Aula Síncrona - 29/04 - Computação

Def: i) $T(u+v) = T(u) + T(v)$ ✓

ii) $T(\alpha u) = \alpha \cdot T(u)$ ✓

Ex 1) $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (x-y, x)$

T é linear, pois considere

$u = (x_1, y_1)$, $v = (x_2, y_2) \in \mathbb{R}^2$, $\alpha \in \mathbb{R}$

i) $T(u+v) = T(x_1+x_2, y_1+y_2)$

$$= (x_1+x_2 - 1(y_1+y_2), x_1+x_2)$$
$$= (x_1+x_2-y_1-y_2, x_1+x_2)$$
$$= (x_1-y_1, x_1) + (x_2-y_2, x_2)$$
$$= T(x_1, y_1) + T(x_2, y_2)$$
$$= T(u) + T(v)$$

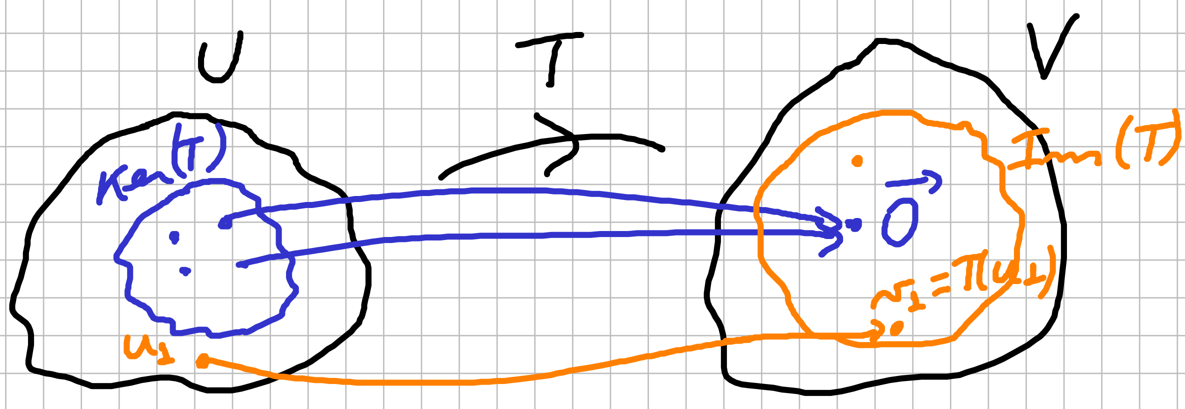
ii) $T(\alpha u) = T(\alpha x_1, \alpha y_1)$

$$= (\underline{\alpha x_1 - \alpha y_1, \alpha x_1})$$

$$\alpha T(u) = \alpha T(x_1, y_1) = \alpha (x_1 - y_1, x_1)$$
$$= (\alpha x_1 - \underline{\alpha y_1, \alpha x_1})$$

$$\Longrightarrow T(\alpha u) = \alpha \cdot T(u)$$

Obs:



Ex 2) $T(x,y) = (x-y, x)$, $T: \mathbb{R}^2 \to \mathbb{R}^2$

T é injetora $\iff$ $Ker(T) = \{\vec{0}\}$

Obs: $Ker(T) = \{ v \in \mathbb{R}^2 \,/\, T(v) = \vec{0} \}$

Note que $T(v) = \vec{0}$, $v = (x,y)$

$$T(x,y) = (0,0)$$
$$(x-y, x) = (0,0)$$
$$\Rightarrow \begin{cases} x-y = 0 \\ x = 0 \end{cases} \Rightarrow -y = 0 \Rightarrow y = 0$$

$\therefore v = (x,y) = (0,0) = \vec{0} \Longrightarrow Ker(T) = \{\vec{0}\}$

Logo, T é injetora

Ex 3) $T(x,y) = (x-y, x)$

$\Rightarrow Im(T) = \{ (x-y, x) \,/\, x, y \in \mathbb{R} \}$

Obs: $(x-y, x) = (x,x) + (-y, 0)$
$$= x(1,1) + y(-1,0)$$
$$\Longrightarrow Im(T) = [(1,1), (-1,0)]$$

