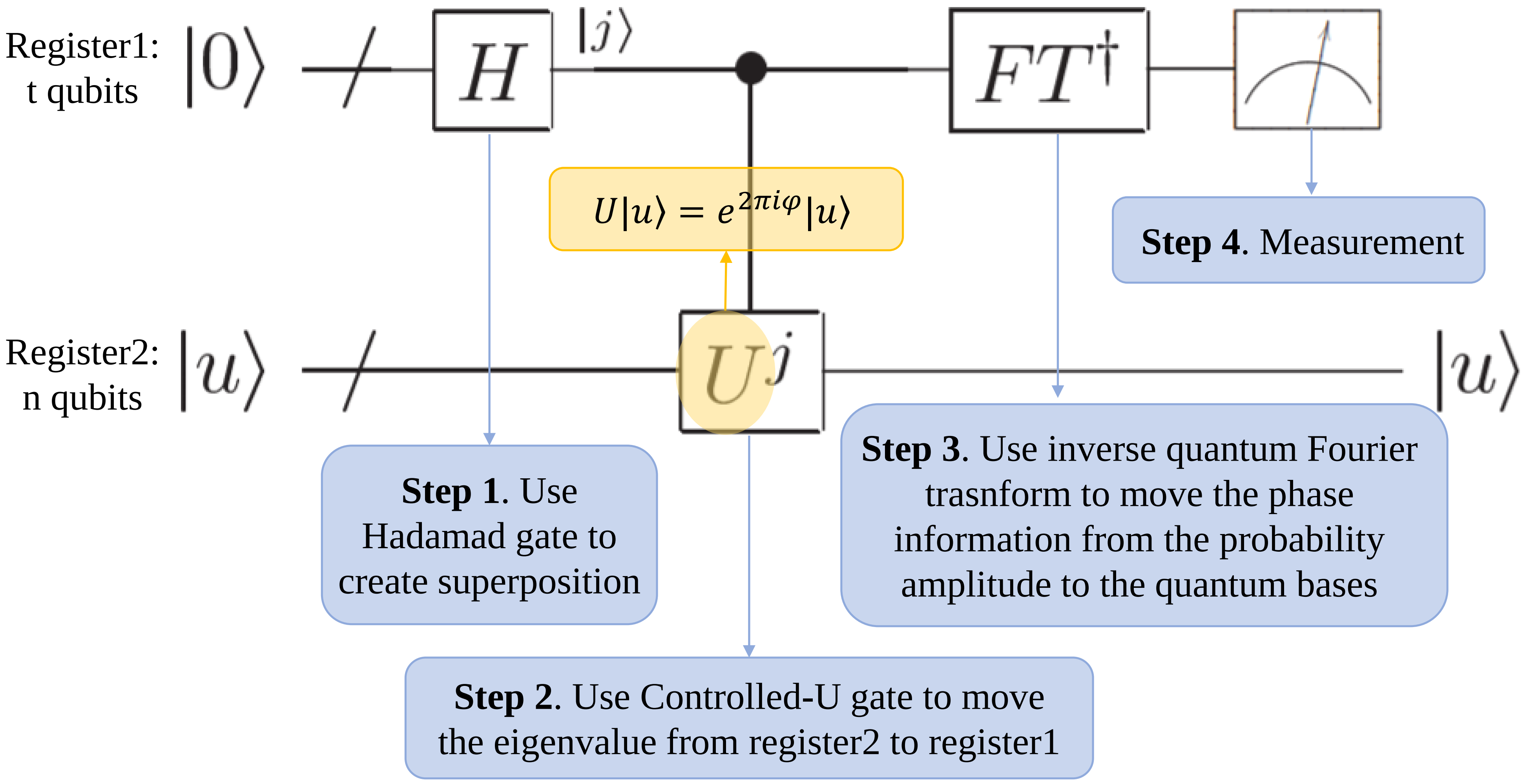
Phase estimation quantum circuit

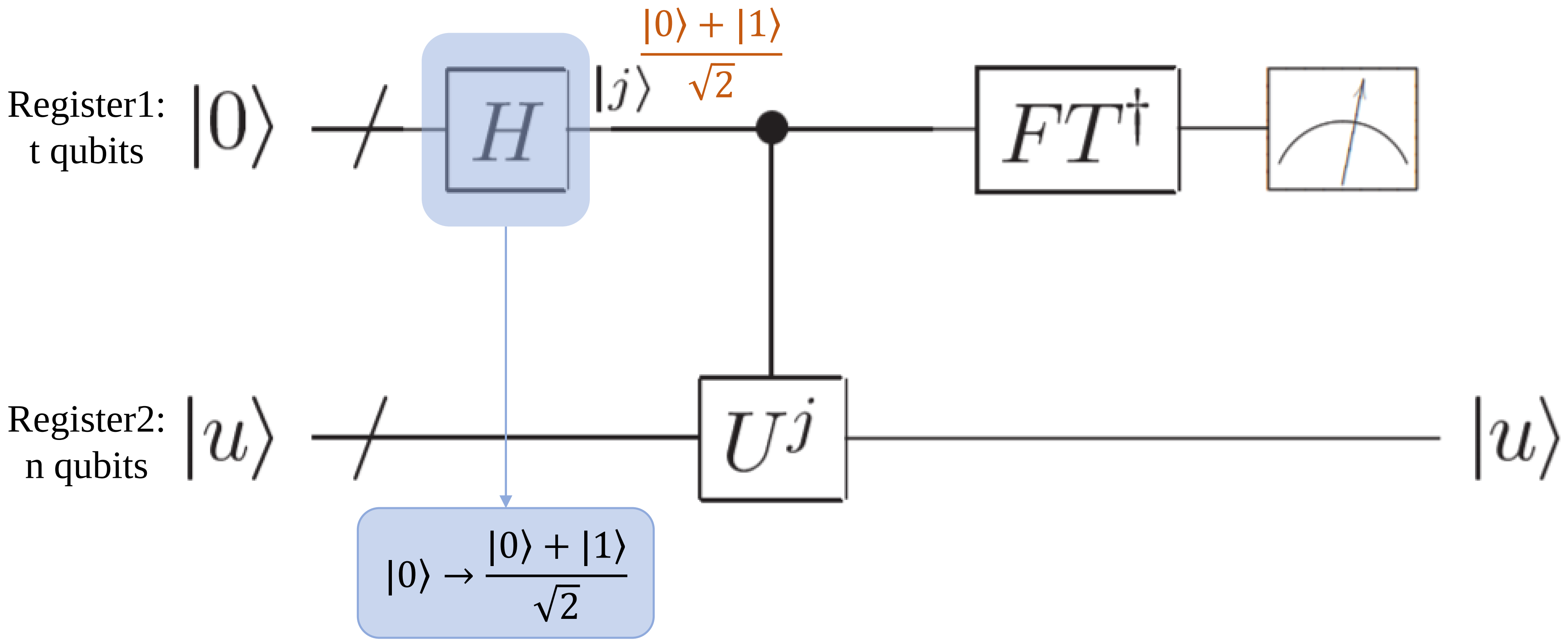
$|u\rangle$ is the eigenvector of U, which is know.
$e^{2\pi i\varphi}$ is the eigenvalue of U, with the phase φ unknown.



Reference & picture: Michael A Nielsen and Isaac Chuang. Quantum computation and quantum information, 2002.

