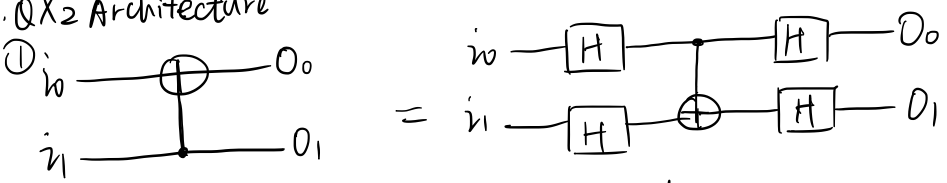
Optimization of Circuits for IBM's five qubit Quantum Computer

A. QX2 Architecture

①



left side:

$$O_0 = \begin{cases} i_0 & \text{if } i_1 = |0\rangle \\ \bar{i_0} & \text{if } i_1 = |1\rangle \end{cases}$$

$$O_1 = i_1$$

right side: (4 cases ①~④)

① if $i_0 = i_1 = |0\rangle$; $O_0 = O_1 = 0$

$$\left.\begin{array}{l} i_0 \to H \to \frac{|0\rangle+|1\rangle}{\sqrt{2}} \\ i_1 \to H \to \frac{|0\rangle+|1\rangle}{\sqrt{2}} \end{array}\right\} \to CNOT \to \left\{\begin{array}{l} \frac{|0\rangle+|1\rangle}{\sqrt{2}} \to H \to |0\rangle \\ \frac{|0\rangle+|1\rangle}{\sqrt{2}} \to H \to |0\rangle \end{array}\right.$$

$\Downarrow$

$$\left[\begin{array}{l} CNOT\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \\ = CNOT\left(\frac{1}{2}\left(|00\rangle+|01\rangle+|10\rangle+|11\rangle\right)\right) \\ = \frac{1}{2}\left(|00\rangle+|01\rangle+|11\rangle+|10\rangle\right) \\ = \frac{|0\rangle+|1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle+|1\rangle}{\sqrt{2}} \end{array}\right]$$

② if $i_0 = 0, i_1 = 1$; $O_0 = 1, O_1 = 1$

$$\left.\begin{array}{l} i_0 \to H \to \frac{|0\rangle+|1\rangle}{\sqrt{2}} \\ i_1 \to H \to \frac{|0\rangle-|1\rangle}{\sqrt{2}} \end{array}\right\} \to CNOT \to \left\{\begin{array}{l} \frac{|0\rangle-|1\rangle}{\sqrt{2}} \to H \to |1\rangle \\ \frac{|0\rangle-|1\rangle}{\sqrt{2}} \to H \to |1\rangle \end{array}\right.$$

$\downarrow$

$$\left[\begin{array}{l} CNOT\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) \\ = CNOT\left(\frac{1}{2}\left(|00\rangle-|01\rangle+|10\rangle-|11\rangle\right)\right) \\ = CNOT\left(\frac{1}{2}\left(|00\rangle-|01\rangle+|11\rangle-|10\rangle\right)\right) \\ = \frac{|0\rangle-|1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle-|1\rangle}{\sqrt{2}} \end{array}\right]$$

③ if $i_0 = 1, i_1 = 0$; $O_0 = 1, O_1 = 0$

$$\left.\begin{array}{l} i_0 \to H \to \frac{|0\rangle-|1\rangle}{\sqrt{2}} \\ i_1 \to H \to \frac{|0\rangle+|1\rangle}{\sqrt{2}} \end{array}\right\} \to CNOT \to \left\{\begin{array}{l} \frac{|0\rangle-|1\rangle}{\sqrt{2}} \to H \to |1\rangle \\ \frac{|0\rangle+|1\rangle}{\sqrt{2}} \to H \to |0\rangle \end{array}\right.$$

