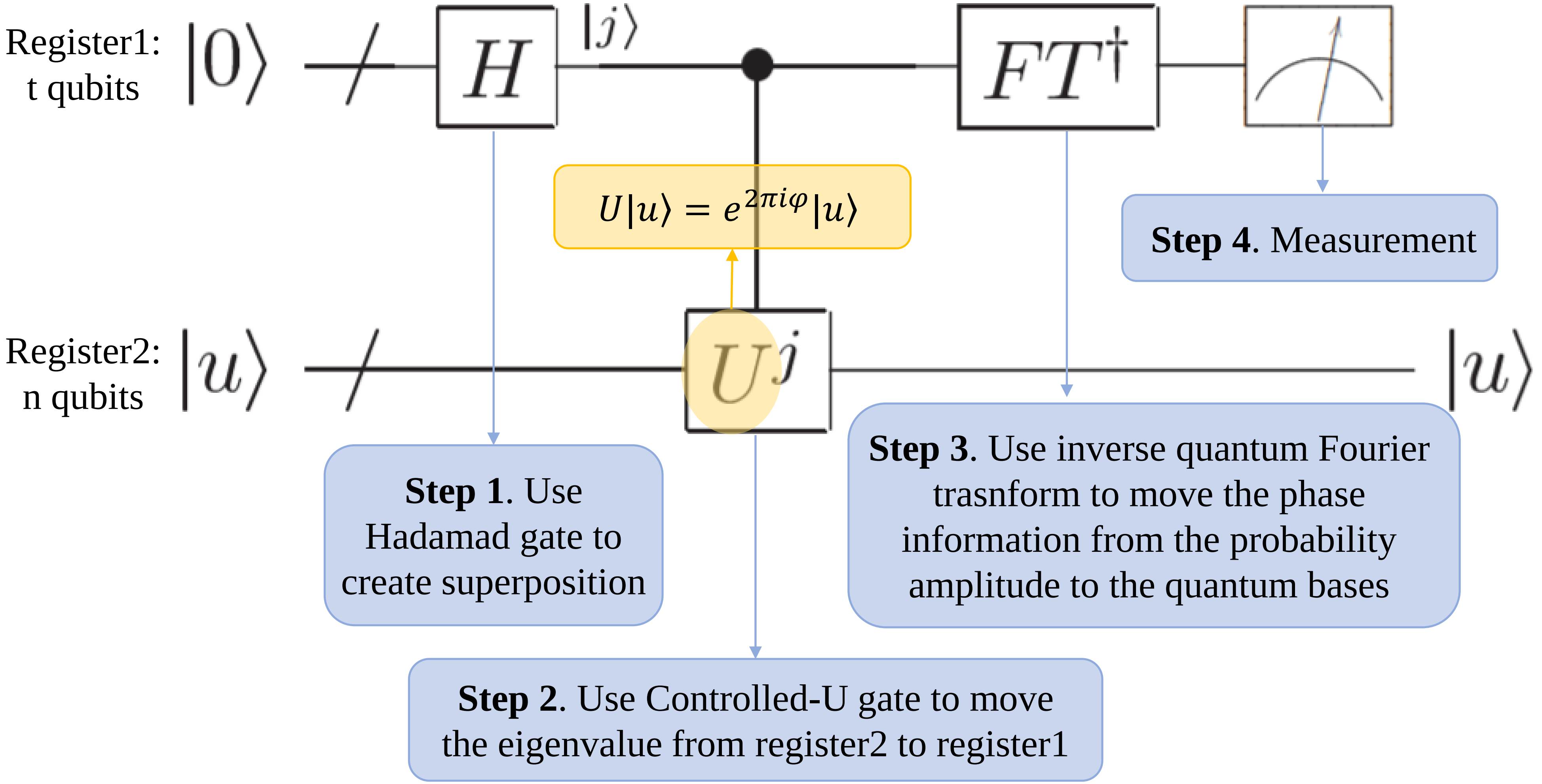
# Phase estimation quantum circuit

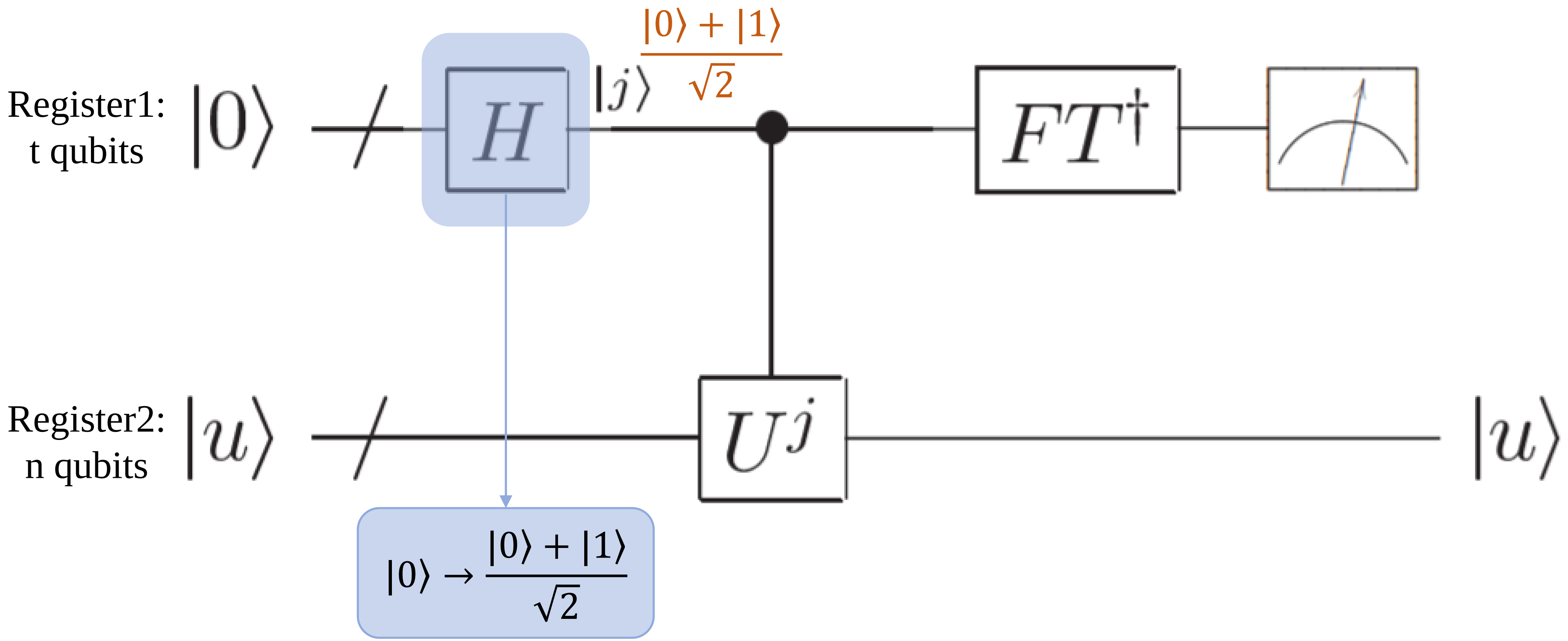
$|u\rangle$ is the eigenvector of U, which is know.
$e^{2\pi i\varphi}$ is the eigenvalue of U, with the phase $\varphi$ unknown.




