Derivação Implícita

$$3x^2 + 10y^3 = e^y$$

Considerando y em função de x

$$\frac{d}{dx}(3x^2 + 10y^3) = \frac{d(e^y)}{dx}$$

$$\frac{3\,d(x^2)}{dx} + \frac{10\,d(y^3)}{dx} = e^y \cdot \frac{dy}{dx}$$

$$6x + 30y^2 \cdot \frac{dy}{dx} = e^y \frac{dy}{dx}$$

$$30y^2 \frac{dy}{dx} - e^y \frac{dy}{dx} = -6x$$

$$(30y^2 - e^y)\frac{dy}{dx} = -6x$$

$$\frac{dy}{dx} = \frac{-6x}{30y^2 - e^y}$$

$$xy^3 + 3x = 5 - 2y$$

Derivada $\frac{dx}{dy}$

$$\frac{d}{dy}\left(xy^3 + 3x\right) = \frac{d}{dy}\left(5 - 2y\right)$$

$$\frac{d}{dy}\left(x \cdot y^3\right) + \frac{d}{dy}\left(3x\right) = -2$$

$$\frac{dx}{dy} \cdot y^3 + x\frac{d(y^3)}{dy} + 3\frac{dx}{dy} = -2$$

$$\frac{dx}{dy} \cdot y^3 + x \cdot 3y^2 + 3\frac{dx}{dy} = -2$$

$$\left(y^3 + 3\right)\frac{dx}{dy} = -2 - 3xy^2$$

$$\frac{dx}{dy} = \frac{-(2+3xy^2)}{y^3+3}$$

1)

$$\operatorname{sen}(y)\cdot\cos(x) + 3xy = e^{y^2}$$

$$\frac{d}{dx}\left(\operatorname{sen}(y)\cdot\cos(x)\right) + \frac{d}{dx}(3xy) = \frac{d\left(e^{y^2}\right)}{dx}$$

$$\frac{d}{dx}(\operatorname{sen}(y))\cdot\cos(x) + \operatorname{sen}(y)\cdot\frac{d(\cos(x))}{dx} + 3\frac{dx}{dx}\cdot y + 3x\frac{dy}{dx} = e^{y^2}\cdot\frac{d(y^2)}{dx}$$

$$\cos(y)\cos(x)\cdot\frac{dy}{dx} + \operatorname{sen}(y)(-\operatorname{sen}(x)) + 3y + 3x\frac{dy}{dx} = e^{y^2}\cdot 2y\cdot\frac{dy}{dx}$$

$$\cos(y)\cdot\cos(x)\frac{dy}{dx} + 3x\frac{dy}{dx} - 2ye^{y^2}\frac{dy}{dx} = \operatorname{sen}(y)\operatorname{sen}(x) - 3y$$

$$\left(\cos(y)\cos(x) + 3x - 2ye^{y^2}\right)\frac{dy}{dx} = \operatorname{sen}(y)\operatorname{sen}(x) - 3y$$

$$\frac{dy}{dx} = \frac{\operatorname{sen}(y)\cdot\operatorname{sen}(x) - 3y}{\cos(y)\cdot\cos(x) + 3x - 2ye^{y^2}}$$

$$\sqrt{x} + \sqrt{y} = \frac{x}{y}$$

$$\frac{d}{dx}\left(x^{\frac{1}{2}}\right) + \frac{d}{dx}\left(y^{\frac{1}{2}}\right) = \frac{d}{dx}\left(\frac{x}{y}\right)$$

$f(x) = x$

$g(x) = y$

$\left(\frac{f}{g}\right)'$

$\frac{f' \cdot g - f \cdot g'}{g^2}$

$$\frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{2} y^{-\frac{1}{2}} \cdot \frac{dy}{dx} = \frac{y - x \cdot \frac{dy}{dx}}{y^2}$$

$$\frac{y^2 \cdot x^{-\frac{1}{2}}}{2} + \frac{y^2 \cdot y^{-\frac{1}{2}}}{2} \cdot \frac{dy}{dx} = y - x\frac{dy}{dx}$$

$$x\frac{dy}{dx} + \frac{y^{\frac{3}{2}}}{2} \cdot \frac{dy}{dx} = y - \frac{y^2}{2\sqrt{x}}$$

