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INTEGRAIS IMPRÓPRIAS

$\int_a^b f(x)dx \therefore f$ SEJA CONTINUA EM $[a,b]$

1- O INTERVALO DE INTEGRAÇÃO NÃO É LIMITADO

2- A FUNÇÃO POSSUI UMA DESCONTINUIDADE INFINITA NO INTERVALO $[a,b]$

INTEGRAIS IMPRÓPRIAS

$$\int_1^b \frac{1}{x^2}\,dx$$

$$= -\frac{1}{x}\Big|_1^b = -\frac{1}{b} + 1 = 1 - \frac{1}{b}$$

$b = 2 \therefore 1 - \frac{1}{2} = 0{,}5$

$b = 10 \therefore 1 - \frac{1}{10} = 0{,}9$

$b = 1000 \therefore 1 - \frac{1}{1000} = 0{,}999$

$$\int_1^{+\infty} \frac{1}{x^2}\, dx = \lim_{b \to +\infty} \int_1^{b} \frac{1}{x^2}\, dx$$

$$= \lim_{b \to +\infty} \left(1 - \frac{1}{b}\right) = 1$$

INTEGRAIS IMPRÓPRIAS
(LIMITES INFINITOS DE INTEGRAÇÃO)

① f CONTÍNUA EM $[a, +\infty)$

$$\int_a^{+\infty} f(x)\, dx = \lim_{b \to +\infty} \int_a^{b} f(x)\, dx$$

② f CONTÍNUA EM $(-\infty, b]$

$$\int_{-\infty}^{b} f(x)\, dx = \lim_{a \to -\infty} \int_a^{b} f(x)\, dx$$

③ f CONTÍNUA EM $(-\infty, +\infty)$

$$\int_{-\infty}^{+\infty} f(x)\, dx = \int_{-\infty}^{c} f(x)\, dx + \int_c^{+\infty} f(x)\, dx$$

PARA QUALQUER $c \in \mathbb{R}$

EXEMPLO: $\int_1^{+\infty} \frac{1}{x}\,dx$

$$= \lim_{b \to +\infty} \int_1^{b} \frac{1}{x}\,dx$$

$$= \lim_{b \to +\infty} \left[\ln(x) \Big|_1^b \right]$$

$$= \lim_{b \to +\infty} \left[\ln(b) - \ln(1) \right]$$

$$= \lim_{b \to +\infty} \ln(b) = +\infty$$

EXEMPLO: $\int_{-\infty}^{1} e^{2x}\,dx$

$$= \lim_{a \to -\infty} \int_a^{1} e^{2x}\,dx$$

$$* \int e^{2x}\,dx = \int e^{u}\,\frac{du}{2} = \frac{1}{2}e^{u} = \frac{1}{2}e^{2x}$$

$$\begin{cases} u = 2x \\ du = 2dx \end{cases}$$

$$= \lim_{a \to -\infty} \left[\frac{1}{2}e^{2x} \Big|_a^1 \right]$$

$$= \lim_{a \to -\infty} \left[\frac{1}{2}e^{2\cdot 1} - \frac{1}{2}e^{2a} \right] = \frac{e^2}{2} - \lim_{a \to -\infty} \frac{e^{2a}}{2}$$

$$= \frac{e^2}{2} - 0 = \frac{e^2}{2}$$

EXEMPLO: $\int_0^{+\infty} 2x e^{-x^2} dx$

$= \lim_{b \to +\infty} \int_0^b 2x e^{-x^2} dx$

* $\int 2x e^{-x^2} dx = \int e^u (-du) = -\int e^u du = -e^u = -e^{-x^2}$

$\begin{cases} u = -x^2 \\ du = -2x dx \end{cases}$

$= \lim_{b \to +\infty} \left[-e^{-x^2} \Big|_0^b \right] = \lim_{b \to +\infty} \left[-e^{-b^2} + e^{-0^2} \right]$

