Gate-model Quantum Computing.

Software Stack

A famous NP (non-deterministic) hard problem

Problem Definition e.g. traveling salesman problem

↓

Quantum Algorithm e.g. QAOA — Quantum Approximate optimization algorithm

↓

Quantum circuit gates & unitary operators

↓

Quantum Compiler → actual set of gates
→ connectivity (2 qubits are not physically connected, but have some interactions between)

QPU Simulator

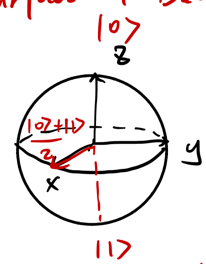
on laptop, we can simulate 20 to 22 qubits.
on super computer, around 50.
Then we will run out of the classical compute power.

Solovay-Kitaev theorem ⇒ Finite set of gates can approximate any unitary operation.

The gate model is universal, because it can transform any quantum states/qubits into any other quantum gates/qubits.

Quantum Circuits

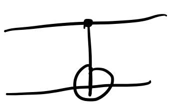
A single-qubit gate moves a point on the surface of Bloch Sphere.



① X-gate $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ —[X]—

② Hadamard gate $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ —[H]—

③ CNOT gate $\begin{bmatrix} 1 & 0 & & \\ 0 & 1 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{bmatrix}$ → when control qubit is 1, then applies NOT.

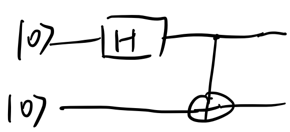


when control qubit is 0, do nothing

$CNOT|00\rangle = |00\rangle$

$CNOT|01\rangle = |11\rangle$

Create $|\phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$:



$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Quantum Annealing for Optimization.

1. Adiabatic Quantum Computing
绝热的

Unitary evaluation and the Hamiltonian

Classical Ising model : $H = -\sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z \rightarrow$ Hermitian

The Hamiltonian is always a Hermitian operator (it is equal to its own conjugate transpose)

Energy expectation value: $\langle H \rangle = \langle \psi | H \psi \rangle$

Evolution: $U|\psi\rangle$

Schrödinger equation: $i\hbar \frac{d}{dt} |\psi(t)\rangle = H|\psi(t)\rangle$

$\hbar$: Plank Constant

The temperal evolution of the system is described by the Hamiltonian applied to the time-dependent state

The fact that Hamiltonian is Hermitian implies that operater U is unitary
⇓
Every Hamiltonian implies unitary operator.

Solution for time-independent H: $U = \exp(iHt/\hbar) \longrightarrow$ unitary

Every gate has an underlying Hamiltonian.

($\bar{A}$: conjugate of A. A^T transpose of A)

summary
- Hermitian matrix: $\bar{A}^T = A \Leftarrow$ the Hamiltonians (H)
- Unitary matrix: $\bar{A}^T = A^{-1} \Leftarrow$ the time-evolution operater U.

The Adiabatic Theorem

2 Hamiltonians
$$\begin{cases} H_0 = \sum_i \sigma_i^x & \rightarrow \text{transverse field} \rightarrow \\ H_1 = -\sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z \end{cases}$$

The ground state, the lowest energy state of this, is equal superposition

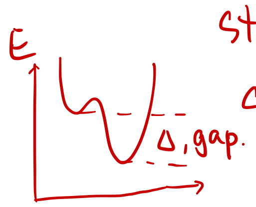
$H(t) = (1-t)H_0 + tH_1$, $t \in [0,1]$

If we change the time t slowly, and start from the ground state of H_0, end at the ground state of H_1.

In classical Ising model, we can easily get stuck at local optimum.

Solution: Adiabatic transition

Stay in ground state (lowest energy) throughout the change.



Speed limit: $\sim \frac{1}{\min(\Delta(t))^2}$, Δ: gap, difference between the ground state and the first excited state.

For different t, we have different gap $\Delta(t)$

However, if the gap is very small, the speed limit will be very bad.

$\therefore$ It is not true to say we can solve a NP-hard problem faster or exponentially faster, because those problems have very small gap.

Adiabatic Quantum Computing

$$H = \underbrace{-\sum_{\langle ij\rangle} J_{ij}\sigma_i^z\sigma_j^z - \sum_i h_i\sigma_i^z}_{\text{classical Ising model}} \underbrace{-\sum_{\langle ij\rangle} g_{ij}\sigma_i^x\sigma_j^x}_{\text{interaction between transverse field.}}$$

(not transverse field ising model, which don't have interactions)

This is universal!

(if it's able to implement a specific Hamiltonian