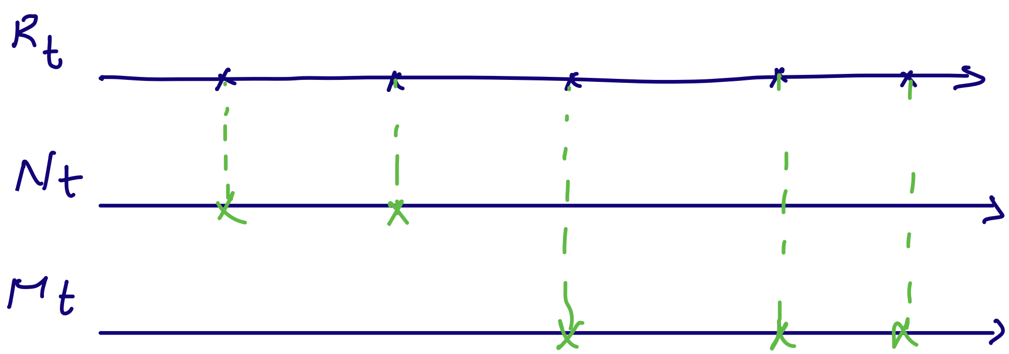


Let $X_1, X_2, \dots$ be indep exponential random variables with parameter λ and let $K \sim \text{Geom}(p)$ and indep of $\{X_i\}$.

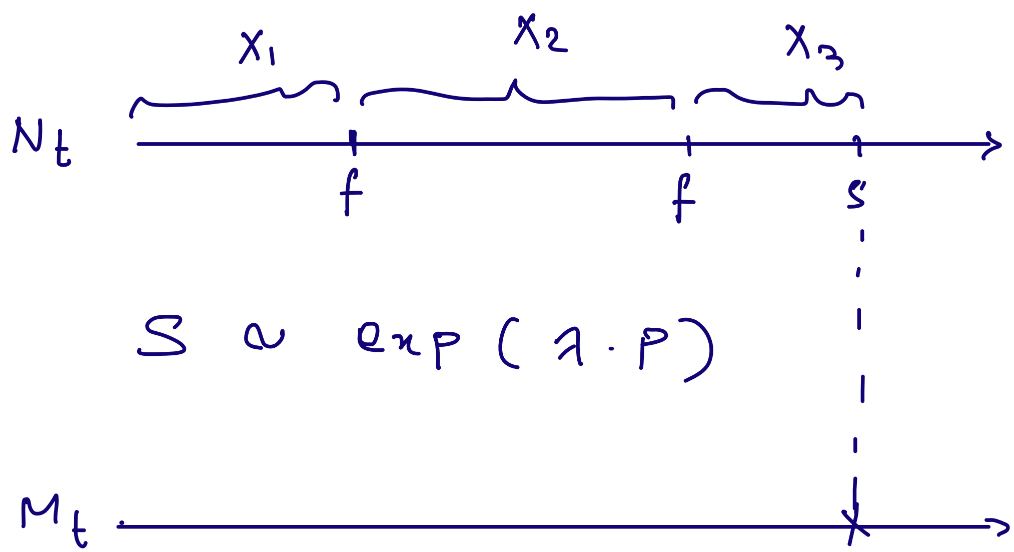
$$S = \sum_{i=1}^{K} X_i$$

Let $\{N_t\}$ be a poisson process with rate λ.

Let T_k be the k-th arrival. Note that

$$X_k = T_k - T_{k-1}$$

let us attach to every arrival an indep random variable $\{Y_k\}$ such that $\begin{cases} P(Y_k = s) = p \\ P(Y_k = f) = 1-p \end{cases}$



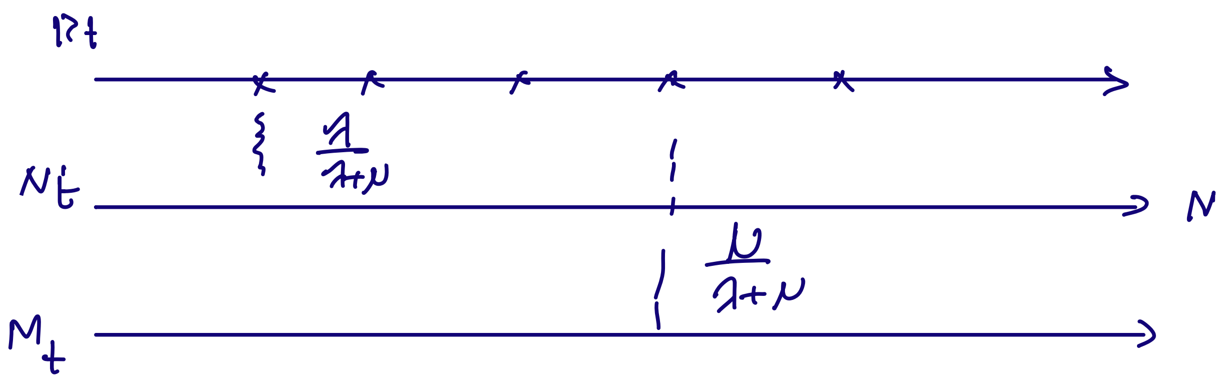
$S \sim \exp(\lambda \cdot p)$

EX (Waiting for the bus). Students arrive to a bus stop according to a poisson proces with rate λ. The shuttle arrives at time $T \sim \exp(\mu)$.