$\downarrow$

$$\left[\begin{array}{l} CNOT\left(\frac{|00\rangle-|10\rangle+|01\rangle-|11\rangle}{2}\right) \\ = \frac{|00\rangle-|11\rangle+|01\rangle-|10\rangle}{2} = \frac{|0\rangle-|1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle+|1\rangle}{\sqrt{2}} \end{array}\right]$$

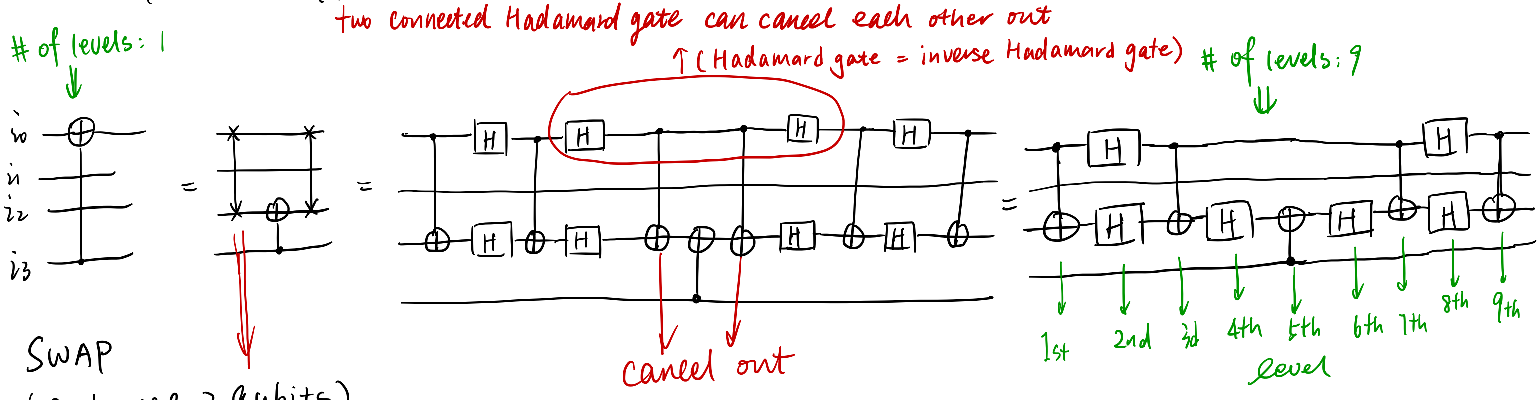
④ if $i_0 = 1, i_1 = 1$; $O_0 = 0, O_1 = 1$

$$\left.\begin{array}{l} i_0 \to H \to \frac{|0\rangle-|1\rangle}{\sqrt{2}} \\ i_1 \to H \to \frac{|0\rangle-|1\rangle}{\sqrt{2}} \end{array}\right\} \to CNOT \to \left\{\begin{array}{l} \frac{|0\rangle+|1\rangle}{\sqrt{2}} \to H \to |0\rangle \\ \frac{|0\rangle-|1\rangle}{\sqrt{2}} \to H \to |1\rangle \end{array}\right.$$

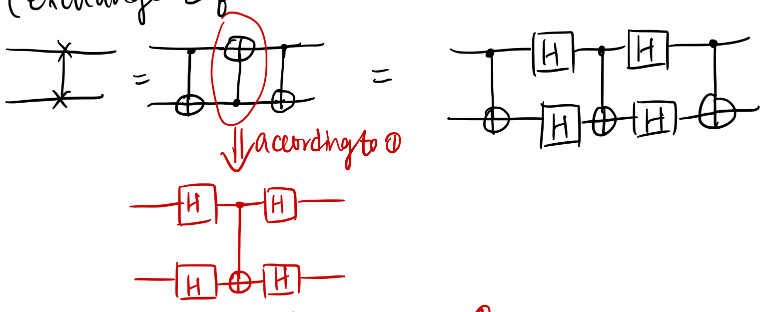
$\Downarrow$

$$\left[\begin{array}{l} CNOT\left(\frac{1}{2}\left(|00\rangle-|01\rangle-|10\rangle+|11\rangle\right)\right) = \frac{1}{2}\left(|00\rangle-|01\rangle-|11\rangle+|10\rangle\right) \\ = \frac{|0\rangle+|1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle-|1\rangle}{\sqrt{2}} \end{array}\right]$$

