Electron Spin Measurement:
Nuclear Magnetic Resonance, the background discovery behind MRI*

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(Dated: September 28, 2023)

In this experiment, we explore the fundamentals of Nuclear Magnetic Resonance (NMR). In a few words, NMR is the phenomenon in which a particle's quantum spin gets fixed to a specific direction by passing through a strong magnetic field. By utilizing a magnetic field-generating device, and placing an oil sample in it, we measured the electrical response of the induced magnetic field through the oil, caused by the machine-generated magnetic field. Tracking precise changes in the frequency of the input magnetic field, we were able to observe a change in behavior for the atom response at the resonant frequency.

I. INTRODUCTION

An electron has a magnetic field due to its spin. When electrons that have opposite spins are put together, there is no net magnetic field because the positive and negative spins cancel each other out. Because electrons of the same spin cancel each other out, the one unpaired electron in the atom will determine the spin. There is a high likelihood for either spin due to the large number of electrons, so when it went through the magnetic field it split into two beams. Even though this theory seemed to explain to the behavior of electrons, and atoms, it wasn't till 1945 that three physicists were able to measure this behavior through experiments. [1]

It was a cold winter morning in 1945 when, for the first time, Robert Pound, Edward Purcell, and Henry Torrey agreed to meet early in the morning to make measurements, in the seek for elusive resonance. After putting paraffin through a magnetic field all day, meticulously increasing the strength of the magnetic field, the physicists had a serendipity moment in which they decided to turn the magnetic field all the way up, to observe an increase in amplitude for the paraffin's response, giving birth to Nuclear Magnetic Resonance [2].

Nuclear Magnetic Resonance (NMR) is the basis of why MRI (Magnetic Resonance Imaging) can save lives daily, through imaging of water in the body, protein links, and neuronal scans. Furthermore, NMR could potentially provide a solution to Moore's Law, by making transistors more effective by fixing their electrons' spin.

To observe the response, spectrometers measure the nuclear magnetic resonance signals. These signals have $\approx 10^19$ nuclear spins, where the detection is by observation of a γ -ray anisotropy €, and the pertinent quantity is the number of γ -photons counted [3].

NMRs induce a magnetic field through a body, and can momentarily fix the direction of electrons in the body and capture an image of this. When electrons are fixed to a specific direction, it is easier to observe the behavior of atoms in the body put through the magnetic field, as they are now organized and have greater predictability.

A. Response Time

We can observe magnetic resonance when both a magnetic moment and angular momentum are present in the stable nuclei. An example of this is the Hydrogen atom, the most common nuclei in the universe, which is a single proton. This proton can be classically thought of as a spinning bar magnet. The behavior of the proton is described by the following equations:

$$\mu = \gamma J = \gamma \hbar \bar{I} \tag{1}$$

$$U = -\mu \dot{B} \rightarrow U = -\mu_z B_o = -\gamma \hbar I_z B_o \tag{2}$$

Where the allowed values for I_z are negative and positive one-half. To find the magnetic resonance of a proton, we use the following relationship

$$f_0(MHz) = 4.258 B_o(kilogauss) \tag{3}$$

We define the thermal equilibrium magnetization as M_o, and the nuclear magnetization as M_z (given that we choose it to be in the z-direction). N_μ represents the number of spins per unit volume . These magnetizaions are defined by the following formulas:

$$M_o = N_\mu \tanh(\frac{\mu B}{kT_1}) \approx N\frac{\mu^2 B}{kT} \tag{4}$$

$$M_z = (N_1 - N_2)\mu \tag{5}$$

Where N is the number of magnetic moments and T_1 is the spin-lattice relaxation time. If at the initial time, M_z is equal to zero, the nuclear magnetization and thermal

* A footnote to the article title

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equilibrium magnetization relationship is given by the following equation:

$$M_z(t) = M_o(1 - e^{-\frac{t}{T_1}}) \tag{6}$$

We can observe the plotted Eq.I A in Fig.1. In this equation, M_o is the initial magnetization of the sample, and T_1 is the time-growth exponential constant.

