Determine o conjunto solução da inequação $\frac{ax-2}{5} < \frac{7}{2}$, com $a<0$.

Solução:

$$\frac{ax-2}{5} < \frac{7}{2}$$

$$ax-2 < \frac{35}{2}$$

$$ax < \frac{35}{2} + \frac{2}{1}$$

$$ax < \frac{35+4}{2} = \frac{39}{2}$$

Como $a<0$ então

$$x > \frac{39}{2a}$$

$$S = \{x \in \mathbb{R} \mid x > \frac{39}{2a}\}$$

$$S = \left(\frac{39}{2a}, +\infty\right)$$

Mostrar que f e g são uma inversa da outra, $f(g(x)) = x$ e $g(f(x)) = x$.

b) $f(x) = 16 - x^2$, $x > 0$

$g(x) = \sqrt{16 - x}$, $x < 16 \Rightarrow 0 < 16 - x$

$D(f) = (0, +\infty)$ e $D(g) = (-\infty, 16)$

$D(f) \cap D(g) = (0, 16)$

$$f(g(x)) = f\left(\sqrt{16-x}\right)$$

$$= 16 - \left(\sqrt{16-x}\right)^2$$

$$= 16 - |16 - x| = 16 - (16 - x)$$

$$= \cancel{16} - \cancel{16} + x = x$$

$f(x) = 16 - x^2$

$f(w) = 16 - w^2$

$f(☺) = 16 - (☺)^2$

$$g(f(x)) = g\left(16 - x^2\right)$$

$$= \sqrt{16 - (16 - x^2)}$$

$$= \sqrt{\cancel{16} - \cancel{16} + x^2} = \sqrt{x^2} = |x| = x$$

$g(x) = \sqrt{16 - x}$

$g(w) = \sqrt{16 - w}$

$g(☺) = \sqrt{16 - (☺)}$

pois $x \in (0, 16)$, ou seja, $x > 0$

Se $f(x) = x^2 - 2x + 1$, encontre a função $g(x)$ tal que $f \cdot g(x) = x - 1$.

$$f \cdot g(x) = f(x) \cdot g(x) = (x^2 - 2x + 1) \cdot g(x)$$

$$(x^2 - 2x + 1) \cdot g(x) = x - 1$$

$$(x-1)^2 \cdot g(x) = x - 1$$

$x \neq 1$

$$g(x) = \frac{(x-1)^1}{(x-1)^2} = \frac{1}{x-1}$$

$$\to \frac{1}{(x-1)^2 \cdot (x-1)^{-1}} = \frac{1}{(x-1)^{2-1}} = \frac{1}{x-1}$$

$$f(x) = \begin{cases} 3x + 1, & \text{se } x \geq 0 \\ 2x - 1 & \text{se } x < 0 \end{cases}$$

$D(f) = \mathbb{R}$

$$g(x) = |4x - 5|$$

$D(g) = \mathbb{R}$

$$g(x) = \begin{cases} 4x - 5 & \text{se } 4x - 5 \geq 0 \\ -(4x - 5) & \text{se } 4x - 5 < 0 \end{cases}$$

$$g(x) = \begin{cases} 4x - 5 & \text{se } 4x \geq 5 \\ -4x + 5 & \text{se } 4x < 5 \end{cases}$$

$$g(x) = \begin{cases} 4x-5 & \text{se } x \geq \frac{5}{4} \\ -4x+5 & \text{se } x < \frac{5}{4} \end{cases}$$

$$f(x) = \begin{cases} 3x+1 & \text{se } x \geq 0 \\ 2x-1 & \text{se } x < 0 \end{cases}$$

Solução a)

$$(f+g)(x) = f(x)+g(x) = \begin{cases} \overbrace{2x-1}^{f(x)} + \overbrace{-4x+5}^{g(x)} & \text{se } x<0 \\ \overbrace{3x+1}^{f(x)} + \overbrace{-4x+5}^{g(x)} & \text{se } 0 \leq x < \frac{5}{4} \\ \overbrace{3x+1}^{f(x)} + \overbrace{4x-5}^{g(x)} & \text{se } x \geq \frac{5}{4} \end{cases}$$

$$(f+g)(x) = \begin{cases} -2x+4 & \text{se } x<0 \\ -x+6 & \text{se } 0 \leq x < \frac{5}{4} \\ 7x-4 & \text{se } x \geq \frac{5}{4} \end{cases}$$

$$f(x) = \begin{cases} 3x+1 & \text{se } x \geq 0 \\ 2x-1 & \text{se } x < 0 \end{cases}$$

$$g(x) = \begin{cases} 4x-5 & \text{se } x \geq \frac{5}{4} \\ -4x+5 & \text{se } x < \frac{5}{4} \end{cases}$$

$$\frac{f}{g}(x) = \begin{cases} \dfrac{2x-1}{-4x+5} & \text{se } x<0 \\ \dfrac{3x+1}{-4x+5} & \text{se } 0 \leq x < \frac{5}{4} \\ \dfrac{3x+1}{4x-5} & \text{se } x > \frac{5}{4} \end{cases}$$

Cuidado !!! $\Longrightarrow$ D

① $g(x) \neq 0$

$|4x-5| \neq |0|$

$|a| = |b|$

$a = b$ ou $a = -b$

$4x - 5 \neq 0$ ou $4x - 5 \neq -0$

$x \neq \frac{5}{4}$ ou $x \neq \frac{5}{4}$

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② $x = \frac{5}{4}$, $g\left(\frac{5}{4}\right) = 4 \cdot \frac{5}{4} - 5 = 5 - 5 = 0$

$$D\left(\frac{f}{g}\right) = \mathbb{R} - \left\{\frac{5}{4}\right\}$$

$$f(x) = 3x - 2 \quad \| \quad g(x) = |x|$$

$(f+g)(x) = f(x) + g(x) = 3x - 2 + |x|$

$(f-g)(x) = f(x) - g(x) = 3x - 2 - |x|$

$(f \cdot g)(x) = f(x) \cdot g(x) = (3x-2)|x|$ $\leftarrow$ ok

$= 3x|x| - 2|x|$ $\rightarrow$ encher linguiça

$\frac{f}{g}(x) = \frac{3x-2}{|x|}$ $\quad x \neq 0$

$f \circ g(x) = f(g(x)) = f(|x|) = 3|x| - 2$