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LISTA 5

(2) a) $B \cap C = \emptyset$
b) $A \cup C = \{p, q, r, s, t, u\}$
c) $\sim C = \{q, r, v, w\}$
d) $A \cap B \cap C = \emptyset$
e) $B - C = \{r, v\}$
f) $\sim(A \cup B) = \{u, w\}$
g) $A \times B =$
h)

(3) $2^4 = 16$

(4) $2^0 = 1$

(5) $P(A) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$

TEM 2 ELEMENTOS

(6) a) V
b) V
c) F
d) V
e) F
f) V
g) F
h)
i)
j) F
k) F
l) V

⑦

a) $A \cup \emptyset = \emptyset \cup A = A$

$\forall x \left[\left[(x \in A \cup \emptyset \rightarrow x \in \emptyset \cup A) \wedge (x \in \emptyset \cup A \rightarrow x \in A \cup \emptyset) \right] \rightarrow x \in A \right] \wedge \left[x \in A \rightarrow \left[(x \in A \cup \emptyset \rightarrow x \in \emptyset \cup A) \wedge (x \in \emptyset \cup A \rightarrow x \in A \cup \emptyset) \right] \right]$

$\forall x \; a \wedge b$

$a: \left[(x \in A \cup \emptyset \rightarrow x \in \emptyset \cup A) \wedge (x \in \emptyset \cup A \rightarrow x \in A \cup \emptyset) \right] \rightarrow x \in A$

$\left[((x \in A \vee x \in \emptyset) \rightarrow (x \in \emptyset \vee x \in A)) \wedge ((x \in \emptyset \vee x \in A) \rightarrow (x \in A \vee x \in \emptyset)) \right] \rightarrow x \in A$

$\left[((x \notin A \wedge x \notin \emptyset) \vee x \in \emptyset \vee x \in A) \wedge ((x \notin \emptyset \wedge x \notin A) \vee x \in A \vee x \in \emptyset) \right] \rightarrow x \in A$

$\left[\left(((x \notin A \wedge x \notin \emptyset) \vee x \in \emptyset) \vee x \in A \right) \wedge \left(((x \notin \emptyset \wedge x \notin A) \vee x \in A) \vee x \in \emptyset \right) \right] \rightarrow x \in A$

$\left[\left(((x \in \emptyset \vee x \notin A) \wedge (x \in \emptyset \vee \cancel{x \notin \emptyset})) \vee x \in A \right) \wedge \left(((x \in A \vee x \notin \emptyset) \wedge (x \in A \vee \cancel{x \notin A})) \vee x \in \emptyset \right) \right] \rightarrow x \in A$

$\left[\left((x \in \emptyset \vee \cancel{x \notin A}) \vee x \in A \right) \wedge \left((x \in A \vee \cancel{x \notin \emptyset}) \vee x \in \emptyset \right) \right] \rightarrow x \in A$

$(\text{verdadeiro}) \rightarrow x \in A$

$\text{Falso} \vee x \in A$

$\boxed{x \in A}$

$b: x \in A \rightarrow \left[(x \in A \cup \emptyset \rightarrow x \in \emptyset \cup A) \wedge (x \in \emptyset \cup A \rightarrow x \in A \cup \emptyset) \right]$

$x \in A \rightarrow \left[((x \in A \vee x \in \emptyset) \rightarrow (x \in \emptyset \vee x \in A)) \wedge ((x \in \emptyset \vee x \in A) \rightarrow (x \in A \vee x \in \emptyset)) \right]$

$x \in A \rightarrow \left[((x \notin A \wedge \cancel{x \notin \emptyset}) \vee x \in \emptyset \vee x \in A) \wedge ((x \notin \emptyset \wedge \cancel{x \notin A}) \vee x \in A \vee x \in \emptyset) \right]$ verdadeiro

$x \in A \rightarrow$ verdadeiro $\therefore x \notin A \vee$ verdadeiro

verdadeiro

$\boxed{\forall x \; x \in A}$ //

c.4d

b) $A \cup A = A$

$\forall x \left[((x \in A \cup A) \rightarrow x \in A) \wedge (x \in A \rightarrow (x \in A \cup A)) \right]$

$\forall x \left[((x \in A \vee x \in A) \rightarrow x \in A) \wedge (x \in A \rightarrow (x \in A \vee x \in A)) \right]$

$\forall x \left[((x \notin A \wedge x \notin A) \vee x \in A) \wedge (x \notin A \vee \underbrace{x \in A \vee x \in A}_{\text{verdadeiro}}) \right]$

$\forall x \left[(x \in A \vee x \notin A) \wedge (x \in A \vee x \notin A) \right.$

VERDADEIRO

c.q.d ∎ //

c) $A \cup B = B \cup A$

$\forall x \left[(x \in A \cup B \rightarrow x \in B \cup A) \wedge (x \in B \cup A \rightarrow x \in A \cup B) \right]$

$\forall x \left[((x \in A \vee x \in B) \rightarrow (x \in B \vee x \in A)) \wedge ((x \in B \vee x \in A) \rightarrow (x \in A \vee x \in B)) \right]$

$\forall x \left[((x \notin A \wedge x \notin B) \vee \overbrace{x \in B \vee x \in A}^{\text{verdadeiro}}) \wedge ((x \notin B \wedge x \notin A) \vee \overbrace{x \in A \vee x \in B}^{\text{verdadeiro}}) \right]$

