Funções Contínuas

f contínua em a

$$\lim_{x \to a} f(x) = f(a)$$

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f é contínua

$$\forall\, b \in D(f) \Rightarrow \lim_{x \to b} f(x) = f(b)$$

$$f(x) = \begin{cases} 0, & x \leq 0 \\ x, & x > 0 \end{cases}$$

Solução:

$$a < 0 \Rightarrow f(a) = 0$$

$$\lim_{\substack{x \to a \\ a<0 \\ x<0}} f(x) = \lim_{x \to a} 0 = 0 = f(a)$$

$$b > 0 \Rightarrow f(b) = b$$

$$\lim_{\substack{x \to b \\ b>0 \\ x>0}} f(x) = \lim_{x \to b} x = b = f(b)$$

$$c = 0 \Rightarrow f(0) = 0$$

$$\lim_{x \to 0} f(x) = ?$$

$$\lim_{\substack{x \to 0^+ \\ x>0}} f(x) = \lim_{x \to 0^+} x = 0 = f(0)$$

$$\lim_{\substack{x \to 0^- \\ x<0}} f(x) = \lim_{x \to 0^-} 0 = 0 = f(0)$$

Concluindo $\lim_{x \to 0} f(x) = 0 = f(0)$.

Portanto f é contínua.

Calcular p de modo que

$$f(x) = \begin{cases} e^{2x}, & x \neq 0 \\ p^3 - 7, & x = 0. \end{cases}$$

seja contínua

Solução: Se $a \neq 0$ então

$$f(a) = e^{2a}$$

$$\lim_{x \to a} f(x) = \lim_{x \to a} e^{2x} = e^{\left(\lim_{x \to a} 2x\right)} = e^{2a} = f(a)$$

f é contínua $\forall\, a \neq 0$.

$a = 0$: Se f é contínua em 0 então

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} e^{2x} = p^3 - 7$$

$$e^{2 \cdot 0} = p^3 - 7$$

$$e^0 = p^3 - 7$$

$$1 + 7 = p^3$$

$$p^3 = 8$$

$$p = \sqrt[3]{8} = 2$$

ou seja, f é contínua para $p = 2$.

e a função f é

$$f(x) = \begin{cases} e^{2x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

$$\lim_{x \to 1} \operatorname{arc\,sen}\left(\frac{1-\sqrt{x}}{1-x}\right)$$

$$= \operatorname{arc\,sen}\left(\lim_{x \to 1} \frac{1-\sqrt{x}}{1-x}\right)$$

$$= \operatorname{arc\,sen}\left(\lim_{x \to 1} \frac{1-\sqrt{x}}{\underbrace{-(\sqrt{x}+1)(\sqrt{x}-1)}}\right)$$

$$= \operatorname{arc\,sen}\left(\lim_{x \to 1} \frac{\cancel{(1-\sqrt{x})}}{(\sqrt{x}+1)\cancel{(1-\sqrt{x})}}\right)$$

$$= \operatorname{arc\,sen}\left(\lim_{x \to 1} \frac{1}{\sqrt{x}+1}\right) =$$

$$= \operatorname{arc\,sen}\left(\frac{1}{\sqrt{1}+1}\right) = \operatorname{arc\,sen}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\boxed{\begin{array}{c} x \geq 0 \\ \sqrt{x} = t \\ \dfrac{1-t}{1-t^2} \\ \shortparallel \\ \dfrac{1-t}{-(t^2-1)} \\ \shortparallel \\ \dfrac{1-t}{-(t+1)(t-1)} \end{array}}$$

$t \to 1$

$$\frac{1-t}{1-t^2} = \frac{1-t}{-(t^2-1)} = \frac{1-t}{-(t+1)(t-1)} = \frac{\cancel{1-t}}{(t+1)\cancel{(1-t)}} = \frac{1}{t+1}$$

$t = \sqrt{x}$

$t^2 = x$

$x \to 1$

$$\frac{1-\sqrt{x}}{1-x} = \frac{1-\sqrt{x}}{-(x-1)} = \frac{1-\sqrt{x}}{-(\sqrt{x}+1)(\sqrt{x}-1)} = \frac{\cancel{1-\sqrt{x}}}{(\sqrt{x}+1)\cancel{(1-\sqrt{x})}} = \frac{1}{\sqrt{x}+1}$$

$$\lim_{x \to -1} \frac{\sqrt[3]{x+2}-1}{x+1} \qquad \frac{0}{0}$$

Solução:

$\Rightarrow u = \sqrt[3]{x+2} \Leftrightarrow u^3 = x+2 \Leftrightarrow u^3 - 2 = x$

$x \to -1$

$u \to 1$

$$\lim_{u \to 1} \frac{u-1}{u^3-1}$$

$$\begin{array}{r|l} u^3 + 0u^2 + 0u - 1 & u-1 \\ \cline{2-2} -(u^3 - u^2) & u^2 + u + 1 \\ \hline 0 + u^2 + 0u & \\ -(u^2 - u) & \\ \hline 0 + u - 1 & \\ -u - 1 & \\ \hline 0 & \end{array}$$

$$= \lim_{u \to 1} \frac{\cancel{u-1}}{\cancel{(u-1)}(u^2+u+1)} = \lim_{u \to 1} \frac{1}{u^2+u+1}$$

$$= \frac{1}{3}$$

Portanto $\displaystyle \lim_{x \to -1} \frac{\sqrt[3]{x+2} - 1}{x+1} = \frac{1}{3}$

$$\lim_{x \to 1} \frac{(\sqrt{3-x^3})^4 - 16}{x^3 - 1}$$

$u = 3 - x^3 \Rightarrow u - 3 = -x^3 \Rightarrow x^3 = 3 - u$

$x \to 1 \Rightarrow u \to 2$

$$\lim_{u \to 2} \frac{u^4 - 16}{2 - u} = \lim_{u \to 2} \frac{u^4 - 16}{-(u-2)}$$

$= (*)$

$$
\begin{array}{l}
u^4 + 0u^3 + 0u^2 + 0u - 16 \quad \underline{|u-2} \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad u^3+2u^2+4u+8 \\
-(u^4 - 2u^3) \\
\hline
0 \quad 2u^3 + 0u^2 \\
\quad -(2u^3 - 4u^2) \\
\hline
\quad 0 \quad +4u^2 + 0u \\
\quad\quad -(4u^2 - 8u) \\
\hline
\quad\quad 0 \quad +8u - 16 \\
\quad\quad\quad 8u - 16 \\
\hline
\quad\quad\quad 0
\end{array}
$$

$$(x) = \lim_{u \to 2} \frac{\cancel{(u-2)}(u^3+2u^2+4u+8)}{-\cancel{(u-2)}}$$

$$= \lim_{u \to 2} -(u^3+2u^2+4u+8)$$

$$= -(2^3+2\cdot 2^2+4\cdot 2+8)$$

$$= -(8+8+8+8) = -32$$

Portanto $\lim_{x \to 1} \frac{(3-x^3)^4-16}{x^3-1} = -32$