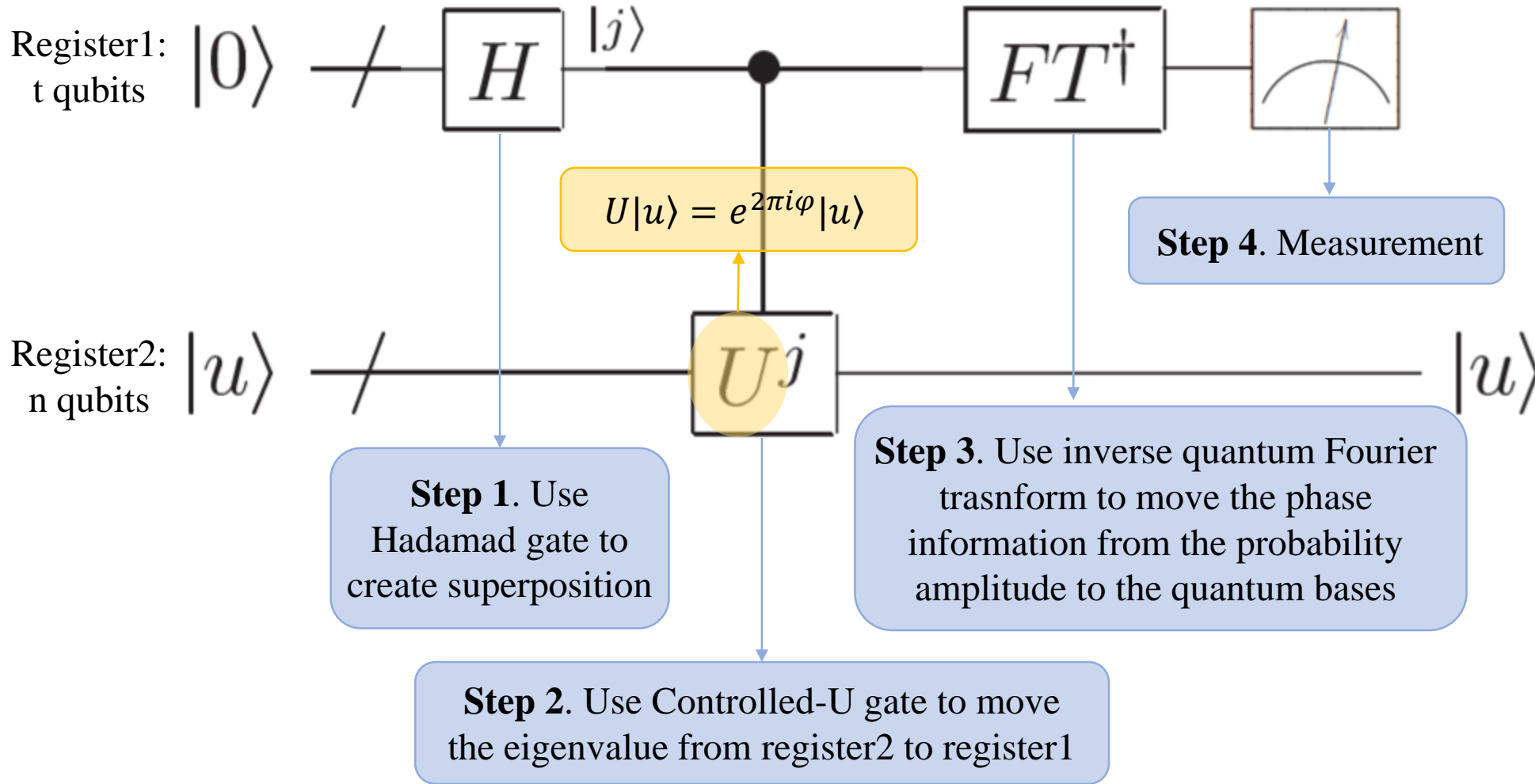


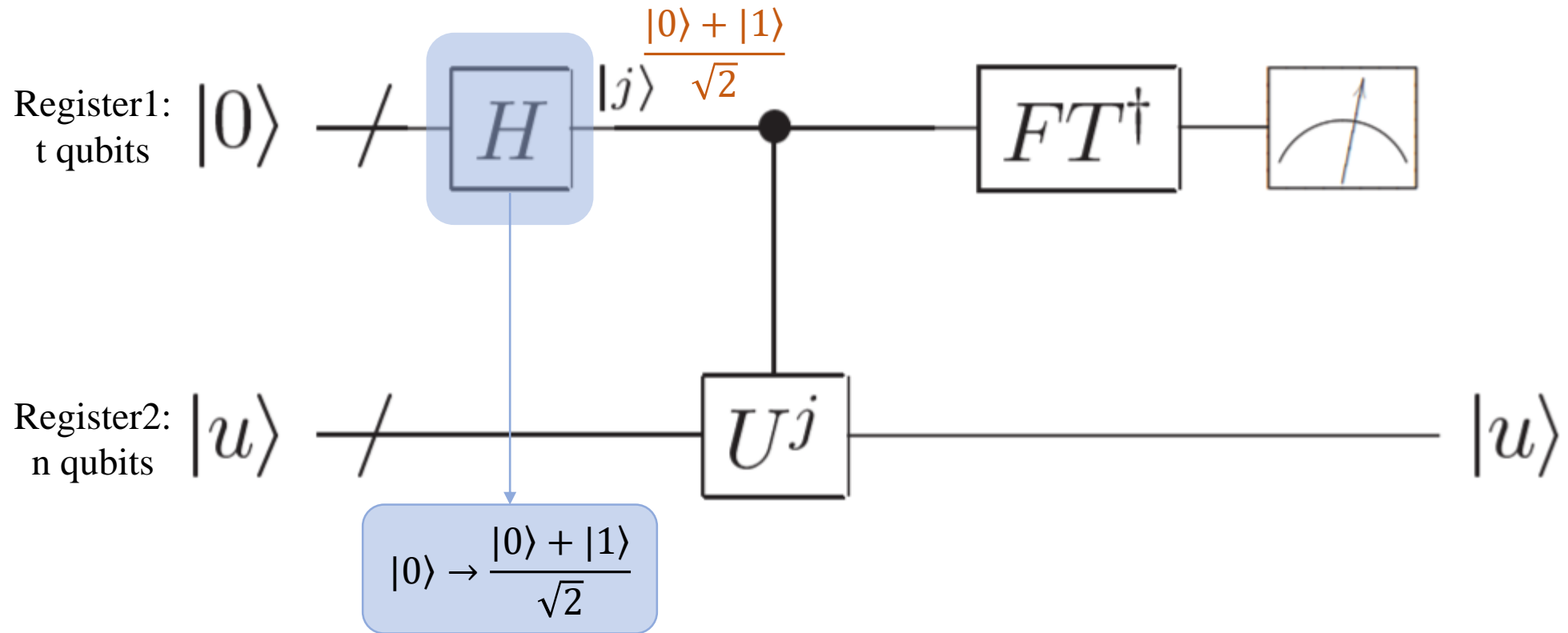
# Phase estimation quantum circuit

$|u\rangle$  is the eigenvector of  $U$ , which is known.  
 $e^{2\pi i\varphi}$  is the eigenvalue of  $U$ , with the phase  $\varphi$  unknown.



