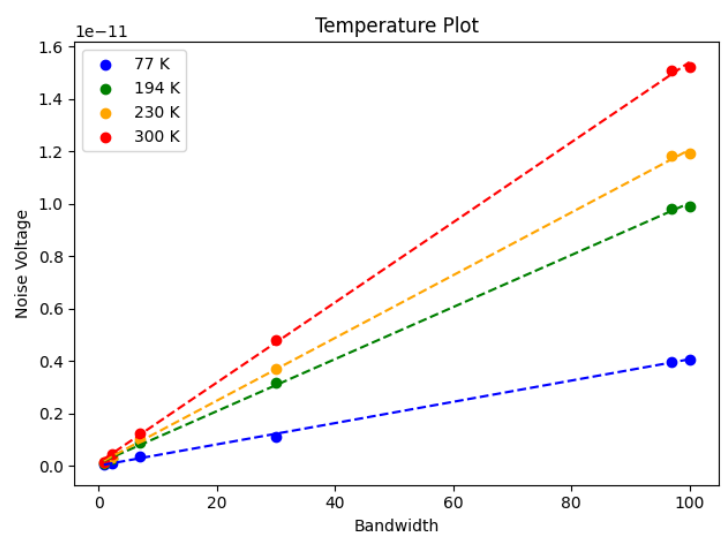

FIG. 8. $10k\Omega$ Johnson Noise Voltage vs Temperature Measurements

exponential relationship between Johnson's noise voltage ($< Vj^2 >$) on the y-axis, and the low-pass frequency filter, on the x-axis.

D. Johnson Noise vs Temperature

Finally, we observed the behavior of Johnson Noise Voltage for 4 different temperatures, taking 6 measurements per temperature. The temperatures used were $77°K$, $194°K$, $230°K$, and $300°K$. The Bandwidths used were $0.9kHz$, $2.3kHz$, $7kHz$, $30kHz$, $97kHz$, and $100kHz$.

In Fig. 8 we can observe a linear relationship between the bandwidth and the Johnson Noise Voltage. The second plot demonstrates the residuals for these measurements. In this case we had to calculate the HLE and LLE amplified gains, and substract them from our original measuremend of DC voltage, given that the expected Johnson Noise measurements were all above the expected value. This was done through grid graph observation.

For the first plot we can also observe a linear relationship between temprature and Johnson Noise. For greater temperatures, we get larger Johnson Noise Voltage outputs.

With the data from this plots, by plugging in temperature, Johnson Noise Voltage, and bandwidth values for each case, and with a constant, resistance value of $R = 9981\Omega$ (measured with the DMM), we can solve for Boltzmann's constant in II B. We used code to solve for Boltzmann's constant for each different temperature and bandwidth case, and got an average of the 24 measurements. We found a Boltzmann constant of $1.136869637e-23JK^-1$.

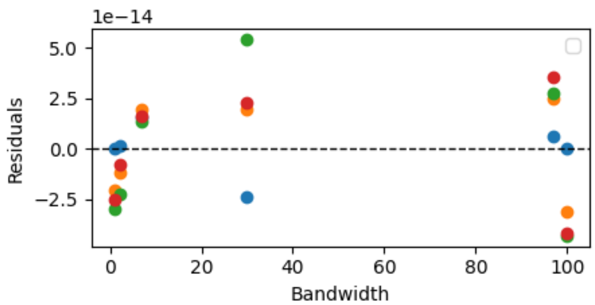

FIG. 9. $10k\Omega$ Johnson Noise Voltage vs Temperature Residuals

IV. UNCERTAINTY

In this experiment, the main error was given by the digital multimeter. Because of the accuracy of the LLE and HLE, their error can be neglected compared to the DMM's. The digital multimeter had an uncertainty of $\pm 0.0005V$. The gains appearing in II B were given by the high-level electronics device manual section, providing an extra $\pm 0.2\%$ uncertainty in each case. Given that the DMM voltage was measured in volts with a resolution of 0.001, the Johnson noise voltage is related to the DMM output voltage by a scale of $1.0e-08V$, and the gains provide a 0.2% uncertainty, we get a total uncertainty of $\pm 0.0005V * 1.0e-08V = \pm 0.5e-12V$ for the DMM, and a total experiment uncertainty of $0.5e-12V$ $+(0.5e-12)*0.002V = \pm 0.51e^{-}12V$.

When it comes to calculating Boltzmann's constant, the error provided by the digital multimeter was multiplied by 24, which is the number of measurements taken to find it. Because we are solving for Boltzmann's constant average value with respect to all of our measurements, we have to account for the error provided by each specific measurement. In this case, we found a DC Voltage uncertainty of $\pm 0.012V$, which translates to a maximum error of $\pm 2e-26V$ for the calculated total average Boltzmann's constant.

V. CONCLUSION

Given a total uncertainty of $\pm 0.51e-12V$ we can effectively conclude that Johnson's noise voltage has a decaying logarithmic relationship with respect to the high-level electronics' gain, a exponentially increasing relationship with respect to the HLE low-pass filter frequency, and a linear relationship with respect to bandwidth and temperature.

When it comes to Johsnon's Noise Voltage with respect to resistance, we expect to see a linear relationship instead of a sigmoid relationship. This error was produced by the bandwidth at which the 10Ω and 100Ω measurements were taken.

Finally, analyzing the temperature and bandwidth relationship to Johsnon Noise voltage, our measurements suggest a successful relationship between this, following a linear relationship provided by II B. On the other hand, when calculating Boltzmann's constant, we found it to be $1.1368696369495446 \pm 0.002e-23JK^{-}1$. The expected value for Boltzmann's constant is of $1.38JK^{-}1$, which suggests that there is an error in our measurements, or uncertainty calculations. The expected Boltzmann constant value does not fall in the uncertainty range for our calculations. Another reason for this error could be calculating a larger than expected value for HLE and LLE amplified gain voltage.

Overall, the experiment was partially successful in helping us understand the Johsnon noise voltage relationships with resistance, frequency, gain, bandwidth and temperature. It is important to know how these relationships work because it lets us design efficient devices. The lower the Johnson noise, the less interference between signals, and the more predictable the output voltages are for different parts on an electronic device. When designing electronics, we can visually understand that we want to keep the low-pass frequency filter in these the lowest, the gain at the highest values possible, and the resistance, bandwidth and temperature in the device at its lowest possible values as well. This means that to achieve a low Johnson noise interference, we will have to find a perfect balance between these values to achieve the lowest presence of Johnson Noise Voltage at the cheapest price possible.

With the information in this paper we can get a better understanding of the hustles of electronics' design, and the challenges we are facing for faster, better technology in the future.

Appendix A: Appendixes

[1] M. Sei Susuki, Chapter 10s johnson-nyquist noise, SUNY at Binghamton (2011).
[2] N. A. Romero, Johnson noise, power **2**, 4 (1998).
[3] *LLE: Low-level electronics device.
[4] **HLE: High-level electronics device.