Aula Síncrona - 01/04 - Computação

Ex 1) $r: (x,y) = (1,2) + t(3,1)$, $t \in \mathbb{R}$

$P = P_0 + t\,v$

↳ eq. vetorial da reta.

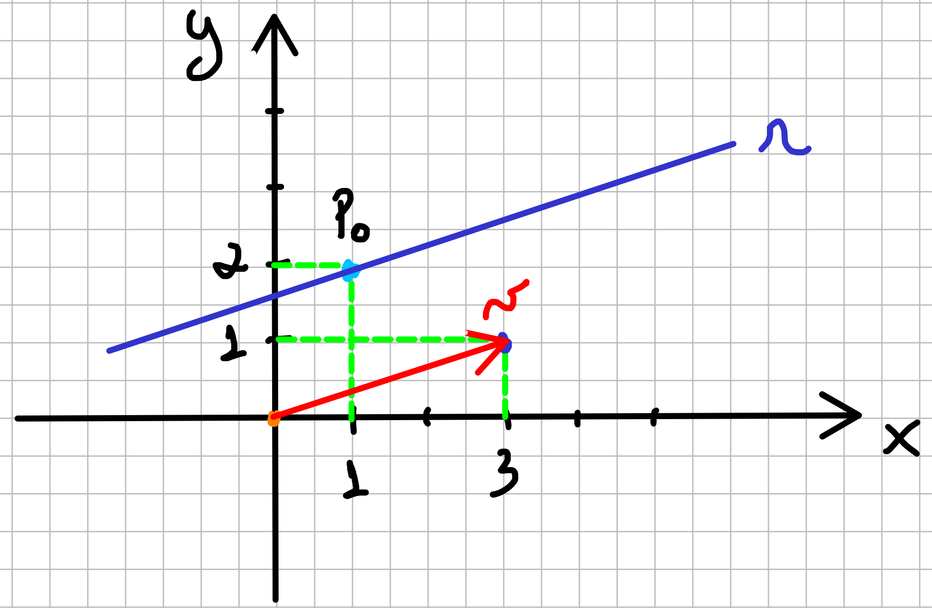
Logo, $P_0 = (1,2)$ é um ponto da reta
$v = (3,1)$ é um vetor diretor

Outra forma: $t = 1 \Rightarrow (x,y) = (1,2) + (3,1) = (4,3)$

$\Rightarrow P_1 = (4,3) \in r$

$P_0 = (1,2)$

$v = (3,1)$



$r: (x,y) = (1,2) + t(3,1)$, $t \in \mathbb{R}$

↳ eq. vetorial da reta

$(x,y) = (1,2) + (3t, t) = (1+3t, 2+t)$

$\Rightarrow r: \begin{cases} x = 1 + 3t \\ y = 2 + t \end{cases}$, $t \in \mathbb{R}$

↳ eq. paramétricas da reta

1º) $x = 1 + 3t \Rightarrow x - 1 = 3t \Rightarrow \frac{x-1}{3} = t$

$2^{\underline{o}})$ $y = 2 + t \Rightarrow y - 2 = 1 \cdot t \Rightarrow \dfrac{y-2}{1} = \underline{t}$

$$\Rightarrow \frac{x-1}{3} = t = \frac{y-2}{1}$$

$$\Rightarrow r: \frac{x-1}{3} = \frac{y-2}{1}$$

$\dfrac{x-x_0}{a} = \dfrac{y-y_0}{b}$

$P_0 = (x_0, y_0)$ —

↳ eq simétricas da reta $v = (a,b)$

Obs: $3(y-2) = 1 \cdot (x-1) \Rightarrow 3y - 6 = x - 1$

$$\Rightarrow 3y = x + 5 \Rightarrow \underline{y = \frac{1}{3}x + \frac{5}{3}}, \; x \in \mathbb{R}$$

Obs 2: $y = \dfrac{1}{3}x + \dfrac{5}{3}, \; x \in \mathbb{R}$

Tome $t = x \Rightarrow y = \dfrac{1}{3}t + \dfrac{5}{3}$

$$\Rightarrow r: \begin{cases} x = t \\ y = \frac{1}{3}t + \frac{5}{3} \end{cases}, \; t$$

$P_1 = (0, 5/3), \; v_1 = (1, 1/3)$

$v = (3,1) = 3 v_1$

Ex 2) $r_1: (x,y,z) = \overbrace{(2,-1,0)}^{P_1} + t\overbrace{(4,1,1)}^{v_1}, \; t \in \mathbb{R}$

$r_2: (x,y,z) = \underbrace{(-1,-1,-1)}_{P_2} + t\underbrace{(3,2,1)}_{v_2}, \; t \in \mathbb{R}$

$$\Rightarrow \overrightarrow{P_1P_2} = P_2 - P_1 = (-3, 0, -1)$$

→ coplanares $v_1 \cdot (v_2 \times \overrightarrow{P_1P_2}) = 0$?

→ reversos $v_1 \cdot (v_2 \times \overrightarrow{P_1P_2}) \neq 0$

→ paralelos : $r_1 // r_2 \iff v_1 // v_2$
$v_1 = \underline{\underline{k}} v_2,\ k \in \mathbb{R}$

ou

concorrentes : $\exists\, P \in r_1 \cap r_2$

Note que: $v_1 \cdot (v_2 \times \overrightarrow{P_1P_2}) = \begin{vmatrix} 4 & 1 & 1 \\ 3 & 2 & 1 \\ -3 & 0 & -1 \end{vmatrix} = -2 \neq 0$

$\therefore$ r_1 e r_2 são retas reversas

Ex 3) ✓ paralelos
✓ perpendiculares (90°)
✓ concorrentes
✗ ~~reversos~~

Obs: 1) reta : $\cos\theta = \dfrac{|v_1 \cdot v_2|}{\|v_1\|\,\|v_2\|}$, $0 \leq \theta \leq 90°$

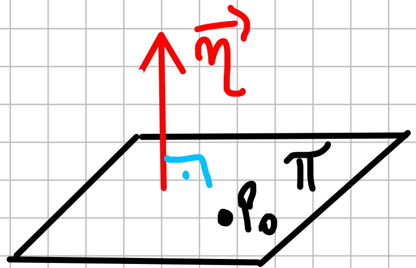
$r_1: P = P_1 + t v_1$
$r_2: P = P_2 + t v_2$
$t \in \mathbb{R}$

2) plano $\Longrightarrow$ $\cos\theta = \dfrac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\|\,\|\vec{n}_2\|}$, $0 \leq \theta \leq 90°$

$\pi_1: \underline{a_1}x + \underline{b_1}y + \underline{c_1}z + d_1 = 0$

$\pi_2: a_2x + b_2y + c_2z + d_2 = 0$

$\Rightarrow \vec{n}_1 = (a_1, b_1, c_1)$



Ex 4) Verdadeiro.

$$d(P_1, \pi) = \frac{|a x_0 + b y_0 + c z_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

$P_1 = (x_0, y_0, z_0)$

$\vec{n} = (a, b, c)$

$$d(r, \pi) = d(P_1, \pi) \quad , \; P_1 \in r$$

$$d(\pi_1, \pi) = d(P_2, \pi) \quad , \; P_2 \in \pi_1$$