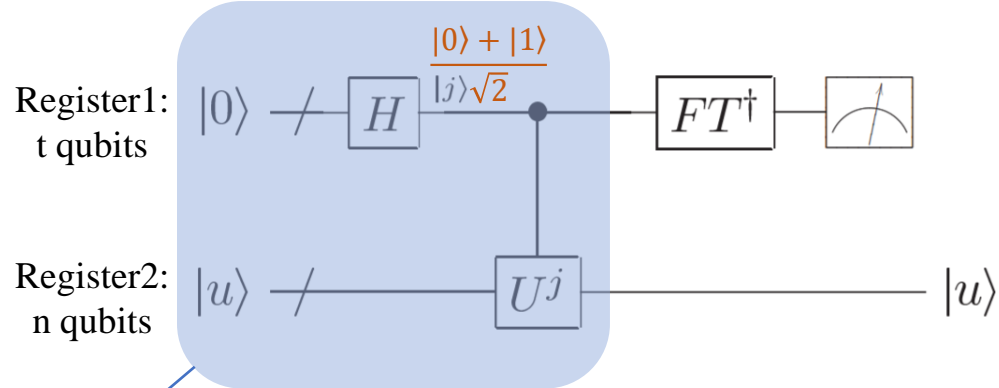
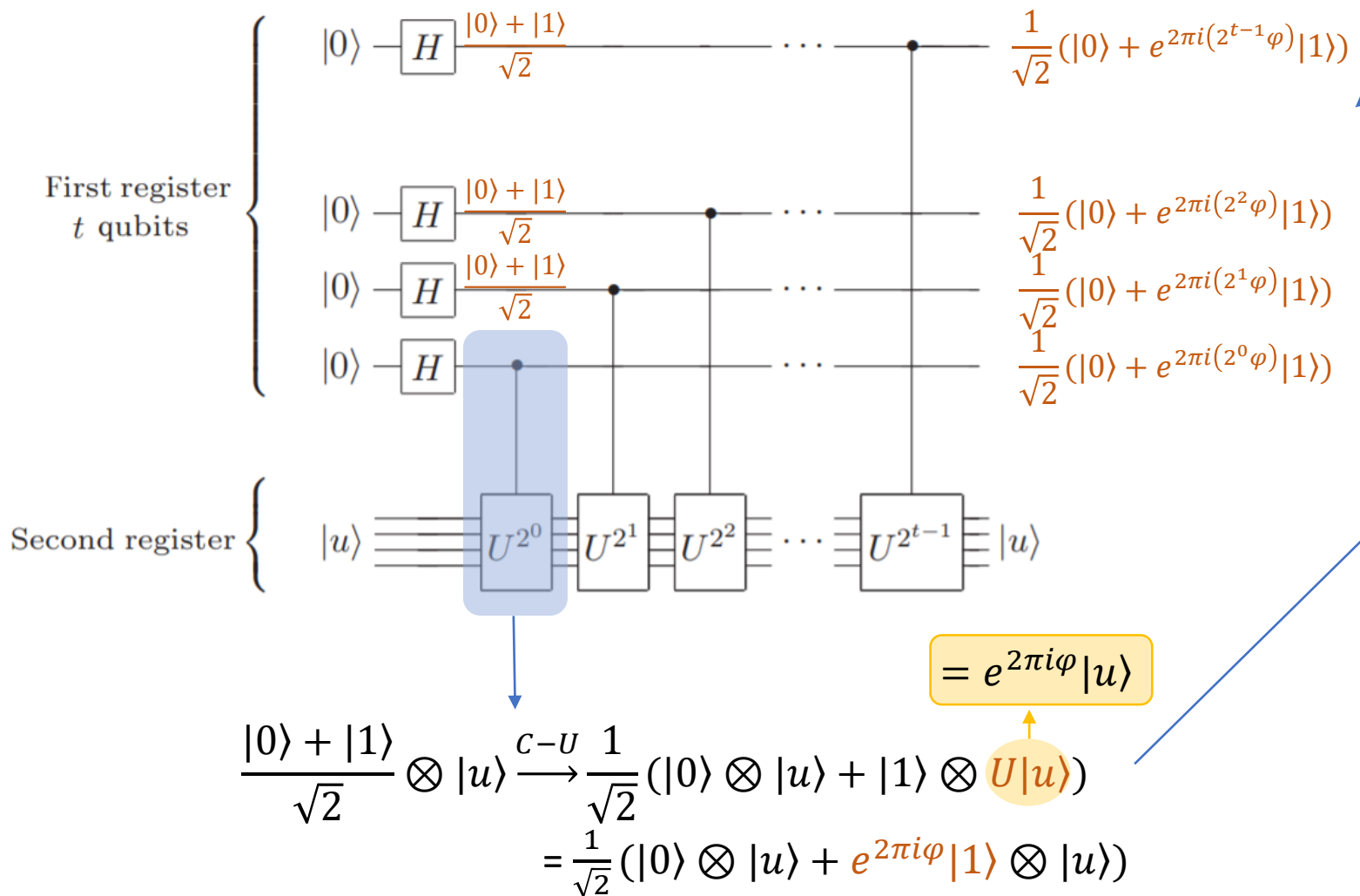
# The procedure for quantum phase estimation

