ENAE311H Final Project: "Double-Cone Waverider"

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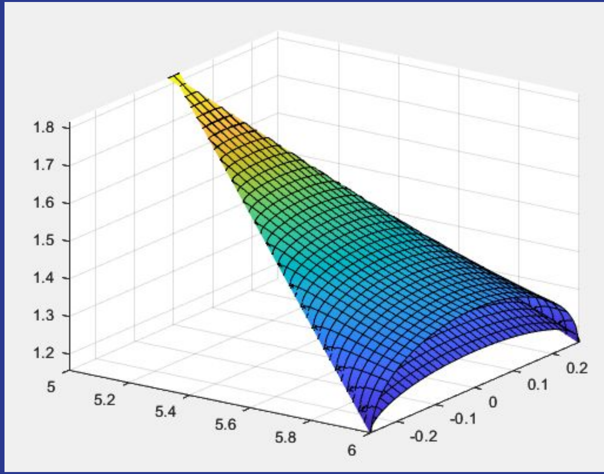


Overview

Objective: optimise a waverider shape in order to reach maximum glide distance given a starting altitude of 40km and initial mach number of 10.

Constraints:

- Volume = 0.01m^3 or less
- Length at most 1m
- Density 2500 kg/m^3

Assumptions:

- Air is an inviscid ideal gas.
- Standard atmospheric model

Shape - Leading Edge

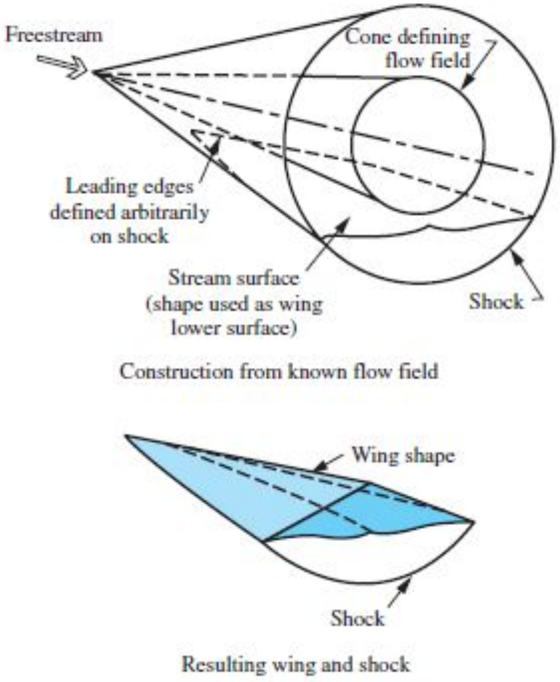
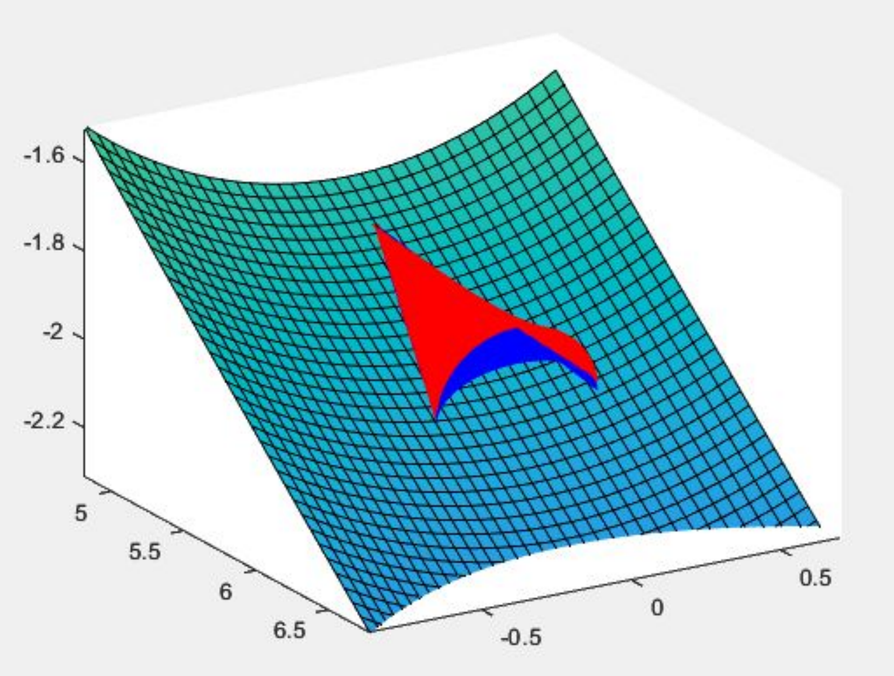


Figure 14.25 Cone flow wing.

