Limites Laterais

$$f(x) = \frac{|x|}{x}$$

$$\lim_{x \to 2} \frac{|x|}{x} = \frac{|2|}{2} = \frac{2}{2} = 1$$

$$\lim_{x \to -1} \frac{|x|}{x} = \frac{|-1|}{-1} = \frac{1}{-1} = -1$$

$$\frac{0}{0}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{|x|}{x} = ?$$

$$\lim_{x \to 0^+} f(x) = \lim_{\substack{x \to 0^+ \\ x > 0}} \frac{|x|}{x} = \lim_{x \to 0^+} \left(\frac{x}{x}\right)$$

$$= \lim_{x \to 0^+} 1 = 1$$

$$\lim_{\substack{x \to 0^- \\ x < 0}} f(x) = \lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \left(\frac{-x}{x}\right)$$

$$= \lim_{x \to 0^-} -1 = -1$$

$$\lim_{x \to 0^+} f(x) = 1 \neq -1 = \lim_{x \to 0^-} f(x)$$

então $\displaystyle\lim_{x \to 0} f(x) = \nexists$

$$h(x) = \begin{cases} 4 - x^2 & \text{se } x \leq 1 \\ 2 + x^2 & \text{se } x > 1 \end{cases}$$

$$\lim_{\substack{x \to 0 \\ x \leq 1}} h(x) = \lim_{x \to 0} 4 - x^2 = 4 - 0^2 = 4$$

$$\lim_{\substack{x \to 2 \\ x > 1}} h(x) = \lim_{x \to 2} 2 + x^2 = 2 + 2^2 = 6$$

$$\lim_{x \to 1} h(x) = ?$$

$$\lim_{\substack{x \to 1^- \\ x < 1}} h(x) = \lim_{x \to 1^-} 4 - x^2 = 4 - (1)^2 = 3$$

$$\lim_{\substack{x \to 1^+ \\ x > 1}} h(x) = \lim_{x \to 1^+} 2 + x^2 = 2 + (1)^2 = 3$$

Como $\displaystyle\lim_{x \to 1^-} h(x) = 3 = \lim_{x \to 1^+} h(x)$

então $\lim_{x \to 1} h(x) = 3$

$$f(x) = \begin{cases} x+1 & \text{se } -2 \leq x < 0 \\ x^2 & \text{se } 0 \leq x \leq 2 \end{cases}$$

$D(f) = [-2, 2]$ $\qquad f(0) = 0^2 = 0$

$$\lim_{x \to -2} f(x) = \nexists$$

pois $\lim_{x \to -2^-} f(x)$ não está definido na função

$$\lim_{\substack{x \to -2^+ \\ x > -2}} f(x) = \lim_{x \to -2^+} x+1 = -2+1 = -1$$

$$\lim_{x \to 0} f(x) = ?$$

$$\lim_{\substack{x \to 0^+ \\ x > 0}} f(x) = \lim_{x \to 0^+} x^2 = 0^2 = 0$$

$$\lim_{\substack{x \to 0^- \\ x < 0}} f(x) = \lim_{x \to 0^-} x + 1 = 0 + 1 = 1$$

Como $\lim_{x \to 0^+} f(x) = 0 \neq 1 = \lim_{x \to 0^-} f(x)$,

Portanto $\lim_{x \to 0} f(x) = \nexists$.

$$\lim_{\substack{x \to 2^- \\ x < 2}} f(x) = \lim_{x \to 2^-} x^2 = 2^2 = 4$$

$$\lim_{x \to -3^+} \frac{\sqrt{x+3}}{x^2+1}$$

$x > -3 \Rightarrow x + 3 > -3 + 3 = 0$

$$= \frac{\lim_{x \to -3^+} \sqrt{x+3}}{\lim_{x \to -3^+} x^2 + 1} = \frac{\sqrt{-3+3}}{(-3)^2+1}$$

$$= \frac{\sqrt{0}}{9+1} = \frac{0}{10} = 0$$

$$\lim_{x \to -3^-} \frac{\sqrt{x+3}}{x^2+1} = \nexists$$

pois $x \to -3^- \Rightarrow x+3<0$

$\sqrt{x+3}$ não é definida para $x+3<0$

Portanto $$\lim_{x \to -3} \frac{\sqrt{x+3}}{x^2+1} = \nexists$$