I. Introduction

The goal of this lab was to learn about how reflections of passive and reactive loads impact the output of a circuit, as a result of the time-varying impedance presented by reactive loads, resulting in reflection coefficients that vary with time. To do so, we measured the input and output impedance of 5 different circuits (C, RC, RL, parallel RC, and parallel RL), and simulated their equivalent circuits using Advanced Design Systems.

II. Theory

When terminating a transmission line with a conductive or inductive load, we get time-varying reflected waveforms of different shapes than the incident waves, as a consequence of the time-varying impedance of such loads. Circuits with differing loads have a reflection coefficient that varies their impedance.

The following equations explain the time-dependent reflection coefficient, reflection coefficient at the corresponding transmission line delay time, time decay constant, and impedance to resistance, capacitance or inductance relationship for R, C, L, series RL, parallel RL, series RC, and parallel RC circuits. The variables and constants used for all equations are as follows:

$t_d - delay\ time\ of\ transmission\ line$
$Z_0 - characteristic\ impedance\ of\ tranmission\ line\ (standard\ Z_0 = 50\Omega)$
$\Gamma(\mathrm{t}) - reflection\ coeffcient$
$R - resistor\ value\ in\ \Omega$
$\tau - time\ decay\ constant$
$C - capacitor\ value\ in\ F$
$L - inductor\ value\ in\ H$

The reflection coefficient of a transmission line circuit with a solely resistive load can be calculated using the following expression:

$$R = \left(\frac{1 + \Gamma(\mathrm{t})}{1 - \Gamma(\mathrm{t})}\right) Z_0$$

Equation 1: Reflection coefficient of a transmission line with an **exclusively resistive** impedance

$$\Gamma(t) = \Gamma(\infty) + \big(\Gamma(t_d) - \Gamma(\infty)\big) e^{-\frac{t}{\tau}}$$

Equation 2: Time-dependent reflection coefficient of a transmission line with an **exclusively capacitive** load

$$\tau = -\frac{t - t_d}{\ln\left[\frac{\Gamma(t) - \Gamma(\infty)}{\Gamma(t_d) - \Gamma(\infty)}\right]}$$

Equation 3: Time decay constant of a transmission line with an **exclusively capacitive** load

$$C = \frac{\tau}{Z_0}$$

Equation 4: Transmission line impedance relationship with **exclusively capacitive** load

$$\Gamma(t) = \Gamma(\infty) + \big(\Gamma(t_d) - \Gamma(\infty)\big)e^{-\frac{t}{\tau}}$$

Equation 5: Time-dependent coefficient of a transmission line with an **exclusively inductive** load

$$\tau = -\frac{t - t_d}{\ln\left[\frac{\Gamma(t) - \Gamma(\infty)}{\Gamma(t_d) - \Gamma(\infty)}\right]}$$

Equation 6: Time decay constant of a transmission line with an **exclusively inductive** load

$$L = \tau Z_0$$

Equation 7: Transmission line impedance relationship with **exclusively inductive** load

$$\Gamma(t) = \Gamma(\infty) + \big(\Gamma(t_d) - \Gamma(\infty)\big)e^{-\frac{t-t_d}{\tau}} = \frac{R - Z_0}{R + Z_0} + \left(1 - \left(\frac{R - Z_0}{R + Z_0}\right)\right)e^{-\frac{t-t_d}{\tau}}$$

Equation 8: Time-dependent reflection coefficient of a transmission line with an inductive and resistive load in series

$$\tau = -\frac{t - t_d}{\ln\left[\frac{\Gamma(t) - \Gamma(\infty)}{\Gamma(t_d) - \Gamma(\infty)}\right]}$$

Equation 9: Time decay constant of a transmission line with an inductive and resistive load in series

$$\Gamma(t_d) = \frac{\infty - Z_0}{\infty + Z_0} \cong 1 \qquad (ideally)$$

Equation 10: Reflection coefficient when at the delay time of a transmission line with an inductive and resistive load in series for an ideal circuit

$$\Gamma(\infty) = \frac{R - Z_0}{R + Z_0} \rightarrow R = \left(\frac{1 + \Gamma(\infty)}{1 - \Gamma(\infty)}\right)Z_0$$

Equation 11: Reflection coefficient after a long period of time (t->∞) for a transmission line with and inductive and resistive load in series

$$L = \tau(R + Z_0)$$

Equation 12: Transmission line impedance relationship with an inductive and resistive load in series

$$\Gamma(t) = \Gamma(\infty) + \big(\Gamma(t_d) - \Gamma(\infty)\big)e^{-\frac{t}{\tau}} = -1 + \left(\left(\frac{R - Z_0}{R + Z_0}\right) - (-1)\right)e^{-\frac{t-t_d}{\tau}}$$

Equation 13: Time-dependent reflection coefficient of a transmission line with an inductive and resistive load in parallel

$$\Gamma(\infty) = \frac{0 - Z_0}{0 + Z_0} \cong -1 \qquad (ideally)$$

Equation 14: Time decay constant of a transmission line with an ideal inductive and resistive load in parallel

$$\Gamma(t_d) = \frac{R - Z_0}{R + Z_0} \rightarrow R = \left(\frac{1 + \Gamma(\mathrm{t_d})}{1 - \Gamma(\mathrm{t_d})}\right)Z_0$$

Equation 15: Reflection coefficient when at the delay time of a transmission line with an inductive and resistive load in parallel

$$L = \frac{\tau}{R^{-1} + Z_0^{-1}}$$

Equation 16: Transmission line impedance relationship with an inductive and resistive load in parallel

$$\Gamma(t) = \Gamma(\infty) + \big(\Gamma(t_d) - \Gamma(\infty)\big)e^{-\frac{t}{\tau}} = 1 + \left(\left(\frac{R - Z_0}{R + Z_0}\right) - 1\right)e^{-\frac{t-t_d}{\tau}}$$

Equation 17: Time-dependent reflection coefficient of transmission line a capacitive and resistive load in series

$$\Gamma(\infty) = \frac{\infty - Z_0}{\infty + Z_0} \cong 1 \qquad (ideally)$$

Equation 18: Reflection coefficient after a long period of time (t->∞) for a transmission line with an ideal capacitive and resistive load in series

$$\Gamma(t_d) = \frac{R - Z_0}{R + Z_0} \rightarrow R = \left(\frac{1 + \Gamma(\mathrm{t_d})}{1 - \Gamma(\mathrm{t_d})}\right) Z_0$$

Equation 19: Reflection coefficient when at the delay time of a transmission line with a capacitive and resistive load in series

$$C = \frac{\tau}{R + Z_0}$$

Equation 20: Transmission line impedance relationship with a capacitive and resistive load in series

$$\Gamma(t) = \Gamma(\infty) + \left(\Gamma(t_d) - \Gamma(\infty)\right) e^{-\frac{t}{\tau}} = \frac{R - Z_0}{R + Z_0} + \left(-1 - \left(\frac{R - Z_0}{R + Z_0}\right)\right) e^{-\frac{t - t_d}{\tau}}$$

Equation 21: Time-dependent reflection coefficient of transmission line a capacitive and resistive load in parallel

$$\Gamma(t_d) = \frac{0 - Z_0}{0 + Z_0} \cong -1 \qquad (ideally)$$

Equation 22: Reflection coefficient when at the delay time of a transmission line with an ideal capacitive and resistive load in parallel

$$\Gamma(\infty) = \frac{R - Z_0}{R + Z_0} \rightarrow R = \left(\frac{1 + \Gamma(\infty)}{1 - \Gamma(\infty)}\right) Z_0$$

Equation 23: Reflection coefficient after a long period of time (t->∞) for a transmission line with capacitive and resistive load in parallel

$$C = \tau(R^{-1} + Z_0^{-1})$$

Equation 24: Transmission line impedance relationship with a capacitive and resistive load in parallel

III. Methodology

Pre-Lab

iii. Load is a 130 Ω resistor.

1)

$$\Gamma(0) = \frac{R_L - Z_0}{R_L + Z_0} = \frac{130 - 50}{130 + 50} = 0.444 \tag{1}$$

$$\Gamma(\infty) = \frac{R_L - Z_0}{R_L + Z_0} = \frac{130 - 50}{130 + 50} = 0.444 \tag{2}$$

2) The sketch of the reflection coefficient is a straight line with a constant value of 0.444.

3) No time decaying constant is necessary for a non-reactive load.

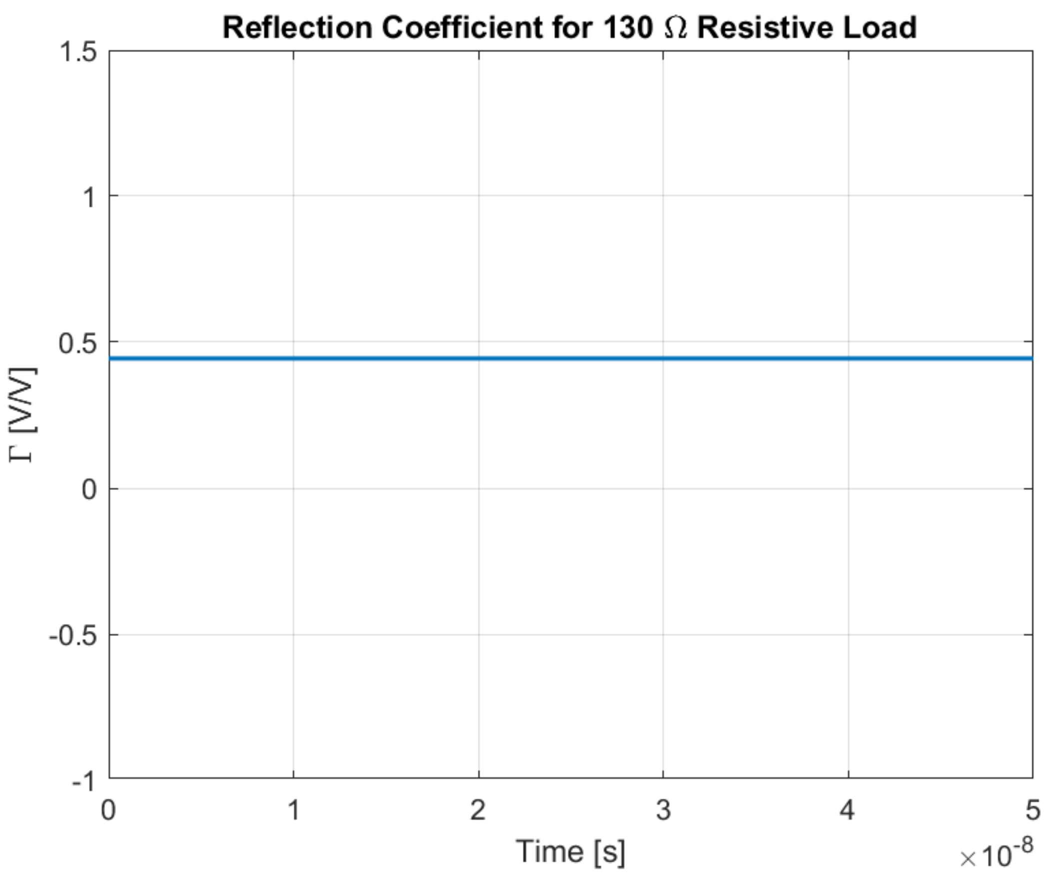

Figure 1: 130 ohm resistor load theoretically expected reflection coefficient

iv. Load is a 220 pF capacitor.