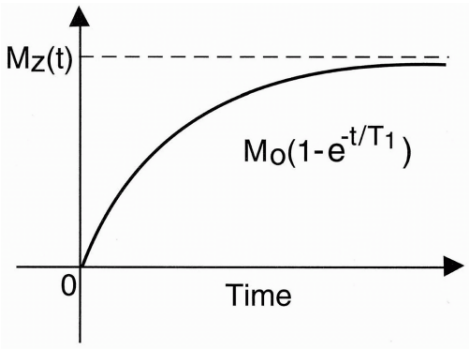

FIG. 1. Thermal Equilibrium Response

B. Decay Time

The net magnetization processing about the constant magnetic field can only be observed after the NMR's transmitter pulse is over. Because the thermal equilibrium magnetization rotates into the x-y plane, where it will process about $B_o\hat{k}$, the decay time for the nuclear magnetization is given in the x-axis and y-axis. These provide an equation for the time it takes for the nuclear magnetization to decay. At peak, the nuclear magnetization is equal to the thermal equilibrium magnetization. In the following figures, we can observe the decaying behavior of the thermal magnetization and the complete pulse sequence.

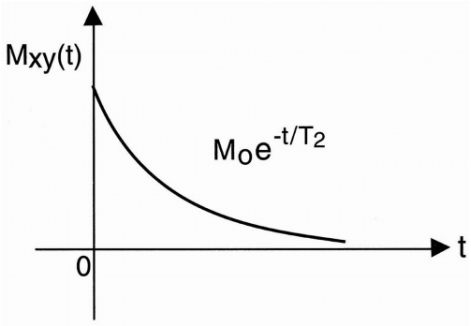

FIG. 2. Thermal Equilibrium Decay

The thermal equilibrium decaying has a mangetization component, M_o, and a decaying-time constant, T_2. For this graph, the y-axis represents the magnetization in the oil sample, and the x-axis represents time.

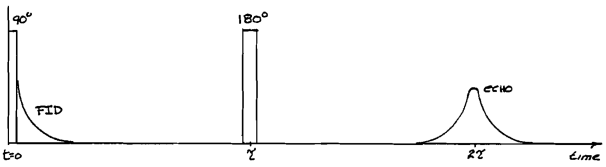

FIG. 3. Pulse Sequence

For the pulse sequence, we can observe an immediate pulse reaction to the input magnetic field, followed by a decaying magnetization, described in Fig.3. This happens at 90° from the input magnetic pulse. In MRIs, this lets a doctor observe the water molecules in a patient's body by releasing an image that shows the amount of presence of water in the body. At 180° from the input magnetic pulse, a second image is released, with higher resolution, because it takes twice the time to reproduce the image of the amount of water in the patient's body. For each millisecond that the MRI machine reproduces the input magnetic pulse, the 180° will have twice the amount of measurements as that of the 90° pulse response.

II. METHOD

A. Setup

For this experiment, we used two devices. The first device was a PSA-1 spectrometer. The second device was an oscilloscope. The spectrometer was connected to the oscilloscope, where the input magnetic field signal and the induced magnetic field signal were displayed. The PSA-1 spectrometer has 2 coils acting as a transformer, generating a magnetic field that runs parallel to the coils' direction, labeled "Permanent Magnets" in Fig. 4.

The NMR diagram (Fig. 4 displays 8 parts:

The Permanent Magnet, being the only part that is physically separated from the NMR device, has two coils on the side, and a space between them to place the sample. The coils generate a magnetic field that lets the pulse travel through the sample.

The Pulse Programmer generates an electric pulse that can have its period and frequency adjusted.

The RF synthesized oscillator turns the pulse signal into a sine signal 90° out of phase. The oscillator also allows us to change the frequency of the pulse, adjusting it before having it go through the sample.

The Mixer adds signals together and shows beats when they are not at resonance

The RF Amplifier amplifies the sinusoidal input signal from the RF synthesized oscillator.

The Receiver intakes the sample-induced magnetic field and transforms it to an RF signal.

The RF Amplifier Detector receives the RF output signal and transforms it into a pulse response signal.

