Limites Infinitos

Limites no Infinito

$$\lim_{x \to +\infty} \left(\frac{x^7 + x^4 + 2}{3x^7 + x + 1} \right) \qquad \frac{+\infty}{+\infty}$$

$$\lim_{x \to +\infty} \frac{1}{x^n} = 0 \qquad n \in \mathbb{N}$$

$$= \lim_{x \to +\infty} \left[\frac{\cancel{x^7}\left(1 + \frac{1}{x^3} + \frac{2}{x^7}\right)}{\cancel{x^7}\left(3 + \frac{1}{x^6} + \frac{1}{x^7}\right)} \right]$$

$$= \lim_{x \to +\infty} \frac{1 + \frac{1}{x^3} + \frac{2}{x^7}}{3 + \frac{1}{x^6} + \frac{1}{x^7}}$$

$$= \frac{1 + \lim\limits_{x \to +\infty} \frac{1}{x^3} + \lim\limits_{x \to +\infty} \frac{2}{x^7}}{3 + \lim\limits_{x \to +\infty} \frac{1}{x^6} + \lim\limits_{x \to +\infty} \frac{1}{x^7}}$$

$$= \frac{1 + 0 + 0}{3 + 0 + 0} = \frac{1}{3} \quad //$$

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$$\lim_{x \to +\infty} \frac{x^3 + 3x - 1}{-2x^2 + x + 1}$$

$$\lim_{x \to +\infty} \frac{x^3 \left(1 + \frac{3}{x^2} - \frac{1}{x^3}\right)}{x^2 \left(-2 + \frac{1}{x} + \frac{1}{x^2}\right)}$$

$$\lim_{x \to +\infty} x \cdot \left[\frac{1 + \frac{3}{x^2} - \frac{1}{x^3}}{-2 + \frac{1}{x} + \frac{1}{x^2}}\right]$$

$$\left[\lim_{x \to +\infty} x\right] \cdot \left[\frac{1 + \lim\limits_{x \to +\infty} \frac{3}{x^2} \to 0 - \lim\limits_{x \to +\infty} \frac{1}{x^3} \to 0}{-2 + \lim\limits_{x \to +\infty} \frac{1}{x} \to 0 + \lim\limits_{x \to +\infty} \frac{1}{x^2} \to 0}\right]$$

$$= \left[\underbrace{\lim_{x \to +\infty} x}_{\to +\infty}\right] \underbrace{\left(-\frac{1}{2}\right)}_{-\frac{1}{2} < 0} = -\infty$$

$$\lim_{x \to -\infty} \frac{3x^6 + 2x^5 + 3}{4x^7 - 2x + 1}$$

$$= \lim_{x \to -\infty} \left[\frac{\cancel{x^7}\left(\frac{3}{x} + \frac{2}{x^2} + \frac{3}{x^7}\right)}{\cancel{x^7}\left(4 - \frac{2}{x^6} + \frac{1}{x^7}\right)} \right]$$

$$= \lim_{x \to -\infty} \frac{\frac{3}{x} + \frac{2}{x^2} + \frac{3}{x^7}}{4 - \frac{2}{x^6} + \frac{1}{x^7}}$$

$$= \frac{\overset{\to 0}{\lim_{x \to -\infty} \frac{3}{x}} + \overset{\to 0}{\lim_{x \to -\infty} \frac{2}{x^2}} + \overset{\to 0}{\lim_{x \to -\infty} \frac{3}{x^7}}}{4 - \overset{\to 0}{\lim_{x \to -\infty} \frac{2}{x^6}} + \overset{\to 0}{\lim_{x \to -\infty} \frac{1}{x^7}}}$$

$$= \frac{0 + 0 + 0}{4 - 0 + 0} = 0$$

$$\lim_{x \to 1} \frac{1}{x-1} = ?$$

$$\lim_{x \to 1^+} \frac{1}{x-1}$$

$$x \to 1^+ \Rightarrow 1 < x \Rightarrow 0 < x-1$$

$$\lim_{x \to 1^+} (x-1) = 0$$

então $$\lim_{x \to 1^+} \frac{1}{x-1} = +\infty$$

$$\lim_{x \to 1^-} \frac{1}{x-1} =$$

$$x \to 1^- \Rightarrow x < 1 \Rightarrow \underline{x-1 < 0}$$

$$\underline{\lim_{x \to 1^-} x-1 = 0}$$

então $$\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$$

$$\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$$

$$\lim_{x \to 1^+} \frac{1}{x-1} = +\infty$$

$\ln +\infty$ $$\lim_{x \to 1} \frac{1}{x-1} = \nexists$$

$$\lim_{x \to 1^+} \frac{x^3 - 1}{x^2 - 2x + 1} \qquad \frac{0}{0}$$

$$= \lim_{x \to 1^+} \frac{\cancel{(x-1)}(x^2+x+1)}{(x-1)^2}$$

$$= \lim_{x \to 1^+} \frac{x^2+x+1}{x-1}$$

$$= \left(\lim_{x \to 1^+} \frac{1}{x-1}\right)\left(\lim_{x \to 1^+} x^2+x+1\right)$$