$= \lim_{b \to +\infty} \left[-e^{-b^2} + 1 \right] = 0 + 1 = \underline{1}$

EXEMPLO: $\int_{-\infty}^{+\infty} x e^{-x^2} dx$

$= \int_{-\infty}^{0} x e^{-x^2} dx + \int_0^{+\infty} x e^{-x^2} dx$

* $\int x e^{-x^2} dx = -\int e^u \frac{du}{2} = -\frac{1}{2} \int e^u du = -\frac{e^u}{2} = -\frac{e^{-x^2}}{2}$

$u = -x^2$
$du = -2x dx$

$\int_{-\infty}^{0} x e^{-x^2} dx = \lim_{a \to -\infty} \left[-\frac{e^{-x^2}}{2} \Big|_a^0 \right]$

$= \lim \left[-\frac{e^{-0^2}}{2} + \frac{e^{-a^2}}{2} \right]$

$= -\frac{1}{2} + \lim_{a \to -\infty} \frac{e^{-a^2}}{2}$

$= -\frac{1}{2}$ //

$$\int_0^{+\infty} x e^{-x^2} dx = \lim_{b \to +\infty} \left[-\frac{e^{-x^2}}{2} \Big|_0^b \right]$$

$$= \lim_{b \to +\infty} \left[-\frac{e^{-b^2}}{2} + \frac{e^{-0^2}}{2} \right]$$

$$= \left[\lim_{b \to +\infty} -\frac{e^{-b^2}}{2} \right] + \frac{1}{2}$$

$$= 0 + \frac{1}{2} = 1/2 \;//$$

PODEMOS CONCLUIR

$$\int_{-\infty}^{+\infty} x e^{-x^2} dx = \int_{-\infty}^{0} x e^{-x^2} dx + \int_{0}^{+\infty} x e^{-x^2} dx$$

$$= -\frac{1}{2} + \frac{1}{2} = 0$$

INTEGRAIS COM INTEGRANDOS INFINITOS

1. f CONTINUA EM $[a, b)$

$$\int_a^b f(x)\, dx = \lim_{c \to b^-} \int_a^c f(x)\, dx$$

2. f CONTINUA EM $(a, b]$

$$\int_a^b f(x)\, dx = \lim_{c \to a^+} \int_c^b f(x)\, dx$$

③ f CONTINUA EM $[a,b]$ EXCETO PARA ALGUM $c \in (a,b)$, ONDE f TENDE AO INFINITO COM VALORES PRÓXIMOS A c,

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

EXEMPLO: $\displaystyle\int_1^2 \frac{1}{\sqrt[3]{x-1}}dx$

$$= \lim_{c \to 1^+} \int_c^2 (x-1)^{-1/3} dx$$

$$= \lim_{c \to 1^+} \left[\frac{(x-1)^{-1/3+1}}{-\frac{1}{3}+1} \Big|_c^2 \right]$$

$$= \lim_{c \to 1^+} \left[\frac{3(x-1)^{2/3}}{2} \Big|_c^2 \right]$$

$$= \lim_{c \to 1^+} \left[\frac{3(2-1)^{2/3}}{2} - \frac{3(c-1)^{2/3}}{2} \right]$$

$$= \frac{3}{2} - \lim_{c \to 1^+} \frac{3(c-1)^{2/3}}{2}$$

$$= \frac{3}{2}$$

EXEMPLO: $\int_{-1}^{2} x^{-3}\,dx$

$$= \int_{-1}^{0} x^{-3}\,dx + \int_{0}^{2} x^{-3}\,dx$$

$$\int_{-1}^{0} x^{-3}\,dx = \lim_{c \to 0^-} \int_{-1}^{c} x^{-3}\,dx$$

$$= \lim_{c \to 0^-} \left[\frac{x^{-2}}{-2} \Big|_{-1}^{c} \right]$$

$$= \lim_{c \to 0^-} \left[-\frac{1}{2c^2} - \frac{1}{-2(1^2)} \right]$$

$$= \lim_{c \to 0^-} \left[-\frac{1}{2c^2} + \frac{1}{2} \right]$$

$$= -\infty$$

COMO $\int_{-1}^{0} x^{-3}\,dx = -\infty$, PODEMOS CONCLUIR QUE $\int_{-1}^{2} x^{-3}\,dx$ É DIVERGENTE.