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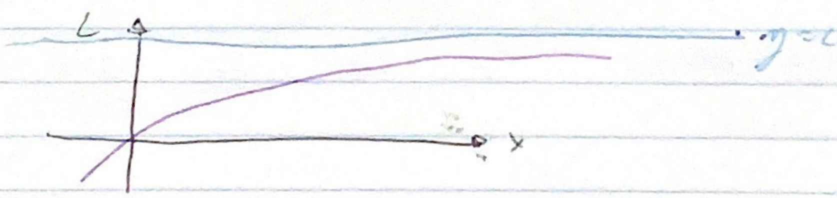
LIMITES NO INFINITO E LIMITES INFINITO

$$\lim_{x \to -\infty} e^x = 0$$

$y = 0$ É ASSINTOTA HORIZONTAL DA FUNÇÃO e^x.

QUANDO $\lim_{x \to +\infty} f(x) = L$



CHAMAMOS A RETA $y = L$ COMO ASSINTOTA HORIZONTAL

$$\begin{cases} \lim\limits_{x \to +\infty} \operatorname{sen}(x) = \nexists \\ \\ \lim\limits_{x \to -\infty} \operatorname{sen}(x) = \nexists \end{cases}$$

ANALOGO À $\cos(x)$

$$\begin{cases} \lim\limits_{x \to \pi/2^-} Tg(x) = +\infty \\ \\ \lim\limits_{x \to \pi/2^+} Tg(x) = -\infty \\ \\ \lim\limits_{x \to \pi/2} |tg(x)| = +\infty \end{cases}$$

$$\begin{cases} \lim\limits_{x \to 0^+} \ln(x) = -\infty \\ \\ \lim\limits_{x \to +\infty} \ln(x) = +\infty \end{cases}$$

tilibra

SE $n \in \mathbb{N}$ E $c \in \mathbb{R}^*$, ENTÃO

$$\begin{cases} \lim_{x \to +\infty} \frac{c}{x^n} = 0 \\ \lim_{x \to -\infty} \frac{c}{x^n} = 0 \end{cases}$$

EXEMPLOS

① $\lim_{x \to +\infty} \frac{x^2+2x+3}{x^3+5x}$

$$= \lim_{x \to +\infty} \frac{\frac{1}{x^3}(x^2+2x+3)}{\frac{1}{x^3}(x^3+5x)}$$

$$= \lim_{x \to +\infty} \frac{\cancel{1/x}^{\,0} + \cancel{2/x^2}^{\,0} + \cancel{3/x^3}^{\,0}}{1+5/x^2} = 0 \;//$$

COMO $\lim_{x \to \infty} (1/x + 2/x^2 + 3/x^3) = 0$

,

$\lim (1+5/x^2) = 1+0 = 1$

(2) $\lim_{x \to +\infty} \dfrac{3x^2 - 2}{x^2 + 5x + 1}$

$= \lim_{x \to +\infty} \dfrac{3 - 2/x^2}{1 + 5/x + 1/x} = \dfrac{3}{1} = 3$ //

SABEMOS QUE $\lim_{x \to +\infty} (3 - 2/x^2) = 3$

$\lim_{x \to +\infty} (1 + 5/x + 1/x^2) = 1$

(4) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 + \sqrt{1 - x}}}{x + 1}$

QUANDO $x \to -\infty$ SEI QUE OS VALORES DE $x < 0$

$\lim_{x \to -\infty} \dfrac{\boxed{\dfrac{\sqrt{x^2 + \sqrt{1-x}}}{x}}}{\dfrac{x+1}{x}} =$

$= \lim_{x \to -\infty} \dfrac{\dfrac{\sqrt{x^2 + \sqrt{1-x}}}{-\sqrt{x^2}}}{1 + 1/x}$

DE $x \to -\infty \therefore x < 0$. $|x| = -x$

$x = -|x| = -\sqrt{x^2}$

$$\lim_{x \to -\infty} -\sqrt{\frac{x^2 + \sqrt{1-x}}{x^2}}$$

$$1 + \frac{1}{x}$$

$$\lim_{x \to -\infty} \frac{-\sqrt{1 + \sqrt{1-x}/x^2}}{1 + 1/x}$$

$$\Rightarrow x^2 = x^{4/2} = \sqrt{x^4}$$

$$= \lim_{x \to -\infty} \frac{-\sqrt{1 + \sqrt{1 - x/x^4}}}{1 + 1/x}$$

$$= \lim_{x \to -\infty} = \frac{-\sqrt{1 + \sqrt{1/x^4 - 1/x^3}}}{1 + 1/x}$$

$$\Rightarrow \lim_{x \to -\infty} -\sqrt{1 + \sqrt{1/x^4 - \frac{1}{x^3}}} = -1$$

$$\Rightarrow \lim_{x \to -\infty} 1 + 0 = 1$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 + \sqrt{1-x}}}{x + 1} \to \frac{-1}{1} = -1$$

