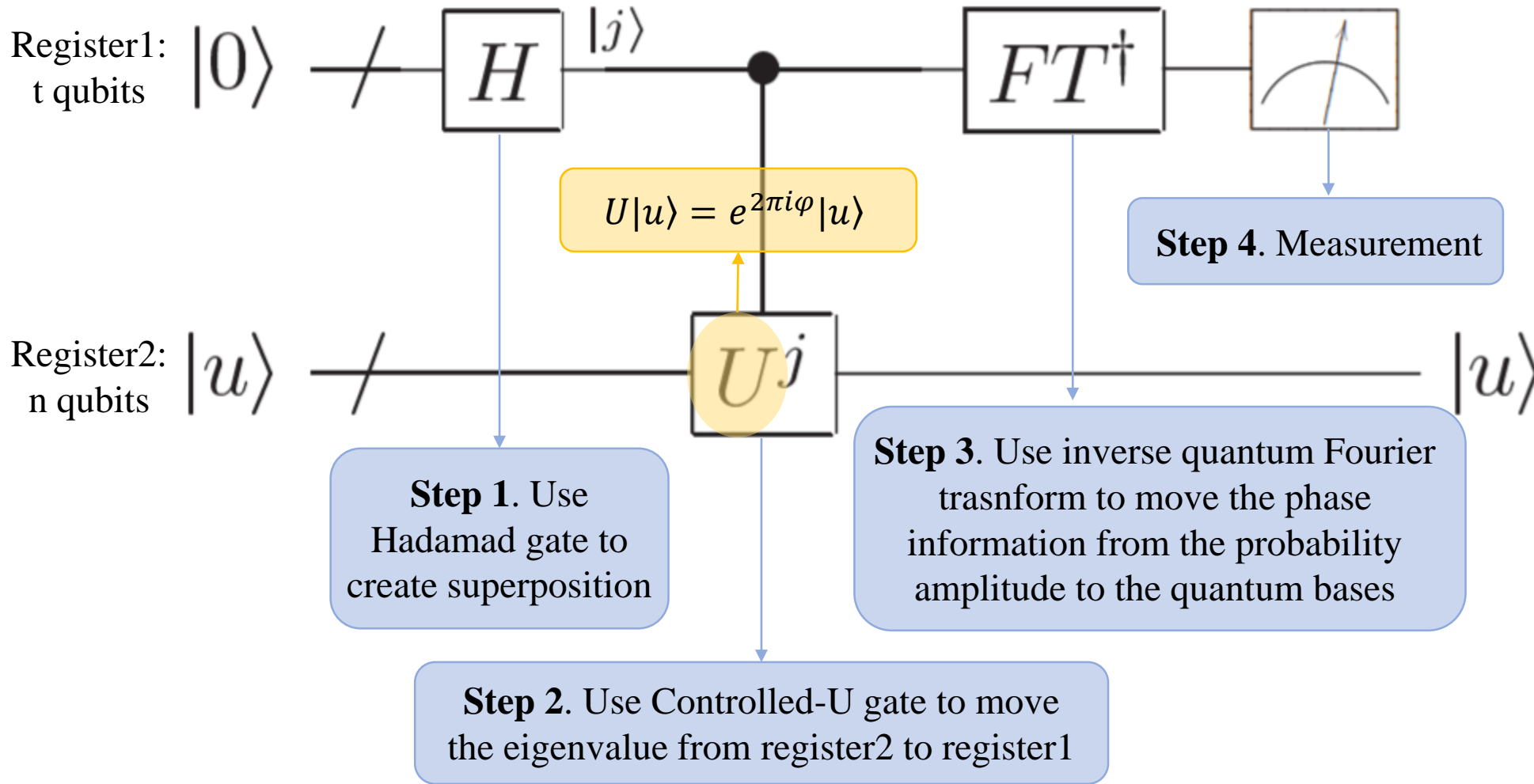


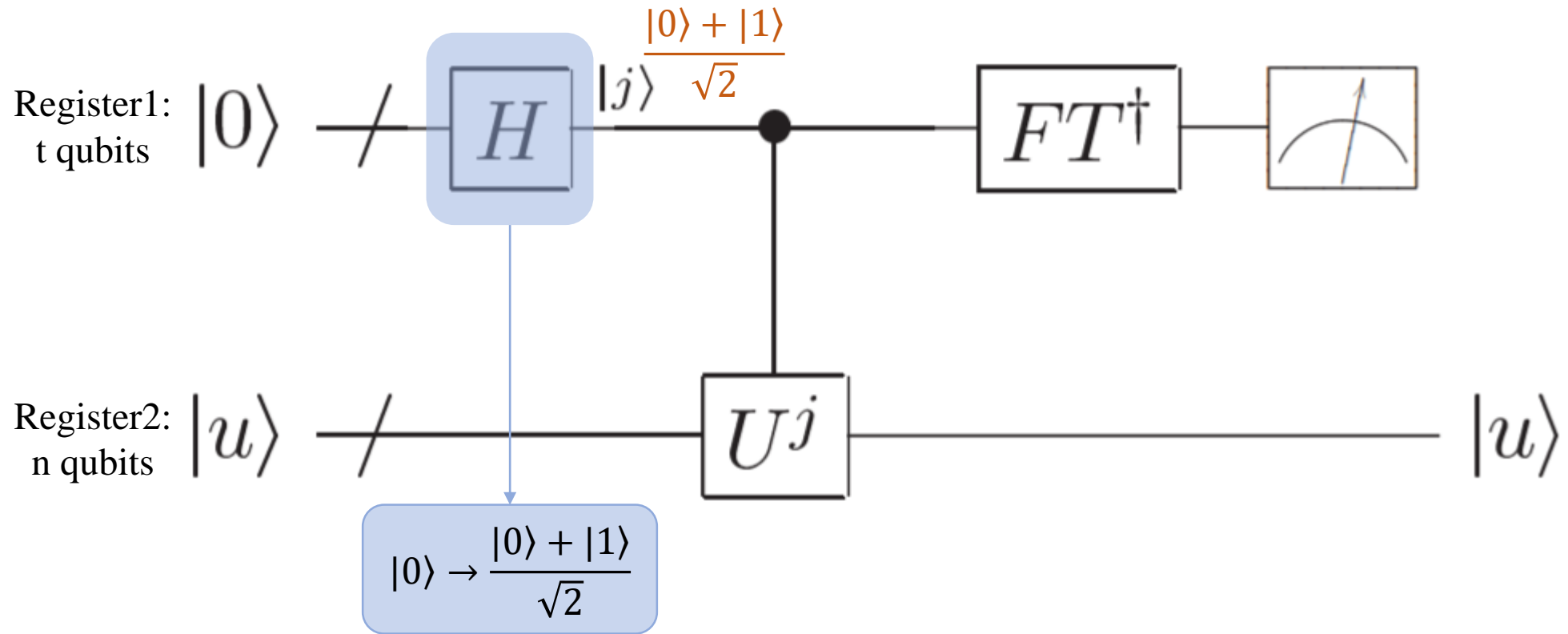
Phase estimation quantum circuit

$|u\rangle$ is the eigenvector of U , which is known.
 $e^{2\pi i\varphi}$ is the eigenvalue of U , with the phase φ unknown.
This circuit is to estimate the phase φ .



