Section 2.1 The Bisection Method

**Question 2**

**Use the Bisection method to find solutions accurate to within , for**

> f = function(x) {x^4-2\*x^3-4\*x^2+4\*x+4}

1. **[-2, -1]**

> bisection (-2, -1, f)

[1] -1.414214

> bisection (-2, -1, f, iter=true)

Midpt (p) LeftB (a) RightB (b) err (b-a)

1 -1.500000 -2.000000 -1.000000 5.000000e-01

2 -1.250000 -1.500000 -1.000000 2.500000e-01

3 -1.375000 -1.500000 -1.250000 1.250000e-01

4 -1.437500 -1.500000 -1.375000 6.250000e-02

5 -1.406250 -1.437500 -1.375000 3.125000e-02

6 -1.421875 -1.437500 -1.406250 1.562500e-02

7 -1.414062 -1.421875 -1.406250 7.812500e-03

8 -1.417969 -1.421875 -1.414062 3.906250e-03

9 -1.416016 -1.417969 -1.414062 1.953125e-03

10 -1.415039 -1.416016 -1.414062 9.765625e-04

11 -1.414551 -1.415039 -1.414062 4.882812e-04

12 -1.414307 -1.414551 -1.414062 2.441406e-04

13 -1.414185 -1.414307 -1.414062 1.220703e-04

14 -1.414246 -1.414307 -1.414185 6.103516e-05

15 -1.414215 -1.414246 -1.414185 3.051758e-05

16 -1.414200 -1.414215 -1.414185 1.525879e-05

17 -1.414207 -1.414215 -1.414200 7.629395e-06

18 -1.414211 -1.414215 -1.414207 3.814697e-06

19 -1.414213 -1.414215 -1.414211 1.907349e-06

20 -1.414214 -1.414215 -1.414213 9.536743e-07

21 -1.414214 -1.414214 -1.414213 4.768372e-07

22 -1.414213 -1.414214 -1.414213 2.384186e-07

23 -1.414214 -1.414214 -1.414213 1.192093e-07

24 -1.414214 -1.414214 -1.414214 5.960464e-08

To find the answer with 2 decimal accuracy, I choose P7 = 1.414062, so the final P = 1.41

1. **[0, 2]**

> bisection (0, 2, f)

[1] 1.414214

> bisection (0, 2, f, iter=true)

Midpt (p) LeftB (a) RightB (b) err (b-a)

1 1.000000 0.000000 2.000000 1.000000e+00

2 1.500000 1.000000 2.000000 5.000000e-01

3 1.250000 1.000000 1.500000 2.500000e-01

4 1.375000 1.250000 1.500000 1.250000e-01

5 1.437500 1.375000 1.500000 6.250000e-02

6 1.406250 1.375000 1.437500 3.125000e-02

7 1.421875 1.406250 1.437500 1.562500e-02

8 1.414062 1.406250 1.421875 7.812500e-03

9 1.417969 1.414062 1.421875 3.906250e-03

10 1.416016 1.414062 1.417969 1.953125e-03

11 1.415039 1.414062 1.416016 9.765625e-04

12 1.414551 1.414062 1.415039 4.882812e-04

13 1.414307 1.414062 1.414551 2.441406e-04

14 1.414185 1.414062 1.414307 1.220703e-04

15 1.414246 1.414185 1.414307 6.103516e-05

16 1.414215 1.414185 1.414246 3.051758e-05

17 1.414200 1.414185 1.414215 1.525879e-05

18 1.414207 1.414200 1.414215 7.629395e-06

19 1.414211 1.414207 1.414215 3.814697e-06

20 1.414213 1.414211 1.414215 1.907349e-06

21 1.414214 1.414213 1.414215 9.536743e-07

22 1.414214 1.414213 1.414214 4.768372e-07

23 1.414213 1.414213 1.414214 2.384186e-07

24 1.414214 1.414213 1.414214 1.192093e-07

25 1.414214 1.414214 1.414214 5.960464e-08

To find the answer with 2 decimal accuracy, I choose P8 = 1.414062

1. **[2, 3]**

> bisection (2, 3, f)

[1] 2.732051

> bisection (2, 3, f, iter=true)

Midpt (p) LeftB (a) RightB (b) err (b-a)