② SWAP Gate Approach:



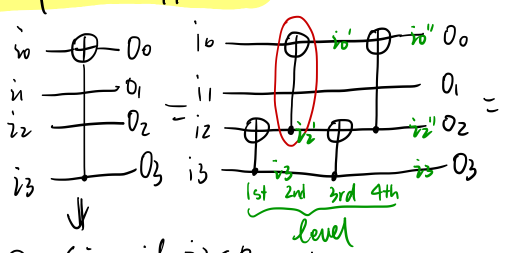
SWAP
(exchange 2 qubits)

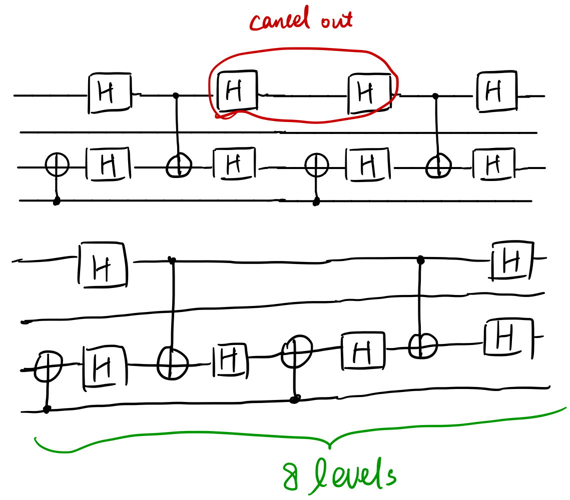


∴ the # of levels increases by at most 8.

Template Approach:

according to ①





∴ the # of levels increase by at most 7.

$O_0 = \begin{cases} i_0 & \text{if } i_3 = 0 \\ \bar{i_0} & \text{if } i_3 = 1 \end{cases}$

$O_3 = i_3$

After 1st level, $i_2' = \begin{cases} i_2 & \text{if } i_3=0 \\ \bar{i_2} & \text{if } i_3=1 \end{cases}$

After 2nd level, $i_0' = \begin{cases} i_0 & \text{if } i_2'=0 \\ \bar{i_0} & \text{if } i_2'=1 \end{cases} = \begin{cases} i_0 & \text{if } i_3=0, i_2=0, \text{ or } i_3=1, i_2=1 \\ \bar{i_0} & \text{if } i_3=0, i_2=1, \text{ or } i_3=1, i_2=0 \end{cases}$

After 3rd level, $i_2'' = \begin{cases} i_2' & \text{if } i_3=0 \\ \overline{i_2'} & \text{if } i_3=1 \end{cases} = i_2$

After 4th level, $i_0'' = \begin{cases} i_0' & \text{if } i_2=0 \\ \overline{i_0'} & \text{if } i_2=1 \end{cases} = \begin{cases} i_0 & \text{if } i_3=0, i_2=0 \\ \bar{i_0} & \text{if } i_3=1, i_2=1 \\ i_0 & \text{if } i_3=0, i_2=1 \\ \bar{i_0} & \text{if } i_3=1, i_2=0 \end{cases} = \begin{cases} i_0 & \text{if } i_3=0 \\ \bar{i_0} & \text{if } i_3=1 \end{cases}$

$\therefore O_0 = i_0'' = \begin{cases} i_0 & \text{if } i_3=0 \\ \bar{i_0} & \text{if } i_3=1 \end{cases}$

$O_3 = i_3$

B. QX4 Architecture : Similar to A