Integrais

Exercícios

$$\int \frac{\ln(ax+b)}{\sqrt{ax+b}}\,dx$$

$$\int u\,dv = u\cdot v - \int v\,du$$

$u = \ln(ax+b)$

$du = \frac{a}{ax+b}\,dx$

$dv = \frac{dx}{\sqrt{ax+b}}$

$v = \int \frac{dx}{\sqrt{ax+b}} = \int (ax+b)^{-\frac{1}{2}}dx$

$\begin{cases} w = ax+b \\ dw = a\,dx \Rightarrow \frac{dw}{a} = dx \end{cases}$

$v = \int w^{-\frac{1}{2}}\frac{dw}{a}$

$= \frac{w^{-\frac{1}{2}+1}}{a(-\frac{1}{2}+1)} = \frac{w^{\frac{1}{2}}}{a(\frac{1}{2})}$

$v = \frac{2}{a}\cdot w^{\frac{1}{2}} = \frac{2}{a}(ax+b)^{\frac{1}{2}}$

Assim

$$\int \frac{\ln(ax+b)}{\sqrt{ax+b}}\,dx = \ln(ax+b)\cdot\frac{2}{a}\sqrt{ax+b} - \int \frac{\cancel{2}}{\cancel{a}}(\sqrt{ax+b})^{\frac{1}{2}}\frac{\cancel{(a)}}{(ax+b)^{1}}dx$$

$$\int \frac{\ln(ax+b)}{\sqrt{ax+b}}\,dx = \frac{2\sqrt{ax+b}\cdot\ln(ax+b)}{a} - 2\int (ax+b)^{-\frac{1}{2}}dx$$

$$= \frac{2\sqrt{ax+b}\cdot\ln(ax+b)}{a} - 2\cdot\left(\frac{2\cdot(ax+b)^{\frac{1}{2}}}{a}\right) + C$$

$$\Rightarrow = \frac{2\sqrt{ax+b}\,\ln(ax+b)}{a} - 4\frac{\sqrt{ax+b}}{a} + C$$

$$\Rightarrow = 2\sqrt{ax+b}\left(\frac{\ln(ax+b) - 2}{a}\right) + C$$

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h

$$\int \cos(\ln(x))\,dx$$

$u = \cos(\ln(x))$ | $dv = dx$

$du = -\operatorname{sen}(\ln(x))\cdot\frac{1}{x}dx$ | $v = x$

$$\int \cos(\ln(x))dx = x\cdot\cos(\ln(x)) - \int x\cdot(-\operatorname{sen}(\ln(x))\cdot\frac{1}{x}dx$$

$$= x\cos(\ln(x)) + \int \operatorname{sen}(\ln(x))\,dx \quad (*)$$

Como

$$\int \operatorname{sen}(\ln(x))\,dx = x\cdot\operatorname{sen}(\ln(x)) - \int x\cos(\ln(x))\frac{1}{x}dx$$

$u = \operatorname{sen}(\ln(x))$ | $dv = dx$

$du = \cos(\ln(x))\cdot\frac{1}{x}dx$ | $v = x$

$$= x\operatorname{sen}(\ln(x)) - \int \cos(\ln(x))dx$$

$$\int \cos(\ln(x))dx = x\cos(\ln(x)) + x\operatorname{sen}(\ln(x)) - \int \cos(\ln(x))dx$$

$$2\int \cos(\ln(x))dx = x\cos(\ln(x)) + x\operatorname{sen}(\ln(x))$$

$$\int \cos(\ln(x))dx = \frac{x\left(\cos(\ln(x)) + \operatorname{sen}(\ln(x))\right)}{2} + C$$

$$\int x^{-3}e^{x^{-1}}dx = \int \underbrace{x^{-1}}_{u}\cdot\underbrace{x^{-2}e^{x^{-1}}dx}_{dv}$$

$u = x^{-1} \Rightarrow du = -x^{-2}dx$

$dv = x^{-2}e^{x^{-1}}dx \Rightarrow v = \int x^{-2}e^{x^{-1}}dx = \int e^{w}\,(-dw) = -e^{w}$

