S&DS 351 / S&DS 551 / MATH 251: Stochastic Processes
Assignment 2: Hints for P1 and P5

Due: 11:59 PM EST, Tuesday, February 14, 2023

Problem 1 For Problem 1, we suggest you to take a look at the Catalan Numbers. The Catalan number C_n describes the number of shortest paths (only horizontally and vertically) on a $\mathbb{R}^2$ plane starting from $(0, 0)$ and ending at (n, n) which stays in the region $y \leqslant x$.

Problem 5 We would like to provide some intermediate steps for Problem 5. For a branching process $\{G_t\}$ with $G_0 = 1$, we can define the probability generating function of G_t to be Ψ_t, which is:

$$\Psi_t(x) = \mathbb{E}\left[x^{G_t}\right] = \sum_{k=0}^{\infty} x^k \cdot \mathbb{P}\{G_t = k\}.$$

1. In the first step, we need to show that for $\Psi(x) := \Psi_1(x)$, it holds that $\Psi_{t+1}(x) = \Psi(\Psi_t(x))$.
2. Next, prove that $\mathbb{P}\{G_t = 1\} = \Psi_t'(0)$.
3. By using the chain's rule of derivative, try to get a simpler expression of

$$\rho_t := \frac{\mathbb{P}\{G_{t+1} = 1\}}{\mathbb{P}\{G_t = 1\}}$$

4. Finally, calculate the limit of ρ_t.