VERDADEIRO

c.q.d ∎ //

8) $A \cup (B \cup C) = (A \cup B) \cup C$

$x \in A \cup (B \cup C) \Leftrightarrow (x \in A) \vee (x \in B \cup C)$

$\Leftrightarrow (x \in A) \vee (x \in B \vee x \in C)$

$\Leftrightarrow (x \in A \vee x \in B) \vee x \in C$

$\Leftrightarrow (x \in A \cup B) \vee x \in C$

$\Leftrightarrow x \in (A \cup B) \cup C$

//

(9)

a) $\boxed{A \cap U = U \cap A} = A$

$$[(x \in A \cap U \to x \in U \cap A) \wedge (x \in U \cap A \to x \in A \cap U)] \to x \in A$$
$$\wedge \; [x \in A \to [(x \in A \cap U \to x \in U \cap A) \wedge (x \in U \cap A \to x \in A \cap U)]]$$

$\forall x \;\; a \wedge b$

a: $[(x \in A \cap U \to x \in U \cap A) \vee (x \in U \cap A \to x \in A \cap U)] \to x \in A$

$[((x \in A \wedge x \in U) \to (x \in U \wedge x \in A)) \vee ((x \in U \wedge x \in A) \to (x \in A \wedge x \in U))] \to x \in A$

$[(x \notin A \vee x \notin U \vee (x \in U \wedge x \in A)) \vee (x \notin U \vee x \notin A \vee (x \in A \wedge x \in U))] \to x \in A$

VERDADERO $\to x \in A$

FALSO $\vee \; x \in A$

$x \in A$

b: $x \in A \to [(x \in A \cap U \to x \in U \cap A) \wedge (x \in U \cap A \to x \in A \cap U)]$

$x \in A \to [((x \in A \wedge x \in U) \to (x \in U \wedge x \in A)) \wedge ((x \in U \wedge x \in A) \to (x \in A \wedge x \in U))]$

$x \in A \to [(x \notin A \vee x \notin U \vee (x \in U \wedge x \in A)) \wedge (x \notin U \vee x \notin A \vee (x \in A \wedge x \in U))]$

$x \in A \to$ VERDADERO

$x \notin A \vee$ VERDADERO

VERDADERO

$\forall x \;\; a \wedge b$

$\forall x \;\; x \in A \wedge$ VERDADERO

$\boxed{\forall x \;\; x \in A}$ //

c.q.d ■ //

b) $A \cap A = A$

$\forall x \left[(x \in A \cap A \to x \in A) \wedge (x \in A \to x \in A \cap A) \right]$

$\forall x \left[((x \in A \wedge x \in A) \to x \in A) \wedge (x \in A \to (x \in A \wedge x \in A)) \right]$

$\forall x \; (x \notin A \vee \cancel{x \notin A} \vee x \in A) \wedge (x \notin A \vee (x \in A \wedge x \in A))$

$\forall x \left[(x \notin A \vee x \in A) \wedge (x \notin A \vee x \in A) \right]$

VERDADEIRO

c.q.d ■ //

c) $A \cap B = B \cap A$

$\forall x \left[(x \in A \cap B \to x \in B \cap A) \wedge (x \in B \cap A \to x \in A \cap B) \right]$

$\forall x \left[((x \in A \wedge x \in B) \to (x \in B \wedge x \in A)) \wedge ((x \in B \wedge x \in A) \to (x \in A \wedge x \in B)) \right]$

$\forall x \left[(x \notin A \vee \cancel{x \notin B} \vee (x \in B \wedge x \in A)) \wedge (x \notin B \vee \cancel{x \notin A} \vee (x \in A \wedge x \in B)) \right]$

VERDADEIRO

c.q.d ■ //

d) $A \cap (B \cap C) = (A \cap B) \cap C$

$$
\begin{aligned}
x \in A \cap (B \cap C) &\Leftrightarrow (x \in A) \wedge (x \in B \cap C) \\
&\Leftrightarrow (x \in A) \wedge (x \in B \wedge x \in C) \\
&\Leftrightarrow (x \in A \wedge x \in B) \wedge x \in C \\
&\Leftrightarrow (x \in A \cap B) \wedge x \in C \\
&\Leftrightarrow x \in (A \cap B) \cap C \quad /
\end{aligned}
$$

(10) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$x \in A \cup (B \cap C) \Leftrightarrow (x \in A) \vee (x \in B \wedge x \in C)$

$(x \in A \vee x \in B) \wedge (x \in A \vee x \in C)$

$x \in (A \cup B) \cap (A \cup C)$ //

(11)

a) $A \cap B = \sim(\sim A \cup \sim B)$

$x \in \sim(\sim A \cup \sim B) \Leftrightarrow \sim[\sim(x \in A) \vee \sim(x \in B)]$

$\Leftrightarrow \sim[x \notin A \vee x \notin B]$

$\Leftrightarrow x \in A \wedge x \in B$

$\Leftrightarrow x \in A \cap B$ //

b) $A \cup B = \sim(\sim A \cap \sim B)$

$x \in \sim(\sim A \cap \sim B) \Leftrightarrow \sim[\sim(x \in A) \wedge \sim(x \in B)]$

$\Leftrightarrow \sim[x \notin A \wedge x \notin B]$

$\Leftrightarrow x \in A \vee x \in B$

$\Leftrightarrow x \in A \cup B$ //

(12)

a) $(A \cup B) \cap \sim A = B \cap \sim A$

$x \in (A \cup B) \cap \sim A \Leftrightarrow (x \in A \vee x \in B) \wedge x \notin A$

$\Leftrightarrow (x \notin A \wedge \overset{F}{x \in A}) \vee (x \notin A \wedge x \in B)$

$\Leftrightarrow x \notin A \wedge x \in B$

$\Leftrightarrow x \in B \wedge x \notin A$

$\Leftrightarrow x \in B \cap \sim A$ //