Boa tarde a todos!

Iniciaremos as 13:50hrs

Ex 1) $\det(A) = \begin{vmatrix} 3 & 1 & -1 \\ 2 & -2 & -4 \\ 1 & -3 & 5 \end{vmatrix}$

Cofatores 1ª linha: $\det(A) = 3 \cdot C_{11} + 1 \cdot C_{12} + (-1) C_{13}$ ($C_{11} = -22$, $C_{12} = -14$, $C_{13} = -4$)

$$C_{ij} = (-1)^{i+j} M_{ij}$$

Note que:

$$C_{11} = (-1)^2 M_{11} = M_{11} = \begin{vmatrix} -2 & -4 \\ -3 & 5 \end{vmatrix} = -10 - 12$$

$$\Rightarrow C_{11} = -22$$

$$C_{12} = (-1)^3 M_{12} = (-1) \cdot \begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = (-1) \cdot (10 - (-4))$$

$$\Rightarrow C_{12} = -14$$

$$C_{13} = (-1)^4 M_{13} = M_{13} = \begin{vmatrix} 2 & -2 \\ 1 & -3 \end{vmatrix} = -6 - (-2)$$

$$\Rightarrow C_{13} = -4$$

Logo, $\det(A) = 3 \cdot (-22) + 1 \cdot (-14) + (-1) \cdot (-4)$

$$\Rightarrow \det(A) = -76 \neq 0$$

$\exists A^{-1}$!

Ex 2)

Ex 2) $(A \mid I) \sim \cdots\cdots \sim (I \mid A^{-1})$

$$\begin{pmatrix} 3 & 1 & -1 & 1 & 0 & 0 \\ 2 & -2 & -4 & 0 & 1 & 0 \\ 1 & -3 & 5 & 0 & 0 & 1 \end{pmatrix} \sim \begin{matrix} \\ -2+ \\ \\ \end{matrix} \begin{pmatrix} 1 & -3 & 5 & 0 & 0 & 1 \\ 2 & -2 & -4 & 0 & 1 & 0 \\ 3 & 1 & -1 & 1 & 0 & 0 \end{pmatrix}$$

$$\sim \begin{matrix} -3 \\ + \\ \\ \end{matrix} \begin{pmatrix} 1 & -3 & 5 & 0 & 0 & 1 \\ 0 & 4 & -14 & 0 & 1 & -2 \\ 3 & 1 & -1 & 1 & 0 & 0 \end{pmatrix} \sim \frac{1}{4} \begin{pmatrix} 1 & -3 & 5 & 0 & 0 & 1 \\ 0 & 4 & -14 & 0 & 1 & -2 \\ 0 & 10 & -16 & 1 & 0 & -3 \end{pmatrix}$$

$$\sim \begin{matrix} \\ + \\ 3 \\ \end{matrix} \begin{pmatrix} 1 & -3 & 5 & 0 & 0 & 1 \\ 0 & 1 & -7/2 & 0 & 1/4 & -1/2 \\ 0 & 10 & -16 & 1 & 0 & -3 \end{pmatrix} \sim \begin{matrix} \\ -10 \\ + \\ \end{matrix} \begin{pmatrix} 1 & 0 & -11/2 & 0 & 3/4 & -1/2 \\ 0 & 1 & -7/2 & 0 & 1/4 & -1/2 \\ 0 & 10 & -16 & 1 & 0 & -3 \end{pmatrix}$$

$$\sim \begin{matrix} \\ \\ \frac{1}{19} \\ \end{matrix} \begin{pmatrix} 1 & 0 & -11/2 & 0 & 3/4 & -1/2 \\ 0 & 1 & -7/2 & 0 & 1/4 & -1/2 \\ 0 & 0 & 19 & 1 & -5/2 & 2 \end{pmatrix} \begin{matrix} + \\ + \\ \frac{11}{2}\ \frac{7}{2} \end{matrix} \begin{pmatrix} 1 & 0 & -\frac{11}{2} & 0 & 3/4 & -\frac{1}{2} \\ 0 & 1 & -\frac{7}{2} & 0 & 1/4 & -1/2 \\ 0 & 0 & 1 & 1/19 & -\frac{5}{38} & \frac{2}{19} \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 11/38 & 1/38 & 3/38 \\ 0 & 1 & 0 & 7/38 & -4/19 & -5/38 \\ 0 & 0 & 1 & 1/19 & -5/38 & 2/19 \end{pmatrix}$$

$$-\frac{55}{76} + \frac{3}{4} = \frac{1}{38}$$

$$\frac{11}{19} - \frac{1}{2} = \frac{3}{38}$$

$$\therefore A^{-1} = \frac{1}{38} \begin{pmatrix} 11 & 1 & 3 \\ 7 & -8 & -5 \\ 2 & -5 & 4 \end{pmatrix}$$

Verificar $A \cdot A^{-1} = I$

$A^{-1} A = I$

$$\det(A \cdot A^{-1}) = \det(I)$$

$\det(A) \cdot \det(A^{-1}) = 1$ ($\det(A) \neq 0$)

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & -2 & -4 \\ 1 & -3 & 5 \end{pmatrix} \overset{-3,\ -2}{\sim} \begin{pmatrix} 1 & -3 & 5 \\ 2 & -2 & -4 \\ 3 & 1 & -1 \end{pmatrix} \sim \frac{1}{2}\begin{pmatrix} 1 & -3 & 5 \\ 0 & 4 & -14 \\ 0 & 10 & -16 \end{pmatrix}$$

$\times x \longrightarrow \times -x \longrightarrow \times -x$

Super $\det(A) = x$

$$\sim \begin{pmatrix} 1 & -3 & 5 \\ 0 & 2 & -7 \\ 0 & 10 & -16 \end{pmatrix} \overset{-5}{\sim} \begin{pmatrix} 1 & -3 & 5 \\ 0 & 2 & -7 \\ 0 & 0 & 19 \end{pmatrix} = A'$$

$-\frac{1}{2}x \longrightarrow -\frac{1}{2}x$

$\det(A') = 1 \cdot 2 \cdot 19 = 38 = -\frac{1}{2}x \iff x = -76$

$\therefore \det(A) = -76$

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$$

onde $\operatorname{adj}(A) = (\operatorname{Cof}(A))^t$