Limites

$$\lim_{x \to 3} x^2 \cdot (2x-1)$$

1º maneira

$$= \left(\lim_{x \to 3} x^2\right) \cdot \left(\lim_{x \to 3} (2x-1)\right)$$

$$= (3)^2 \cdot (2 \cdot (3) - 1) = 9 \cdot (6-1)$$

$$= 9 \cdot 5 = 45 //$$

2º maneira

$$= \lim_{x \to 3} (2x^3 - x^2) = 2 \cdot (3)^3 - (3)^2$$

$$= 2 \cdot 27 - 9 = 54 - 9 = 45 //$$

$$\lim_{x \to -2} (5x+7)^4 = \left(\lim_{x \to -2} (5x+7)\right)^4$$

$$= (5 \cdot (-2) + 7)^4$$

$$= (-10+7)^4 = (-3)^4 = 81 //$$

$$\lim_{x \to 2} \frac{x}{-3x+1}$$

Como $-3.(2)+1 = -6+1 = -5 \neq 0$

$$= \frac{\lim_{x \to 2} x}{\lim_{x \to 2} (-3x+1)} = \frac{2}{-5} = -\frac{2}{5} = \frac{-2}{5}$$

$$\lim_{x \to 4} \sqrt[3]{\frac{x}{-7x+1}}$$

$$= \sqrt[3]{\lim_{x \to 4} \frac{x}{-7x+1}} = \sqrt[3]{\frac{4}{-7(4)+1}}$$

$$= \sqrt[3]{\frac{4}{-27}} = -\sqrt[3]{\frac{4}{27}}$$

$$= -\frac{\sqrt[3]{4}}{\sqrt[3]{27}} = -\frac{\sqrt[3]{4}}{3}$$

$$\lim_{x \to 1} \sqrt{\frac{x^3+2x+1}{x^2+2}}$$

$$= \sqrt{\frac{\lim_{x \to 1} (x^3+2x+1)}{\lim_{x \to 1} (x^2+2)}} = \sqrt{\frac{(1^3+2(1)+1)}{(1^2+2)}}$$

$$= \sqrt{\frac{4}{3}} = \frac{\sqrt{4}}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}} \cdot 1 = \frac{2}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2\sqrt{3}}{3} \;//$$

$$\lim_{x \to 1} \sqrt{\frac{x^3+2x+1}{x^2+2}} = \lim_{x \to 1} \frac{\sqrt{x^3+2x+1}}{\sqrt{x^2+2}}$$

$$= \frac{\lim_{x \to 1} \sqrt{x^3+2x+1}}{\lim_{x \to 1} \sqrt{x^2+2}} = \frac{\sqrt{1^3+2 \cdot 1+1}}{\sqrt{1^2+2}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$$

$$\lim_{x \to 5} \frac{x^2 - 5^2}{x - 5} = \lim_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}}$$

$$= \lim_{x \to 5} x+5 = 5+5 = 10$$

$a>0$

$$x^2 - a = (x+\sqrt{a})(x-\sqrt{a})$$

$$x^2 - a^2 = (x+a)(x-a)$$

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} =$$

$$\frac{\sqrt{x}-2}{(x-4)} \cdot \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \frac{x + \cancel{2\sqrt{x}} - \cancel{2\sqrt{x}} - 4}{(x-4)(\sqrt{x}+2)}$$

$$= \frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2}$$

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x \to 4} \frac{1}{\sqrt{x}+2}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$$

$$\lim_{x \to 2} \frac{x^2+8x-20}{x^2-x-2}$$

$$2^2-2-2=4-4=0$$

$$2^2+8\cdot 2-20=4+16-20=0$$

$$\begin{array}{l|l} x^2+8x-20 & x-2 \\ \hline -(x^2-2x) & x+10 \\ \hline 0\ +10x-20 & \\ \quad -(10x-20) & \\ \hline \qquad 0 & \end{array}$$

$$x^2+8x-20=(x-2)(x+10)$$

$$x^2-x-2$$

$a=1$, $b=-1$, $c=-2$

$$\Rightarrow \frac{+1\pm\sqrt{1+8}}{2} = \frac{1\pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$$

$x_1=2$ и $x_2=-1$

$$x^2-x-2=(x-2)(x-(-1))=(x-2)(x+1)$$

$$\lim_{x \to 2} \frac{x^2 - 8x + 20}{x^2 - x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+10)}{\cancel{(x-2)}(x+1)}$$

$$= \lim_{x \to 2} \frac{x+10}{x+1} = \frac{2+10}{2+1} = \frac{12}{3} = 4$$

$$\lim_{x \to -1} \frac{x^2 + 2x + 1}{x+1} = \lim_{x \to -1} \frac{(x+1)^2}{\cancel{(x+1)}}$$

$$= \lim_{x \to -1} x+1 = -1+1 = 0$$

Teorema do Confronto

$$\lim_{x \to 0} x^2 \left|\operatorname{sen}\left(\tfrac{1}{x}\right)\right|$$

$\forall\, x \neq 0 \qquad 0 \leq \left|\operatorname{sen}\left(\tfrac{1}{x}\right)\right| \leq 1$

$$0 \cdot x^2 \leq x^2 \left|\operatorname{sen}\left(\tfrac{1}{x}\right)\right| \leq x^2$$

$$0 \leq x^2 \left|\operatorname{sen}\left(\tfrac{1}{x}\right)\right| \leq x^2$$

$$\lim_{x \to 0} 0 = 0$$

$$\lim_{x \to 0} x^2 = 0^2 = 0$$

Pelo Teorema do Confronto temos

$$\lim_{x \to 0} x^2 \left|\operatorname{sen}\left(\frac{1}{x}\right)\right| = 0$$

$$\lim_{x \to 2} \frac{x}{x^2 - 4x} = \frac{2}{4-8} = \frac{2}{-4} = -\frac{1}{2}$$

$$\lim_{x \to 2} \frac{\cancel{x}}{\cancel{x}}\left(\frac{1}{x-4}\right) = \lim_{x \to 2} \frac{1}{x-4} = \frac{1}{2-4} = \frac{1}{-2}$$

$$= -\frac{1}{2}$$

$$\boxed{\lim_{x \to 0} -x^4 \cdot \cos\left(\frac{1}{x}\right)}$$

$$-1 \leq \cos\left(\frac{1}{x}\right) \leq 1 \qquad x \neq 0$$

$$-x^4 < 0$$
$$x \neq 0$$

$$(-1)\cdot(-x^4) \geq -x^4 \cos\left(\frac{1}{x}\right) \geq 1 \cdot (-x^4)$$

$$x^4 \geq -x^4 \cos\left(\frac{1}{x}\right) \geq -x^4$$

Como $\lim_{x \to 0} x^4 = 0^4 = 0$

$\lim_{x \to 0} -x^4 = -0^4 = 0$

então $\lim_{x \to 0} -x^4 \cos\left(\frac{1}{x}\right) = 0$