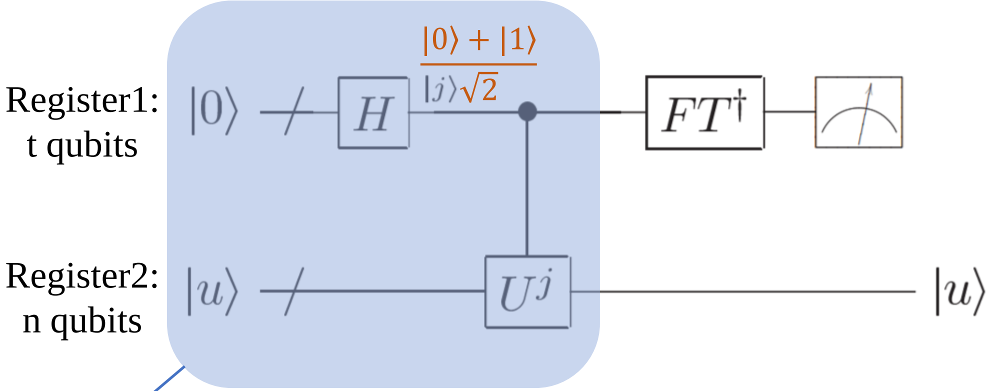
The procedure for quantum phase estimation

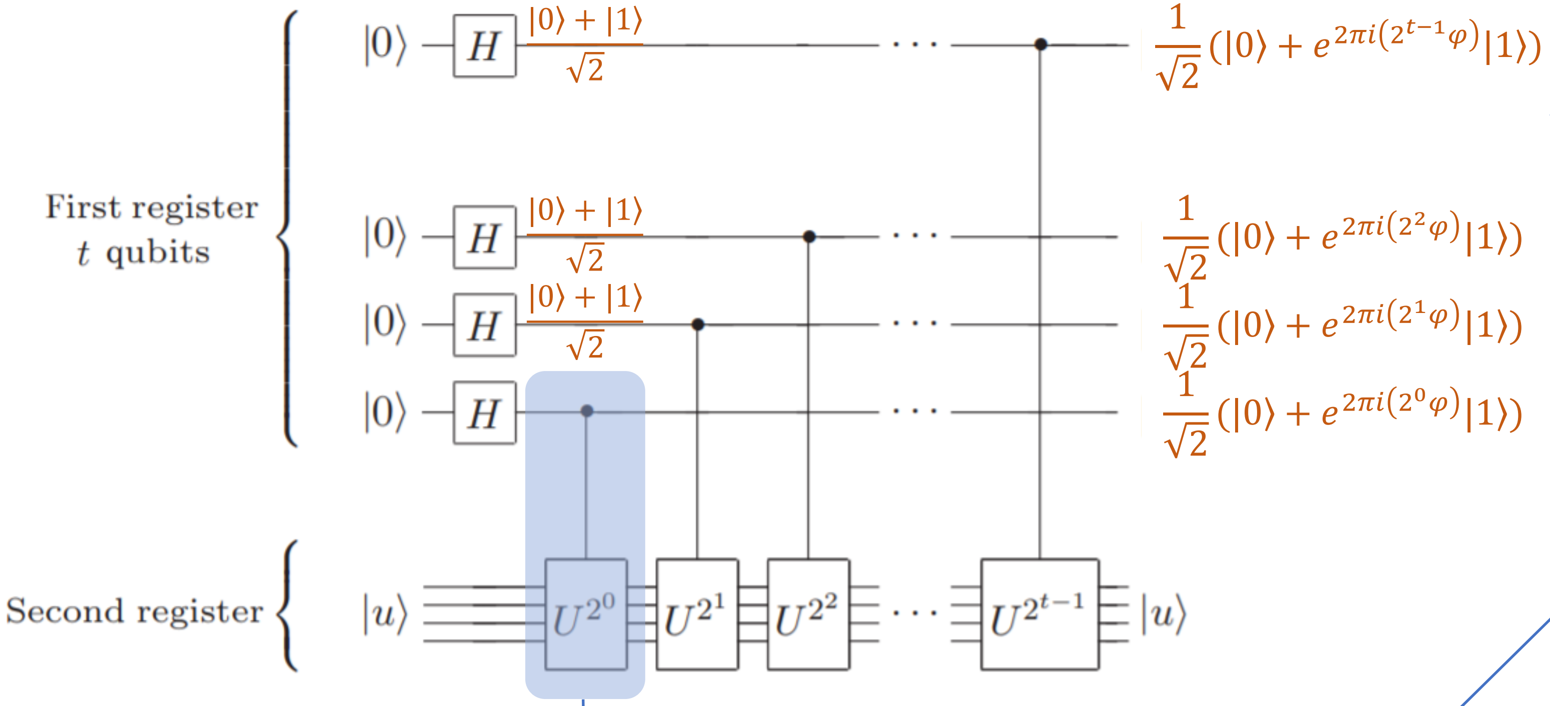
Step 1. Use Hadamad gate to create superposition