$$\frac{dy}{dx}\left(x + \frac{y^{\frac{3}{2}}}{2}\right) = y - \frac{y^2\sqrt{x}}{x}$$

$$\frac{dy}{dx}\left(\frac{2x + y^{\frac{3}{2}}}{2}\right) = \frac{yx - y^2\sqrt{x}}{x}$$

$$\frac{dy}{dx} = \frac{2\left(yx - y^2\sqrt{x}\right)}{x\left(2x + y^{\frac{3}{2}}\right)}$$

$$e^{x+y} = 3x^2 - 2y$$

$$\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}\left(3x^2 - 2y\right)$$

$u = x+y$

$$\frac{d}{dx} \cdot e^{u} = e^{u} \cdot \frac{du}{dx}$$

$$e^{x+y} \cdot \frac{d(x+y)}{dx} = 3\frac{d(x^2)}{dx} - 2\frac{dy}{dx}$$

$$e^{x+y} \cdot \left(1 + \frac{dy}{dx}\right) = 6x - 2\frac{dy}{dx}$$

$$e^{x+y} + e^{x+y} \cdot \frac{dy}{dx} + 2\frac{dy}{dx} = 6x$$

$$\left(e^{x+y} + 2\right)\frac{dy}{dx} = 6x - e^{x+y}$$

$$\frac{dy}{dx} = \frac{6x - e^{x+y}}{e^{x+y} + 2}$$

$$\frac{d}{dx}\left(\log_{10}(x+y)\right) = \frac{d}{dx}(2xy)$$

$f = 2x$
$g = y$

$$\frac{d(f\cdot g)}{dx} = \frac{df}{dx}\cdot g + f\frac{dg}{dx}$$

$$\frac{d}{dx}\left(\log_a(u)\right) = \frac{1}{u}\cdot\frac{du}{dx}\cdot\log_a(e)$$

$$\frac{1}{x+y}\cdot\frac{d(x+y)}{dx}\log_{10}(e) = 2y + 2x\cdot\frac{dy}{dx}$$

$$\frac{\log_{10}(e)}{x+y}\cdot\left(1+\frac{dy}{dx}\right) = 2y + 2x\frac{dy}{dx}$$

$$\frac{\log_{10}(e)}{x+y} + \frac{\log_{10}(e)}{x+y}\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$$

$$\frac{\log_{10}(e)}{x+y}\frac{dy}{dx} - 2x\frac{dy}{dx} = 2y - \frac{\log_{10}(e)}{x+y}$$

$$\left(\frac{\log_{10}(e)}{x+y} - 2x\right)\frac{dy}{dx} = \frac{2y(x+y) - \log_{10}(e)}{x+y}$$

$$\left(\frac{\log_{10}(e) - 2x(x+y)}{(x+y)}\right)\frac{dy}{dx} = \frac{2y(x+y) - \log_{10}(e)}{(x+y)}$$

$$\frac{dy}{dx} = \left(\frac{2y(x+y) - \log_{10}(e)}{\cancel{(x+y)}}\right)\left(\frac{\cancel{(x+y)}}{\log_{10}(e) - 2x(x+y)}\right)$$

$$\frac{dy}{dx} = \frac{2y(x+y) - \log_{10}(e)}{\log_{10}(e) - 2x(x+y)}$$

$$= \frac{2yx + 2y^2 - \log_{10}(e)}{\log_{10}(e) - 2x^2 - 2xy}$$

$a, b \in \mathbb{R}_+^* - \{1\}$

Calcular $\frac{dy}{dx}$ da equação

$$a \sin^2(y) = b^x$$

$$a \frac{d}{dx}(\sin^2(y)) = \frac{d(b^x)}{dx}$$

$$a \cdot 2\sin(y) \cdot \frac{d}{dx}(\sin(y)) = b^x \cdot \ln(b)$$

$$2a \sin(y) \cdot \cos(y) \cdot \frac{dy}{dx} = b^x \ln(b)$$

$$\frac{dy}{dx} = \frac{b^x \cdot \ln(b)}{2a \sin(y) \cdot \cos(y)}$$