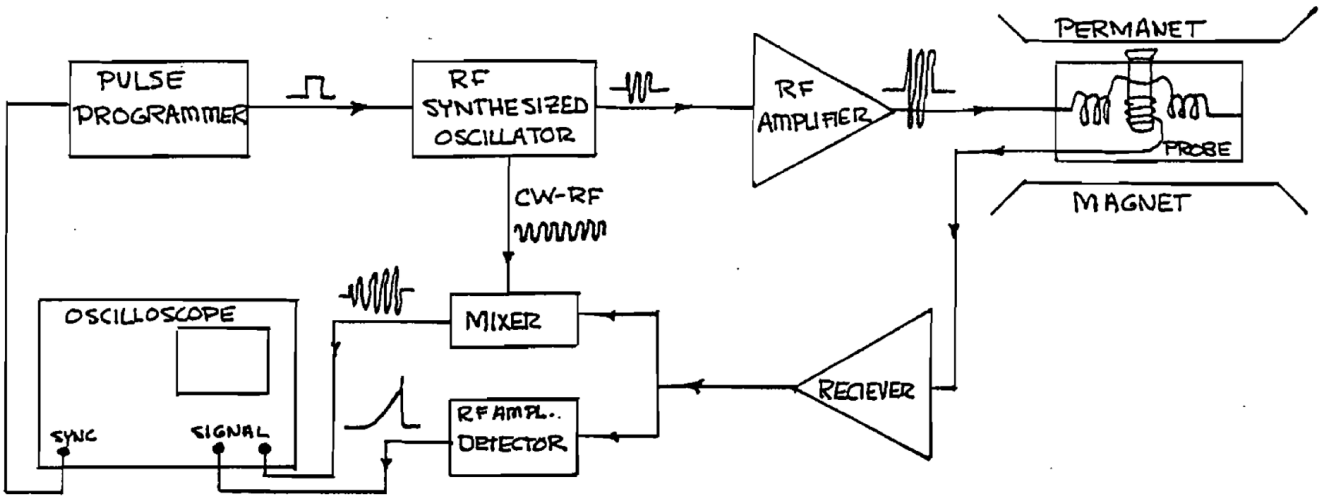

FIG. 4. NMR setup

The Oscilloscope displays imaging of the input pulse signal, the pulse output signal, and the mixer sinusoidal RF signal.

B. Measurement Method

To take measurements, we set the A width to 20%, the mode to "Int", the repetition time to $100ms, at 100\%$, the number of B pulses to zero, turned on the synchronization of A, and off the synchronization of B. We continued to measure the magnetic field flux density with a flux meter to be 3.2 T.

Following this, we repetitively varied the input pulse frequency generated by the spectrometer. At resonant frequency, the output oil frequency response was expected to have the least oscillation. This meant the frequency response was expected to go from underdamped to critically damped (Fig.5) when reaching resonant frequency. This frequency was expected to resonate with the oil sample at its fundamental frequency. Changing the frequency would display a different asymptotic lattice response to the spectrometer-generated pulses. The closer to resonance, the less oscillation the response had. We took different measurements of our observations at different frequencies and eventually found that the resonant frequency of oil is approximately X, at a temperature of Y, air pressure of Z, and inducting magnetic field of W.

To find the resonant frequency, we systematically changed the NMR input frequency applied to the pulse till the output response observed in the oscilloscope had a uniformly decaying shape.

C. Spin-Lattice Relaxation Time

Once we found the resonant frequency, we started to take measurements of the change in frequency as the relaxation time passed. Fig.1 shows the expected behavior of the Spin-lattice relaxation time (T1). Mineral oil reaches thermal equilibrium at 12 ms, meaning the time it takes to come down from the pulse reaction is 12 ms.

III. ERROR

In this experiment, 3 components brought errors in the calculations: the spectrometer's frequency resolution, the oscilloscope's voltage resolution, and the Gaussmeter resolution. The resolutions and errors were given as follows:

Component	Resolution	Uncertainty
Spectrometer (Hz)	$2.88675 * 10^{-6}$	± 0.00001
Oscilloscope (ms)	0.00721688	± 0.025
Gaussmeter (G)	0.0288675	± 0.1
TOTAL	0.00721688	± 0.0358017

TABLE I. Resolution and Uncertainty

For the total resolution, the oscilloscope's measurements have the lowest resolution for the experiment, given that we deal with values of $1ms$ for the relaxation time, while dealing with values of $14 - 16Hz$ for the frequency, and $32000 - 33000G$ for the magnetic field.