**Step 1.** Use Hadamard gate to create superposition



# The procedure for quantum phase estimation

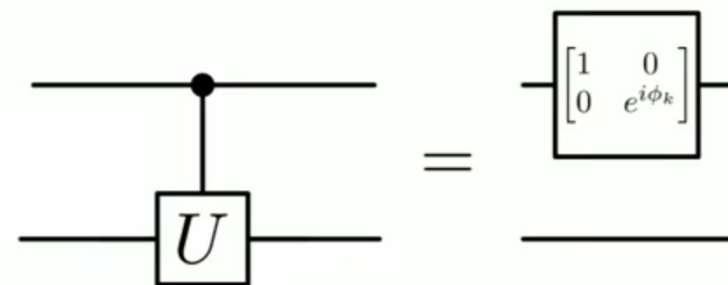
**Step 2.** Use Controlled-U gate to move the eigenvalue from register2 to register1



$\therefore$  The target qubit  $|u\rangle$  does not change, but the control qubit  $|1\rangle$  changes, which is contrary to the ordinary situation.

$\therefore$  We move the eigenvalue  $e^{2\pi i \phi}$  from the  $|u\rangle$  space to the  $|1\rangle$  space.

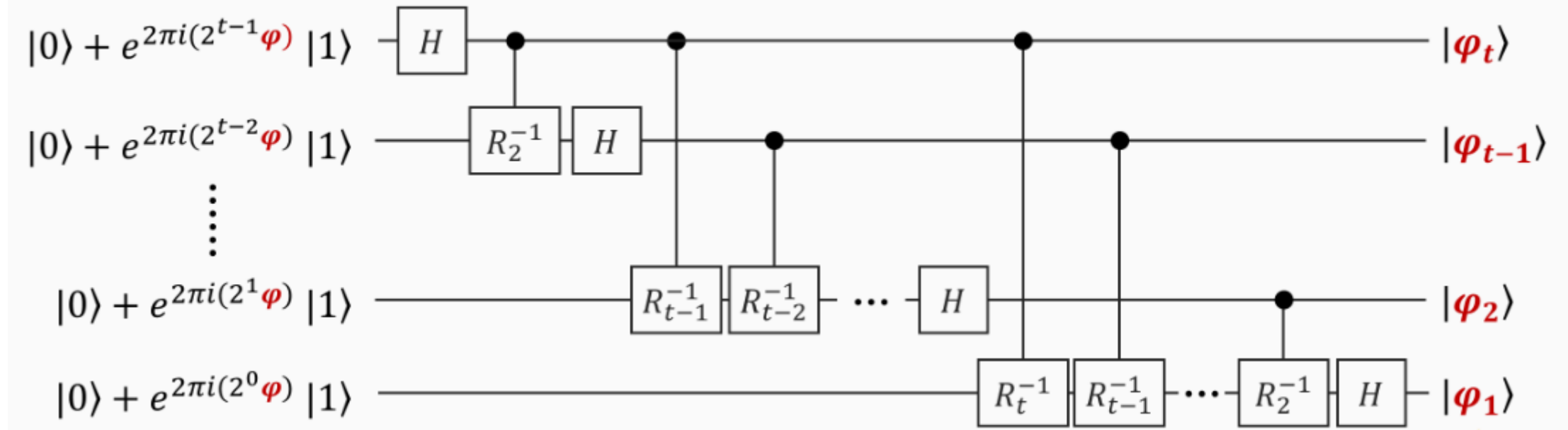
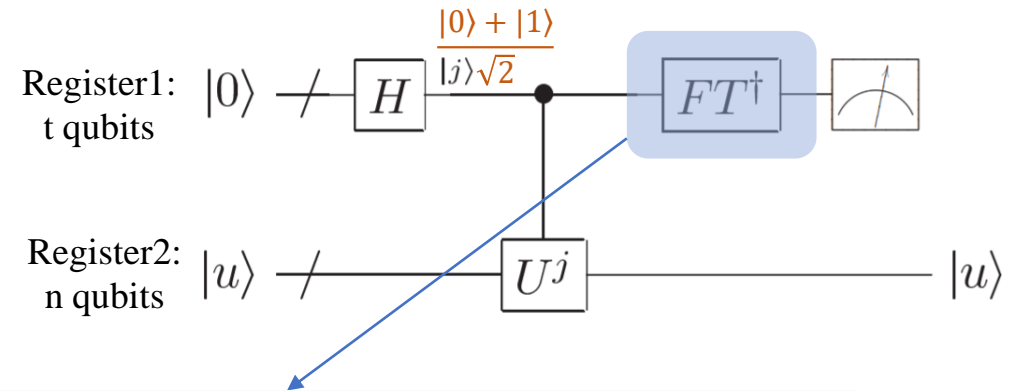
$\therefore U^{2^n}$  denotes by the controlled-U gate, which can be implemented by:



PS. The gate on the right-hand side is the controlled phase gate, which is specified in **Step.3**.

# The procedure for quantum phase estimation

**Step 3.** Use inverse quantum Fourier transform to move the phase information from the probability amplitude to the quantum bases



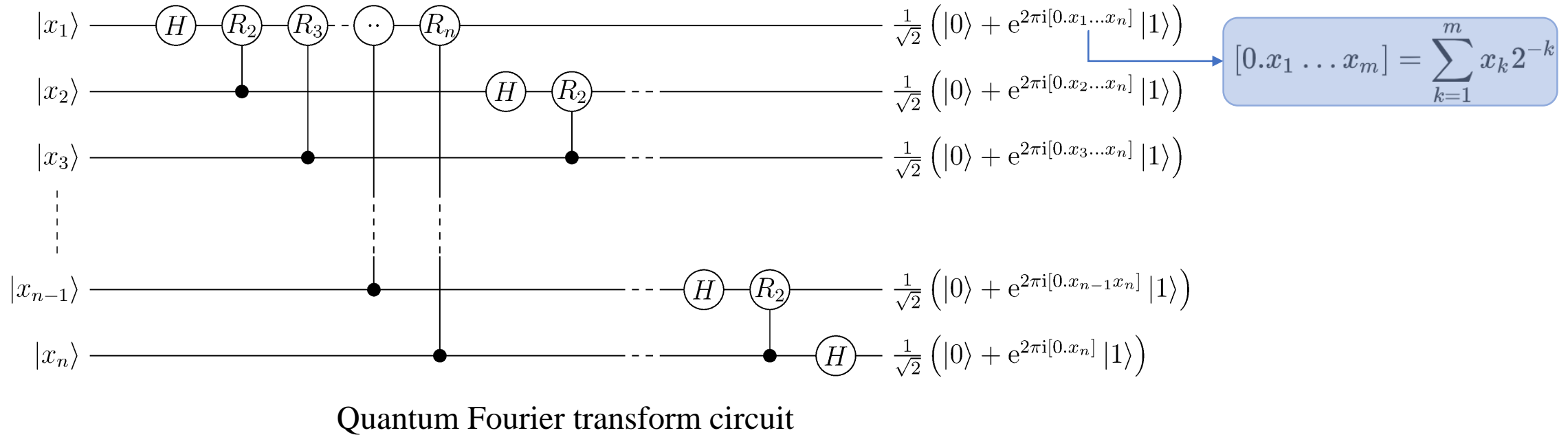
For **inverse** quantum Fourier transform ( $|j\rangle \rightarrow \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{-2\pi i jk/2^n} |k\rangle$ ),

the gate  $R_k^{-1}$  denotes the controlled phase gate  $R_k^{-1} \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{2\pi i}{k}} \end{bmatrix}$ .