$= -e^{x^{-1}}$

$w = x^{-1}$

$dw = -x^{-2}dx$

$$\int x^{-3} e^{x^{-1}} dx = -x^{-1} \cdot e^{x^{-1}} - \int -e^{x^{-1}} (-x^{-2})\, dx$$

$$= -x^{-1} e^{x^{-1}} - \int e^{x^{-1}} \cdot x^{-2}\, dx$$

$$= -x^{-1} e^{x^{-1}} - \left(-e^{x^{-1}}\right) + C$$

$$= -x^{-1} e^{x^{-1}} + e^{x^{-1}} + C$$

$$= e^{x^{-1}} \left(1 - x^{-1}\right) + C$$

$$= e^{\frac{1}{x}} \left(1 - \frac{1}{x}\right) + C = e^{\frac{1}{x}} \left(\frac{x - 1}{x}\right) + C$$

$$\int \frac{dx}{x^3 - 4x^2} = \int \frac{dx}{x^2 (x - 4)}$$

Frações Parciais

$$\frac{1}{x^2 (x - 4)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x - 4}$$

$$1 = A(x - 4) + Bx(x - 4) + Cx^2$$

Para $x = 0$

$$1 = A \cdot (0 - 4) + \cancel{B \cdot 0 (0 - 4)} + \cancel{C \cdot 0^2}$$

$$1 = -4A \Rightarrow A = -\frac{1}{4}$$

Para $x = 4$

$$1 = A \cdot \cancel{(4-4)} + B \cdot \cancel{4(4-4)} + C \cdot 4^2$$

$$1 = 16C \Rightarrow C = \frac{1}{16}$$

Para $x = 1$

$$1 = -\frac{1}{4} \cdot (1-4) + B \cdot (1) \cdot (1-4) + \frac{1}{16} \cdot (1)^2$$

$$1 = \frac{3}{4} - 3B + \frac{1}{16}$$

$$3B = \frac{12}{16} + \frac{1}{16} - \frac{16}{16} = -\frac{3}{16}$$

$$B = -\frac{1}{16}$$

$$\frac{1}{x^2(x-4)} = -\left(\frac{1}{4} \cdot \frac{1}{x^2}\right) - \left(\frac{1}{16} \cdot \frac{1}{x}\right) + \left(\frac{1}{16} \cdot \left(\frac{1}{x-4}\right)\right)$$

$$= -\frac{1}{4x^2} - \frac{1}{16x} + \frac{1}{16(x-4)}$$

$$\int \frac{dx}{x^3 - 4x^2} = \int \frac{dx}{x^2(x-4)} = -\int \frac{dx}{4x^2} - \int \frac{dx}{16x} + \int \frac{dx}{16(x-4)}$$

$$= -\frac{1}{4} \int x^{-2} dx - \frac{1}{16} \int \frac{dx}{x} + \frac{1}{16} \int \frac{dx}{x-4}$$

$$= -\frac{1}{4} \cdot \frac{x^{-2+1}}{(-2+1)} - \frac{1}{16} \ln(x) + \frac{1}{16} \ln(x-4) + C$$

Portanto

$$\int \frac{dx}{x^3-4x^2} = \frac{1}{4x} - \frac{\ln(x)}{16} + \frac{\ln(x-4)}{16} + C$$

$$\int sen^3(2x)\cos^4(2x)\,dx$$

$w = 2x \Rightarrow dw = 2dx$

$$= \int sen^3(w) \cdot \cos^4(w)\frac{dw}{2} = \frac{1}{2}\int sen^3(w)\cos^4(w)dw$$

$$= \frac{1}{2}\int sen^2(w) \cdot sen(w) \cdot \cos^4(w)\,dw$$

$$= \frac{1}{2}\int (1-\cos^2(w)) \cdot \cos^4(w) \cdot sen(w)\,dw$$

$u = \cos(w)$

$du = -sen(w)\,dw$

$$= \frac{1}{2}\int (1-u^2) \cdot u^4 \cdot (-du)$$

$$= \frac{1}{2}\int (u^2-1) \cdot u^4\,du = \frac{1}{2}\int (u^6-u^4)\,du$$

$$= \frac{1}{2}\left(\frac{u^7}{7} - \frac{u^5}{5}\right) + C$$

$$= \frac{1}{2}\left(\frac{\cos^7(w)}{7} - \frac{\cos^5(w)}{5}\right) + C$$

$$= \frac{\cos^7(2x)}{14} - \frac{\cos^5(2x)}{10} + C$$