1)

$$\Gamma(0) = -1 \tag{3}$$

$$\Gamma(\infty) = 1 \tag{4}$$

2) The sketch of the reflection coefficient is a decaying exponential from 1 to -1.

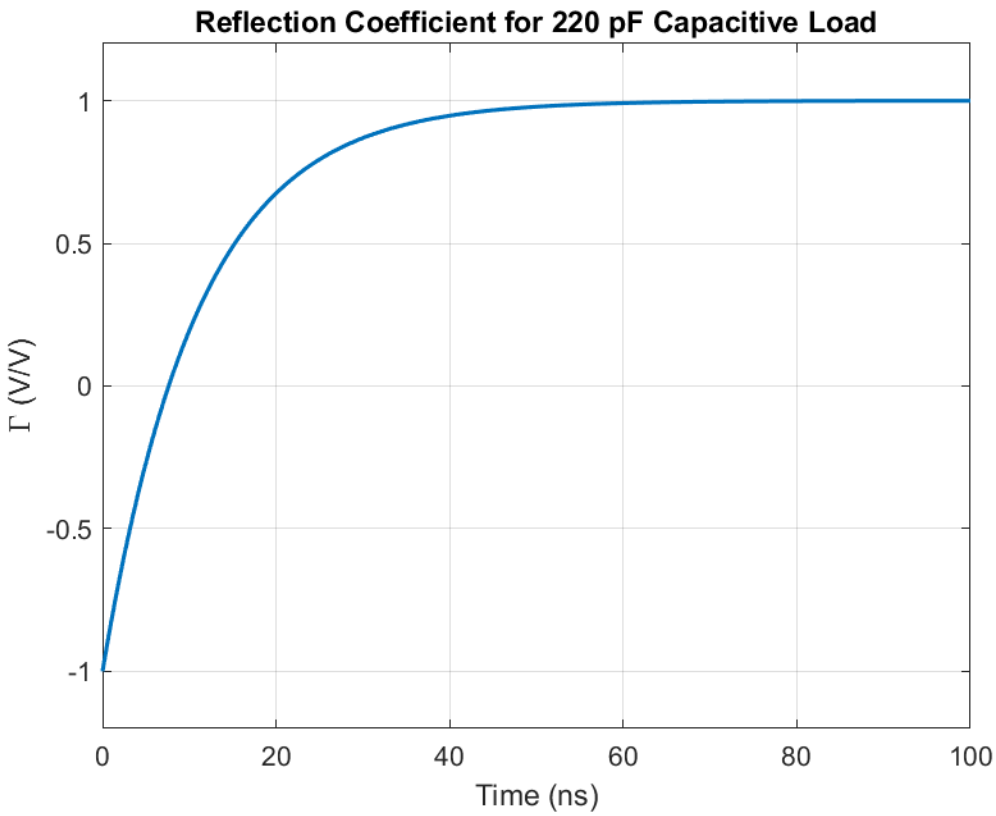

Figure 2: 220 pF capacitor load theoretically expected reflection coefficient

v. Load is a 0.39 μH inductor in series with a 130 Ω resistor.
1)

$$\Gamma(0) = \frac{R_L - Z_0}{R_L + Z_0} = \frac{130 - 50}{130 + 50} = 0.444 \tag{6}$$

$$\Gamma(\infty) = \ 1 \tag{7}$$

2) The sketch of the reflection coefficient is a straight line with a constant value of 0.444.
3) No time decaying constant is necessary for a non-reactive load.

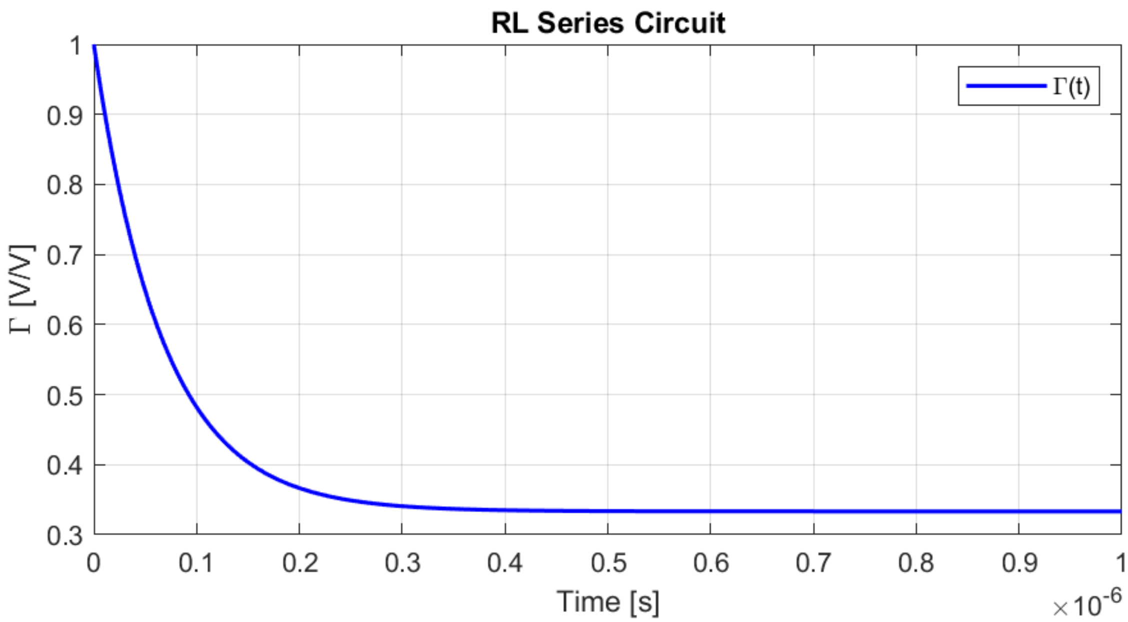

Figure 3: 130 ohm resistor and 0.39 uH inductor in series load theoretically expected reflection coefficient

vi. Load is a 0.39 μH inductor in parallel with a 130 Ω resistor.
1)

$$\Gamma(\infty) = -\ 1 \tag{8}$$

$$\Gamma(0) \ = \frac{R_L - Z_0}{R_L + Z_0} = \frac{130 - 50}{130 + 50} = 0.444 \tag{9}$$

2) The sketch of the reflection coefficient is a decaying exponential from -1 to 0.444.
3)

$$\tau = \frac{L}{R + Z_0} = \frac{0.39 \times 10^{-6}}{130 + 50} \approx \ 2.16 \text{ x } 10^{-9} \ . \tag{10}$$

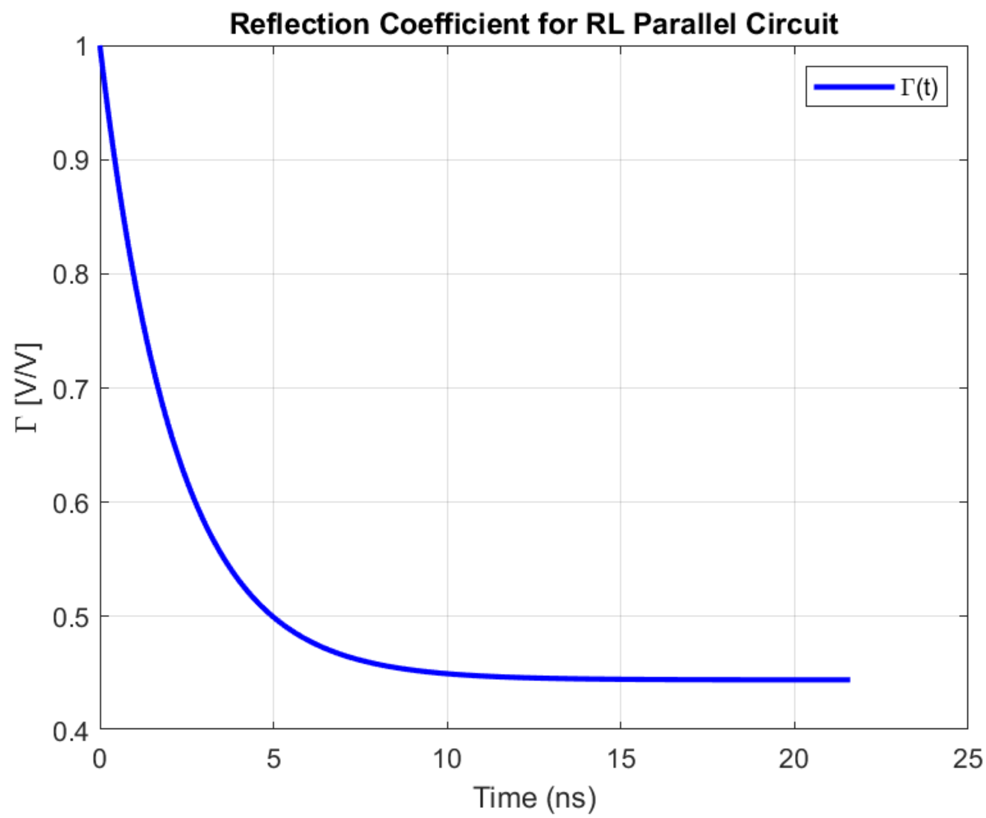

Figure 4: 130 ohm resistor and 0.39 uH inductor in parallel load theoretically expected reflection coefficient

vii. Load is a 220 pF capacitor in series with a 130 Ω resistor.

