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# TEOREMA DE L'HOSPITAL

SEJA $I$ UM INTERVALO ABERTO, $a \in I$, E $f, g$ DIFERENCIÁVEIS EM $I - \{a\}$.

SUPONHAMOS QUE $g(x) \neq 0$ $\forall x \in I - \{a\}$ E $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ OU $\lim_{x \to a} |f(x)| = \lim_{x \to a} |g(x)| = +\infty$

SE $\lim_{x \to a} \frac{f'(x)}{g'(x)} = L$, ENTÃO $\lim_{x \to a} \frac{f(x)}{g(x)} = L$

EXEMPLO: $\lim_{x \to 0} \frac{e^x - 1}{2x}$

$* \lim_{x \to 0} \frac{(e^x - 1)'}{(2x)'} = \lim_{x \to 0} \frac{e^x}{2} = \frac{e^0}{2} = \frac{1}{2}$

$* \lim_{x \to 0} \frac{e^x - 1}{2x} = \frac{1}{2}$ //

EXEMPLO: $\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = \lim_{x \to 0} \frac{\cos x}{1} = 1$ //

EXEMPLO: $\lim_{x \to 0} \frac{\operatorname{sen} x - x}{e^x + e^{-x} - 2}$

$= \lim_{x \to 0} \frac{(\operatorname{sen} x - x)'}{(e^x + e^{-x} - 2)'} = \lim_{x \to 0} \frac{\cos x - 1}{e^x - e^{-x}}$

$= \lim_{x \to 0} \frac{-\operatorname{sen} x}{e^x + e^{-x}} = \frac{-\operatorname{sen} 0}{e^0 + e^{-0}} = 0$ //

EXEMPLO: $\lim_{x \to +\infty} \frac{e^x - 1}{x^3 + 4x}$

$= \lim_{x \to +\infty} \frac{e^x}{3x^2+4} = \lim_{x \to +\infty} \frac{e^x}{6x} = \lim_{x \to +\infty} \frac{e^x}{6} = +\infty$ //

EXEMPLO: $\lim_{x \to 0^+} x^n \ln(x) \quad n > 0 \quad (0 \cdot \infty)$

$= \lim_{x \to 0^+} x^n \ln(x) = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^n}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-n x^{-n-1}}$

$= \lim_{x \to 0^+} \frac{1}{x(-n x^{-n-1})} = \lim_{x \to 0^+} \frac{1}{-n x^{-n}} = \lim_{x \to 0} \frac{x^n}{-n}$

$= \frac{0^n}{-n} = 0$ //

EXEMPLO: $0^0$, $\infty^0$ ou $1^\infty$

$\lim_{x \to \frac{\pi}{2}} \cos x^{\cos x}$

$F(x) = \cos(x)^{\cos(x)} \Rightarrow \ln(F(x)) = \ln(\cos(x)^{\cos(x)})$

$\Rightarrow \ln(F(x)) = \cos(x) \cdot \ln(\cos(x))$

$\lim_{x \to \frac{\pi}{2}^-} F(x) = \lim_{x \to \frac{\pi}{2}} \cos(x)^{\cos(x)}$

$\ln\left(\lim_{x \to \frac{\pi}{2}^-} F(x)\right) = \ln\left(\lim_{x \to \frac{\pi}{2}^-} \cos x^{\cos x}\right)$

$= \lim_{x \to \frac{\pi}{2}^-} \ln(\cos x^{\cos x})$

$= \lim_{x \to \frac{\pi}{2}^-} \underbrace{\cos x}_{0} \cdot \underbrace{\ln(\cos x)}_{-\infty}$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{\ln(\cos x)}{\frac{1}{\cos x}}$$

$$\circ \, (\ln(\cos x))' = \frac{1}{\cos x} (\cos x)' = -\frac{\operatorname{sen} x}{\cos x}$$

$$\circ \left(\frac{1}{\cos x}\right)' = \frac{1' \cos x - \cos x' \, 1}{\cos^2 x} = \frac{\operatorname{sen} x}{\cos^2(x)}$$

$$= \lim_{x \to \frac{\pi}{2}^-} \frac{-\frac{\operatorname{sen} x}{\cos x}}{\frac{\operatorname{sen} x}{\cos^2 x}} = \lim_{x \to \frac{\pi}{2}^-} -\frac{\operatorname{sen} x}{\cos x} \cdot \frac{\cos^2(x)}{\operatorname{sen} x}$$

$$= \lim_{x \to \frac{\pi}{2}^-} \cos = \cos\left(\frac{\pi}{2}\right) = 0$$

$$\circ \, \ln\left(\lim_{x \to \frac{\pi}{2}^-} \cos x^{\cos x}\right) = 0$$

$$\lim_{x \to \frac{\pi}{2}^-} \cos x^{\cos x} = e^0 = 1 \; //$$

Exemplo: $\lim_{x \to 0} |x|^x$ $\quad 0^0$

$$\ln\left(\lim_{x \to 0} |x|^x\right) = \lim_{x \to 0} \ln(|x|^x) = \lim_{x \to 0} x \ln(|x|)$$

$$= \lim_{x \to 0} \frac{\ln(|x|)}{\frac{1}{x}}$$

ERRO:

$$(\ln(|x|))' = \frac{1}{|x|}(|x|)' = \frac{1}{|x|}.111 = \frac{1}{|x|}$$

ABSURDO!

MARGINAL CONTA!

$$(\ln(|x|))' = ? \qquad \ln(|x|) = \begin{cases} \ln(x), & \text{se } x > 0 \\ \ln(-x), & \text{se } x < 0 \end{cases}$$

$$\Rightarrow (\ln(|x|))' = \begin{cases} (\ln(x))', & \text{se } x > 0 \\ (\ln(-x))', & \text{se } x < 0 \end{cases}$$

$$= \begin{cases} 1/x & \text{se } x > 0 \\ \frac{1}{-x}(-x)' = \frac{-1}{-x} = \frac{1}{x} & \text{se } x < 0 \end{cases} \qquad x \neq 0$$

$$\lim_{x \to 0} \frac{\ln(|x|)}{\frac{1}{x}} = \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0} \frac{1}{x} \cdot -x^2$$

$$= \lim_{x \to 0} -x = 0$$

$$\Rightarrow \ln\left(\lim_{x \to 0} |x|^x\right) = 0 \quad \therefore \quad \lim_{x \to 0} |x|^x = e^0 = 1$$