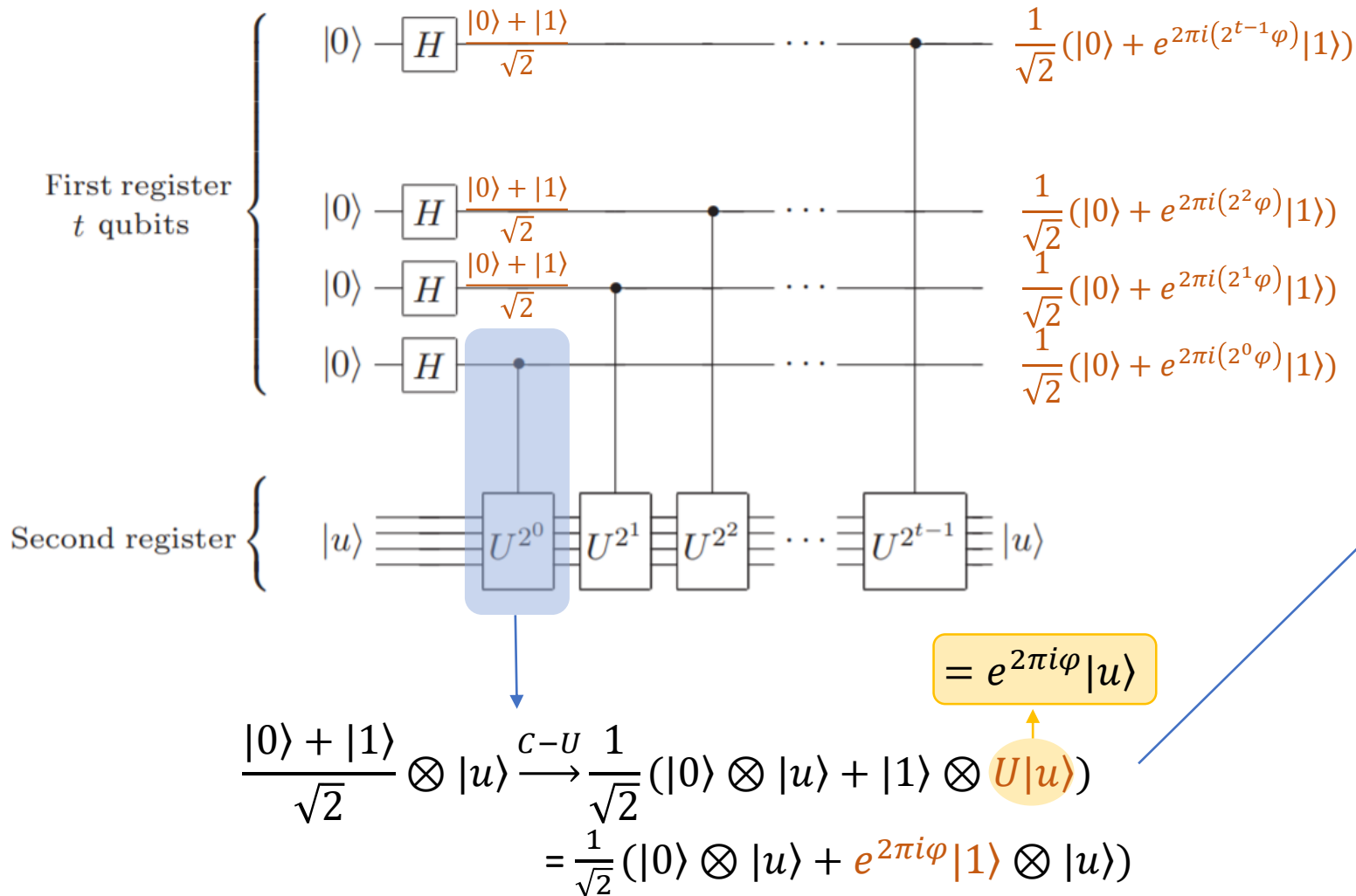
The procedure for quantum phase estimation

Step 1. Use Hadamard gate to create superposition



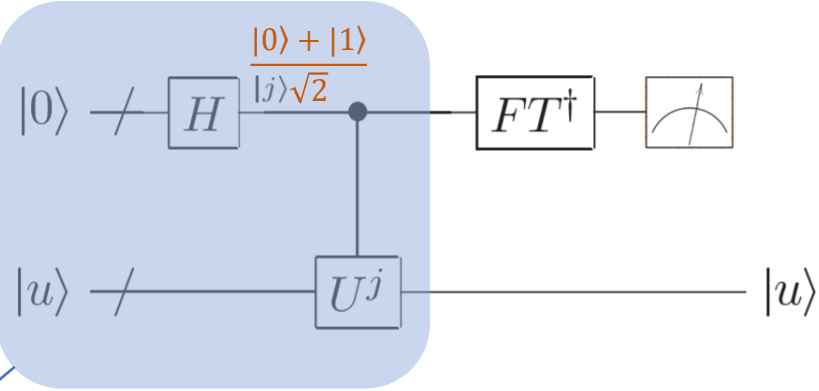
The procedure for quantum phase estimation

