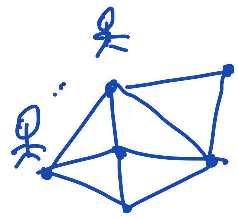
$G = (V, E)$

1) connected

2) Not bipartite



$$\pi_v = \frac{d(v)}{2|E|}$$

suppose that $\{X_n : 0 \leq n \leq N\}$ is an irreducible positive recurrent with the stationary dist π. Also assume that $X_0 \sim \pi$. so $X_n \sim \pi$ for any n. Define the reversed chain Y by $Y_n = X_{N-n}$ for $0 \leq n \leq N$

The sequence Y is a MC with $P(Y_{n+1} = j \mid Y_n = i) = \frac{\pi_j}{\pi_i} P_{ji}$

proof:

$$P(Y_{n+1} = j \mid Y_n = i_n, Y_{n-1} = i_{n-1}, \dots, Y_0 = i_0)$$

$$= \frac{P(Y_k = i_k \text{ for } 0 \leq k \leq n+1)}{P(Y_k = i_k \text{ for } 0 \leq k \leq n)}$$

$$= \frac{P(X_{N-n-1} = i_{n+1}, \dots, X_N = i_0)}{P(X_{N-n} = i_n, \dots, X_N = i_0)}$$

$$= \frac{\pi_{i_{n+1}} \times P_{i_{n+1}, i_n} \times \cdots \times P_{i_1, i_0}}{\pi_{i_n} \times P_{i_n, i_{n-1}} \times \cdots \times P_{i_1, i_0}}$$

$$= \frac{\pi_{i_{n+1}}}{\pi_{i_n}} \times P_{i_{n+1}, i_n}$$

Let $\{X_n : 0 \le n \le N\}$ be an irreducible MC. with stationary dist π. The chain is called reversible if the transition matrices of X & its time-reversed are the sam

$$\pi_i P_{ij} = \pi_j P_{ji} \quad (*)$$

Suppose π satisfies $(*)$ condition. then π is the stationary dist.

$$\sum_i \pi_i P_{ij} = \sum_i \pi_j P_{ji} = \pi_j \sum_i P_{ji} = \pi_j$$

Eg. $P_{ij} = P_{ji} \Rightarrow \pi$ is the uniform dist.

MCMC.

$$\sum_{\theta \in \Theta} g(\theta)\pi(\theta) \approx \frac{1}{n} \sum_{r=0}^{n-1} g(X_r)$$

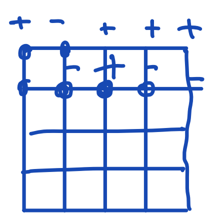
$$\int_\Theta g(\theta)\pi(\theta)\, d\theta$$

Bayesian inference. we have a prior density $\pi(\theta)$ and some data x is collected. what is the posterior $\pi(\theta|x)$

$$\pi(\theta|x) = \frac{f(x|\theta)\,\pi(\theta)}{\int_{\varphi} f(x|\varphi)\,\pi(\varphi)}$$

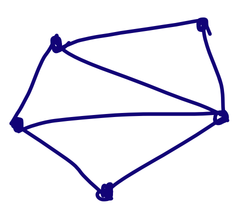
A graph $G = (V,E)$ where each vertex can be in a $+1$ or -1 state. So the state space $\{+1,-1\}^V$

$$\pi(\theta) = \frac{1}{Z}\exp\left(\sum_{v\sim w} \theta_v \theta_w\right)$$



$$Z = \sum_{\theta} \exp\left(\sum_{v\sim w} \theta_v \theta_w\right)$$

we have a graph $G=(V,E)$. An independent set is a set of vertices that do not share an edge



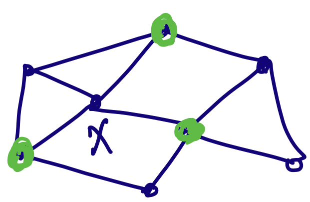
Basic idea: we try to find an ergodic MC whose set of states is the sample space and whose stationary dist is the required sample distribution.

$$X_0, X_1, \ldots, \underbrace{X_r}_{\substack{\text{sample}\\ \text{dist}}} \;\cdots\; \underbrace{X_{2r}}_{\substack{\text{sample}\\ \text{dist}}}, \ldots$$

Let's first construct a uniform dist over the state space Ω.

First we have to design a set of moves to ensure that the chain is irreducible, Let $N(x)$ be all the state that we can reach in one step. We require

$$x \in N(y) \Leftrightarrow y \in N(x)$$



random walk $\rightsquigarrow$ $\dfrac{d(u)}{2|E|}$

For a finite state space Ω and the neighborhood structure $\{N(x), x \in \Omega\}$ Let $N = \max_x |N(x)|$.

Let $M \geq N$. Consider the following MC

$$P_{x,y} = \begin{cases} \frac{1}{M} & \text{if } x \neq y \ \& \ y \in N(x) \\ 0 & y \notin N(x) \\ 1 - \frac{N(x)}{M} & \text{if } x = y \end{cases}$$

$$P_{x,y} = P_{y,x} \implies \pi \text{ is uniform}$$

Let's see how this idea works for sampling from an independent set uniformly.

$G = (V, E)$

① X_0 is an arbitrary indep set in G

② To compute X_{i+1}:

a) choose a node v uniformly at random from V

b) if $v \in X_i$ then $X_{i+1} = X_i \setminus \{v\}$

c) if $v \notin X_i$ and if adding v to X_i still gives an indep set, then $X_{i+1} = X_i \cup \{v\}$

d) otherwise $X_i = X_{i+1}$

$M = |V|$

$X_0 \rightarrow X_1 \rightarrow \cdots X_r$