

15. 1. $\exists x Px \wedge Fx$

$\therefore \exists x Px \wedge Dx$

2. $\forall x Fx \rightarrow Dx$

3. $Pa \wedge Fa$ IE1

4. $Fa \rightarrow Da$ IU2

5. Fa SIMP 3

6. Da MP 4, 5

7. Pa SIMP 3

8. $Pa \wedge Da$ CONJ 6, 7

9. $\exists x Px \wedge Dx$ GE 8

(16)

$$\therefore \exists x \neg Jx$$

1. $\forall x \forall y Cx \wedge Jy \rightarrow Mxy$
Cs

2

3. $\exists x Sx \wedge \neg Mx$

4. $Sa \wedge \neg Ma$ IE 3

5. $\forall x Cx \wedge Ja \rightarrow Mxa$ IUL

6. $Cs \wedge Ja \rightarrow Msa$ IUS

7. $\neg Ma$ IMP 4

8. $\neg Cs \vee \neg Ja$ MT 6, 7

9. $\neg Ja$ SD 2, 8

10. $\exists x \neg Jx$ GES

17)

$\therefore \exists x A x \wedge L x$

1. $\exists x A x \wedge \neg M x$

2. $\forall x O x \rightarrow M x$

3. $\forall x O x \vee L x$

4. $A a \wedge \neg M a$ IE1

5. $O a \rightarrow M a$ IU2

6. $O a \vee L a$ IU3

7. $A a$ SIMP 4

8. $\neg M a$ SIMP 4

9. $\neg O a$ MT 5, 8

10. $L a$ SD 6, 9

11. $A a \wedge L a$ CONJ 7, 10

12. $\exists x A x \wedge L x$ GE11

18)

$$1. \forall x Mx \rightarrow Ix \vee Gx$$

$$2. \forall x Gx \wedge Ax \rightarrow Fx$$

$$3. \quad \neg Ij \wedge Aj \quad \therefore Mj \rightarrow Fj$$

$$4. Mj \rightarrow Ij \vee Gj \quad IU1$$

$$5. Gj \wedge Aj \rightarrow Fj \quad IU2$$

$$6. \neg Mj \vee Ij \vee Gj \quad EQ4$$

$$7. \neg Ij \quad SIMP3$$

$$8. \neg Mj \vee Gj \quad SD 6, 7$$

$$9. \neg Gj \vee \neg Aj \vee Fj \quad EQ5$$

$$10. Aj \quad SIMP3$$

$$11. \neg Gj \vee Fj \quad SD 9, 10$$

$$12. Mj \rightarrow Gj \quad EQ8$$

$$13. Gj \rightarrow Fj \quad EQ11$$

$$14. Mj \rightarrow Fj \quad SM 12, 13$$

19) 1. $\forall x \forall y \quad Ex \wedge Cxy \rightarrow Dy$ $\therefore \exists x Dx$

2. $\exists x \exists y \quad Ex \wedge Cxy$

3. $Ex \wedge Cxy$ $IE2, IE2$

4. $Ex \wedge Cxy \rightarrow Dy$ $II1, II1$

5. Dy $MP3,4$

6. $\exists x Dx$ GES

20) 1. $\forall x \exists y \quad Cx \rightarrow Txy$ $\leftarrow \therefore \exists x Dmx$

2. $\forall x \forall y \quad Txy \rightarrow Dxy$

3. Cm

4. $\forall x \quad Cx \rightarrow Txa \quad \text{IE1}$

5. $Cm \rightarrow Tma \quad \text{IV4}$

6. $Tma \rightarrow Dma \quad \text{IV2, IV2}$

7. $Cm \rightarrow Dma \quad \text{5, 6}$

8. $Dma \quad \text{MP 3, 7}$

9. $\exists x Dmx$

$$21) 1. \exists x \forall y A x \wedge R x y \quad \therefore \exists x \forall y A x \wedge I x y$$

$$2. \forall x \forall y R x y \rightarrow I x y$$

$$3. \forall y A a \wedge R a y \quad \text{IE 1}$$

$$A a \wedge R a b \quad \text{IU 3}$$

$$4. R a b \rightarrow I a b \quad \text{IU 2, IU 2}$$

$$5. R a b \quad \text{SIMP 4}$$

$$6. I a b \quad \text{MP 5, 6}$$

$$7. A a \quad \text{SIMP 4}$$

$$8. A a \wedge I a b \quad \text{CONJ 7, 6}$$

$$9. \exists x \forall y A x \wedge I x y$$

14) 1. $\forall x Px$

2. $\exists x \neg Px$

3. $\neg Pa$ I62

4. Pa IUL

5. F CONJ 3,4

$\exists x Qx$

$\sim \Rightarrow \sim$

$\sim \rightarrow \sim$
T

$F \Rightarrow \text{X}$

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$$12) a) U = \{-5, -3, -1, 1, 3, 5, 7\}$$

$$\exists x P x$$

$$P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5) \vee P(7)$$

$$b) \forall x ((x \neq 1) \rightarrow P x)$$

$$P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5) \wedge P(7)$$

$$c) \exists x ((x \geq 0) \wedge P(x))$$

$$P(1) \vee P(3) \vee P(5) \vee P(7)$$

$$f) \forall x ((x > 0) \rightarrow \neg P(-x)) \wedge \forall x P x$$

$$\neg P(-1) \wedge \neg P(-3) \wedge \neg P(-5) \wedge \neg P(-7) \wedge$$

$$P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5) \wedge P(7)$$

ف

6)

a) $\exists x Gx \wedge Cx \wedge Fx$

b) $\forall x Gx \vee Cx \vee Fx$

c) $\exists x Gx \wedge Fx \wedge \neg Cx$

d) $\neg \exists x Gx \wedge Cx \wedge Fx$

e) $\forall x Gx \wedge Cx \rightarrow \neg Fx$