We opted for a conical shaped wave rider over a wedge design to save weight. Conical shapes tend to have higher volumetric efficiency meaning we can get similar lift and drag coefficients while being lighter.

Shape



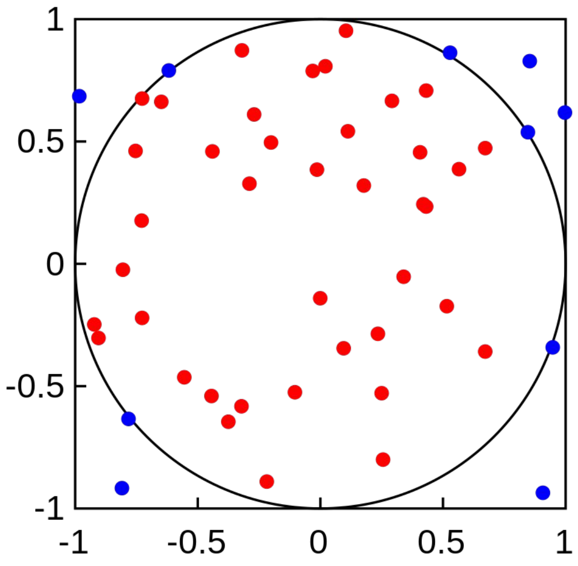
- Volume bound by 2 cones with x constrained from 5 to 6 for a total length of 1m.
- Iterated through multiple shapes by varying eccentricity of both upper and lower surfaces

```
bottomsurface = @(x, y)
tan(theta_s)*(x-dist)*(sqrt((tan(delta)*(x-dist))
^2 - (y/ECC_B)^2) + tan(theta_s)*offset) +
offset;

uppersurface = @(x, y)
tan(theta_s)*(x-dist)*(sqrt((tan(u_ang)*(x-dist))
^2 - (y/ECC_U)^2) + tan(theta_s)*offset) +
offset;

FINAL ECC_B, ECC_U = (1, 0.55)
```

Shape - Volume

Monte Carlo Method:

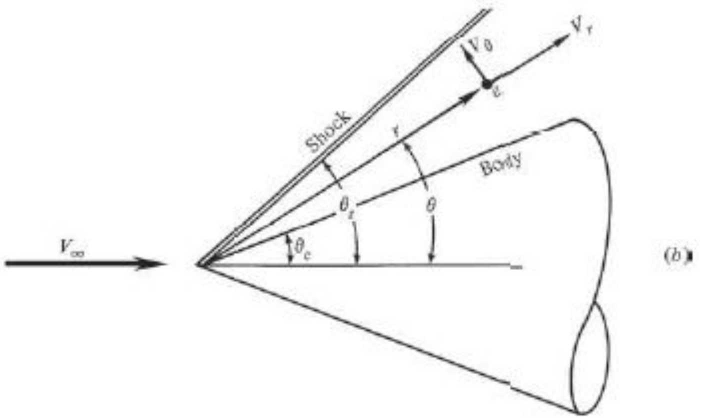
```
%number of guesses
total_guesses=10000;
%number of guesses that land inside the wave rider
inside=0;
for i=1:1:total_guesses

   %generate random points
   x=rand+5; %random number from 5 to 6
   y=rand-0.5; %random value from -0.5 to 0.5
   z=rand+1; %random value from 1 to 2

   syms ybound
   ybound=vpasolve(bottomsurface(x, ybound)-uppersurface(x, ybound), ybound, 0.5);
   if (bottomsurface(x, y)<z) && (z<uppersurface(x, y)) && (ybound*-1<y) && (ybound>y)
       inside=inside+1;
   end
end
vTotal=1*1*1;
vRider=inside/total_guesses*(vTotal)
```



Final volume -
0.0073m^3

Numerically Solving for Angle of Solid Cone

- Using methods described in Anderson 13.9, we can numerically solve for the angle of a solid cone that would be generating a shock wave at some pre-set angle.