**Reference & picture:** Michael A Nielsen and Isaac Chuang. Quantum computation and quantum information, 2002.

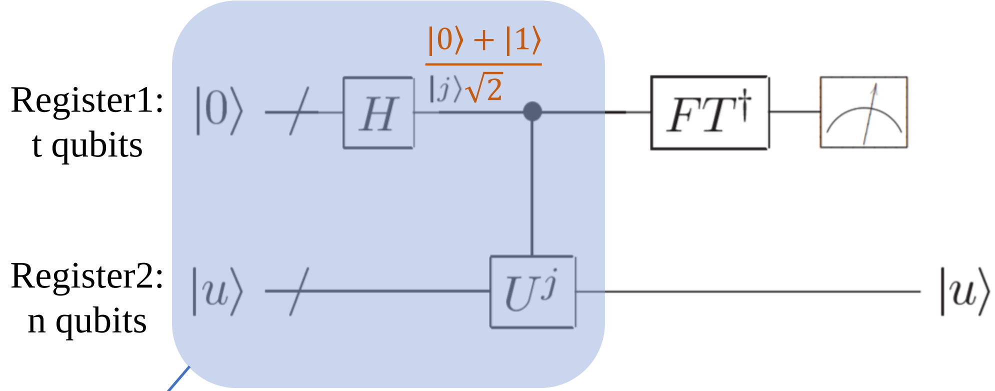
# The procedure for quantum phase estimation

