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QUESTÃO 1: CALCULE A INTEGRAL $\int \frac{e^x}{\sqrt{e^{2x}+1}} dx$

$$\begin{cases} u = e^x \\ du = e^x dx \therefore dx = \frac{du}{e^x} \end{cases}$$

$$\int \frac{e^x}{\sqrt{e^{2x}+1}} dx = \int \frac{\cancel{e^x}}{\sqrt{u^2+1}} \cdot \frac{du}{\cancel{e^x}} = \int \frac{1}{\sqrt{u^2+1}} du = \ln|u + \sqrt{u^2+1}|$$

$$= \ln|e^x + \sqrt{e^{2x}+1}| + C$$

PORTANTO,

$$\int \frac{e^x}{\sqrt{e^{2x}+1}} dx = \ln|e^x + \sqrt{e^{2x}+1}| + C \quad //$$

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QUESTÃO 2: USANDO A TÉCNICA DE FRAÇÕES PARCIAIS, CALCULE A INTEGRAL $\int \frac{x-1}{(x-2)^2(x-3)^2} dx$

$$* \quad \frac{x-1}{(x-2)^2(x-3)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)} + \frac{D}{(x-3)^2}$$

$$\therefore \quad x-1 = A(x-2)(x-3)^2 + B(x-3)^2 + C(x-2)^2(x-3) + D(x-2)^2$$

PARA $x = 2$

$$2-1 = A\cancel{(2-2)}^{0}(2-3)^2 + B(2-3)^2 + C\cancel{(2-2)^2}^{0}(2-3) + D\cancel{(2-2)}^{0\,2}$$

$$1 = B(-1)^2$$

$$\boxed{B = 1}$$

PARA $x=3$

$$3-1 = A(3-2)\cancel{(3-3)}^{0\,2} + B\cancel{(3-3)}^{0\,2} + C(3-2)^2\cancel{(3-3)}^{0} + D(3-2)^2$$

$$2 = D(1)^2 \quad \therefore \quad \boxed{D = 2}$$

PARA $x=0$

$$0-1 = A(0-2)(0-3)^2 + B(0-3)^2 + C(0-2)^2(0-3) + D(0-2)^2$$

$$-1 = A(-2)(-3)^2 + 1\cdot(-3)^2 + C(-2)^2(-3) + 2(-2)^2$$

$$-1 = A(-2)(9) + 1\cdot 9 + C(4)(-3) + 2\cdot 4$$

$$-1 = -18A + 9 - 12C + 8$$

$$18A + 12C = 9+8+1$$

$$18A + 12C = 18 \quad (\div 6)$$

$$3A + 2C = 3$$

## PARA $x=1$

$$1-1 = A(1-2)(1-3)^2 + B(1-3)^2 + C(1-2)^2(1-3) + D(1-2)^2$$

$$0 = A(-1)(-2)^2 + B(-2)^2 + C(-1)^2(-2) + D(-1)^2$$

$$0 = A(-1)(4) + 1\cdot(4) + C(1)(-2) + 2\cdot 1$$

$$0 = -4A + 4 - 2C + 2$$

$$4A + 2C = 6 \qquad \div 2$$

$$2A + C = 3$$

## RESOLVENDO O SISTEMA

$$* \begin{cases} 3A + 2C = 3 \\ 2A + C = 3 \qquad l_1 \leftarrow l_1 - 2l_2 \end{cases}$$

$$\begin{cases} -A + 0 = -3 \therefore \boxed{A = 3} \\ 2A + C = 3 \therefore 2\cdot 3 + C = 3 \therefore C = 3-6 \therefore \boxed{C = -3} \end{cases}$$

$$* \frac{x-1}{(x-2)^2(x-3)^2} = \frac{3}{(x-2)} + \frac{1}{(x-2)^2} - \frac{3}{(x-3)} + \frac{2}{(x-3)^2}$$

## CALCULANDO A INTEGRAL

$$\int \frac{x-1}{(x-2)^2(x-3)^2} = \int \left( \frac{3}{(x-2)} + \frac{1}{(x-2)^2} - \frac{3}{(x-3)} + \frac{2}{(x-3)^2} \right) dx$$

$$= \int \frac{3}{x-2}\,dx + \int \frac{1}{(x-2)^2}\,dx - \int \frac{3}{(x-3)}\,dx + \int \frac{2}{(x-3)^2}\,dx$$

$$= 3\int \frac{1}{x-2}\,dx + \int \frac{1}{(x-2)^2}\,dx - 3\int \frac{1}{x-3} + 2\int \frac{1}{(x-3)^2}\,dx$$

$$\int \frac{1}{x-2}\,dx = \int \frac{1}{u}\,du = \int \frac{du}{u} = \ln|u| = \ln|x-2|$$