Let $\{N_t\}$ & $\{M_t\}$ be indep Poisson processes with paremeters λ, μ.

Let $R_t = N_t + M_t$ $\rightsquigarrow$ total # students & shuttles arrive at the bus stop.

$\rightsquigarrow$ parameter $\mu + \lambda$



P(k passengers get on the shuttle)

$$= \left(\frac{\lambda}{\lambda+\mu}\right)^k \frac{\mu}{\lambda+\mu}$$

Ex (kidney transplants): Patients A & B are in need of a kidney. Kidney from organ donors come according to a poisson process with rate λ. If the paitient A & B do not get a kidney, they will die according to an exponential distribution with para. μ_A & μ_B

Assume that A is on top of the waiting list.

What is the chance of paitient B getting a new kidney befor his old kidney fails?

M_t^A , M_t^B , N_t

$M_t^A \downarrow$ paitient A

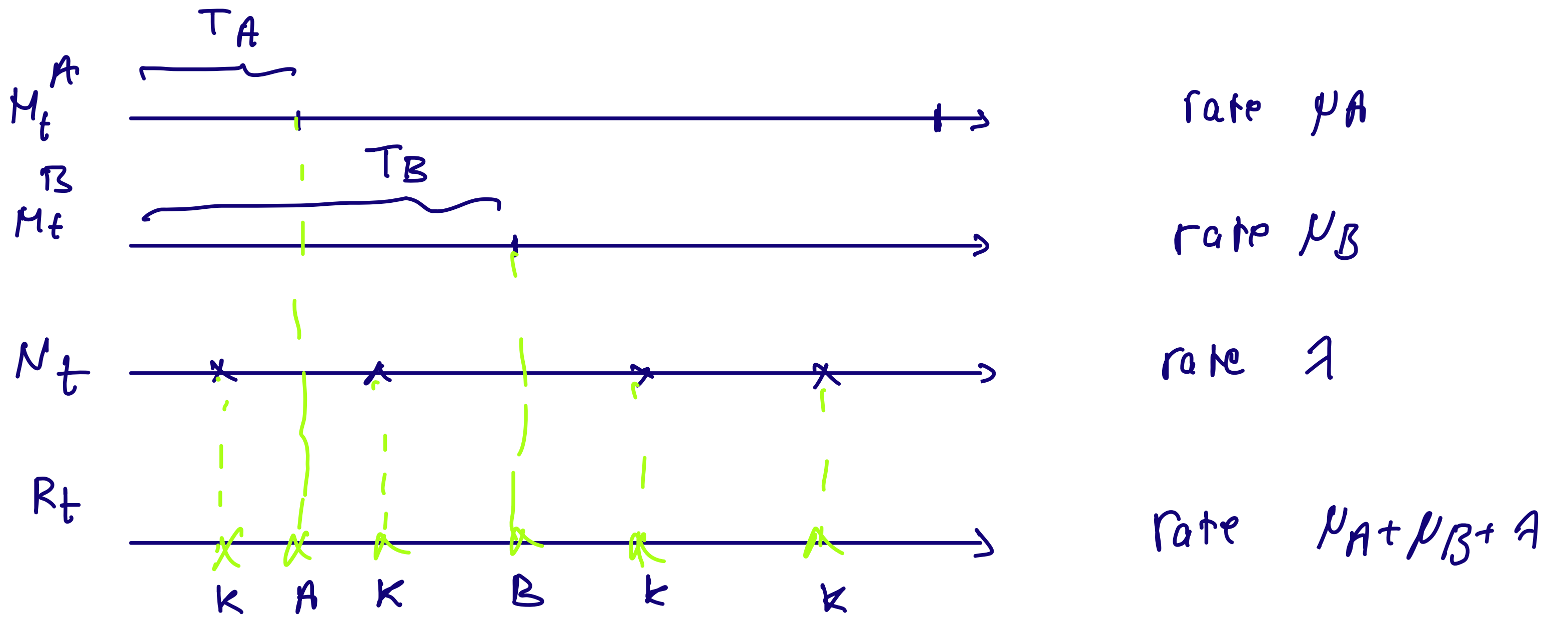
$M_t^B \downarrow$ Patient B

$N_t \downarrow$ kidney

T_A : The lifetime of the old Kidney of patient A

T_B : " " " " " " " B

$R_t = N_t + M_t^A + M_t^B \rightsquigarrow$ rate $\lambda + \mu_A + \mu_B$



$E = \{$ B get a kidney befor he dies $\}$