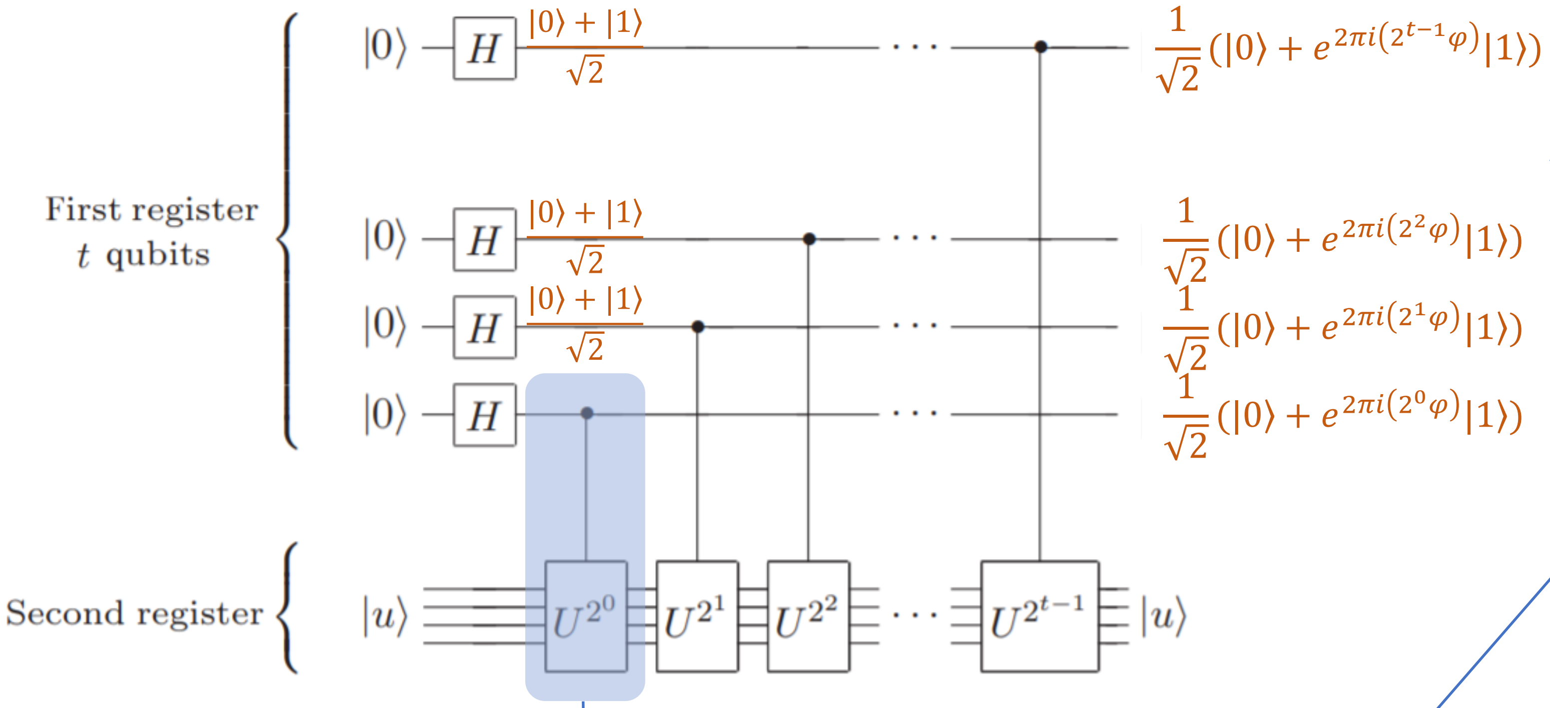
**Step 1**. Use Hadamad gate to create superposition