1)

$$\Gamma(0) = \frac{R_L - Z_0}{R_L + Z_0} = \frac{130 - 50}{130 + 50} = 0.444 \tag{11}$$

$$\Gamma(\infty) = 1 \tag{12}$$

2) The sketch of the reflection coefficient is a decaying exponential from 0.444 to 1.

3)

$$\tau = R_L C = 130 \times 220 \times 10^{-12} \approx 2.86 \times 10^{-8} s. \tag{13}$$

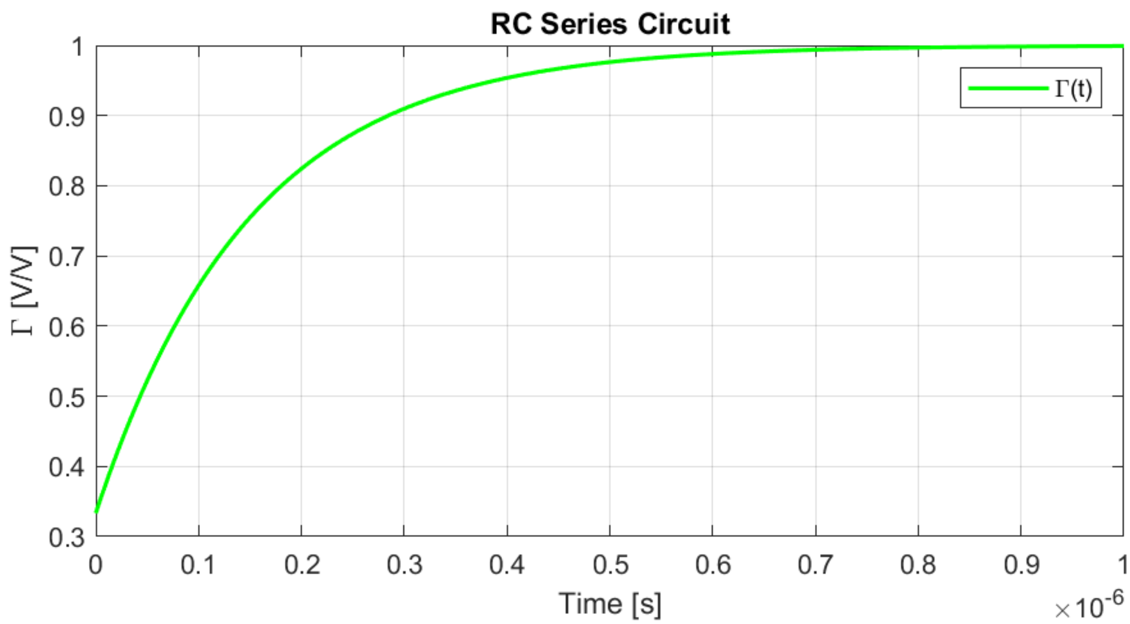

Figure 5: 130 ohm resistor and 220 pF capacitor in series load theoretically expected reflection coefficient

viii. Load is a 220 pF capacitor in parallel with a 130 Ω resistor.

1)

$$\Gamma(0) = -1 \tag{14}$$

$$\Gamma(\infty) = \frac{R_L - Z_0}{R_L + Z_0} = \frac{130 - 50}{130 + 50} = 0.444 \tag{15}$$

2) The sketch of the reflection coefficient is a decaying exponential from -1 to 0.444.

3)

$$\tau = \frac{C}{\frac{1}{R} + \frac{1}{Z_0}} = \frac{220 \times 10^{-12}}{\frac{1}{130} + \frac{1}{50}} \approx 6.16 \times 10^{-9} s. \tag{16}$$

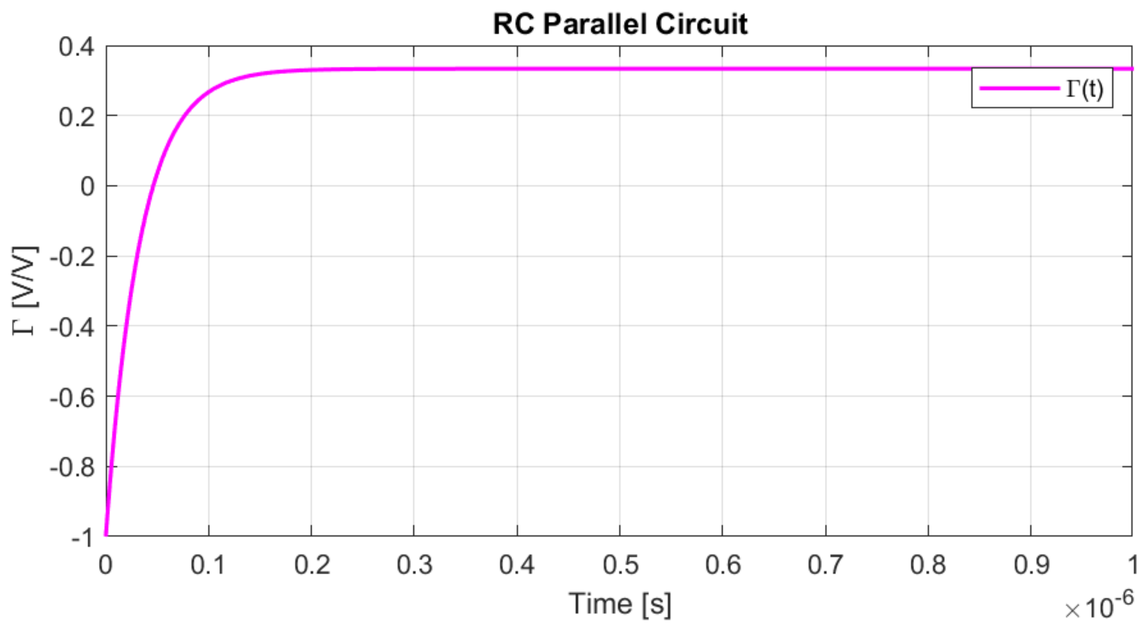

Figure 6: 130 ohm resistor and 220 pF capacitor in parallel load theoretically expected reflection coefficient

This was performed by connecting an open and short circuit equivalent adapter/load at the end of port 1 and 2 cables, as observed in Fig. 2, and finally connecting both channels together through a cable equivalent adapter/load. For each measurement, the Cal kit used was 85031B, averaged to 50, and saved using the "Dump Screen Image" function. For the 3dB measurement, we used the "Marker" function, and moved the torque till reaching 3dB, observing the corresponding frequency.

In-Lab

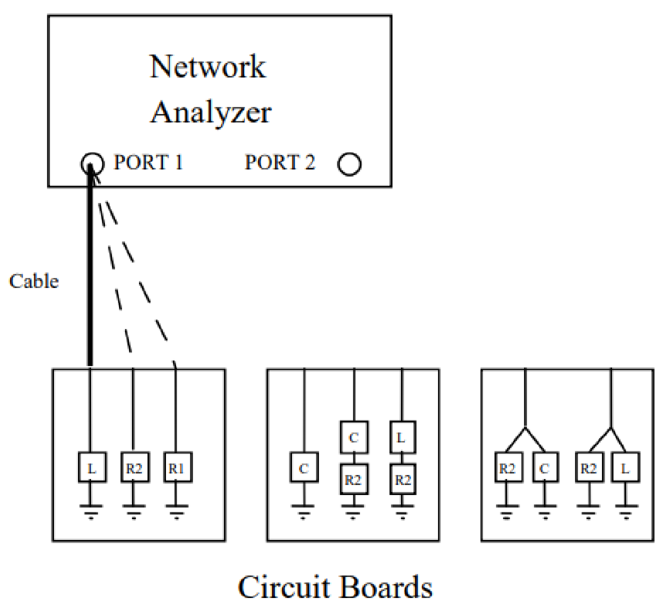

Figure 7: Test setup – Lab 3

The Network Analyzer was set to stop at a frequency of 1.5 GHz, to measure 401 points, and with a low-frequency limit. It was further set to perform a transform response starting at -5 nanoseconds and stopping at 60 nanoseconds, and to view the real part of the reflectance Γ.

The Network Analyzer was further calibrated using an open, short and matched load termination. We continued to connect to each termination of the 3 circuit boards displayed in Figure 7 and save images of the reflections displayed in the Network Analyzer for each load (8 in total).

