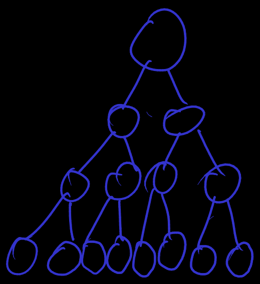
Build Max Heap

```
BUILD-MAX-HEAP(A, n)
1. FOR i = CHAO(n/2) DOWNTO 1 DO
2.     MAX-HEAPIFY(A, i, n)
```

H		Custo
3		$1 \cdot 3 = 1 \cdot H = 3$
2		$2 \cdot 2 = 2 \cdot H = 4$
1		$4 \cdot 1 = 4 \cdot H = 4$
0		0



- Uma heap com n elementos possui altura $\lfloor \lg n \rfloor$.
- Uma heap com n elementos possui no máximo $\left\lceil \frac{n}{2^{H+1}} \right\rceil$ nós em uma altura H.

P/ $|x|<1$,

$$\sum_{k=0}^{\infty} k x^k = \frac{x}{(1-x)^2}$$

$$T(n) = \sum_{H=0}^{\lfloor \lg n \rfloor} \left(\frac{n}{2^{H+1}} \cdot H \right)$$

$$= \frac{n}{2} \sum_{H=0}^{\lfloor \lg n \rfloor} \left(H \cdot \left(\frac{1}{2}\right)^H \right)$$

$$\leq \frac{n}{2} \sum_{H=0}^{\infty} \left(H \cdot \left(\tfrac{1}{2}\right)^H \right)$$

$$\leq \frac{n}{2} \cdot 2 = O(n).$$

$$\frac{1/2}{(1-1/2)^2} = \frac{1/2}{1/4} = \frac{1}{2} \cdot \frac{4}{1} = 2.$$

Análise "inocente" (superestimando o custo do MAX-HEAPIFY na maioria dos nós)

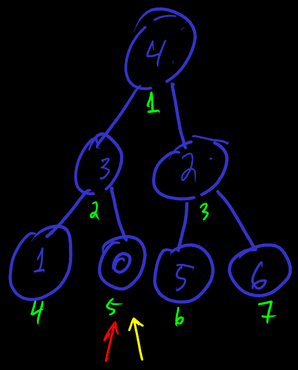
O custo do MAX-HEAPIFY é $\Theta(\lg n)$. Como ele executa $\lfloor n/2 \rfloor$ vezes, $T(n) = \lfloor n/2 \rfloor \cdot \lg(n)$

$$= n/2 \cdot \lg(n)$$

$$= O(n \lg n).$$

HEAPSORT

```
HEAPSORT(A, n)
  BUILD-MAX-HEAP(A, n)          — Θ(n)
  th = n
  FOR i = n DOWNTO 2 DO         ⎫
      troca A[1] e A[i]         ⎬ n-1 vezes
      th = th - 1               ⎪
      MAX-HEAPIFY(A,1,th)       ⎭ — Θ(lg n)
```



$$T(n) = \Theta(n) + n-1(\Theta(\lg n))$$
$$= \Theta(n) + \Theta(n \lg n) - \Theta(\lg n)$$
$$= \Theta(n \lg n).$$