Boa tarde!

Iniciaremos às 13:50 hrs.

Ex 1) * produto escalar $u \cdot v$, $u, v \in \mathbb{R}^n$

produto vetorial $u \times v$, $u, v \in \mathbb{R}^3$

produto misto $u \cdot (v \times w)$, $u, v, w \in \mathbb{R}^3$

Ex 2) $u \cdot v = (1, 2, 1) \cdot (3, 0, 2)$

$$= 1 \cdot 3 + 2 \cdot 0 + 1 \cdot 2$$

$$= 3 + 0 + 2 = 5$$

$\therefore \; u \cdot v = 5$

Obs: $\|v\| = \|(v_1, \dots, v_n)\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

$$v \cdot v = \|v\|^2$$

Ex 3) $v \times w = \left| \begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 3 & 0 & 2 & 3 & 0 \\ -1 & 2 & -3 & -1 & 2 \end{array} \right.$

$$= 0\vec{i} - 2\vec{j} + 6\vec{k} - 0\vec{k} - 4\vec{i} - (-9)\vec{j}$$

$$= -4\vec{i} + 7\vec{j} + 6\vec{k}$$

$\therefore\ v \times w = -4\vec{i} + 7\vec{j} + 6\vec{k}$

ou

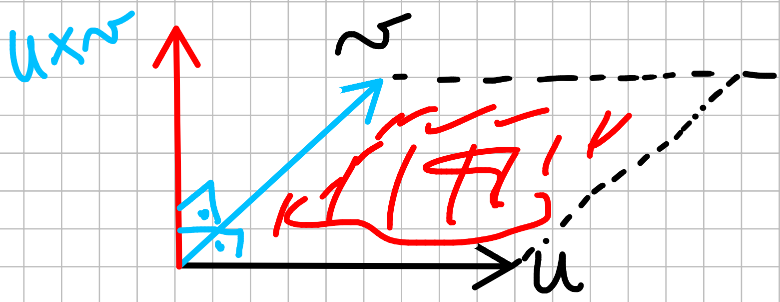
$v \times w = (-4, 7, 6)$

Ex 4) $u \cdot (v \times w) = \left|\begin{array}{ccc|cc} 1 & 2 & 1 & 1 & 2 \\ 3 & 0 & 2 & 3 & 0 \\ -1 & 2 & -3 & -1 & 2 \end{array}\right.$

$= 0 + (-4) + 6 - 0 - 4 - (-18)$

$= 16$

$\therefore\ u \cdot (v \times w) = 16$

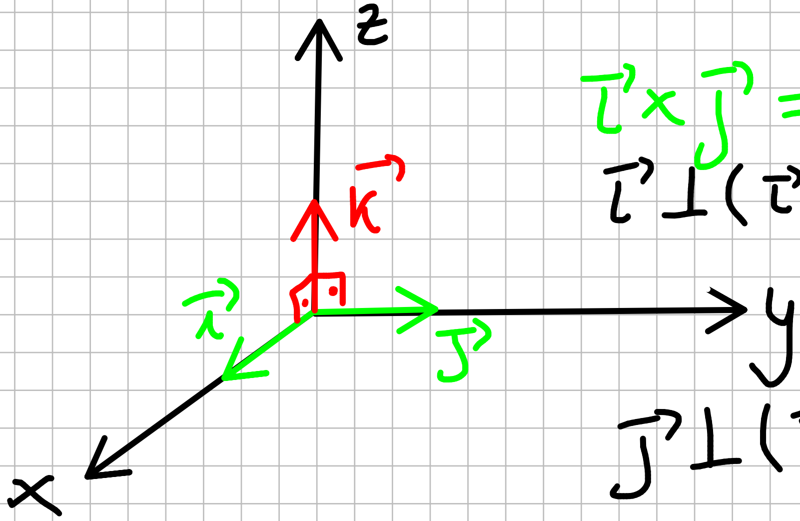


Area = $\|u \times v\|$

$u \perp (u \times v)$

$v \perp (u \times v)$

Obs: $u \perp v \iff u \cdot v = 0$



$\vec{i} \times \vec{j} = \vec{k}$

$\vec{i} \perp (\vec{i} \times \vec{j})$

$\vec{j} \perp (\vec{i} \times \vec{j})$

Ex 5) $\theta = ?$

$$\cos\theta = \frac{\cancel{u\cdot v}\to 5}{\cancel{\|u\|}\to\sqrt{6}\;\;\cancel{\|v\|}\to\sqrt{13}} \iff \theta = \cos^{-1}\left(\frac{u\cdot v}{\|u\|\,\|v\|}\right)$$

Note que: $u\cdot v \overset{ex2}{=} 5$

$$\|u\| = \|(1,2,1)\| = \sqrt{1+4+1} = \sqrt{6}$$

$$\|v\| = \|(3,0,2)\| = \sqrt{9+0+4} = \sqrt{13}$$

Logo, $\cos\theta = \dfrac{5}{\sqrt{6}\,\sqrt{13}} = \dfrac{5}{\sqrt{78}} \simeq 0{,}56$

$$\Longrightarrow \cos\theta = \frac{5}{\sqrt{78}} \iff \theta = \cos^{-1}\left(\frac{5}{\sqrt{78}}\right) \simeq 55{,}3^{\circ}$$

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Ex 6) $\{u, v, w\}$ é ortonormal   Falso!