IV. Results & Discussion

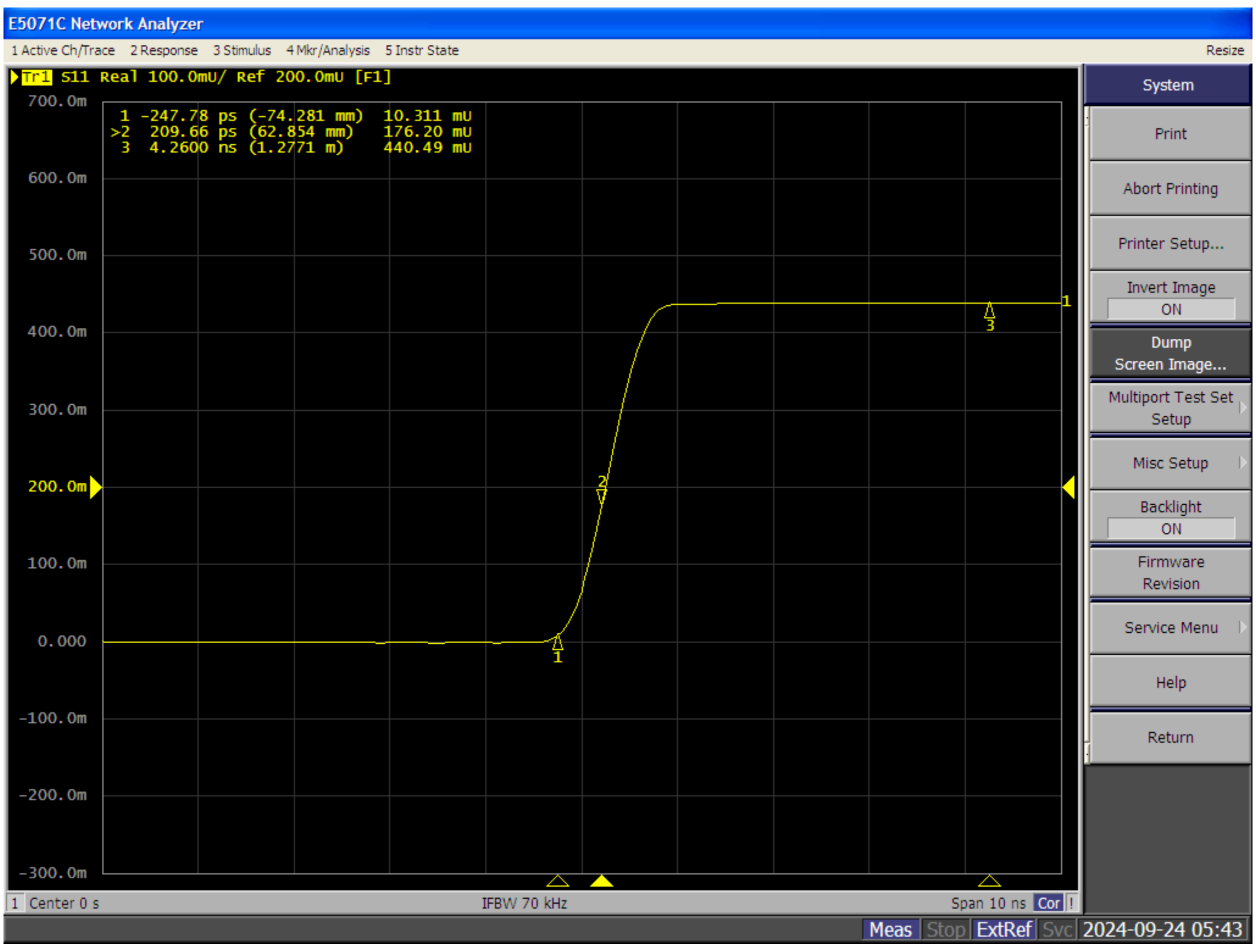

Figure 8: 130 ohm resistor reflection coefficient measurement

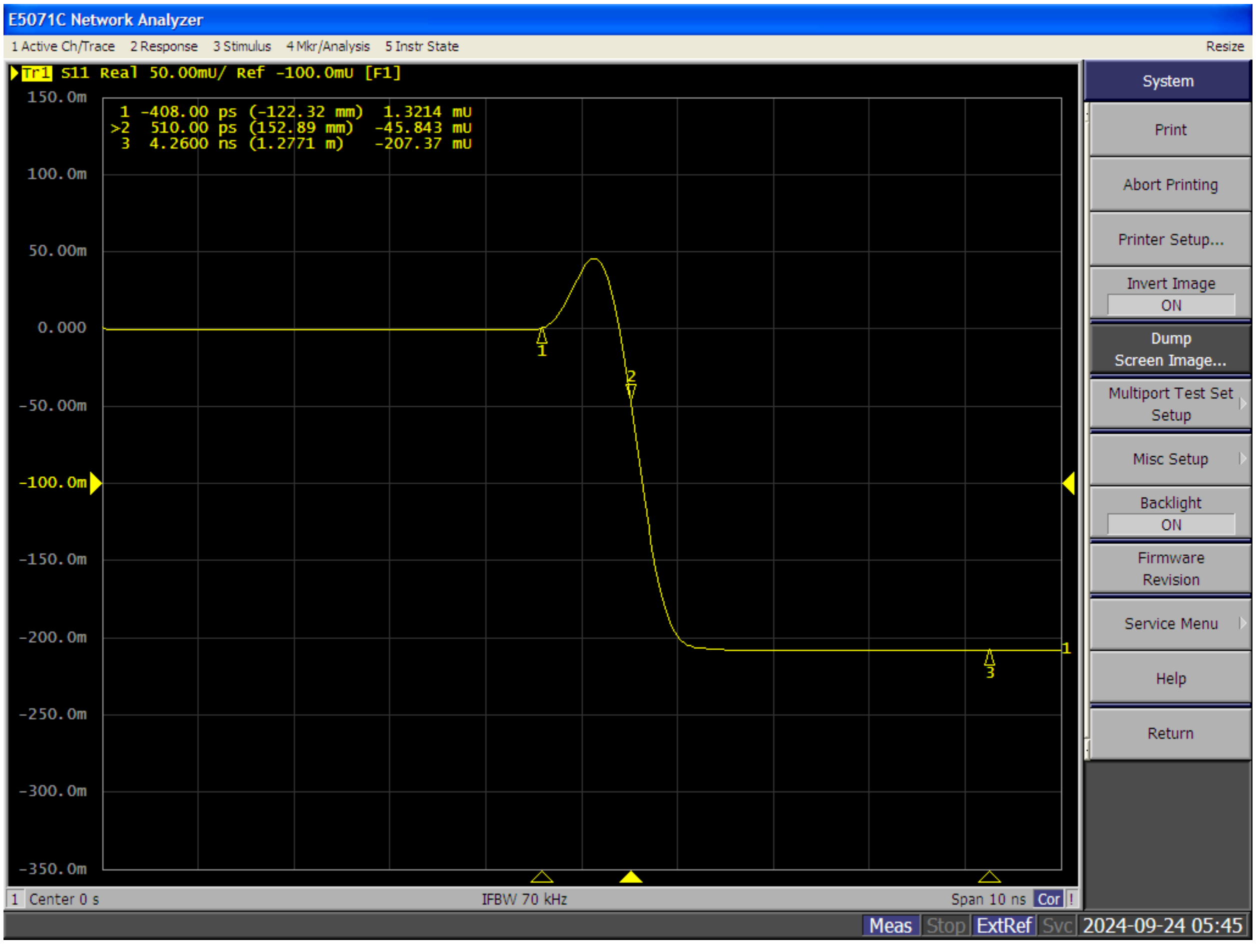

Figure 9: 33 ohm resistor reflection coefficient measurement

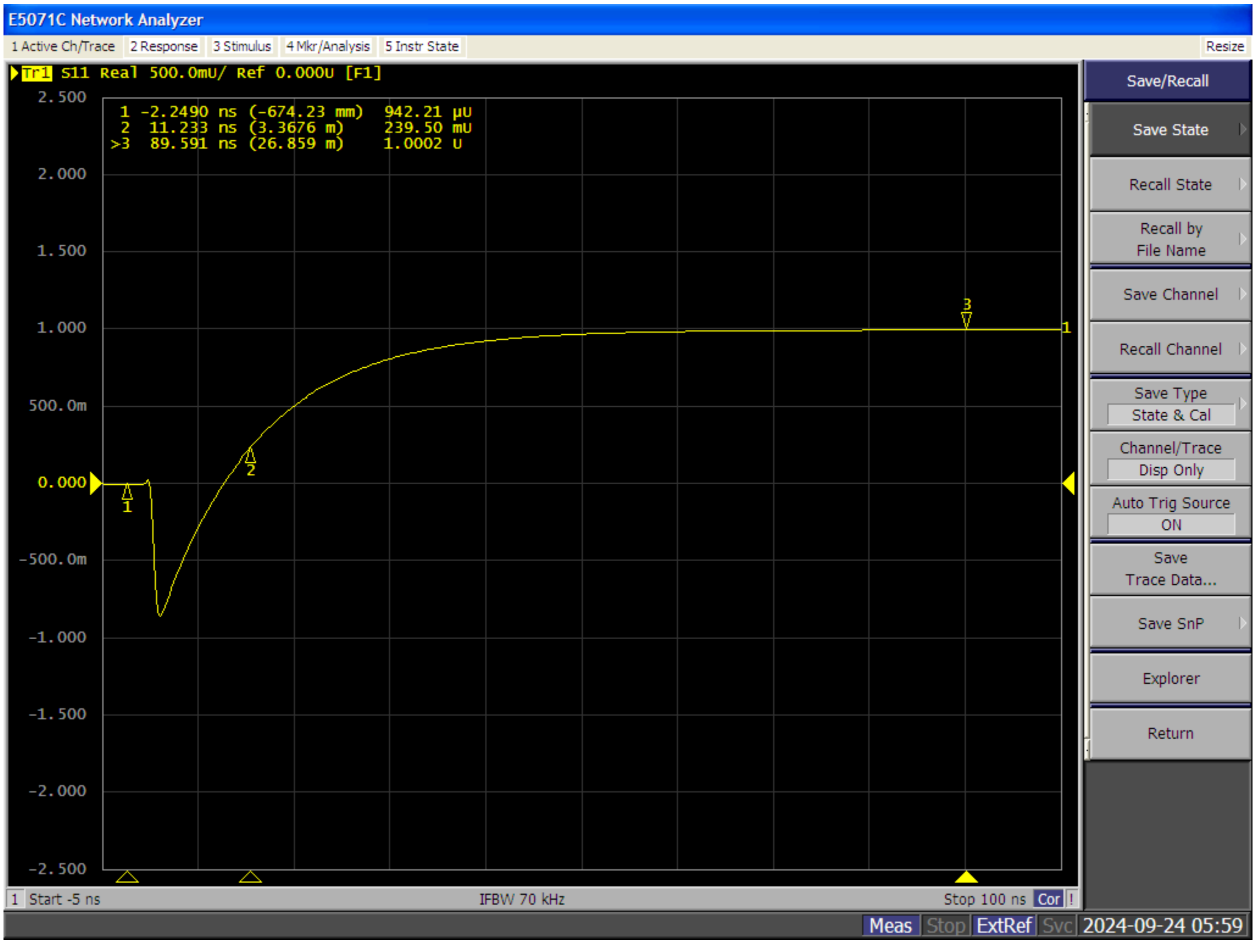

Figure 10: 220 pF Capacitor reflection coefficient measurement

