$q \rightsquigarrow$ original MC

$\pi \rightsquigarrow$ stationary dist

$\Downarrow$

$$P_{x,y} = \begin{cases} q_{x,y} \times \left[\left(\frac{\pi(y)}{\pi(x)} \times \frac{q_{x,y}}{q_{y,x}} \right) \wedge 1 \right] & \text{if } x \neq y \\ 1 - \sum_{z:\, z \neq x} q_{x,z} \left[\left(\frac{\pi(z)}{\pi(x)} \frac{q_{x,z}}{q_{z,x}} \right) \wedge 1 \right] & \text{if } x = y \end{cases}$$

Glauber dynamic / Gibbs sampling

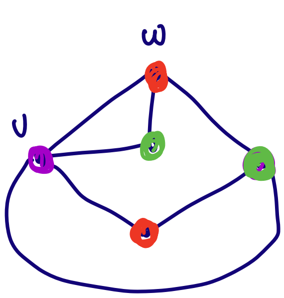
Let V be the vertex set & S is a finite set

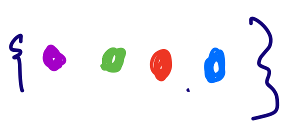
Let S^V be the set of configurations, meaning the set of functions from $V \to S$.

Let π be a probabity dist over the set of config

How can we sample from π?

EX (proper Coloring). A proper coloring of a graph $G(V,E)$ is a configuration x of $\{1, \dots, q\}^V$, such that $x(v) \neq x(w)$ if $\{v,w\}$ is an edge.



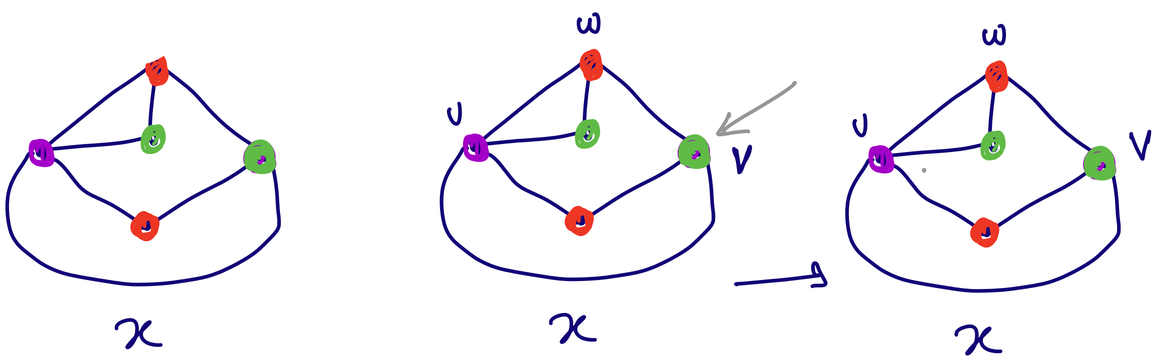


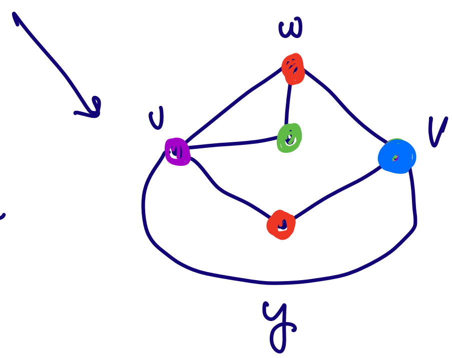
For a given configuration x, call a color j allowable at v if j is

x — different from all color assigned to neighbors of v.

Given a proper coloring x, we can generate a new coloring as follows:

1) select a vertex at random
2) select a color j uniformly at random from allowable colors at v
3) recolor vertex v with color j.





Let $A_v(x)$ be the set of allowable colors at v in configuration x

$$A_v(x) = A_v(y)$$

prob of moving from $x \to y$ is $\frac{1}{|V|} \times \frac{1}{|A_v(x)|}$

prob u a ~ $y \to x$ is $\frac{1}{|V|} \times \frac{1}{|A_r(y)|}$

$P_{x,y} = P_{y,x}$

Gibbs sampler: Let V & S be finite sets.
Let $\Omega \subseteq S^V$ be the state space.
Let π be a distribution on Ω. The Gibbs sample for π is a reversible MC with state space Ω and the transition probability describe below.
The chain moves from state $x \in \Omega$ as follows.
a vertex v is chosen uniformly at random from V and a new state is chosen according to measure π, conditioned on the set of states equal to x at all vertices different from v.

for $x \in \Omega$ and $v \in V$, let

$$\Omega(x,v) = \{ y \in \Omega : y(w) = x(w) \text{ for all } w \neq v \}$$

Define the distribution π conditioned on $\Omega(x,v)$ as

$$\pi^v_{x,y} = \pi(y \mid \Omega(x,v)) = \begin{cases} \frac{\pi(y)}{\pi(\Omega(x,v))} & \text{if } y \in \Omega(x,v) \\ 0 & \text{if } y \notin \Omega(x,v) \end{cases}$$

Consider the following chain on (not necessarily proper) q-coloring. Choose a vertex v uniformly at random from V and a color again uniformly at random from ALL possible q colors.

MH accepts the proposed re-coloring w.p 1 if it yields to a proper coloring, otherwise rejects. If there are a allowable colors at vertex v, then the chance of remaining in the current coloring is $1 - \frac{a-1}{q} = \frac{1+q-a}{q}$

for the Gibbs sampler the chance of staying in the current coloring is $\frac{1}{a}$