Obs: $T: U \longrightarrow V$, B base de U, $\dim U = n$
C base de V, $\dim V = m$

$$[T]_{B,C} = \begin{pmatrix} - & - & \\ - & - & \\ \vdots & \vdots & \\ - & - & \end{pmatrix}_{m \times n}$$

$T(u_1) = w_1 = c_1 v_1 + c_2 v_2 + \dots + c_m v_m$

$T(u_2) = \dots$

Ex 4) $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (x-y, x)$

e $B = \{(1,2), (0,-3)\}$, $[T]_{B,B} = [T]_B = \begin{pmatrix} - & - \\ - & - \end{pmatrix}_{2\times 2}$

Note que:

$T(1,2) = (1-2, 1) = (-1, 1)$ (I)

$T(0,-3) = (0-(-3), 0) = (3,0)$ (II)

Segue que:

I) $T(1,2) = \underline{(-1,1) = c_1(1,2) + c_2(0,-3)}$

$\Rightarrow (-1,1) = (c_1, 2c_1) + (0, -3c_2)$

$\Rightarrow (-1,1) = (c_1, 2c_1 - 3c_2)$

$\Rightarrow \begin{cases} \underline{c_1 = -1} \\ 2c_1 - 3c_2 = 1 \end{cases} \Rightarrow -2 - 3c_2 = 1 \Rightarrow \underline{c_2 = -1}$

Logo, $T(1,2) = (-1,1) = \underline{-1}(1,2) \underline{-1}(0,-3)$

II) $T(0,-3) = \underline{(3,0) = c_4(1,2) + c_5(0,-3)}$

$\Rightarrow (3,0) = (c_4, 2c_4 - 3c_5)$

$\Rightarrow \begin{cases} \underline{c_4 = 3} \\ 2c_4 - 3c_5 = 0 \end{cases} \Rightarrow 6 - 3c_5 = 0 \Rightarrow \underline{c_5 = +2}$

Logo, $T(0,-3) = (3,0) = \underline{3}(1,2) \underline{+2}(0,-3)$

Note que:

$T(1,2) = (-1,1) = \underline{-1}(1,2) \underline{-1}(0,-3)$

$T(0,-3) = (3,0) = \underline{3}(1,2) \underline{+2}(0,-3)$

Logo, $[T]_B = \begin{pmatrix} -1 & 3 \\ -1 & 2 \end{pmatrix}$ é a matriz da transformação linear T.

Exercício: Dado $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (x-y, x)$, determine a matriz de T em relação a base canônica. $C = \{\vec{i}, \vec{j}\} = \{(1,0), (0,1)\}$

Segue que

$T(1,0) = (1,1) = \cancel{c_1}^{1}(1,0) + \cancel{c_2}^{1}(0,1)$

$T(0,1) = (-1,0) = \cancel{c_3}^{-1}(1,0) + \cancel{c_4}^{0}(0,1)$

$$\Longrightarrow [T]_C = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

Obs: $T(x,y) = (x-y, x)$ $\quad [T]_C = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$

↳ base canônica

Obs 2) $T(x,y) = (x-y, x)$

$[T]_C = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$ $\quad [T]_B = \begin{pmatrix} -1 & 3 \\ -1 & 2 \end{pmatrix}$

Outra forma Ex 4

$$[T]_B = [I]_{B,C}^{-1} [T]_C [I]_{B,C}$$

$[I]_{B,C} = \begin{pmatrix} 1 & 0 \\ 2 & -3 \end{pmatrix}$ pois

$I(1,2) = (1,2) = \cancel{c_1}^{1}(1,0) + \cancel{c_2}^{2}(0,1)$

$I(0,-3) = (0,-3) = \cancel{c_3}^{0}(1,0) + \cancel{c_4}^{-3}(0,1)$

Obs: $[I]_{B,C}^{-1} = \begin{pmatrix} 1 & 0 \\ 4/3 & -1/3 \end{pmatrix}$