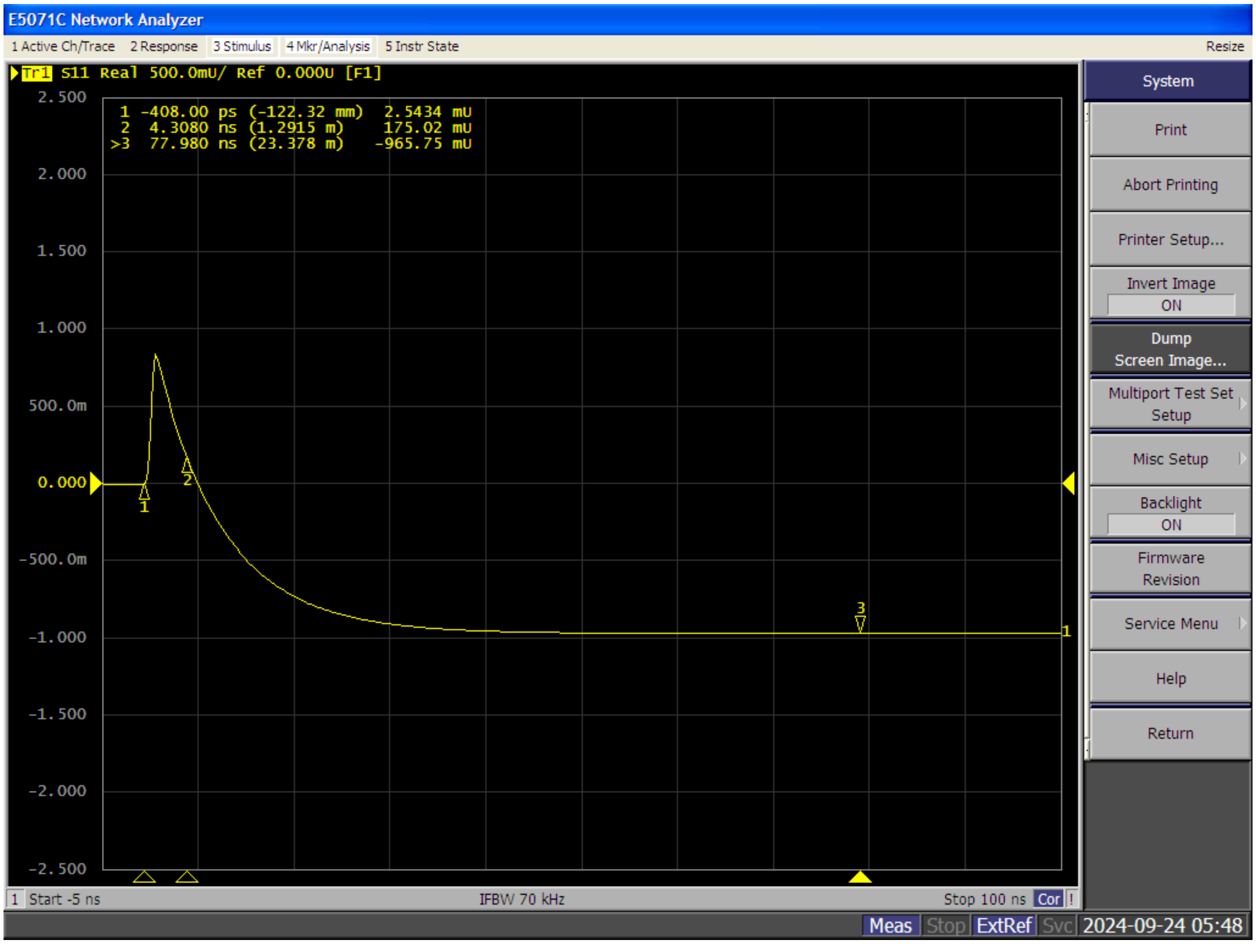

Figure 11: 0.39 uH Inductor reflection coefficient measurement

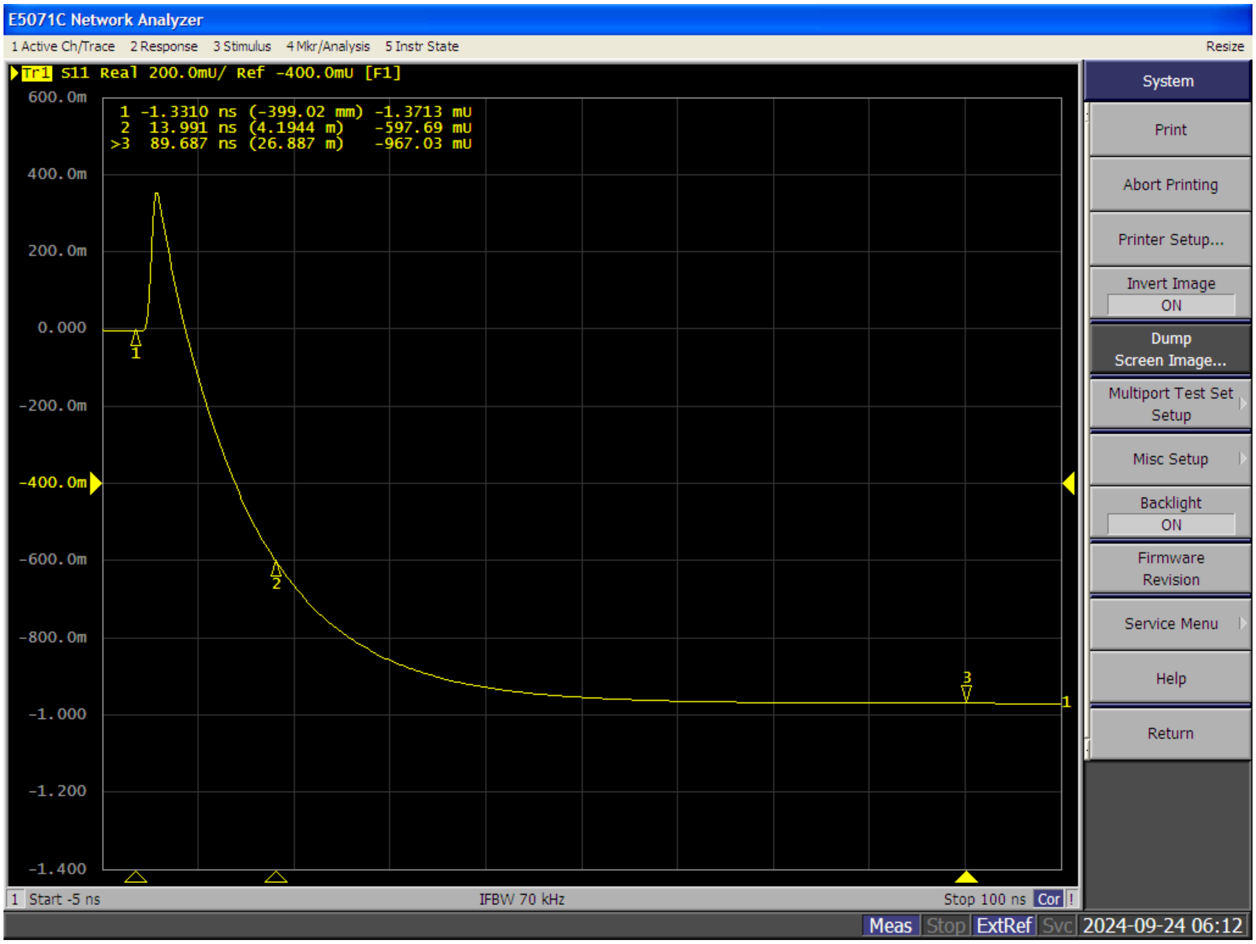

Figure 12: Parallel 0.39 uH and 130 ohm reflection coefficient measurement

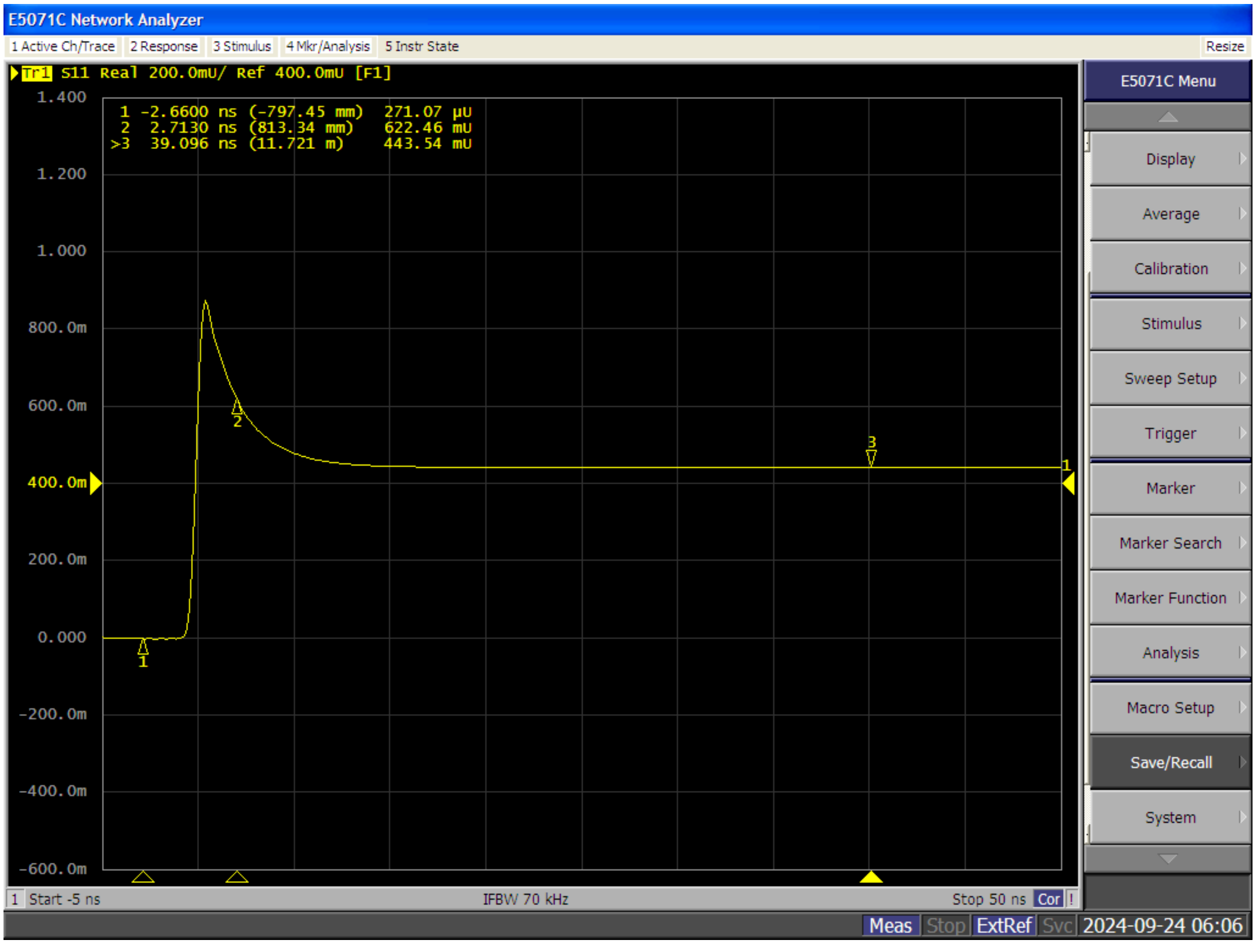

Figure 13: Series 0.39 uH and 130 ohm reflection coefficient measurement

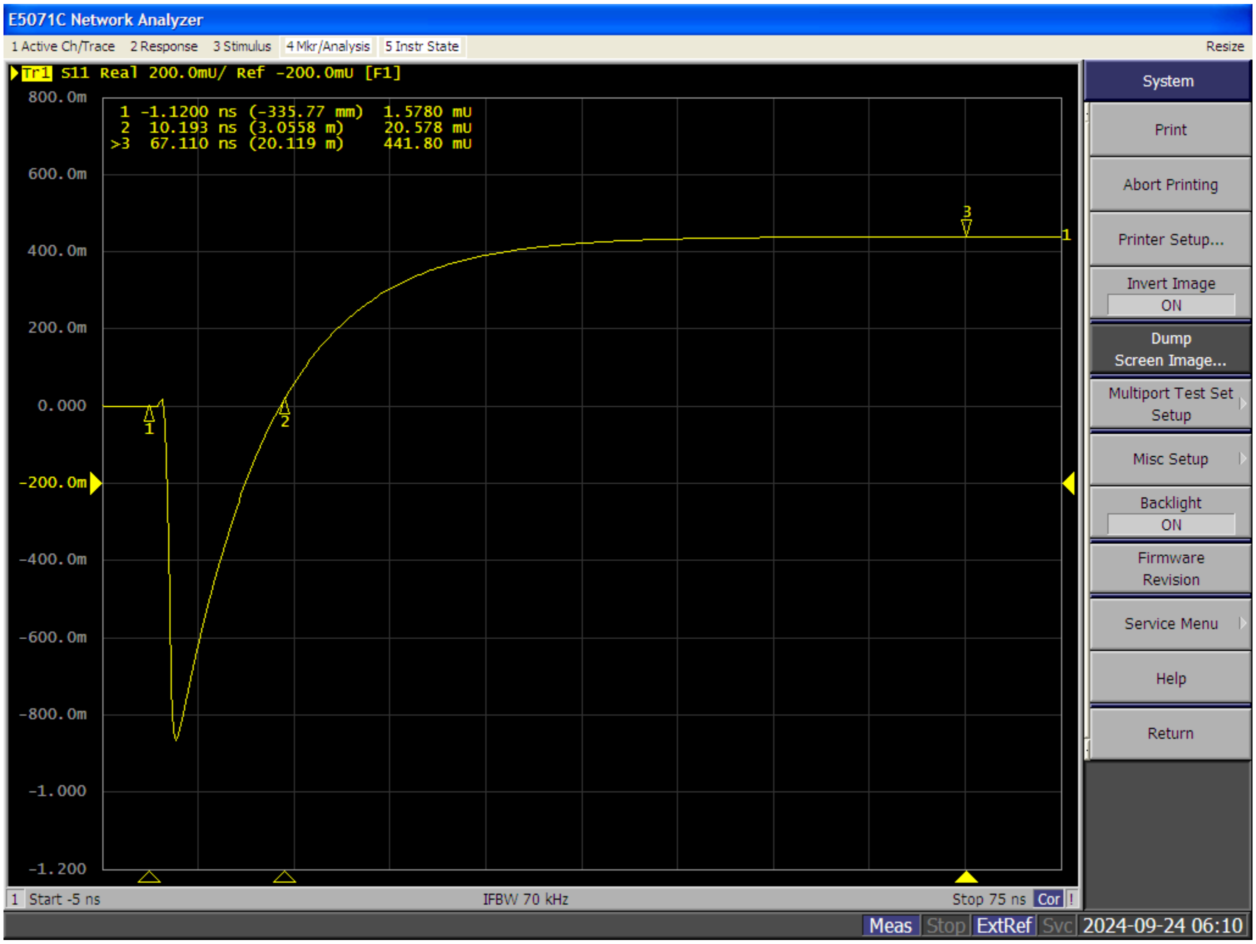

