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INTEGRAIS POR SUBSTITUIÇÃO TRIGONOMÉTRICA

$\sqrt{a^2-x^2}$; $\sqrt{a^2+x^2}$; $\sqrt{x^2-a^2}$

ONDE $a > 0$ E x VARIÁVEL

EXEMPLO: $\int \frac{dx}{x^2\sqrt{9-x^2}}$

$\sqrt{a^2-x^2}$, ONDE $a = 3$

$x = 3\operatorname{sen}\theta \qquad -\pi/2 < \theta < \pi/2$

$\sqrt{9-x^2} = \sqrt{9-9\operatorname{sen}^2\theta} = 3\sqrt{1-\operatorname{sen}^2\theta} = 3\sqrt{\cos^2\theta}$

COMO $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ TEMOS $\cos\theta > 0$

$\sqrt{9-x^2} = 3\cos\theta$

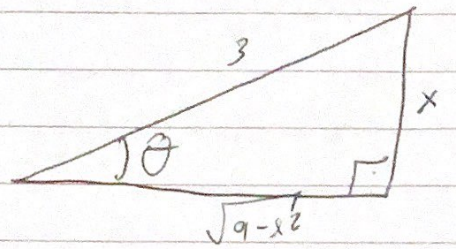
$x = 3\operatorname{sen}\theta \qquad dx = 3\cos\theta\, d\theta$

$$\int \frac{dx}{x^2\sqrt{9-x^2}} = \int \frac{3\cos\theta\, d\theta}{(9\operatorname{sen}^2\theta)\, 3\cos\theta} = \int \frac{d\theta}{9\operatorname{sen}^2\theta}$$

$$= \frac{1}{9}\int \frac{d\theta}{\operatorname{sen}^2\theta} = \frac{1}{9}\int \operatorname{cossec}^2\theta\, d\theta = -\frac{1}{9}\operatorname{cotg}(\theta) + C$$

$\theta = \text{arcsen}(x/3)$

$= \frac{-1}{9} \ \text{catg}\left(\text{arcsen}\left(\frac{x}{3}\right)\right) + C$



$\text{sen}\,\theta = \frac{x}{3}$

$\text{cotg}(\theta) = \frac{\sqrt{9-x^2}}{x}$

$$\int \frac{dx}{x^2\sqrt{9-x^2}} = \frac{-1}{9} \ \frac{\sqrt{9-x^2}}{x} + C$$

$$= -\frac{\sqrt{9-x^2}}{9x} + C$$

Ejemplo: $\int \frac{dx}{\sqrt{16+x^2}}$

$\sqrt{a^2+x^2} \qquad a > 0$

$x = a \cdot tg(\theta)$

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sec(\theta) > 0$

$x = 4\,tg(\theta)$

$dx = 4\sec^2(\theta)\,d\theta$

$\sqrt{16+x^2} = \sqrt{16+16\,tg^2\theta}$

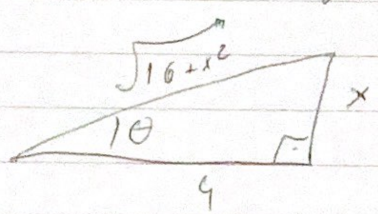
$= \sqrt{16(1+tg^2\theta)} = 4\sqrt{\sec^2(\theta)}$

$= 4\sec\theta$

$$\int \frac{dx}{\sqrt{16+x^2}} = \int \frac{4\sec^2\theta\,d\theta}{4\sec\theta}$$

$$= \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + tg\,\theta| + C$$



$tg\,\theta = \frac{x}{4}$

$\sec(\theta) = \frac{\sqrt{16+x^2}}{4}$

$$\int \frac{dx}{\sqrt{16+x^2}} = \ln\left|\frac{\sqrt{16+x^2}}{4} + \frac{x}{4}\right| + C$$

$$= \ln\left|\frac{\sqrt{16+x^2} + x}{4}\right| + C$$

$= \ln|\sqrt{16+x^2} + x| - \ln(4) + C$

COMO $\sqrt{16+x^2} + x > 0 \quad \forall x$

TOMANDO $D = -\ln(4) + C$

$$\int \frac{dx}{\sqrt{16+x^2}} = \ln\left(\sqrt{16+x^2} + x\right) + D$$

EXEMPLO: $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\, dx$

$\sqrt{x^2-9} = \sqrt{x^2 - 3^2} \qquad a = 3$

$x = 3 \sec\theta$

$dx = 3 \sec\theta \, tg\theta \, d\theta$

$0 \leq \theta < \pi/2$ ou $\pi \leq \theta < 3\pi/2$

$tg\,\theta \geq 0$

$\sqrt{x^2-9} = \sqrt{9\sec^2\theta - 9} = 3\sqrt{\sec^2(\theta) - 1}$

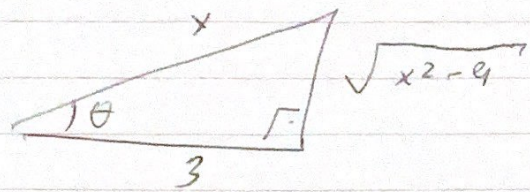
$= 3\sqrt{tg^2\theta} = 3\,tg(\theta)$

$$\int \frac{\sqrt{x^2-9}}{x}\, dx = \int \frac{3\,tg\theta \cdot 3\sec\theta\, tg\theta\, d\theta}{3\sec\theta}$$

$= 3\int tg^2\theta\, d\theta = 3\int (\sec^2\theta - 1)\, d\theta$

$= 3\int \sec^2\theta\, d\theta - 3\int d\theta$

$= 3\,\text{tg}\,\theta - 3\theta + C$



$\sec\theta = x/3$

$\text{tg}\,\theta = \frac{\sqrt{x^2-9}}{3}$

$\theta = \text{arcsen}\left(\frac{x}{3}\right)$

$$\int \frac{\sqrt{x^2-9}}{x}\,dx = 3\,\frac{\sqrt{x^2-9}}{3} - 3\,\text{arcsen}\left(\frac{x}{3}\right) + C$$

$$= \sqrt{x^2-9} - 3\,\text{arcsen}\left(\frac{x}{3}\right) + C$$