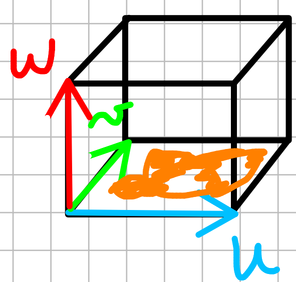
ortogonal + unitários

$\cancel{\|u\|}\to\sqrt{6}$, $\cancel{\|v\|}\to\sqrt{13}$ e $\|w\| \longrightarrow 1$

$\cancel{u\cdot v}\to 5$, $u\cdot w$, $v\cdot w \longrightarrow 0$

Obs: $|u\cdot(v\times w)| \longrightarrow$ volume

$\|u\times v\| \longrightarrow$ área



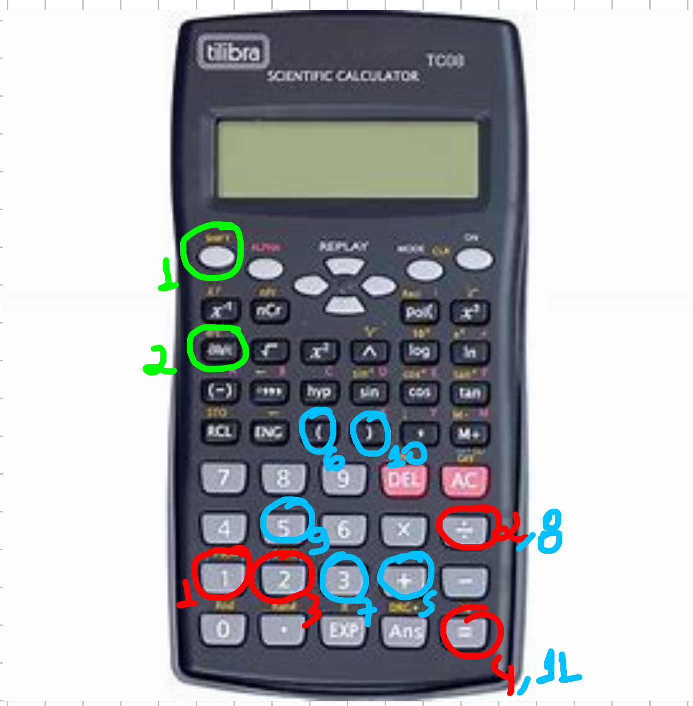
$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{78}}\right) \simeq 55{,}3^\circ$$

Outra forma:

$$\cos\theta = \frac{5}{\sqrt{78}} \Rightarrow \theta \simeq 55{,}3^\circ$$

$$\frac{1}{2}+\frac{3}{5} = \frac{11}{10}$$

$\longrightarrow$ 11⌟10



$$v \times w = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & 2 \\ -1 & 2 & -3 \end{vmatrix} \quad , \quad C_{ij} = (-1)^{i+j} M_{ij}$$

$$= 3 \cdot C_{21} + 0 \cdot C_{22} + 2 \cdot C_{23}$$

Note 1) $C_{21} = (-1)^3 \begin{vmatrix} \vec{j} & \vec{k} \\ 2 & -3 \end{vmatrix} = (-1)(-3\vec{j} - 2\vec{k})$

$$\Rightarrow C_{21} = \underline{+3\vec{j} + 2\vec{k}}$$

2) $C_{23} = (-1)^5 M_{23} = (-1) \begin{vmatrix} \vec{i} & \vec{j} \\ -1 & 2 \end{vmatrix} = (-1)(2\vec{i} + \vec{j})$

$$\Rightarrow C_{23} = \underline{-2\vec{i} - \vec{j}}$$

$$v \times w = 3 \cdot C_{21} + 0 \cdot C_{22} + 2 \cdot C_{23}$$
$$= 3(+3\vec{j} + 2\vec{k}) + 2(-2\vec{i} - \vec{j})$$
$$= +9\vec{j} + 6\vec{k} - 4\vec{i} - 2\vec{j}$$
$$= -4\vec{i} + 7\vec{j} + 6\vec{k}$$