Figure 14: Parallel 220 pF and 130 ohm reflection coefficient measurement

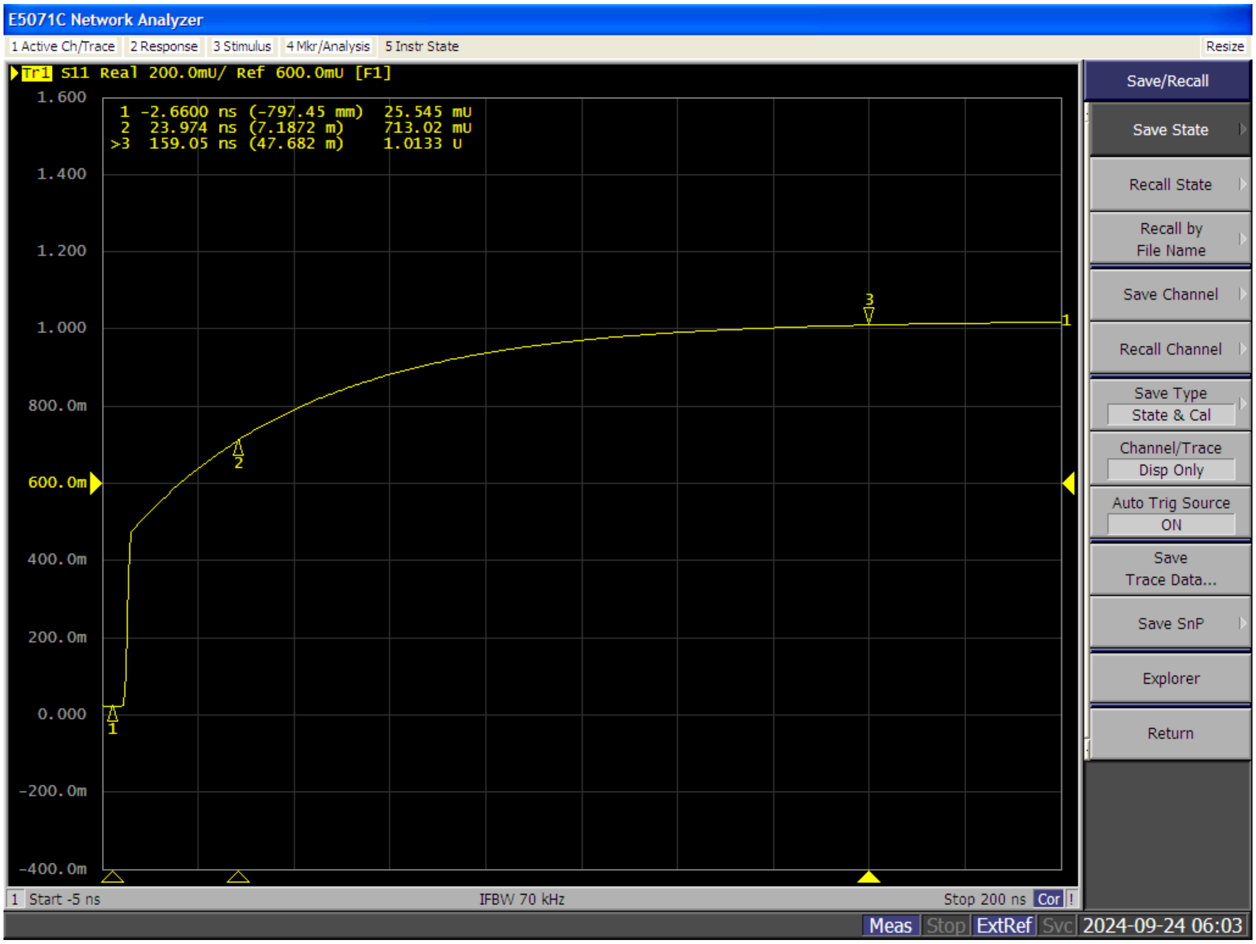

Figure 15: Series 220 pF and 130 ohm reflection coefficient measurement

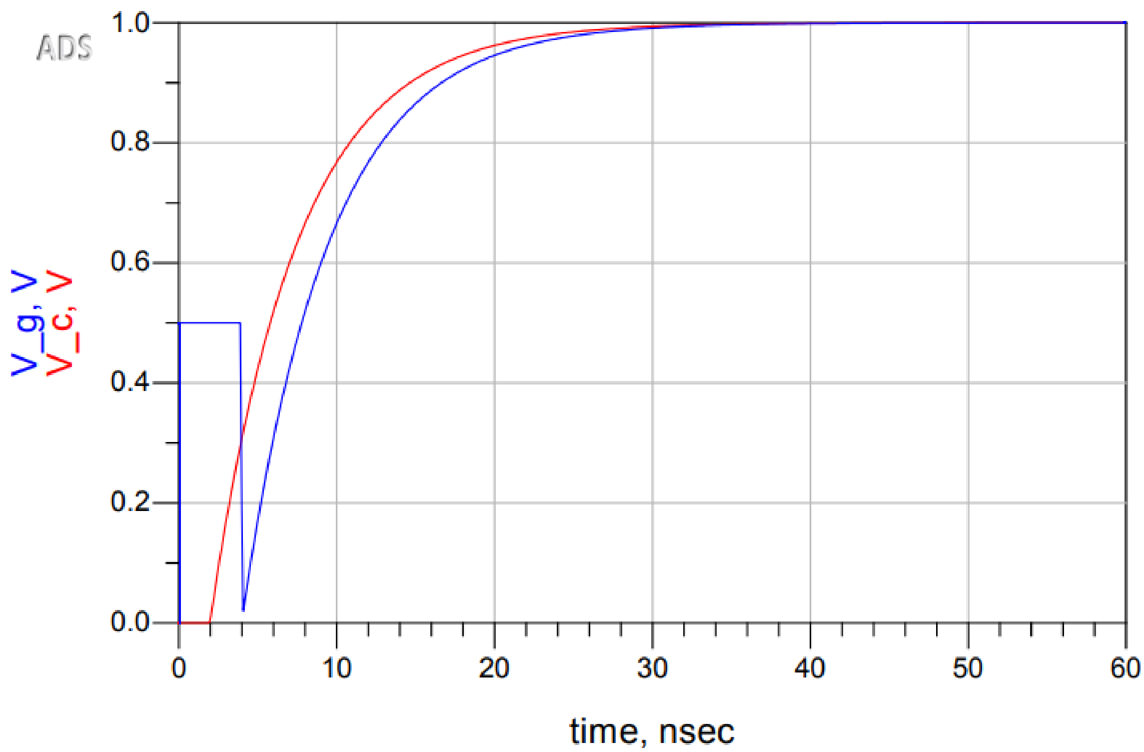

Figure 16: 220 pF Capacitor transmission line load circuit simulation using Advanced Design System