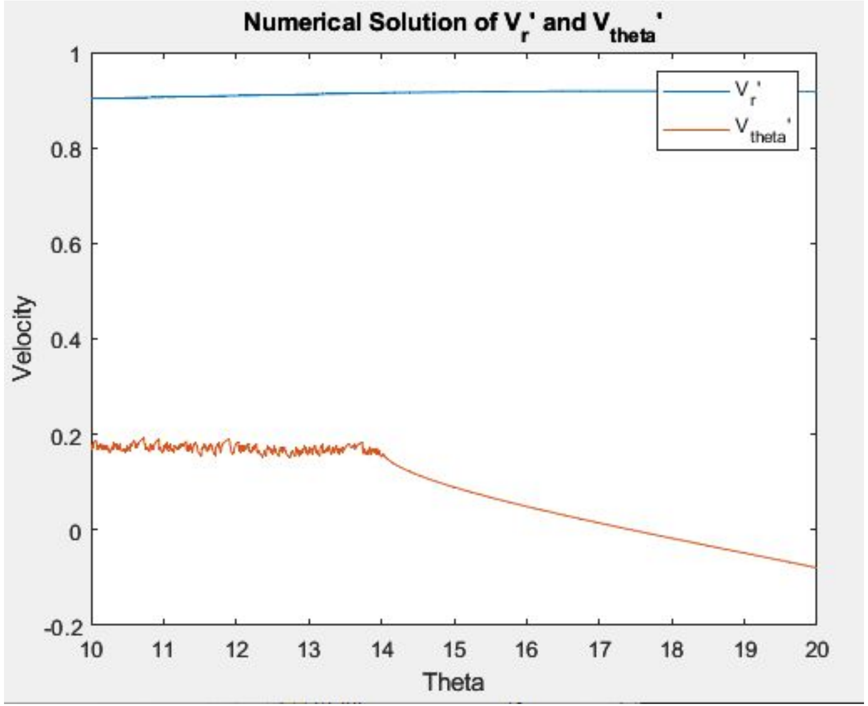
Numerically Solving for Angle of Solid Cone, pt. 2

$$\boxed{\begin{aligned}&\frac{\gamma-1}{2}\left[V_{\max}^2 - V_r^2 - \left(\frac{dV_r}{d\theta}\right)^2\right]\left[2V_r + \frac{dV_r}{d\theta}\cot\theta + \frac{d^2V_r}{d\theta^2}\right]\\ &\quad - \frac{dV_r}{d\theta}\left[V_r\frac{dV_r}{d\theta} + \frac{dV_r}{d\theta}\left(\frac{d^2V_r}{d\theta^2}\right)\right] = 0\end{aligned}} \tag{13.78}$$

$$\begin{aligned}&\frac{\gamma-1}{2}\left[1 - V_r'^2 - \left(\frac{dV_r'}{d\theta}\right)^2\right]\left[2V_r' + \frac{dV_r'}{d\theta}\cot\theta + \frac{d^2V_r'}{d\theta^2}\right]\\ &\quad - \frac{dV_r'}{d\theta}\left[V_r'\frac{dV_r'}{d\theta} + \frac{dV_r'}{d\theta}\left(\frac{d^2V_r'}{d\theta^2}\right)\right] = 0\end{aligned} \tag{13.80}$$

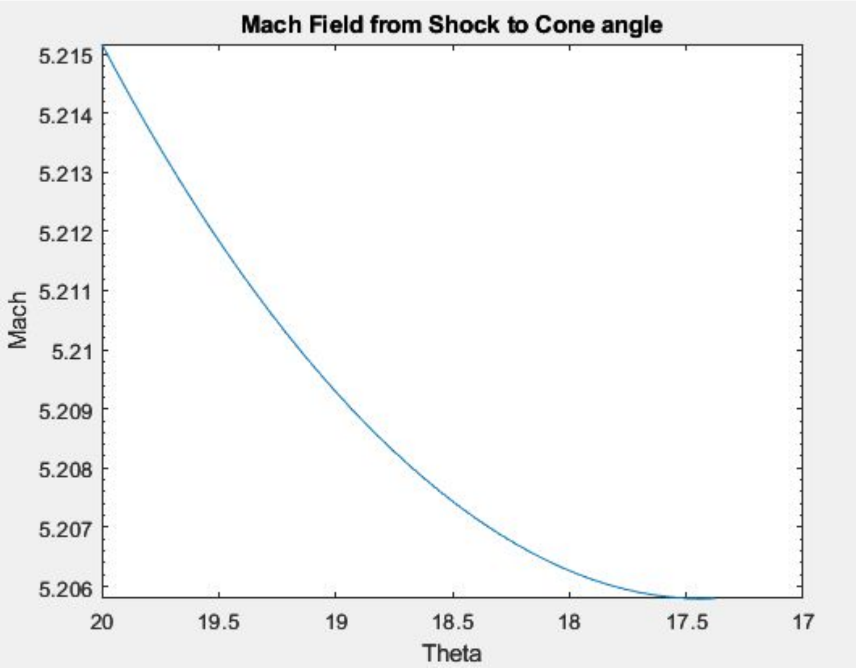
- Taylor Maccoll equation for solution of conical flows.
- Normalize the equation with respect to the V_max, using V’ = V/V_max