The procedure for quantum phase estimation

Step 2. Use Controlled-U gate to move the eigenvalue from register2 to register1





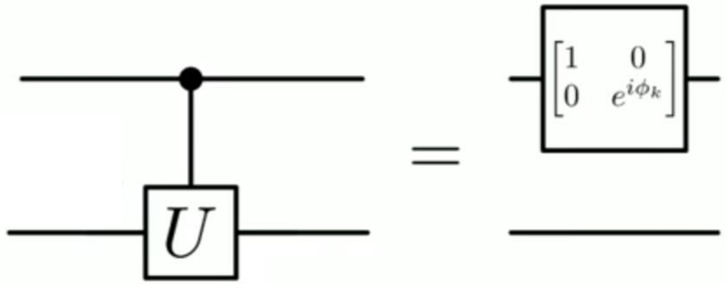
$$\frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes|u\rangle \xrightarrow{C-U} \frac{1}{\sqrt{2}}(|0\rangle\otimes|u\rangle+|1\rangle\otimes \underbrace{U|u\rangle}_{=e^{2\pi i\varphi}|u\rangle})$$

$$=\frac{1}{\sqrt{2}}(|0\rangle\otimes|u\rangle+e^{2\pi i\varphi}|1\rangle\otimes|u\rangle)$$

∴ The target qubit $|u\rangle$ does not change, but the control qubit $|1\rangle$ changes, which is contrary to the ordinary situation.

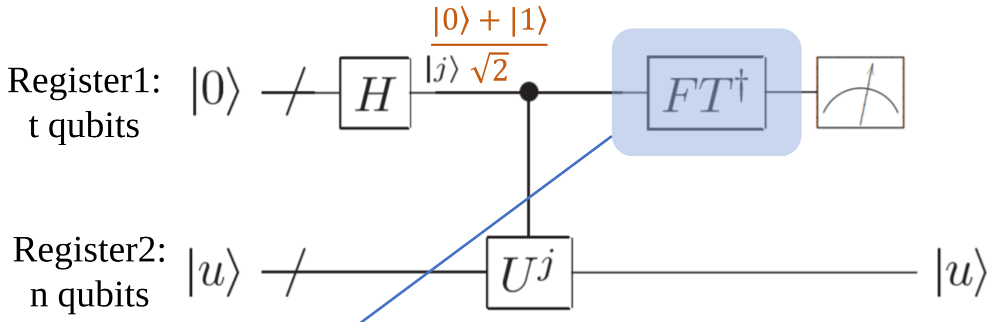
∴ We move the eigenvalue $e^{2\pi i\varphi}$ from the $|u\rangle$ space to the $|1\rangle$ space.

∴ U^{2^n} denotes by the controlled-U gate, which can be implemented by:

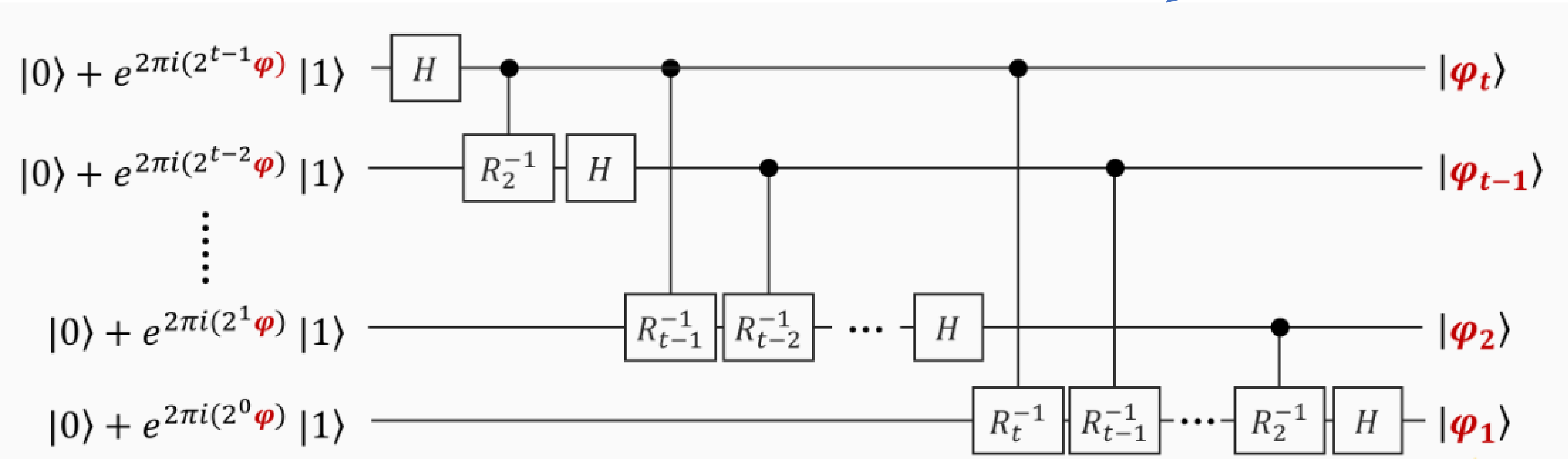


PS. The gate on the right-hand side is the controlled phase gate, which is specified in **Step.3**.

The procedure for quantum phase estimation



Step 3. Use inverse quantum Fourier trasnform to move the phase information from the probability amplitude to the quantum bases



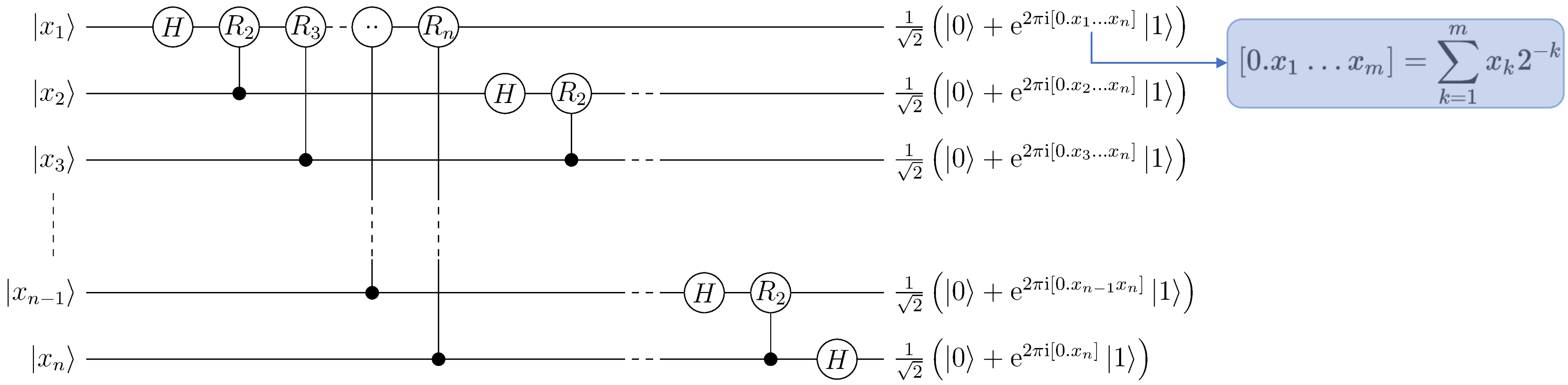
For inverse quantum Fourier trnasform ($|j\rangle \longrightarrow \frac{1}{2^{\frac{n}{2}}}\sum_{k=0}^{2^n-1} e^{-2\pi i jk/2^n}|k\rangle$),

the gate ${R_k}^{-1}$denotes the controlled phase gate ${R_k}^{-1} \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{2\pi i}{k}} \end{bmatrix}$.