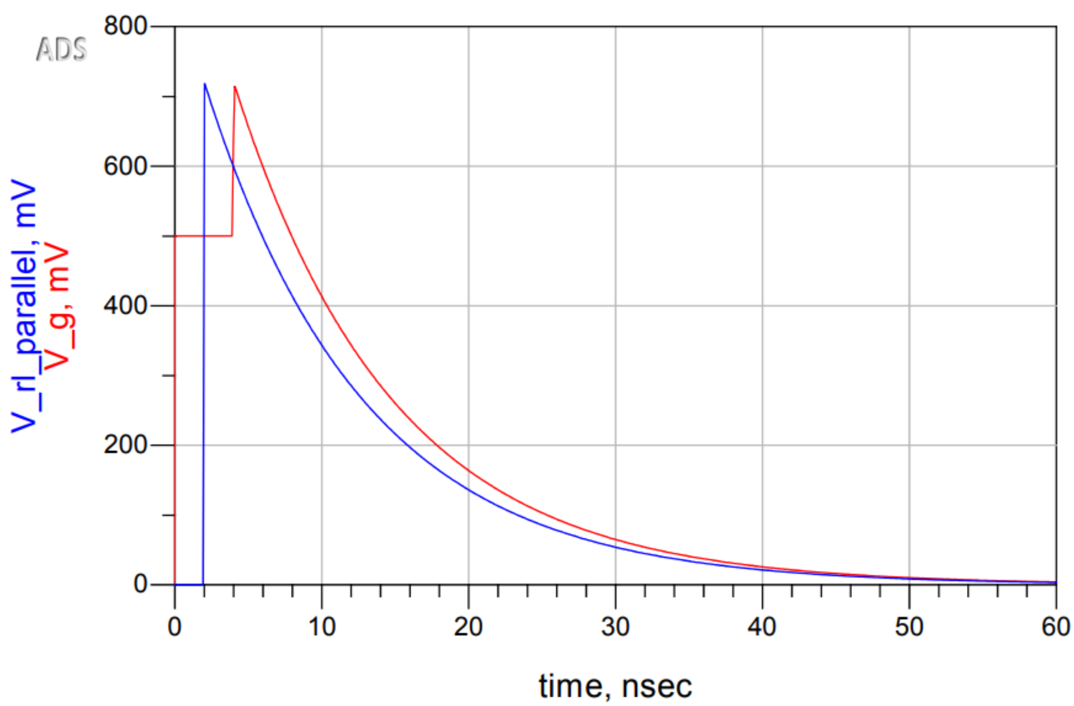

Figure 17: 0.39 uH inductor in parallel with 130 ohm transmission line load circuit simulation using Advanced Design System

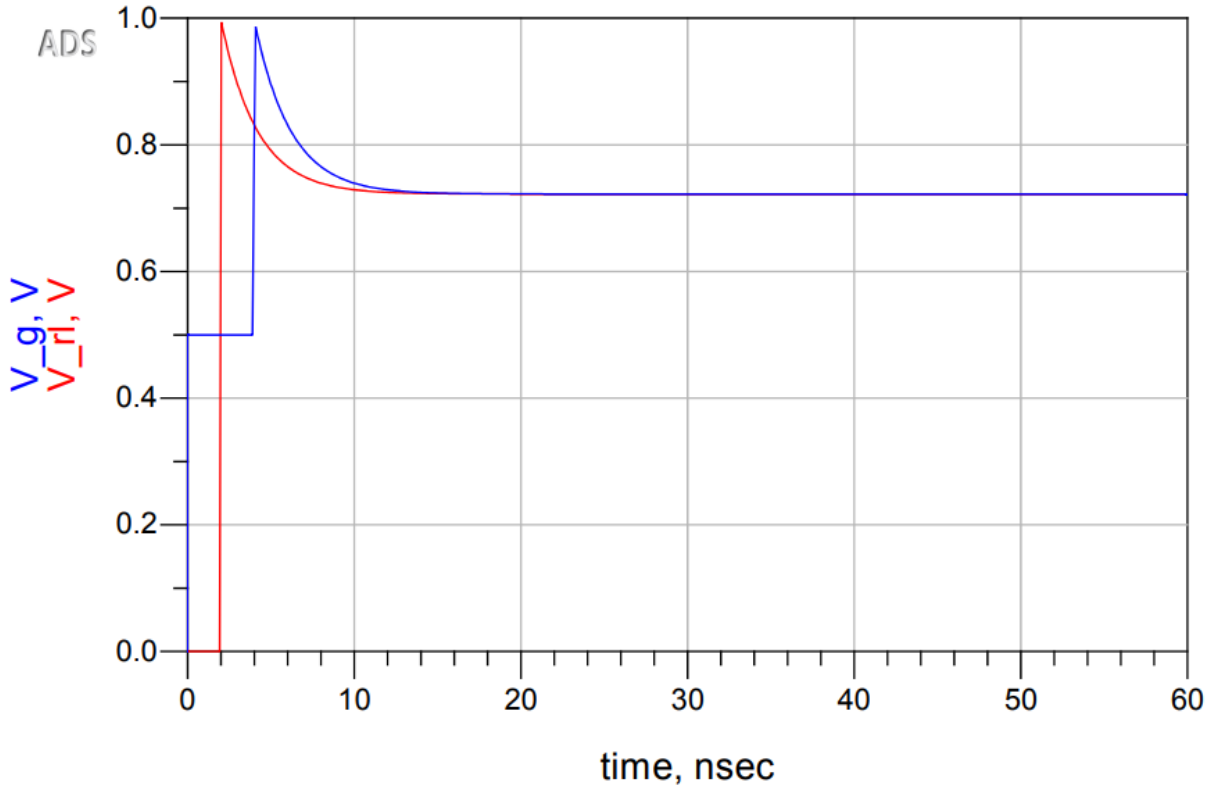

Figure 18: 0.39 uH inductor in series with 130 ohm resistor transmission line load circuit simulation using Advanced Design System

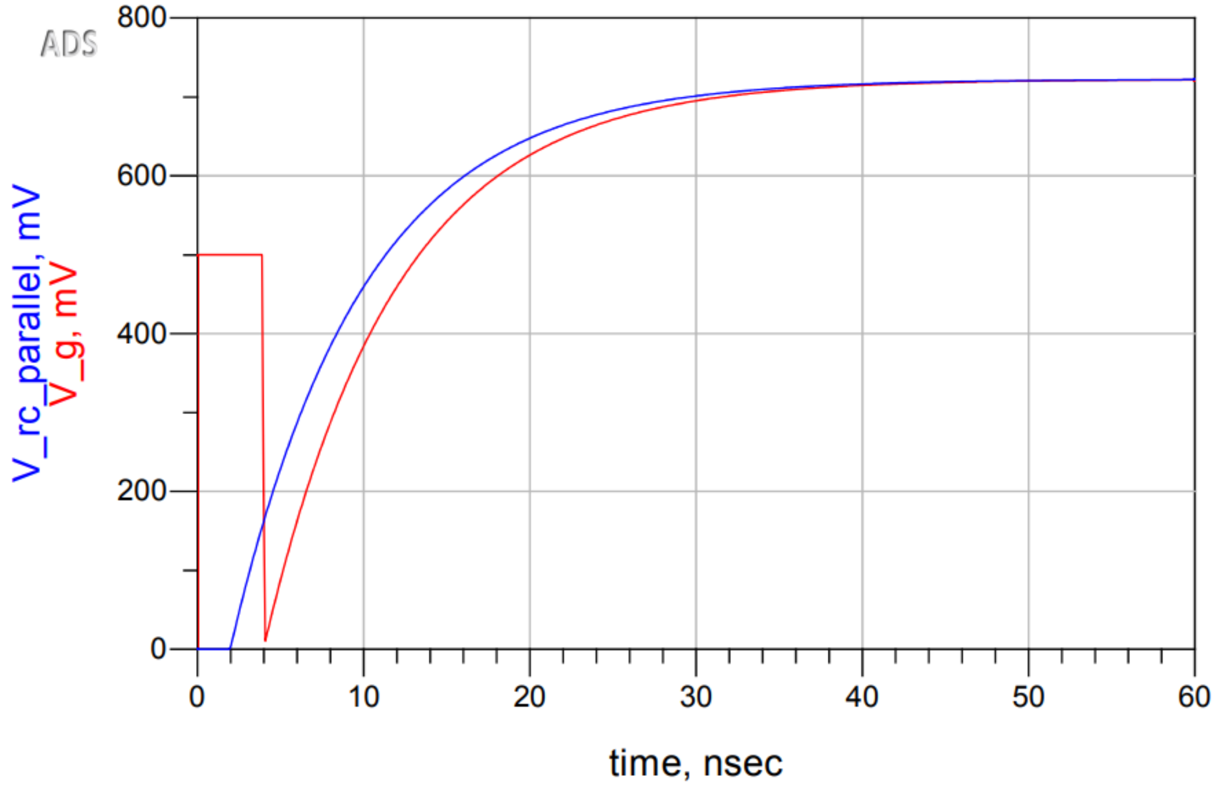

Figure 19: 220 pF capacitor in parallel with 130 ohm inductor transmission line load circuit simulation using Advanced Design System