Creating the Velocity Flow Field



- Used ODE45 to solve for the velocity components across thetas
- Found the zeros of the tangential velocity to find where the flow must stop, indicating the existence of a resulting solid cone

Resulting Mach Field



- Numerically solving for Mach number as a function of the angle from the horizontal
- Mach field spans from directly behind the shock wave to directly on the solid cone

Pressure Distribution and Lift/Drag

- Based on the Mach field, we can find the pressures at each point on the surface of the waverider.
- Using these pressures, we calculate the total force that is exerted on the waverider, which we can decompose into drag and lift forces.
- Condense entire numerical solver for conical shock wave to a function that returns total force given a Mach number and altitude.

```
function f = force_m_alt(M_inf, alt)
```

Equations of Motion for Waverider

$$\ddot{y}=Lift/m-g$$

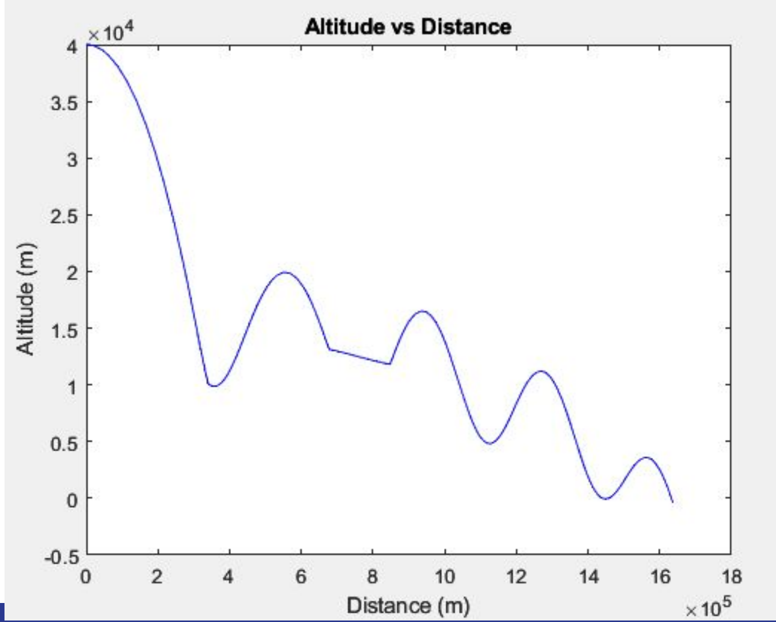
$$\ddot{x}=-Drag/m$$

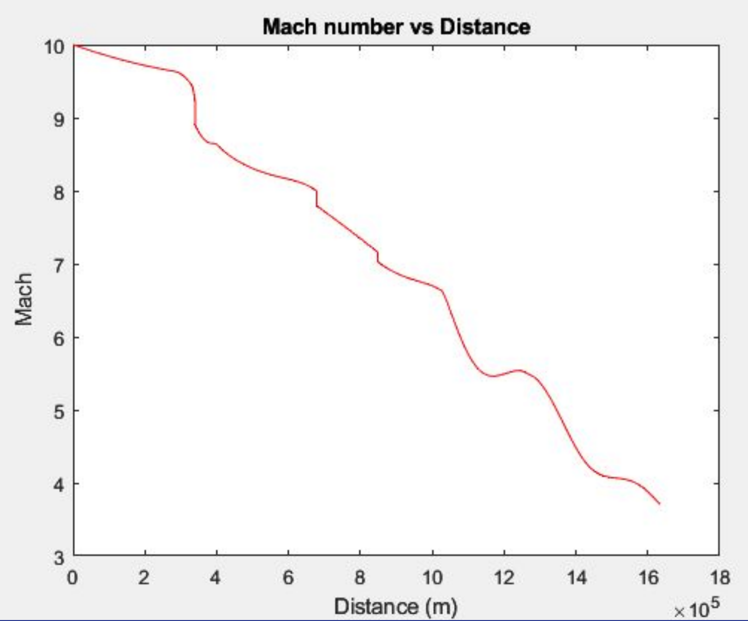
- Assume that the vehicle is always aligned with the flow, thus the AOA = 0 always.

```
[t,y] = ode45(@myodefun2,[0 300] ,[0, 10*sqrt(atmosisa(40000)*R*gam), 40000, 0],[], m, D, L, R, gam, g);

% y = [x dx y dy]
function ydot = myodefun2(t,y,m, D, L, R, gam, g)
   ydot(1,1) = y(2);
   ydot(2,1) = -D(sqrt(y(2)^2+y(4)^2)/sqrt(atmosisa(y(3))*R*gam), y(3))/m;
   ydot(3,1) = y(4);
   ydot(4,1) = L(sqrt(y(2)^2+y(4)^2)/sqrt(atmosisa(y(3))*R*gam), y(3))/m-g;
end
```