Step 2. Use Controlled-U gate to move the eigenvalue from register2 to register1



Register1:
 t qubits

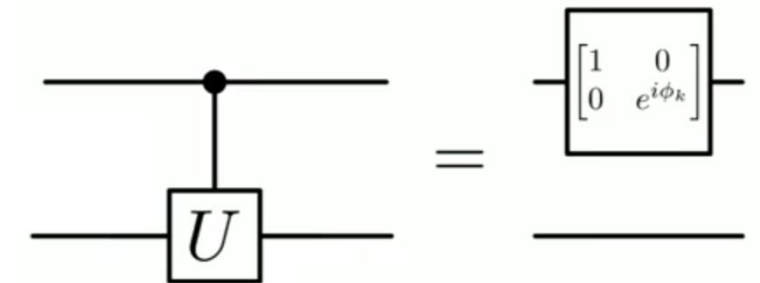
Register2:
 n qubits



\therefore The target qubit $|u\rangle$ does not change, but the control qubit $|1\rangle$ changes, which is contrary to the ordinary situation.

\therefore We move the eigenvalue $e^{2\pi i\phi}$ from the $|u\rangle$ space to the $|1\rangle$ space.

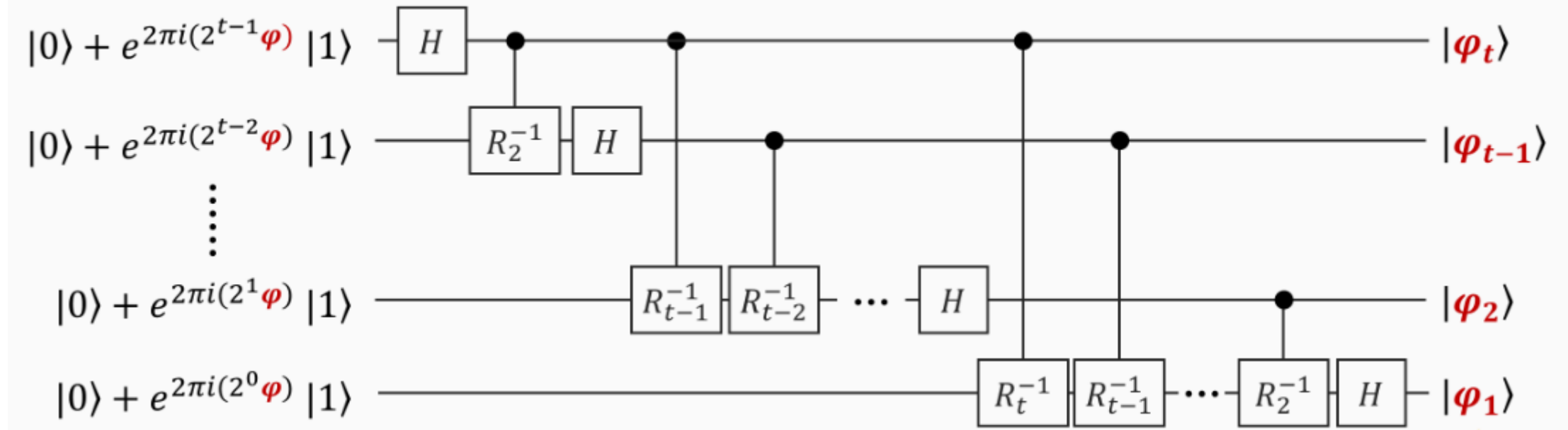
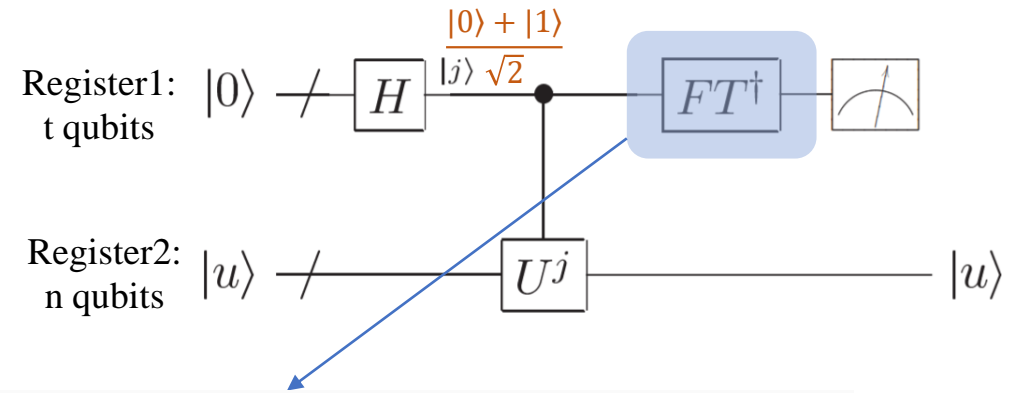
$\therefore U^{2^n}$ denotes by the controlled-U gate, which can be implemented by:



PS. The gate on the right-hand side is the controlled phase gate, which is specified in **Step.3**.

The procedure for quantum phase estimation

Step 3. Use inverse quantum Fourier transform to move the phase information from the probability amplitude to the quantum bases



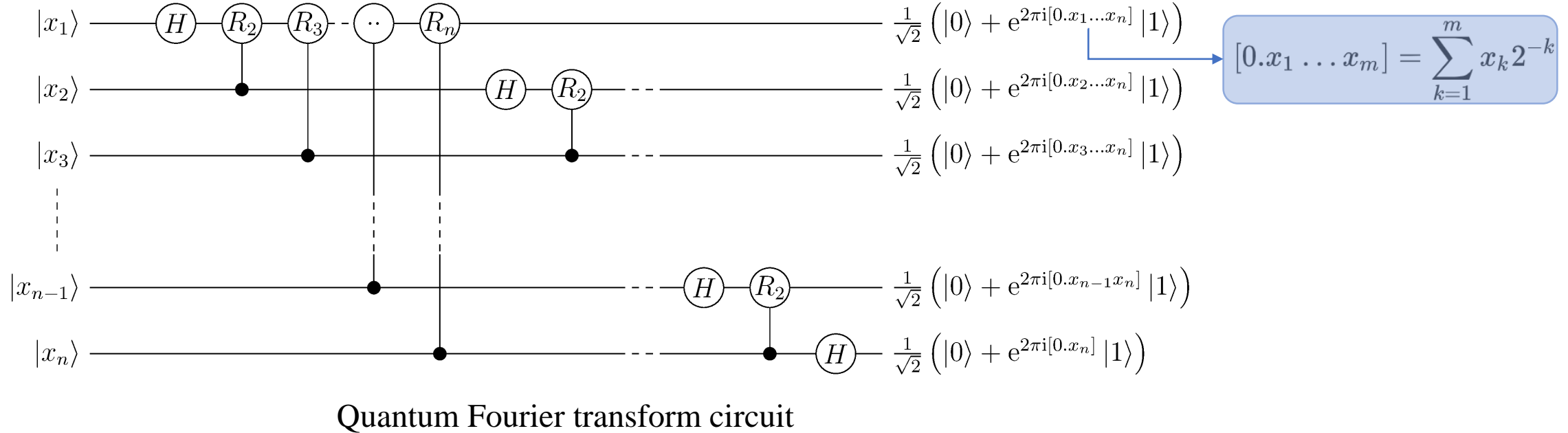
For **inverse** quantum Fourier transform ($|j\rangle \rightarrow \frac{1}{\sqrt{2^n}} \sum_{k=0}^{2^n-1} e^{-2\pi i jk/2^n} |k\rangle$),

the gate R_k^{-1} denotes the controlled phase gate $R_k^{-1} \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{2\pi i}{k}} \end{bmatrix}$.