$$= \left(\lim_{x \to 1^+} \frac{1}{x-1}\right)\left(1^2+1+1\right)$$

$$= 3\left(\lim_{x\to 1^+} \frac{1}{x-1}\right)$$

No exemplo anterior vimos

$$\lim_{x\to 1^+} \frac{1}{x-1} = +\infty \quad \text{e} \quad 3 > 0$$

Então $\lim_{x\to 1^+} \frac{x^3-1}{x^2-2x+1} = +\infty$

$$\lim_{x\to +\infty} \frac{\sqrt{x^2+5x}}{\sqrt{x^2+1}}$$

$$= \lim_{x\to +\infty} \sqrt{\frac{x^2+5x}{x^2+1}}$$

$$= \sqrt{\lim_{x\to +\infty} \frac{x^2\left(1+\frac{5}{x}\right)}{x^2\left(1+\frac{1}{x^2}\right)}}$$

$$x^2 + \frac{5x^2}{x}$$

$$x^2 + 5x$$

$$= \sqrt{\frac{1 + \lim\limits_{x \to +\infty} \frac{5}{x}}{1 + \lim\limits_{x \to +\infty} \frac{1}{x^2}}}$$

$$= \sqrt{\frac{1}{1}} = 1$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2 + \sqrt{1-x}}}{x+5}$$

$$= \lim_{x \to -\infty} \frac{\left(\frac{1}{x}\right)\sqrt{x^2 + \sqrt{1-x}}}{\left(\frac{1}{x}\right)(x+5)}$$

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{x^2 + \sqrt{1-x}}}{x}}{\left(1 + \frac{5}{x}\right)}$$

$x \to -\infty \qquad x < 0 \quad \sqrt{x^2} > 0$

$$x = -\sqrt{x^2} < 0$$

$$= \lim_{x \to -\infty} \frac{\dfrac{\sqrt{x^2+\sqrt{1-x}}}{-\sqrt{x^2}}}{\left(1+\dfrac{5}{x}\right)}$$

$$= \lim_{x \to -\infty} \frac{-\sqrt{\dfrac{x^2+\sqrt{1-x}}{x^2}}}{\left(1+\dfrac{5}{x}\right)}$$

$$= \lim_{x \to -\infty} \frac{-\sqrt{1+\dfrac{\sqrt{1-x}}{x^2}}}{\left(1+\dfrac{5}{x}\right)}$$

$$x \to -\infty$$
$$x^2 = \sqrt{x^4}$$

$$= \lim_{x \to -\infty} \frac{-\sqrt{1+\sqrt{\dfrac{1-x}{x^4}}}}{\left(1+\dfrac{5}{x}\right)}$$

$$= \lim_{x \to -\infty} \frac{-\sqrt{1+\sqrt{\dfrac{1}{x^4}-\dfrac{1}{x^3}}}}{1+\dfrac{5}{x}}$$

$$= \frac{-\sqrt{1+\sqrt{\lim\limits_{x \to -\infty}\dfrac{1}{x^4}-\lim\limits_{x \to -\infty}\dfrac{1}{x^3}}}}{}$$

$$1 + \lim_{x \to -\infty} \frac{5}{x}$$

$$= \frac{-\sqrt{1 + \sqrt{0 - 0}}}{1 + 0}$$

$$= \frac{-\sqrt{1}}{1} = \frac{-1}{1} = -1$$

$$\lim_{x \to -\infty} \frac{x + 7}{\sqrt{2x^2 + 1}}$$

$$= \lim_{x \to -\infty} \frac{x \cdot \left[\frac{(x+7)}{x}\right]}{x \cdot \left(\frac{\sqrt{2x^2+1}}{x}\right)}$$

$$= \lim_{x \to -\infty} \frac{1 + \frac{7}{x}}{\frac{\sqrt{2x^2+1}}{x}}$$

$x \to -\infty$

$x = -\sqrt{x^2}$

$$= \lim_{x \to -\infty} \frac{1 + \frac{7}{x}}{-\sqrt{\frac{2x^2+1}{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{1 + \frac{7}{x}}{-\sqrt{2 + \frac{1}{x^2}}}$$

$$= \frac{1 + \lim_{x \to -\infty} \frac{7}{x}}{-\sqrt{2 + \lim_{x \to -\infty} \frac{1}{x^2}}} = \frac{1 + 0}{-\sqrt{2+0}}$$

$$= \frac{1}{-\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$f(x) = \frac{2x + |x|}{5x - 3|x|}$$

$$\underbrace{\lim_{x \to +\infty}}_{x > 0} f(x) \quad e \quad \underbrace{\lim_{x \to -\infty}}_{x < 0} f(x)$$

$$\lim_{x \to +\infty} \frac{2x + x}{5x - 3x} \quad \Bigg| \quad \lim_{x \to -\infty} \frac{2x - x}{5x + 3x}$$

$$\lim_{x \to +\infty} \frac{3\cancel{x}}{2\cancel{x}}$$

$$\lim_{x \to +\infty} \frac{3}{2}$$

$$= \frac{3}{2}$$

$$\lim_{x \to -\infty} \frac{\cancel{x}}{8\cancel{x}}$$

$$= \lim_{x \to -\infty} \frac{1}{8}$$

$$= \frac{1}{8}$$