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INTEGRAIS TRIGONOMETRICAS

$\int sen^m(x) \cos^n(x)\, dx$ m é um inteiro ímpar

$\int sen^{m-1}(x) \cos(x) \cdot sen(x)\, dx$

$sen^2(x) = 1 - \cos^2(x)$

FAÇA A SUBSTITUIÇÃO

$u = \cos(x) \quad \therefore \quad du = -sen(x)\, dx$

EXEMPLO: $\int sen^5(x) \cos(x)\, dx$

$\int sen^4(x) \cos(x) \cdot sen(x)\, dx$

$= \int (sen^2(x))^2 \cos(x)\, sen(x)\, dx$

TROCANDO $sen^2(x) = 1 - \cos^2(x)$

$= \int (1 - \cos^2(x))^2 \cos(x)\, sen(x)\, dx$

$= \int (1 - 2\cos^2(x) + \cos^4(x)) \cos x\, sen x\, dx$

$= \int (\cos(x) - 2\cos^3(x) + \cos^5(x))\, sen(x)\, dx$

$u = \cos x \quad du = -sen(x)\, dx$

$= \int (u - 2u^3 + u^5) \cdot (-du)$

$= \int (-u + 2u^3 - u^5)\, du$

$= -\int u\, du + 2\int u^3\, du - \int u^5\, du$

$= -\frac{u^2}{2} + 2\frac{u^4}{4} - \frac{u^6}{6} + C$

$= -\frac{\cos^2(x)}{2} + \frac{\cos^4(x)}{2} - \frac{\cos^6(x)}{6} + C$ //

$\int sen^m(x) \cos^n(x)\, dx$ n é INTEIRO ÍMPAR

$\int sen^m(x) \cdot \cos^{n-1}(x) \cdot \cos(x)\, dx$

TROCAR $\cos^{n-1}(x)$ EM TERMOS DE $sen(x)$, COM A IDENTIDADE $\cos^2(x) = 1 - sen^2(x)$

$u = sen(x) \therefore du = \cos(x)\, dx$

EXEMPLO: $\int \cos^3(x)\, sen^4(x)\, dx$

$= \int \cos^2(x)\, sen^4(x) \cos(x)\, dx$

$= \int (1 - sen^2(x))\, sen^4(x) \cos(x)\, dx$

$= \int (sen^4(x) - sen^6(x)) \cos(x)\, dx$

$u = sen(x) \quad du = \cos(x)\, dx$

$= \int (u^4 - u^6)\, du$

$= \int u^4\, du - \int u^6\, du$

$= \frac{u^5}{5} - \frac{u^7}{7} + C = \frac{sen^5(x)}{5} - \frac{sen^7(x)}{7} + C$ //

$\int sen^m(x) \cos(x)\, dx$ ∴ m e n são pares

$sen^2(x) = \frac{1 - \cos(2x)}{2}$ $\cos^2(x) = \frac{1 + \cos(x)}{2}$

Exemplo: $\int sen^2(x) \cos^2(x)\, dx$

$= \int \left(\frac{1 - \cos(2x)}{2}\right)\left(\frac{1 + \cos(2x)}{2}\right) dx$

$= \frac{1}{4} \int (1 - \cos(2x)) (1 + \cos(2x))\, dx$

$= \frac{1}{4} \int (1 + \cos(2x) - \cos(2x) - \cos^2(2x))\, dx$

$= \frac{1}{4} \int (1 - \cos^2(2x))\, dx$

$= \frac{1}{4} \int sen^2(2x)\, dx$

$= \frac{1}{4} \left(- \frac{sen(2x) \cdot \cos(2x)}{4} + \frac{1}{2} \int dx \right)$

$= - \frac{sen(2x) \cos(2x)}{16} + \frac{x}{8} + C$ //

$\int Tg^{m}(x)\, sec^{n}(x)\, dx$ m é inteiro ímpar

$= \int Tg^{m-1}(x)\, sec^{n-1}(x)\, Tg(x)\, sec(x)\, dx$

Trocando $Tg^2(x) = sec^2(x) - 1$

$u = sec(x)$ $du = sec(x)\, Tg(x)\, dx$

Exemplo: $\int Tg^{3}(x)\, sec^{5}(x)\, dx$

$= \int Tg^{2}(x)\, sec^{4}(x)\, Tg(x)\, sec(x)\, dx$

$= \int (sec^{4}(x) - 1)\, sec^{4}(x)\, Tg(x)\, sec(x)\, dx$

$= \int (sec^{6}(x) - sec^{4}(x))\, (Tg(x)\, sec(x))\, dx$

$u = sec(x)$ $du = sec(x)\, Tg(x)\, dx$

$= \int (u^6 - u^4)\, du = \int u^6 du - \int u^4 du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C = \frac{sec^7(x)}{7} - \frac{sec^5(x)}{5} + C$ //

$\int Tg^{m}(x)\, sec^{n}(x)\, dx$ n é inteiro par

$\int Tg^{m}(x)\, sec^{n-2}(x)\, sec^{2}(x)\, dx$

Trocar $sec^2(x) = 1 + Tg^2(x)$

$u = tg(x)$ $\therefore$ $du = sec^2(x)\, dx$

EXEMPLO: $\int tg^2(x)\, sec^4(x)\, dx$

$= \int tg^2(x)\, sec^2(x) \cdot sec^2(x)\, dx$

$= \int tg^2(x)\, (tg^2(x) + 1)\, sec^2(x)\, dx$

$= \int (tg^4(x) + tg^2(x))\, sec^2(x)\, dx$

$u = tg(x) \qquad du = sec^2(x)\, dx$

$\int (u^4 + u^2)\, du = \frac{u^5}{5} + \frac{u^3}{3} + C = \frac{tg^5(x)}{5} + \frac{tg^3(x)}{3} + C$ //

$\int tg^m(x)\, sec^n(x)\, dx$ se m é par e n ímpar

EXEMPLO: $\int tg^2(x)\, sec^3(x)\, dx$

$\int \frac{sen^2(x)}{cos^2(x)} \cdot \frac{1}{cos^3(x)}\, dx$

$= \int \frac{sen^2(x)}{cos^5(x)}\, dx = \int \frac{(1 - cos^2(x))\, dx}{cos^5(x)}$

$= \int \frac{1}{cos^5(x)}\, dx - \int \frac{1}{cos^3(x)}\, dx = \int sec^5(x)\, dx - \int sec^3(x)\, dx$

...

EXEMPLO: $\int Tg^2(x)\, sec(x)\, dx$

$u = Tg(x)$ $\qquad$ $dv = Tg(x)\, sec(x)\, dx$

$du = sec^2(x)\, dx$ $\qquad$ $v = sec(x)$

$= Tg(x)\, sec(x) - \int sec^3(x)\, dx$

...

$\int coTg^m(x)\, cossec^2(x)\, dx$

$cossec^2(x) = coTg^2(x) + 1$

$coTg^2(x) = cossec^2(x) - 1$

$\frac{d}{dx}(cossec(x)) = cossec(x)\, coTg(x)$

$\frac{d}{dx}(coTg(x)) = cossec^2(x)$