Ex 1)

$$\begin{cases} x - 2y - 3z = 0 \\ 4x - 3y = -18 \\ 2y + 5z = -8 \end{cases}$$

$n = 3 \rightarrow$ incógnitas

$AX = B$ $\quad m = 3$ $\hookrightarrow$ equações

$[A \,\vdots\, B]$

$$\begin{array}{l} -4 \\ +L \end{array} \begin{pmatrix} 1 & -2 & -3 & | & 0 \\ 4 & -3 & 0 & | & -18 \\ 0 & 2 & 5 & | & -8 \end{pmatrix} \sim \begin{array}{l} -\frac{2}{5} \\ +L \end{array} \begin{pmatrix} 1 & -2 & -3 & | & 0 \\ 0 & 5 & 12 & | & -18 \\ 0 & 2 & 5 & | & -8 \end{pmatrix}$$

$-\frac{24}{5} + 5 =$

$-\frac{24}{5} + \frac{25}{5}$

$\frac{36}{5} - 8$

$\frac{36}{5} - \frac{40}{5}$

$$\sim \begin{pmatrix} 1 & -2 & -3 & | & 0 \\ 0 & 5 & 12 & | & -18 \\ 0 & 0 & 1/5 & | & -4/5 \end{pmatrix}$$

Note que $P_c = 3$ e $P_\alpha = 3$

Segue que $P_c = P_\alpha = 3 = n \overset{\text{Teo}}{\Longrightarrow}$ sistema é compatível determinado.

Obs: $\det(A) = \begin{vmatrix} 1 & -2 & -3 \\ 4 & -3 & 0 \\ 0 & 2 & 5 \end{vmatrix} = 1 \neq 0$

Logo, podemos utilizar os 4 métodos estudados para obter a solução do sistema.

Teo: $P_c = P_\alpha = n \Longrightarrow$ compatível determinado

$P_c = P_\alpha < n \Longrightarrow$ compatível indeterminado

$P_c \neq P_\alpha \Longrightarrow$ incompatível

Resolução:

$$\begin{cases} x - 2y - 3z = 0 \\ 4x - 3y = -18 \\ 2y + 5z = -8 \end{cases}$$

→ Regra Cramer

$$x = \frac{D_x}{\det(A)} \; ; \; y = \frac{D_y}{\det(A)} \; ; \; z = \frac{D_z}{\det(A)}$$

Recorde que $\det(A) = 1$

$$D_x = \begin{vmatrix} 0 & -2 & -3 \\ -18 & -3 & 0 \\ -8 & 2 & 5 \end{vmatrix} = 0 \implies x = \frac{0}{1} = 0$$

$$D_y = \begin{vmatrix} 1 & 0 & -3 \\ 4 & -18 & 0 \\ 0 & -8 & 5 \end{vmatrix} = 6 \implies y = \frac{6}{1} = 6$$

$$D_z = \begin{vmatrix} 1 & -2 & 0 \\ 4 & -3 & -18 \\ 0 & 2 & -8 \end{vmatrix} = -4 \implies z = \frac{-4}{1} = -4$$

$\therefore$ $(0, 6, -4)$ é a solução do sistema.

→ Método da Matriz Inversa

$$\det(A) = 1 \neq 0 \implies X = A^{-1} B$$

$$\implies \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 0 \\ -18 \\ -8 \end{pmatrix} \qquad A^{-1} = ?$$

Confira que: $A^{-1} = \begin{pmatrix} -15 & 4 & -9 \\ -20 & 5 & -12 \\ 8 & -2 & 5 \end{pmatrix}$

Logo, $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -15 & 4 & -9 \\ -20 & 5 & -12 \\ 8 & -2 & 5 \end{pmatrix}_{3\times 3} \cdot \begin{pmatrix} 0 \\ -18 \\ -8 \end{pmatrix}_{3\times 1}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -15\cdot 0 + 4\cdot(-18) + (-9)\cdot(-8) \\ -20\cdot 0 + 5\cdot(-18) + (-12)\cdot(-8) \\ 8\cdot 0 + (-2)\cdot(-18) + 5\cdot(-8) \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ -4 \end{pmatrix}$$

$\therefore$ $(0, 6, -4)$ é a solução do sistema.

Ex 2) $\begin{cases} 2x + 2y + z = 2 \\ -x - y = 0 \\ x + y + z = 5 \end{cases}$

$$\left(\begin{array}{ccc|c} 2 & 2 & 1 & 2 \\ -1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ -1 & -1 & 0 & 0 \\ 2 & 2 & 1 & 2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & -1 & -8 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & -3 \end{array}\right)$$

Note que! $P_c = 2$ e $P_{ac} = 3$

Segue que:

$P_c = 2 \neq 3 = P_\alpha \xRightarrow{Teo}$ sistema é incompatível.

Obs:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & -3 \end{array}\right) \Rightarrow \begin{cases} x+y+z=5 \\ z=5 \\ 0=-3 \end{cases}$$ Absurdo!

Ex 3) $\begin{cases} 2x+4y+6z=-6 \\ 3x-2y-4z=-38 \\ x+2y+3z=-3 \end{cases}$

$$\tfrac{1}{2}\left(\begin{array}{ccc|c} 2 & 4 & 6 & -6 \\ 3 & -2 & -4 & -38 \\ 1 & 2 & 3 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & -3 \\ 3 & -2 & -4 & -38 \\ 1 & 2 & 3 & -3 \end{array}\right) \quad (-1,\ -3)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & -3 \\ 0 & -8 & -13 & -29 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Note que $P_c = 2$, $P_\alpha = 2$ e $n = 3$

Segue que $P_c = P_\alpha = 2 < 3 = n$

$\xRightarrow{Teo}$ sistema é compatível indeterminado.

→ Método Gauss

$$\begin{cases} 2x+4y+6z=-6 \\ 3x-2y-4z=-38 \\ x+2y+3z=-3 \end{cases}$$

$$\begin{array}{c} \tfrac{1}{2} \\ \\ \\ \end{array}\left(\begin{array}{ccc|c} 2 & 4 & 6 & -6 \\ 3 & -2 & -4 & -38 \\ 1 & 2 & 3 & -3 \end{array}\right) \sim \begin{array}{c} -1 \quad -3 \\ + \\ + \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 3 & -3 \\ 3 & -2 & -4 & -38 \\ 1 & 2 & 3 & -3 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & -3 \\ 0 & -8 & -13 & -29 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Longrightarrow \begin{cases} x + 2y + 3z = -3 & (II) \\ \quad -8y - 13z = -29 & (I) \end{cases}$$

$$(I) \quad -8y = 13z - 29 \iff y = -\frac{13}{8} z + \frac{29}{8}$$

substituindo na eq II obtemos

$$x = -2y - 3z - 3 = -2\left(-\frac{13}{8} z + \frac{29}{8}\right) - 3z - 3$$

$$x = \frac{13}{4} z - \frac{29}{4} - 3z - 3 = \frac{1}{4} z - \frac{41}{4}$$

$\therefore \left(\frac{1}{4} z - \frac{41}{4}, -\frac{13}{8} z + \frac{29}{8}, z\right), z \in \mathbb{R}$, é a solução do sistema.