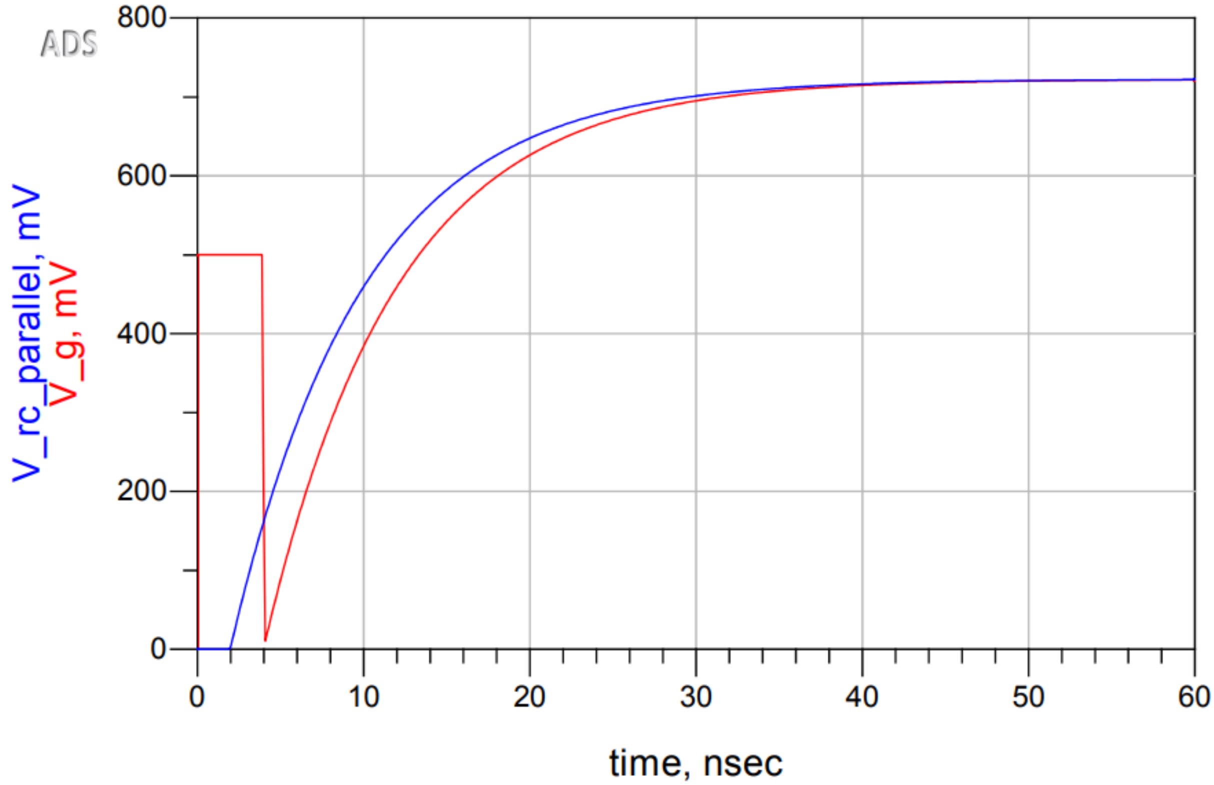

Figure 20: 220 pF capacitor in parallel with 130 ohm resistor transmission line load circuit simulation using Advanced Design System

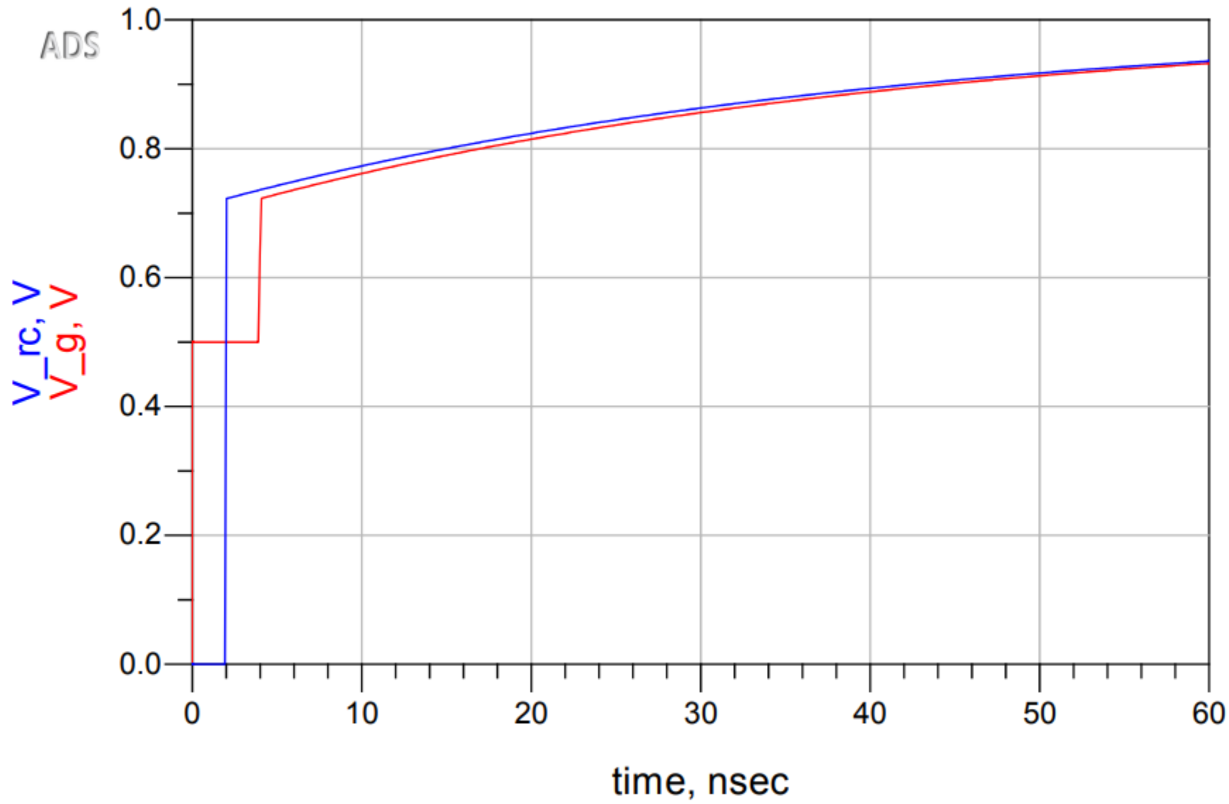

Figure 20: 220 pF capacitor in series with 130 ohm resistor transmission line load circuit simulation using Advanced Design System

We can observe that when applying Eq. 1 to circuit iii, we get an expected reflective index of 0.444 at both initial and final times. In Fig. 8 we can observe that the reflective index at a final time is of 0.440, while the initial index is at 0, as a result of the delay produced by the micro-strip line connecting the SMA connector and the load.

For the 33 ohm resistor, we observed a negative reflective index at the final times, with a reflection coefficient of 0.207. Applying Eq. 1 we can confirm that this is the corresponding reflection for a 32.8 ohm resistor.

In Fig. 10 we observe an inversed decaying exponential waveform, with an initial reflective index of 0.00094221, approximately zero, and a final reflection coefficient of 1.0002 as expected when applying Eq. 2 to circuit iv. Fig. 10 agrees with the Advanced Design System simulation displayed in Fig. 16. The capacitor acts as a short circuit at time zero, and as an open circuit at time infinity.

In Fig. 11 we observe a decaying exponential waveform, with an initial reflection coefficient of ~0.75 after the delay caused by the micro-strip, and a final reflective coefficient of -0.96575, which correlates with our expected values for Eq. 5, 6 and 7 when applied to circuit iv Opposite to the capacitor, the inductor acts as an open circuit at time zero, and as a short circuit at time infinity. Further, we can see this is the theoretically expected behavior for a capacitive load in Fig. 2.

In Fig. 12 we observe a decaying exponential waveform, with an initial reflection coefficient of ~0.375 after the delay caused by the micro-strip, and a final reflective coefficient of -0.96703, which correlates with our expected values when applying Eq. 13, 14, 15 and 16 to circuit vi. This RL parallel circuit behavior assimilates to Fig. 17's graph, but our ADS has a different initial and final reflective coefficient.

In Fig. 13 we observe the reflection coefficient to behaves as a rapidly decaying exponential from ~0.880 to 0.44354. This agrees with our calculated expected values using Eq. 17, 18, 19 and 20, applied to circuit v. The NA displays the expected graph from the theoretically simulated values from Fig. 3, and those in Fig. 18, but with a lower peak initial reflective coefficient, and a lower final reflective coefficient compared to Fig. 18's ADS simulated circuit.

In Fig. 14 we can observe an inverse decaying exponential with initial reflective coefficient of ~ -0.880 and a final reflective coefficient of 0.44180. This agrees with our calculated reflection coefficients for circuit viii, and display an equivalent behavior with Fig. 6. and Fig. 19. When comparing to Fig. 6, our results display a higher value for the initial reflection coefficient. For the simulated ADS Fig. 19, the simulation suggests a higher value for the final reflection coefficient.

In Fig. 15 we observe an inverse decaying exponential, with an initial reflection coefficient of ~0.444, and a final reflection coefficient of 1, which correlates with the expected behavior for an RC series circuit for case vii, and agrees with Fig. 5 (although the simulation starts at a lower reflection coefficient value, suggesting a code error). These results further agree with the graph behavior of Fig. 20, although the ones measured have a higher initial reflection coefficient of ~0.5, and we cannot observe the exact final reflection coefficient, although it appears to damp to 1 as expected.

Conclusion

The goal of the lab was to learn about the behavior of transmission lines' reflection coefficients as a function of time for capacitive and inductive loads. We were able to understand that the reflection coefficients for transmission lines with a single capacitive or inductive load, and with or without a resistive load, all have a decaying exponential behavior with respect to time.

We further learnt the differences in behavior and values between the theoretically simulated circuits, the real, measured circuits with a Network Analyzer, and the Advanced Design System circuit simulations. The theoretical circuit simulations appear to have greater accuracy in the final reflection coefficient values, but do not display the initial delay caused by the microstrip in transmission lines, and tend to have a calculate a greater value for the initial reflection coefficient. On the other hand, the ADS simulated circuits are able to display a behavior closer to the ones found in the real measurements, but often display greater reflection coefficient values for both initial and final times.

Furthermore, some mistakes observed in the lab were the initial value error for the code we created to graph the expected RC series circuit, which should be corrected in the future by creating more accurate codes; and not measuring the component values with a DMM and a AC meter, which improves reliability of labs.

All in all, the lab was partially successful, as we were able to get an accurate understanding of the differences between theory, simulation, and real circuit time-varying reflection coefficients for transmission lines with time-dependent loads.