**Picture:** <https://www.qtumist.com/info/PE-H5/index.html>

# Introduction to quantum Fourier transform

This slide is not part of the quantum phase estimation algorithm, but just to compare iQFT and QFT.



PS. For quantum Fourier transform ( $|j\rangle \rightarrow \frac{1}{2^{n/2}} \sum_{k=0}^{2^n-1} e^{2\pi i j k / 2^n} |k\rangle$ ),

the gate  $R_k$  denotes the controlled phase gate  $R_k \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{k}} \end{bmatrix}$ ;

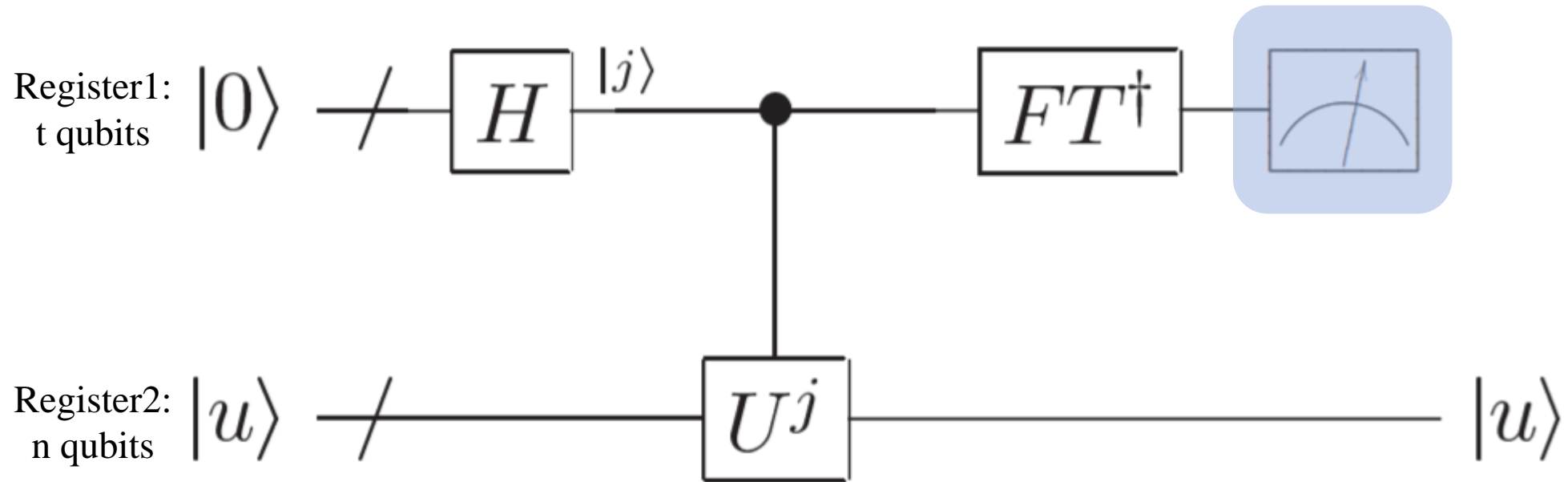
**Picture:** [https://www.wikiwand.com/en/Quantum\\_Fourier\\_transform](https://www.wikiwand.com/en/Quantum_Fourier_transform)

# The procedure for quantum phase estimation

## Step 4. Measurement

If  $U$  is an exact binary fraction, we can measure its phase with probability 1;

Otherwise, we can measure it with a very high probability close to 1.



After divide the measuring result of register1 (binary string  $\psi_1\psi_2\cdots\psi_t$ ) by  $2^t$ , we can get the phase  $\varphi = 0.\psi_1\psi_2\cdots\psi_t$ .  
e.g. Our result for measuring the register1 is 0001 ( $t=4$ ). After divided by  $2^4$ , we get the answer 0.0001, which in decimal is 1/16. Thus the phase is 1/16.