(5) $\lim_{x \to +\infty} \frac{\sqrt{16x^2+1}}{5x-2}$

$\lim_{x \to +\infty} \frac{\frac{\sqrt{16x^2+1}}{x}}{\frac{5x-2}{x}} =$

$x \to +\infty \therefore x > 0 \therefore |x| = x \therefore x = \sqrt{x^2}$

$= \lim_{x \to +\infty} \frac{\frac{\sqrt{16x^2+1}}{\sqrt{x^2}}}{5 - \frac{2}{x}}$

$= \lim_{x \to +\infty} \frac{\sqrt{16 + 1/x^2}}{5 - 2/x}$

$\begin{cases} \lim_{x \to +\infty} \sqrt{16 + 1/x^2} = 4 \\ \lim_{x \to -\infty} 5 - 2/x = 5 \end{cases}$

$\lim \frac{\sqrt{16x^2+1}}{5x-2} = 4/5$

6) $\lim_{x \to +\infty} sen\left(\frac{1}{x}\right)$

$= sen\left(\lim_{x \to +\infty} 1/x\right) = sen(0) = 0$

OU

$\lim_{x \to +\infty} sen(1/x) = \lim_{t \to 0} sen(t) = sen\,0 = 0$ //

$t = 1/x \therefore x \to +\infty \therefore t \to 0$

7) $\lim_{x \to 0^-} e^{1/x} =$

SABEMOS QUE $\lim_{x \to 0^-} \frac{1}{x} = -\infty$

CHAMANDO $t = \frac{1}{x} \therefore x \to 0^- \therefore t \to -\infty$

$\Rightarrow \lim_{t \to -\infty} e^{t} = 0$ //

⑧ $\lim_{x \to -\infty} \dfrac{x^5 + 3x^2 + 2x}{x^3 - 7x^2}$

$= \lim_{x \to -\infty} \dfrac{\dfrac{x^5 + 3x^2 + 2x}{x^5}}{\dfrac{x^3 - 7x^2}{x^5}}$

$= \lim_{x \to -\infty} \boxed{\dfrac{1 + 3/x^3 + 2/x^4}{\dfrac{1}{x^2} - \dfrac{7}{x^3}}}$

SABEMOS QUE $\dfrac{1}{x^2} \to 0$, $x \to -\infty$

$\dfrac{-7}{x^3} \to 0$ $x \to -\infty$

$\dfrac{1}{x^2} - \dfrac{7}{x^3} \to 0$ QUANDO $x \to -\infty$

E $\lim_{x \to -\infty} \left(1 + \frac{3}{x^3} + \frac{2}{x^4}\right) = 1$

PORTANTO $\lim_{x \to -\infty} \dfrac{x^5 + 3x^2 + 2x}{x^3 - 7x^2} = +\infty$!

(9) $\lim\limits_{x \to 0^-} \dfrac{\cos x}{x} =$

Como $\lim\limits_{x \to 0^-} \cos x = \cos 0 = 1 > 0$

$\lim\limits_{x \to 0^-} \dfrac{1}{x^-} = -\infty$

$\lim \dfrac{\cos x}{x} = \lim \cos x \cdot \left(\dfrac{1}{x}\right) = -\infty$

(10) $\lim\limits_{x \to +\infty} \dfrac{2x^3 + 5}{3x^2 + x + 2}$

$\lim\limits_{x \to +\infty} 2x^3 + 5 = +\infty$

$\lim\limits_{x \to +\infty} 3x^2 + x + 2 = +\infty$

$\lim\limits_{x \to +\infty} \dfrac{\frac{2x^3+5}{x^2}}{\frac{3x^2+x+2}{x^2}} = \lim\limits_{x \to +\infty} \dfrac{2x + 5/x^2}{3 + 1/x + 2/x^2} = +\infty$

$\Rightarrow \lim\limits_{x \to +\infty} 2x + 5/x^2 = +\infty$

$\Rightarrow \lim\limits_{x \to +\infty} 3 + \frac{1}{x} + \frac{2}{x^2} = 3$

$\to 0$ $\qquad\qquad\qquad$ $\to 3 > 0$

$$\lim_{x \to +\infty} \left(2x + \frac{5}{x^2} \right) \cdot \left(\frac{1}{3 + \frac{1}{x} + \frac{2}{x^2}} \right)$$

Portanto $\displaystyle \lim_{x \to +\infty} \frac{2x^3 + 5}{3x^2 + x + 2} = +\infty$