# The procedure for quantum phase estimation

**Step 2**. Use Controlled-U gate to move the eigenvalue from register2 to register1



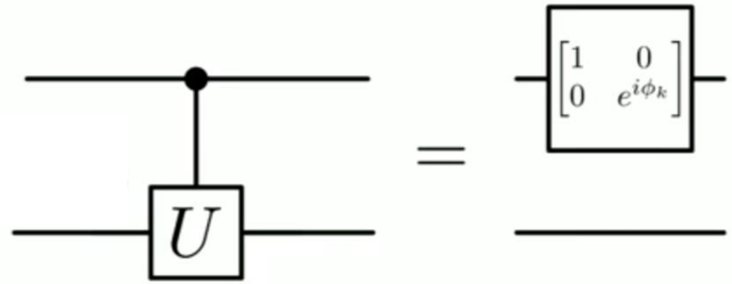

$$= e^{2\pi i\varphi}|u\rangle$$

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}} \otimes |u\rangle \xrightarrow{C-U} \frac{1}{\sqrt{2}}(|0\rangle \otimes |u\rangle + |1\rangle \otimes U|u\rangle)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle \otimes |u\rangle + e^{2\pi i\varphi}|1\rangle \otimes |u\rangle)$$

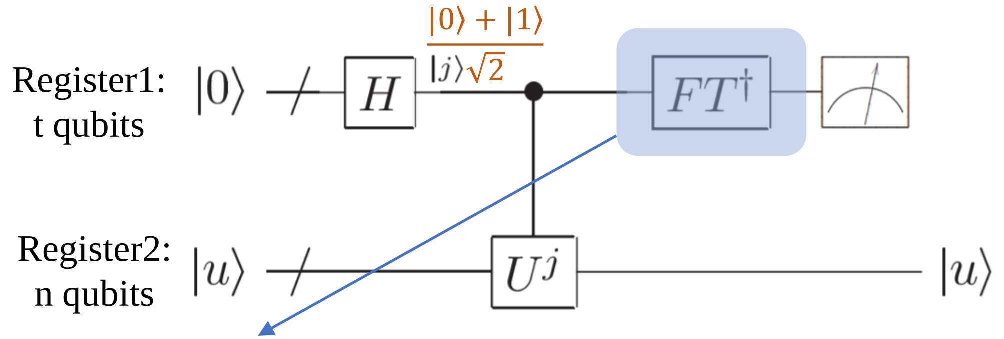
∴ We move the eigenvalue $e^{2\pi i\varphi}$ from the $|u\rangle$ space to the $|1\rangle$ space.

∴ $U^{2^n}$ denotes by the controlled-U gate, which can be implemented by:

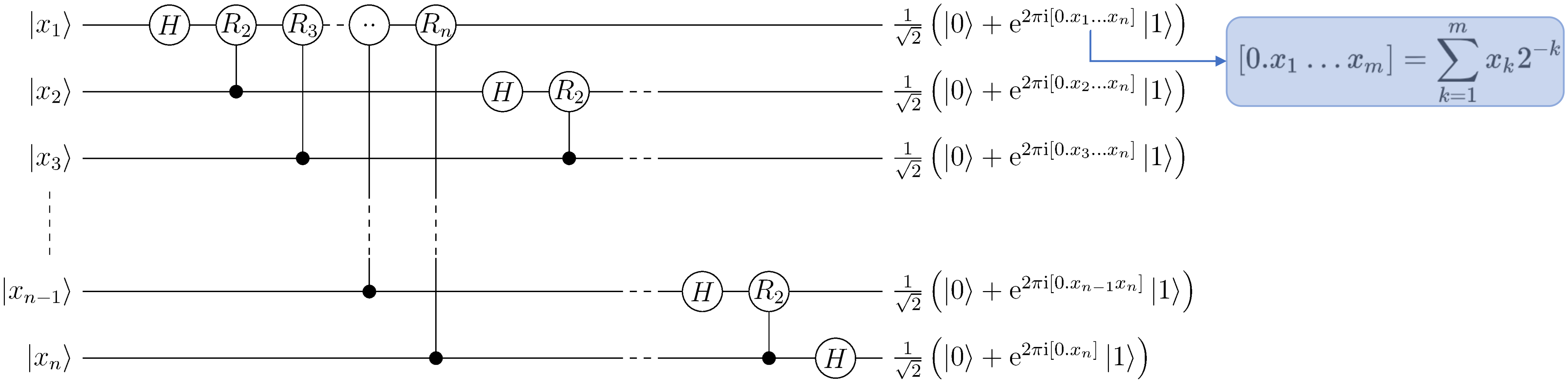



PS. The gate on the right-hand side is the controlled phase gate, which is defined in **Step.3**.