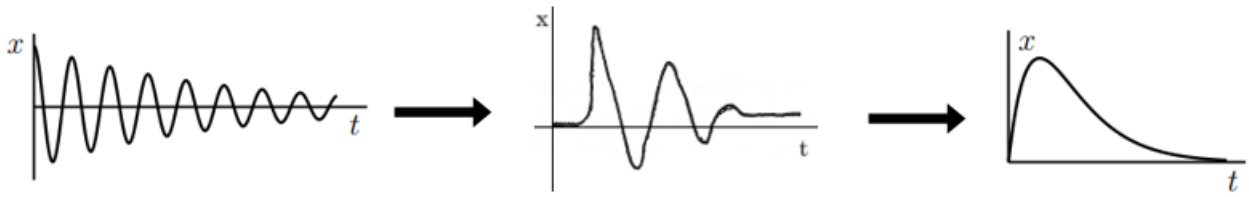

FIG. 5. Output frequency response change as we reach resonance (from underdamped to critically damped.

In this figure, we can see the expected behavior of the magnetization voltage response change as we reach resonant frequency. The closer to the sample's resonant frequency, the less fluctuation in the magnetization voltage response

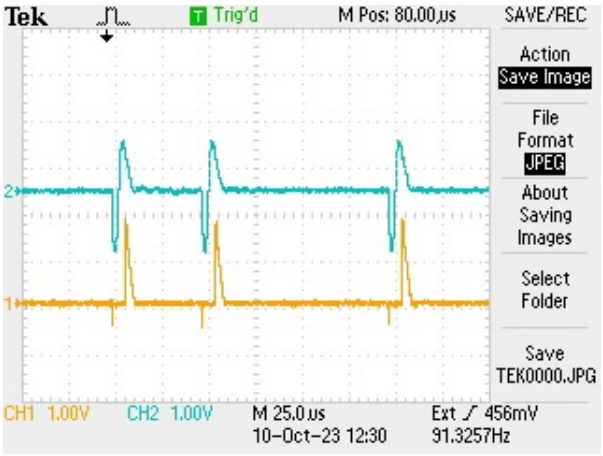

FIG. 6. Frequency pulse input vs output response

In this figure we can observe a 90° response after $\tau = 50\mu s$ followed by a 180° response after $2\tau = 100\mu s$ with respect to the first response

IV. CONCLUSION

A. Results

After switching the frequency input values of the Nuclear Magnetic Resonator, we found a critically damped response (observe Fig.5) at a frequency between $15.34718MHz$ to $15.36018MHz$. This frequency range is a result of the temperature affecting the input magnetic field strength. After finding the resonant frequency, we were able to take measurements for the spin-lattice T-1 relaxation time, and the spin-spin T-2 relaxation time.

In Fig. 7 graph we can observe the results for our eight magnetization voltage measurements through time for the increasing pulse. The data aligns with a logarithmic fit, which helps determine a relaxation time of $8ms$. The residual on the graph below the fit helps visualize the accuracy of the measurements, yielding a $\pm 0.03V$ error from the measurements to the expected data. We found a $\chi^2 = 4.99$ error for these measurements.