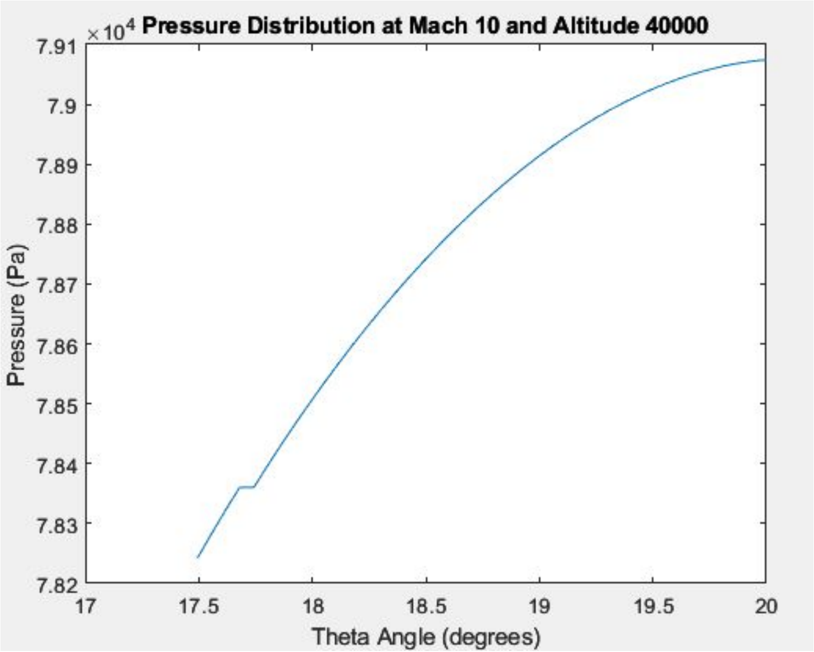
Range

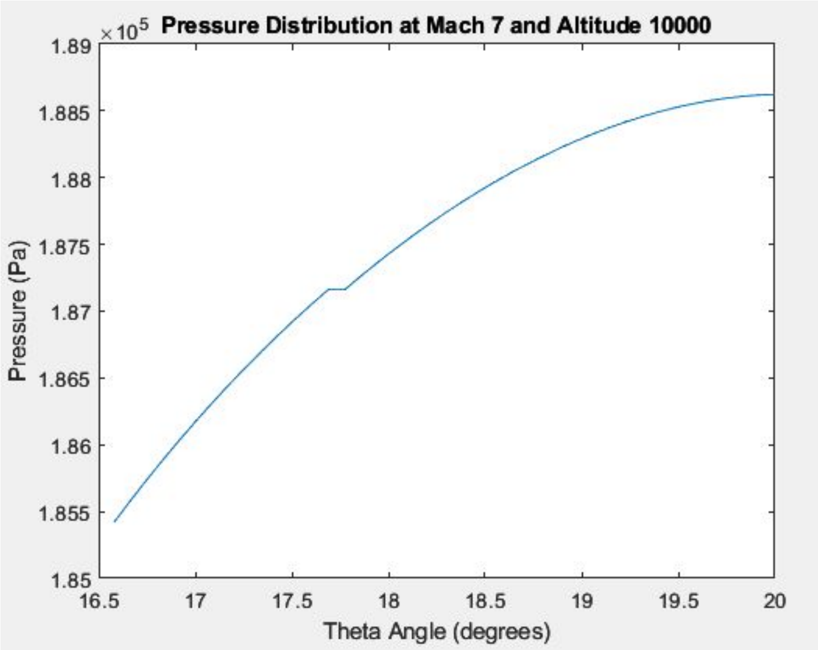
- Numerically solving ODE that describes the distance traveled and the altitude based on EOMs that are varied with respect to Mach number and altitude.





Pressure Distributions at Points in Trajectory





Results

- Max distance: 1637 km
- Final Mach number: 3.7
- Descent time: 13.65 minutes