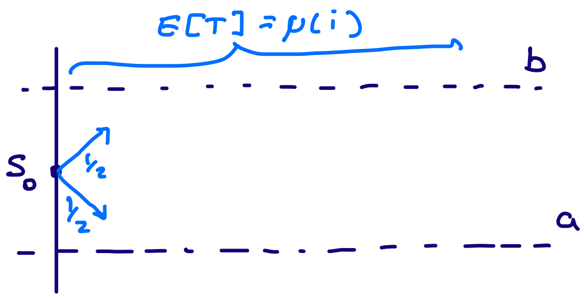


$$\mu(i) = E[T \mid S_0 = i]$$

↗ Expected hitting time

$$\mu(i) = \frac{1}{2}\left(\mu(i+1)+1\right) + \frac{1}{2}\left(\mu(i-1)+1\right)$$

$$= 1 + \frac{1}{2}\mu(i+1) + \frac{1}{2}\mu(i-1)$$

$$\mu(a) = \mu(b) = 0$$

$$\mu(i+1) - \mu(i) = \mu(i) - \mu(i-1) - 2$$

$$\mu(a+1) - \mu(a) = d$$

$$\cancel{\mu(a)} = 0$$

$$\cancel{\mu(a+1)} - \cancel{\mu(a)} = d$$

$$\cancel{\mu(a+2)} - \cancel{\mu(a+1)} = d - 2$$

$$\mu(a+3) - \cancel{\mu(a+2)} = d - 4$$

$$\vdots$$

$$\mu(a+i) - \mu(a+i-1) = d - 2(i-1)$$

Add LHS & RHS

$$\mu(a+i) = i \times d - 2(0+1+\cdots+(i-1))$$
$$= i \times d - i(i-1)$$
$$= i(d-i+1)$$

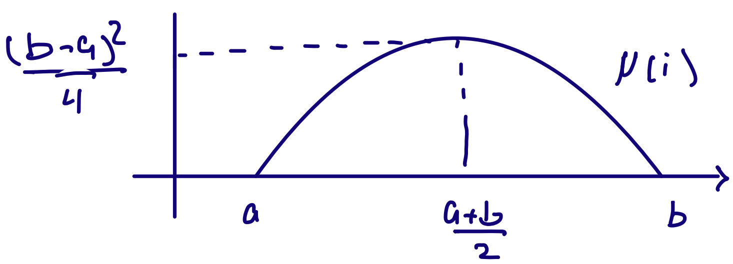
$$0 = \mu(b) = \mu(a+(b-a)) = (b-a)(d-b+a+1)$$

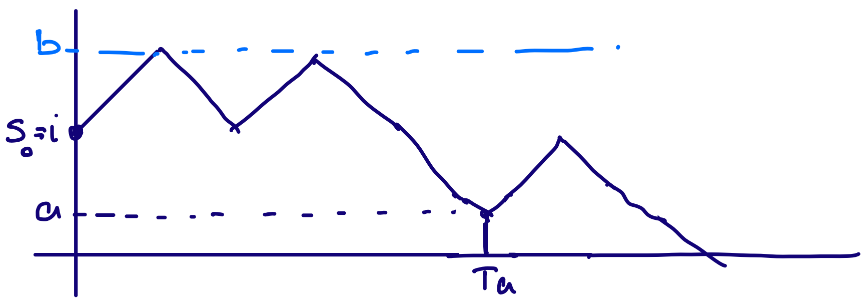
$$\Rightarrow d = b-a-1$$

so $\mu(a+i) = i(b-a-i)$

equivalently

$$\mu(i) = E[T \mid S_0 = i] = \mu(i) = (i-a)(b-i) \quad a \leq i \leq b$$





$T_a = \min(n : S_n = a)$

whether $T_0 < \infty$?

$T = \min(n : S_n = 0 \text{ or } S_n = b)$

we know that w.p 1, $T < \infty$

$$S_T = 0 \Rightarrow T_0 = T < \infty$$

$$\{ S_T = 0 \} \subseteq \{ T_0 < \infty \}$$

Therefore

$$\underbrace{P(T_0 < \infty \mid S_0 = i)}_{*} \geq P(S_T = 0 \mid S_0 = i) = 1 - \frac{i}{b}$$

$$1 \geq * \geq 1 \Rightarrow P(T_0 < \infty) = 1$$

So, the gambler will go bankrupt!!!

$$T = T_0 \wedge T_b = \min(T_0, T_b)$$

In order to reach b, you need to win at least $b - S_0$

$$T_b \geq b - S_0$$

$$b \to \infty \Rightarrow T_b \to \infty$$

$$\lim_{b\to\infty} T = T_0 \wedge \lim_{b\to\infty} T_b = T_0$$

$$E[T_0 \mid S_0 = i] = \lim_{b\to\infty} E[T \mid S_0 = i] = \lim_{b\to\infty} i(b-i)$$

$$= \begin{cases} 0 & \text{if } i = 0 \\ \infty & \text{if } i > 0 \end{cases}$$