Integrais

$$\int \left(\frac{1}{3} e^{3x} + sen(3x) \right) dx$$

$$= \int \frac{1}{3} e^{3x} dx + \int sen(3x)\, dx$$

$$\begin{cases} u = 3x \\ du = 3dx \Rightarrow \frac{du}{3} = dx \end{cases}$$

$$= \int \frac{1}{3} e^{u} \cdot \frac{du}{3} + \int sen(u) \frac{du}{3}$$

$$= \frac{1}{9} \int e^{u} du + \frac{1}{3} \int sen(u)\, du$$

$$= \frac{1}{9} e^{u} - \frac{1}{3} \cos(u) + C$$

$$= \frac{1}{9} e^{3x} - \frac{1}{3} \cos(3x) + C \,//$$

$$\int \underbrace{\left(5^x + e^{-1}\right)} dx \qquad \int a^x dx$$

$$= \int 5^x dx + \int e^{-1} dx$$

$$= \boxed{\frac{5^x}{\ln(5)} + e^{-1}x + C}$$

$$\int e^{-1} dx = e^{-1} \underbrace{\int dx}$$

$$= e^{-1}(x + C)$$

$$= e^{-1}x + e^{-1}C$$

$$= e^{-1}x + C_1$$

$$\int \frac{\cos(x) + \sec(x)}{\cos(x)}\, dx$$

$$= \int \left(\frac{\cos(x)}{\cos(x)} + \frac{\sec(x)}{\cos(x)} \right) dx$$

$$= \int \left(1 + \sec^2(x) \right) dx$$

$$= \int dx + \int \sec^2(x)\, dx = x + \operatorname{tg}(x) + C$$

$$\int \operatorname{sen}(2x) \sqrt{5 + \operatorname{sen}^2(x)}\, dx$$

Dica: $\operatorname{sen}(2x) = 2\operatorname{sen}(x) \cdot \cos(x)$

$$= \int 2\operatorname{sen}(x) \cdot \cos(x) \cdot \sqrt{5 + \operatorname{sen}^2(x)}\, dx$$

$$\begin{cases} u = 5 + \operatorname{sen}^2(x) \\ du = 2 \cdot \operatorname{sen}(x) \cdot \cos(x)\, dx = \operatorname{sen}(2x)\, dx \end{cases}$$

$$= \int \sqrt{u}\, du = \int u^{\frac{1}{2}} du$$

$$= \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \frac{2}{3} \left(5 + sen^2(x)\right)^{\frac{3}{2}} + C$$

$$= \frac{2}{3} \sqrt{\left(5 + sen^2(x)\right)^3} + C$$

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1^2 - x^2}}\, dx$$

$$\int \frac{1}{\sqrt{a^2 - u^2}}\, du = arc\, sen\left(\frac{u}{a}\right) + C$$

$$u^2 < a^2$$

$$= arc\, sen\left(\frac{x}{1}\right) \Big|_0^{\frac{1}{2}} = arc\, sen\left(\frac{1}{2}\right) - arc\, sen(0)$$

$$= \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

$$\int_1^4 \frac{1+x}{\sqrt{x}}\,dx = \int_1^4 \left(\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}}\right) dx$$

$$= \int_1^4 \left(x^{-\frac{1}{2}} + x^{\frac{1}{2}}\right) dx$$

$$= \int_1^4 x^{-\frac{1}{2}}\,dx + \int_1^4 x^{\frac{1}{2}}\,dx$$

$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_1^4 + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Bigg|_1^4$$

$$= 2x^{\frac{1}{2}}\Big|_1^4 + \frac{2}{3}x^{\frac{3}{2}}\Big|_1^4$$

$$= 2\cdot\sqrt{4} - 2\sqrt{1} + \frac{2}{3}\,4^{\frac{3}{2}} - \frac{2}{3}\,1^{\frac{3}{2}}$$

$$= 4 - 2 + \frac{2}{3}\sqrt[2]{64} - \frac{2}{3}$$

$$= 2 + \frac{2}{3}\cdot 8 - \frac{2}{3} = 2 + \frac{16}{3} - \frac{2}{3}$$

$$= 2 + \frac{14}{3} = \frac{6+14}{3} = \frac{20}{3}$$

$$\int_0^1 \frac{1}{1+t^2}\,dt = \operatorname{arctg}(t)\Big|_0^1$$

$$= \operatorname{arctg}(1) - \operatorname{arctg}(0)$$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

$$\int_0^8 \sqrt[3]{x}\,dx = \int_0^8 x^{\frac{1}{3}}\,dx$$

$$= \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\Big|_0^8 = \frac{x^{\frac{4}{3}}}{\frac{4}{3}}\Big|_0^8$$

$$= \frac{3}{4}\sqrt[3]{x^4}\Big|_0^8 = \frac{3}{4}\sqrt[3]{8^4} - \frac{3}{4}\sqrt[3]{0^4}$$

$$= \frac{3}{4}\sqrt[3]{4096} = \frac{3}{4}\cdot 16$$

$$= 3.4 = 12 \,//$$

$$\int_{-1}^{1} (x^7 + x^3 + x)\,dx$$

$$= \int_{-1}^{1} x^7 dx + \int_{-1}^{1} x^3 dx + \int_{-1}^{1} x\, dx$$

$$= \frac{x^8}{8}\Big|_{-1}^{1} + \frac{x^4}{4}\Big|_{-1}^{1} + \frac{x^2}{2}\Big|_{-1}^{1}$$

$$= \frac{1^8}{8} - \frac{(-1)^8}{8} + \frac{1^4}{4} - \frac{(-1)^4}{4} + \frac{1^2}{2} - \frac{(-1)^2}{2}$$

$$= \cancel{\frac{1}{8}} - \cancel{\frac{1}{8}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} = 0 \,//$$

$$\int \operatorname{sen}^3(x)\,dx$$

$$\int \operatorname{sen}^n(au)\,du = -\frac{\operatorname{sen}^{n-1}(au)\cdot\cos(au)}{a\cdot n} + \left(\frac{n-1}{n}\right)\int \operatorname{sen}^{n-2}(au)\,du$$

$n=3$; $a=1$; $u=x$

$$= -\frac{\operatorname{sen}^{3-1}(x)\cdot\cos(x)}{3} + \left(\frac{3-1}{3}\right)\cdot\int \operatorname{sen}^{3-2}(x)\,dx$$

$$= -\frac{\operatorname{sen}^2(x)\cos(x)}{3} + \frac{2}{3}\int \operatorname{sen}(x)\,dx$$

$$= -\frac{\operatorname{sen}^2(x)\cos(x)}{3} - \frac{2}{3}\cos(x) + C$$

$$= -\frac{1}{3}\cos(x)\left(\operatorname{sen}^2(x)-2\right) + C$$