Limites Fundamentais

$$\lim_{t \to 0} \frac{sen(t)}{t} = 1$$

$$\lim_{x \to 0} \frac{sen(5x)}{x} = \lim_{u \to 0} \frac{sen(u)}{\frac{u}{5}}$$

$$\frac{sen\, u}{1} \cdot \frac{5}{u}$$

$u = 5x \Rightarrow x \to 0 \Rightarrow u \to 0$

$\hookrightarrow \quad x = \frac{u}{5}$

$$= \lim_{u \to 0} 5 \cdot \frac{sen(u)}{u}$$

$$= 5 \cdot \lim_{u \to 0} \frac{sen(u)}{u} = 5 \cdot 1 = 5$$

$$\lim_{t\to 0} \frac{\sin(2t)}{\sin(7t)} = \lim_{t\to 0} \frac{\frac{\sin(2t)}{2t}\cdot 2t}{\frac{\sin(7t)}{7t}\cdot 7t}$$

$$= \lim_{t\to 0} \frac{2\cancel{t}}{7\cancel{t}} \cdot \left[\frac{\frac{\sin(2t)}{2t}}{\frac{\sin(7t)}{7t}}\right]$$

$$= \frac{2}{7} \cdot \left[\frac{\lim_{t\to 0} \frac{\sin(2t)}{2t}}{\lim_{t\to 0} \frac{\sin(7t)}{7t}}\right]$$

$u = 2t \Rightarrow t \to 0 \quad u \to 0$

$v = 7t \Rightarrow t \to 0 \quad v \to 0$

$$= \frac{2}{7} \left[\frac{\lim_{u\to 0} \frac{\sin(u)}{u}}{\lim_{v\to 0} \frac{\sin(v)}{v}}\right] = \frac{2}{7} \cdot \frac{1}{1} = \frac{2}{7}$$

$$\lim_{x \to 0} \frac{\tan(x)}{x} = \lim_{x \to 0} \frac{\frac{\sin(x)}{\cos(x)}}{x}$$

$$= \lim_{x \to 0} \left[\frac{1}{\cos(x)} \cdot \frac{\sin(x)}{x} \right]$$

$$= \lim_{x \to 0} \frac{1}{\cos(x)} \cdot \lim_{x \to 0} \frac{\sin(x)}{x}$$

$$= \frac{1}{\cos(0)} \cdot 1 = \frac{1}{1} \cdot 1 = 1$$

$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to -\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to 0^+} (1 + kx)^{\frac{1}{x}} \qquad k > 0$$

$$kx = \frac{1}{t} \Leftrightarrow kt = \frac{1}{x} \Leftrightarrow t = \frac{1}{kx} \qquad k > 0$$

$$x \to 0^+ \Rightarrow \frac{1}{kx} \to +\infty \Rightarrow t \to +\infty$$

$$\lim_{t \to +\infty} \left(1 + \frac{1}{t}\right)^{kt}$$

$$= \lim_{t \to +\infty} \left[\left(1 + \frac{1}{t}\right)^{t}\right]^{k} = \left[\underbrace{\lim_{t \to +\infty} \left(1 + \frac{1}{t}\right)^{t}}\right]^{k}$$

$$= e^{k}$$

$$\lim_{x \to +\infty} \left(1 + \frac{5}{x-1}\right)^{x+7}$$

$$\frac{1}{u} = \frac{5}{x-1} \iff x - 1 = 5u$$

$$\iff x = 5u + 1$$

$$x \to +\infty \implies u \to +\infty$$

$$\lim_{u \to +\infty} \left(1 + \frac{1}{u}\right)^{5u+1+7}$$

$$= \lim_{u \to +\infty} \left(1 + \frac{1}{u}\right)^{5u+8}$$

$$= \lim_{u \to +\infty} \left[\left(1 + \frac{1}{u}\right)^{5u} \cdot \left(1 + \frac{1}{u}\right)^{8}\right]$$

$$= \lim_{u \to +\infty} \left[\left(1 + \frac{1}{u}\right)^u \right]^5 \cdot \lim_{u \to +\infty} \left(1 + \frac{1}{u}\right)^8$$

$$= \left[\lim_{u \to +\infty} \left(1 + \frac{1}{u}\right)^u \right]^5 \cdot \left(\lim_{u \to +\infty} \left(1 + \frac{1}{u}\right) \right)^8$$

$$= e^5 \cdot (1 + 0)^8 = e^5.$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln(a)$$

$(0 < a \neq 1)$

$$\lim_{x \to 1} \frac{e^{x-1} - a^{x-1}}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{e^{x-1} - a^{x-1} + 1 - 1}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{(e^{x-1} - 1) - (a^{x-1} - 1)}{(x+1) \cdot (x-1)}$$

$$= \lim_{x \to 1} \left[\frac{1}{(x+1)} \cdot \left(\frac{e^{x-1} - 1 - (a^{x-1} - 1)}{(x-1)} \right) \right]$$

$$= \underbrace{\lim_{x \to 1} \frac{1}{x+1}}_{} \cdot \lim_{x \to 1} \left(\frac{e^{x-1} - 1}{(x-1)} - \frac{(a^{x-1} - 1)}{(x-1)} \right)$$

$u = x - 1 \Rightarrow x \to 1 \Rightarrow u \to 0$

$$= \frac{1}{1+1} \cdot \left[\lim_{u \to 0} \frac{e^{u} - 1}{u} - \lim_{u \to 0} \frac{a^{u} - 1}{u} \right]$$

$$= \frac{1}{2} \cdot \left[\ln(e) - \ln(a) \right]$$

$$= \frac{1}{2} \left[1 - \ln(a) \right]$$

$$\lim_{u \to 0} \frac{a^{u} - b^{u}}{u}$$

$a, b > 0$
$a \neq b \neq 1$

$$\lim_{t \to 0} \frac{k^{t} - 1}{t} = \ln(k)$$

$k > 0$
$k \neq 1$

$$\lim_{u \to 0} \frac{b^{u}\left(\frac{a^{u}}{b^{u}} - 1\right)}{u}$$

$$= \lim_{u \to 0} \left[b^{u} \cdot \frac{\left(\frac{a}{b}\right)^{u} - 1}{u} \right] \qquad k = \frac{a}{b}$$

$$= \lim_{u \to 0} b^{u} \cdot \lim_{u \to 0} \frac{\left(\frac{a}{b}\right)^{u} - 1}{u}$$

$$= b^{0} \cdot \ln\left(\frac{a}{b}\right) = 1 \cdot \ln\left(\frac{a}{b}\right)$$

$$\lim_{x \to -\infty} \frac{\sin(x)}{x^5}$$

$$x \to -\infty$$

$$-1 \leq \sin(x) \leq 1 \qquad \frac{1}{x^5} < 0$$

$$-\frac{1}{x^5} \geq \frac{\sin(x)}{x^5} \geq \frac{1}{x^5}$$

$$\lim_{x \to -\infty} -\frac{1}{x^5} = 0 = \lim_{x \to -\infty} \frac{1}{x^5}$$

$$\Rightarrow \lim_{x \to -\infty} \frac{\sin(x)}{x^5} = 0$$

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g) $$\frac{\left(1 - \cos^4(x)\right)}{x^2} = \frac{\left(1 + \cos^2(x)\right)\left(1 - \cos^2(x)\right)}{x^2}$$

$$= \left(1+\cos^2(x)\right) \cdot \left(\frac{1-\cos^2(x)}{x^2}\right)$$