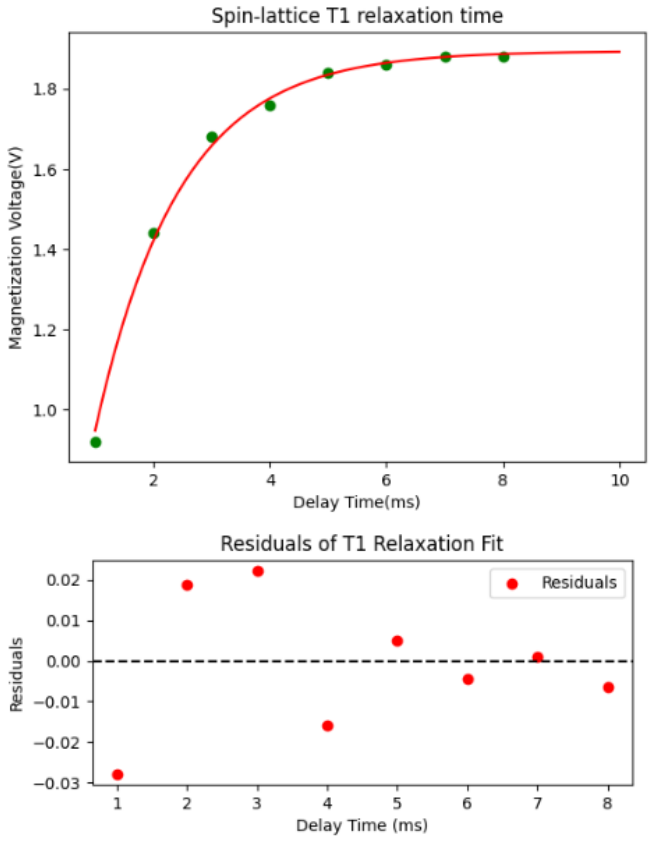

FIG. 7. Spin-lattice T-1 Relaxation Time

In the following Fig. 8 we can observe the results for our seven measurement magnetization measurements through time for the decaying pulse. Because the measurements were not enough to determine the exact decaying relaxation time, our inverse exponential fit helps determine a relaxation time of $35ms$, with a $\pm 0.02V$ error given by the residual graph. We found a $\chi^2 = 3.33$ error for these measurements.

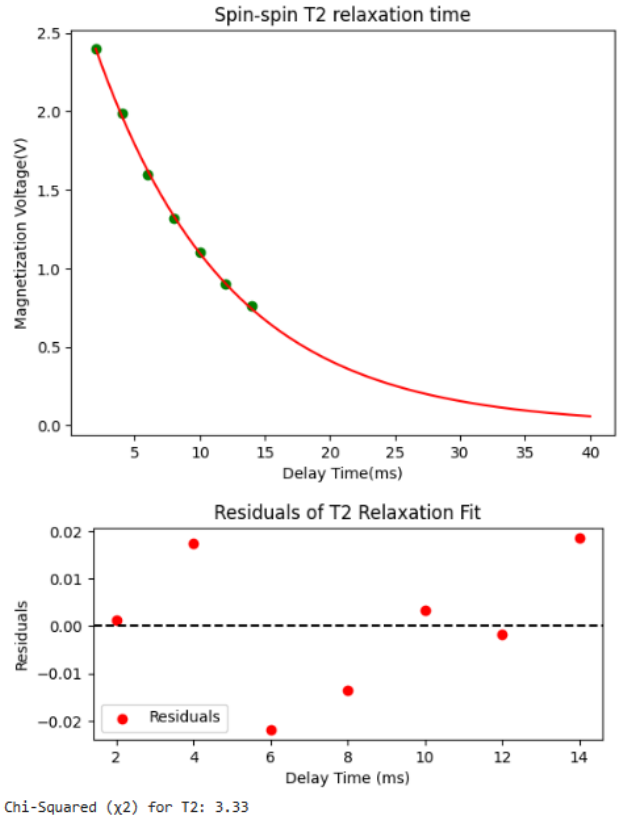

FIG. 8. Spin-spin T2 relaxation time

B. Synthesis and Relevance

The results determine that the resonant frequency for the oil sample is between $15.34718MHz$ to $15.36018MHz$ with a total uncertainty of $\pm 0.0358017MHz$. For this resonant frequency, we found a spin-lattice T1 relaxation time of $8ms$ with a $\chi^2 = 4.99$ error, and a spin-spin T2 relaxation time of $35ms$ with a $\chi^2 = 3.33$ error.

The measurements taken help define the accuracy of the Nuclear Magnetic Resonance for future measurements and get an understanding of how Magnetic Resonance Imaging works.

Appendix A: Appendixes

[1] M. L. U. L. C. U. Liza Chu (UCD), Sharon Wei (UCD), Electron spin, Chemistry LibreTexts (2023).
[2] P. Horowitz, Nuclear magnetic resonance:nbsp; lyman laboratory, december 1945 by paul horowitz, Harvard University (2023).
[3] C. D. Jeffries, Dynamic orientation of nuclei, Annual Review of Nuclear Science **14**, 101 (1964).