$E_1 = \{$ The first arrival of R_t is of type K or A $\}$

$E_2 = \{$ in next arrivals, K arrives befor B $\}$

$$E = E_1 \cap E_2$$

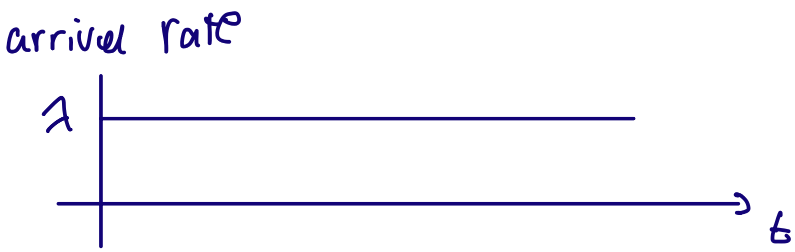
$$P(E) = P(E_1) \times P(E_2)$$

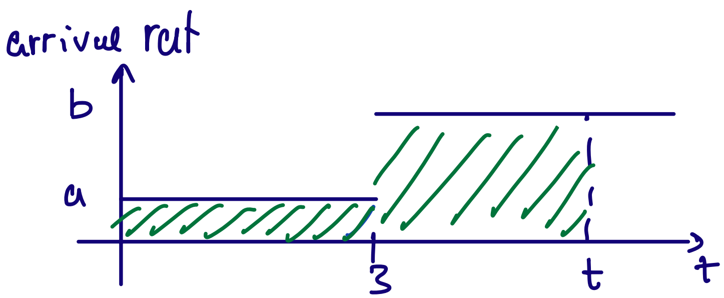
$$P(E_1) = \frac{\lambda}{\lambda+\mu_A+\mu_B} + \frac{\mu_A}{\lambda+\mu_A+\mu_B}$$

$$P(E_2) = pr(\text{ K arrives befor B }) = \frac{\lambda}{\lambda+\mu_B}$$

$$P(E) = \frac{\lambda+\mu_A}{\lambda+\mu_A+\mu_B} \cdot \frac{\lambda}{\lambda+\mu_B}$$

Nonhomogeneous poisson process.





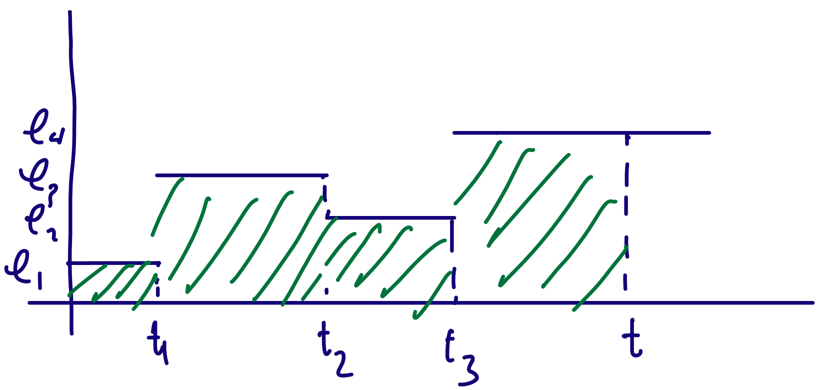
for $t > 3$

$$N_t = N_3 + (N_t - N_3)$$

$\text{Pois}(3a)$

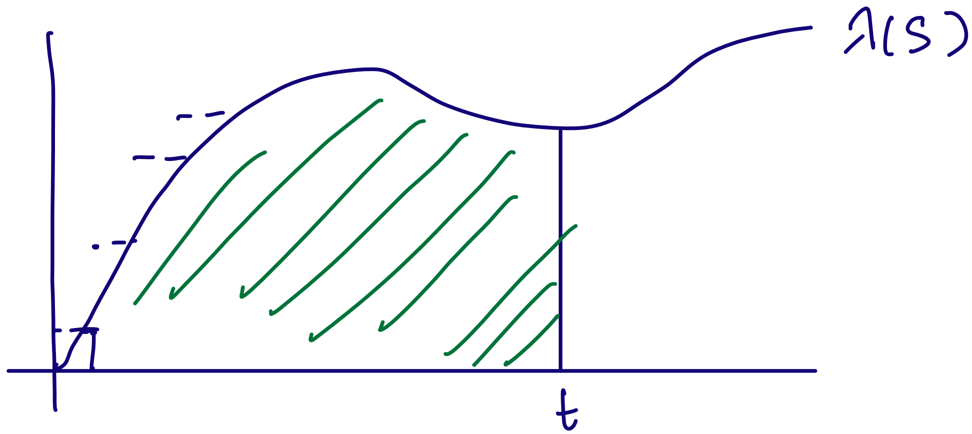
$\text{Pois}((t-3)b)$

$\text{Pois}(3a + b(t-3))$



$$N_t = (N_t - N_{t_3}) + (N_{t_3} - N_{t_2}) + (N_{t_2} - N_{t_1}) + N_{t_1}$$

Pois (with the rate of the surface }



1) $N_t - N_s$ and $\{N_r\}_{r \leq s}$ are inde for $t > s$

2) $N_t - N_s \sim \text{Pois}\left(\int_s^t \lambda(u)\,du \right)$ for $t > s$

$$P(T > t) = P(N_t = 0) = e^{\left(-\int_0^t \lambda(u)\,du\right)}$$