# The procedure for quantum phase estimation




**Step 3**. Use inverse quantum Fourier trasnform to move the phase information from the probability amplitude to the quantum bases




Quantum Fourier transform circuit

PS. For quantum Fourier transform ($|j\rangle \longrightarrow \frac{1}{2^{\frac{n}{2}}}\sum_{k=0}^{2^n-1} e^{2\pi ijk/2^n}|k\rangle$),

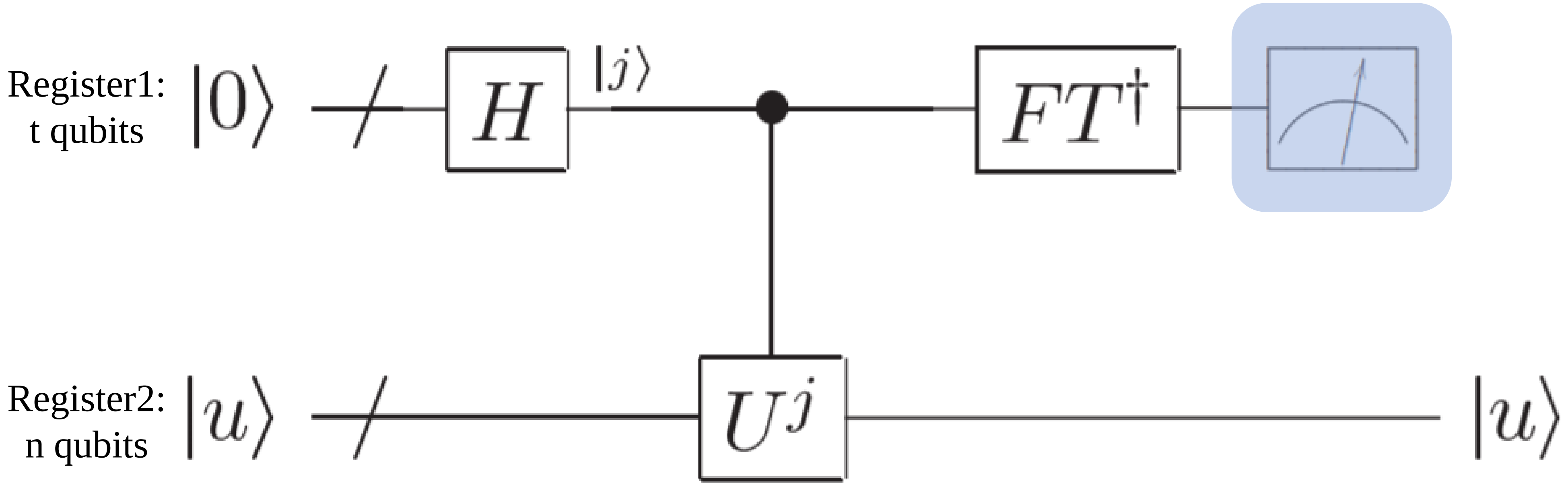
the gate $R_k$ denotes the controlled phase gate $R_k \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{k}} \end{bmatrix}$;

For inverse quantum Fourier trnasform ($|j\rangle \longrightarrow \frac{1}{2^{\frac{n}{2}}}\sum_{k=0}^{2^n-1} e^{-2\pi ijk/2^n}|k\rangle$),

the gate $R_k$ denotes the controlled phase gate $R_k \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{2\pi i}{k}} \end{bmatrix}$.

# The procedure for quantum phase estimation

**Step 4**. Measurement
If U is an exact binary fraction, we can measure its phase with probability 1;
Otherwise, we can measure it with a very high probability close to 1.




After divide the measuring result of register1 ($\psi_1\psi_2\cdots\psi_t$) by $2^t$, we can get the phase $0.\psi_1\psi_2\cdots\psi_t$.
e.g. Our result for measuring the register1 is 0001 (t=4). After divided by $2^4$, we get the answer 0.0001, which in decimal is 1/16. Thus the phase is 1/16.