Picture: https://www.qtumist.com/info/PE-H5/index.html

Introduction to quantum Fourier transform

This slide is not part of the quantum phase estimation algorithm, but just to compare iQFT and QFT.



Quantum Fourier transform circuit

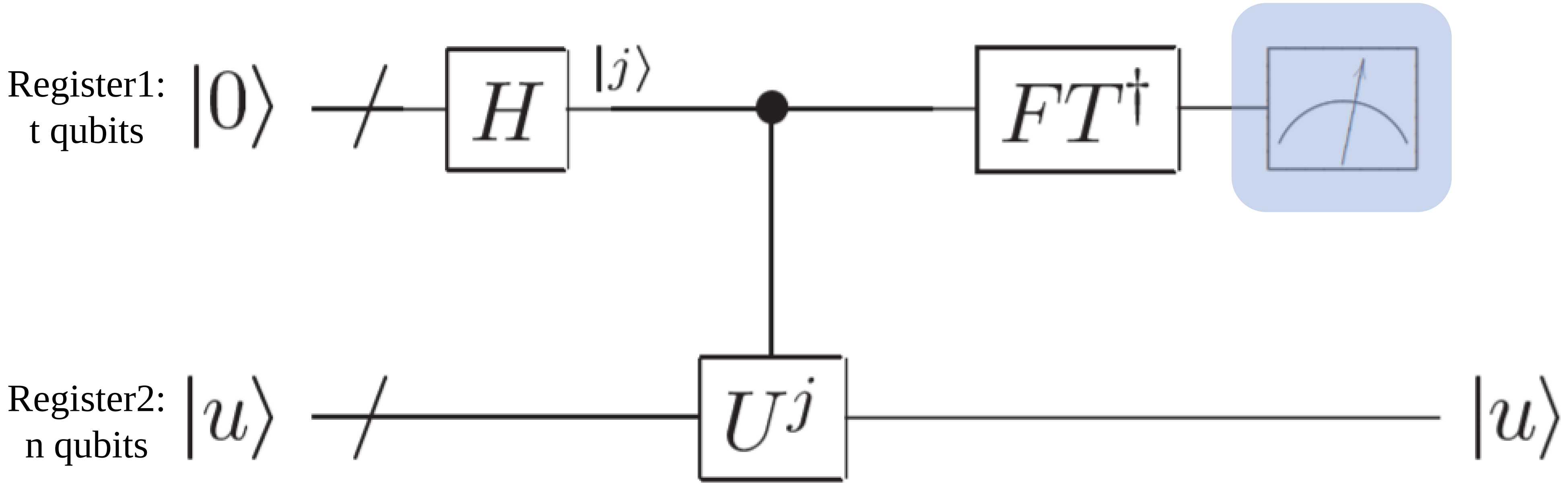
PS. For quantum Fourier transform ($|j\rangle \longrightarrow \frac{1}{2^{\frac{n}{2}}} \sum_{k=0}^{2^n-1} e^{2\pi i jk/2^n} |k\rangle$),

the gate R_k denotes the controlled phase gate $R_k \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{k}} \end{bmatrix}$;

Picture: https://www.wikiwand.com/en/Quantum_Fourier_transform

The procedure for quantum phase estimation

Step 4. Measurement
If U is an exact binary fraction, we can measure its phase with probability 1;
Otherwise, we can measure it with a very high probability close to 1.



After divide the measuring result of register1 (binary string $\psi_1\psi_2\cdots\psi_t$) by 2^t, we can get the phase $\varphi = 0.\psi_1\psi_2\cdots\psi_t$.
e.g. Our result for measuring the register1 is 0001 (t=4). After divided by 2^4, we get the answer 0.0001, which in decimal is 1/16. Thus the phase is 1/16.