$$\begin{cases} u = x-2 \\ du = 1\cdot dx \end{cases}$$

$$\int \frac{1}{(x-2)^2}\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du = \frac{u^{-1}}{-1} = \frac{-1}{u} = \frac{-1}{x-2}$$

$$\begin{cases} u = x-2 \\ du = 1\cdot dx \end{cases}$$

$$\int \frac{1}{x-3}\,dx = \int \frac{1}{u}\,du = \int \frac{du}{u} = \ln|u| = \ln|x-3|$$

$$\begin{cases} u = x-3 \\ du = 1\cdot dx \end{cases}$$

$$\int \frac{1}{(x-3)^2}\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du = \frac{u^{-1}}{-1} = \frac{-1}{u} = \frac{-1}{x-3}$$

$$\begin{cases} u = x-3 \\ du = 1\cdot dx \end{cases}$$

$$= 3\ln|x-2| - \frac{1}{x-2} - 3\ln|x-3| - \frac{2}{x-3} + C \quad //$$

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QUESTÃO 3: CALCULE A INTEGRAL $\int_{-\infty}^{+\infty} e^{-|x+1|}\,dx$

$$|x+1| = \begin{cases} -x-1, & \text{se } x < -1 \\ x+1, & \text{se } x \geq -1 \end{cases}$$

$$\int_{-\infty}^{+\infty} e^{-|x+1|}\,dx = \int_{-\infty}^{-1} e^{-|x+1|}\,dx + \int_{-1}^{+\infty} e^{-|x+1|}\,dx$$

$$= \int_{-\infty}^{-1} e^{-(-x-1)}\,dx + \int_{-1}^{+\infty} e^{-(x+1)}\,dx$$

$$= \int_{-\infty}^{-1} e^{x+1}\,dx + \int_{-1}^{+\infty} e^{-x-1}\,dx$$

$$= \lim_{a \to -\infty} \int_{a}^{-1} e^{x+1}\,dx + \lim_{b \to +\infty} \int_{-1}^{b} e^{-x+1}\,dx$$

$$\int e^{x+1}\,dx = \int e^{1} \cdot e^{x}\,dx = e \int e^{x}\,dx = e \cdot e^{x}$$

$$= e^{x+1}$$

$$\int e^{-x-1}\,dx = \int e^{u}(-du) = -\int e^{u}\,du = -e^{u} = -e^{-x-1}$$

$$\begin{cases} u = -x-1 \\ du = -1 \cdot dx \end{cases}$$

$$= \lim_{a \to -\infty} \left[ e^{x+1} \Big|_{a}^{-1} \right] + \lim_{b \to +\infty} \left[ -e^{-x-1} \Big|_{-1}^{b} \right]$$

$$= \lim_{a \to -\infty} \left[ e^{-1+1} - e^{a+1} \right] + \lim_{b \to +\infty} \left[ -e^{-b-1} + e^{1-1} \right]$$

$$= \lim_{a \to -\infty} \left[ e^{0} - e^{a+1} \right] + \lim_{b \to +\infty} \left[ -e^{-b-1} + e^{0} \right]$$

$$= \lim_{a \to -\infty} \left[ 1 - e^{a+1} \right] + \lim_{b \to +\infty} \left[ -e^{-b-1} + 1 \right]$$

$$\begin{cases} a \to -\infty \therefore e^{a+1} \to 0 \therefore -e^{a+1} \to 0 \\ b \to +\infty \therefore -b \to -\infty \therefore -e^{-b-1} \to 0 \end{cases}$$

$$= 1 + 0 + 0 + 1$$

$$= 2 \; //$$

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QUESTÃO 4: MOSTRE QUE $\int_1^{+\infty} \frac{1+e^{-x}}{x}\,dx$ É DIVERGENTE.

* $\frac{1}{x} < \frac{1}{x} + \frac{e^{-x}}{x}$ , $x \geq 1$

$\frac{1}{x} < \frac{1+e^{-x}}{x}$

* $\int_1^{+\infty} \frac{1}{x}\,dx = \lim_{b \to +\infty} \int_1^{b} \frac{1}{x}\,dx = \lim_{b \to +\infty} \left[ \ln|x| \Big|_1^b \right]$

$= \lim_{b \to +\infty} \left[ \ln|b| - \ln|1| \right] = +\infty$

$b \to +\infty \therefore \ln|b| \to +\infty$

COMO $\int_1^{+\infty} \frac{1}{x}\,dx = +\infty$, ENTÃO A

INTEGRAL $\int_1^{+\infty} \frac{1+e^{-x}}{x}\,dx$ É DIVERGENTE.