Picture: <https://www.qtumist.com/info/PE-H5/index.html>

Introduction to quantum Fourier transform

This slide is not part of the quantum phase estimation algorithm, but just to compare iQFT and QFT.



PS. For quantum Fourier transform ($|j\rangle \rightarrow \frac{1}{2^{n/2}} \sum_{k=0}^{2^n-1} e^{2\pi i jk/2^n} |k\rangle$),

the gate R_k denotes the controlled phase gate $R_k \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{k}} \end{bmatrix}$;

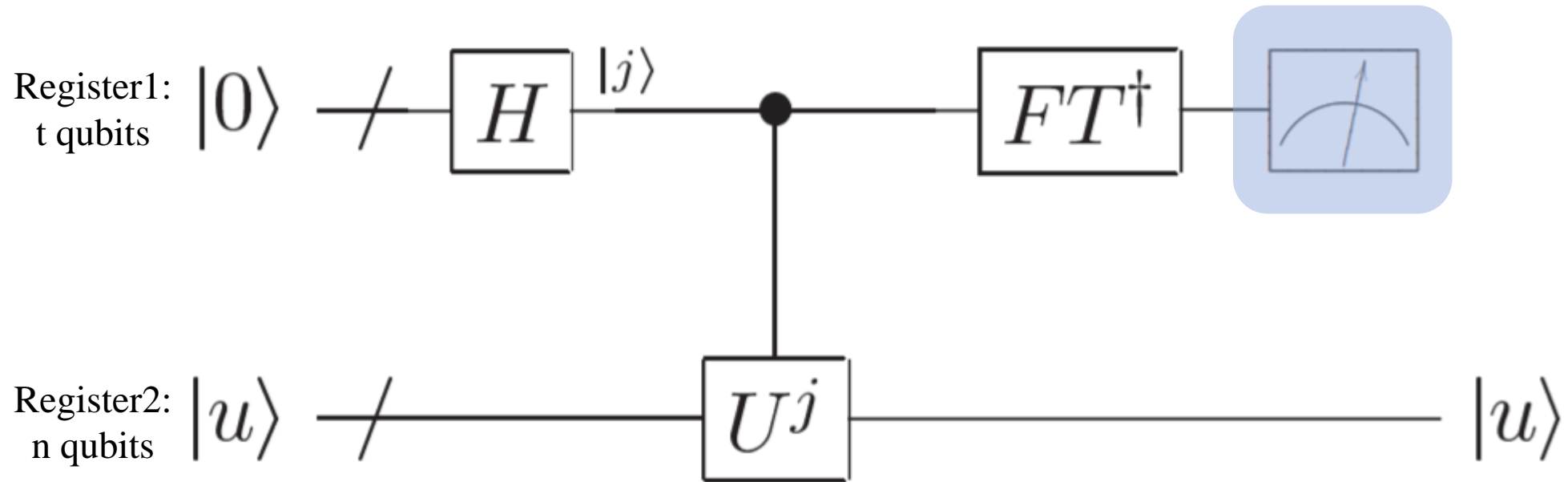
Picture: https://www.wikiwand.com/en/Quantum_Fourier_transform

The procedure for quantum phase estimation

Step 4. Measurement

If U is an exact binary fraction, we can measure its phase with probability 1;

Otherwise, we can measure it with a very high probability close to 1.



After divide the measuring result of register1 (binary string $\psi_1\psi_2\cdots\psi_t$) by 2^t , we can get the phase $\varphi = 0.\psi_1\psi_2\cdots\psi_t$.
e.g. Our result for measuring the register1 is 0001 ($t=4$). After divided by 2^4 , we get the answer 0.0001, which in decimal is 1/16. Thus the phase is 1/16.