1 2.500000 2.000000 3.000000 5.000000e-01

2 2.750000 2.500000 3.000000 2.500000e-01

3 2.625000 2.500000 2.750000 1.250000e-01

4 2.687500 2.625000 2.750000 6.250000e-02

5 2.718750 2.687500 2.750000 3.125000e-02

6 2.734375 2.718750 2.750000 1.562500e-02

7 2.726562 2.718750 2.734375 7.812500e-03

8 2.730469 2.726562 2.734375 3.906250e-03

9 2.732422 2.730469 2.734375 1.953125e-03

10 2.731445 2.730469 2.732422 9.765625e-04

11 2.731934 2.731445 2.732422 4.882812e-04

12 2.732178 2.731934 2.732422 2.441406e-04

13 2.732056 2.731934 2.732178 1.220703e-04

14 2.731995 2.731934 2.732056 6.103516e-05

15 2.732025 2.731995 2.732056 3.051758e-05

16 2.732040 2.732025 2.732056 1.525879e-05

17 2.732048 2.732040 2.732056 7.629395e-06

18 2.732052 2.732048 2.732056 3.814697e-06

19 2.732050 2.732048 2.732052 1.907349e-06

20 2.732051 2.732050 2.732052 9.536743e-07

21 2.732050 2.732050 2.732051 4.768372e-07

22 2.732051 2.732050 2.732051 2.384186e-07

23 2.732051 2.732051 2.732051 1.192093e-07

24 2.732051 2.732051 2.732051 5.960464e-08

P7 = 2.726562

1. **[-1, 0]**

> bisection (-1, 0, f)

[1] -0.7320508

> bisection (-1, 0, f, iter=true)

Midpt (p) LeftB (a) RightB (b) err (b-a)

1 -0.5000000 -1.0000000 0.0000000 5.000000e-01

2 -0.7500000 -1.0000000 -0.5000000 2.500000e-01

3 -0.6250000 -0.7500000 -0.5000000 1.250000e-01

4 -0.6875000 -0.7500000 -0.6250000 6.250000e-02

5 -0.7187500 -0.7500000 -0.6875000 3.125000e-02

6 -0.7343750 -0.7500000 -0.7187500 1.562500e-02

7 -0.7265625 -0.7343750 -0.7187500 7.812500e-03

8 -0.7304688 -0.7343750 -0.7265625 3.906250e-03

9 -0.7324219 -0.7343750 -0.7304688 1.953125e-03

10 -0.7314453 -0.7324219 -0.7304688 9.765625e-04

11 -0.7319336 -0.7324219 -0.7314453 4.882812e-04

12 -0.7321777 -0.7324219 -0.7319336 2.441406e-04

13 -0.7320557 -0.7321777 -0.7319336 1.220703e-04

14 -0.7319946 -0.7320557 -0.7319336 6.103516e-05

15 -0.7320251 -0.7320557 -0.7319946 3.051758e-05

16 -0.7320404 -0.7320557 -0.7320251 1.525879e-05

17 -0.7320480 -0.7320557 -0.7320404 7.629395e-06

18 -0.7320518 -0.7320557 -0.7320480 3.814697e-06

19 -0.7320499 -0.7320518 -0.7320480 1.907349e-06

20 -0.7320509 -0.7320518 -0.7320499 9.536743e-07

21 -0.7320504 -0.7320509 -0.7320499 4.768372e-07

22 -0.7320507 -0.7320509 -0.7320504 2.384186e-07

23 -0.7320508 -0.7320509 -0.7320507 1.192093e-07

24 -0.7320508 -0.7320509 -0.7320508 5.960464e-08

P9 = -0.7324219

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**Question 4**

> f = function(x) {2+cos(exp(x) - 2) - exp(x)}

> bisection (0.5, 1.5, f)

[1] 1.007624

> bisection (0.5, 1.5, f, iter=true)

Midpt (p) LeftB (a) RightB (b) err (b-a)

1 1.000000 0.500000 1.500000 5.000000e-01

2 1.250000 1.000000 1.500000 2.500000e-01

3 1.125000 1.000000 1.250000 1.250000e-01

4 1.062500 1.000000 1.125000 6.250000e-02

5 1.031250 1.000000 1.062500 3.125000e-02

6 1.015625 1.000000 1.031250 1.562500e-02

7 1.007812 1.000000 1.015625 7.812500e-03

8 1.003906 1.000000 1.007812 3.906250e-03

9 1.005859 1.003906 1.007812 1.953125e-03

10 1.006836 1.005859 1.007812 9.765625e-04

11 1.007324 1.006836 1.007812 4.882812e-04

12 1.007568 1.007324 1.007812 2.441406e-04

13 1.007690 1.007568 1.007812 1.220703e-04

14 1.007629 1.007568 1.007690 6.103516e-05

15 1.007599 1.007568 1.007629 3.051758e-05

16 1.007614 1.007599 1.007629 1.525879e-05

17 1.007622 1.007614 1.007629 7.629395e-06

18 1.007626 1.007622 1.007629 3.814697e-06

19 1.007624 1.007622 1.007626 1.907349e-06

20 1.007625 1.007624 1.007626 9.536743e-07

21 1.007624 1.007624 1.007625 4.768372e-07

22 1.007624 1.007624 1.007624 2.384186e-07

23 1.007624 1.007624 1.007624 1.192093e-07

24 1.007624 1.007624 1.007624 5.960464e-08

P10 = 1.006836

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**Question 7**

Since

So, we choose the interval [2, 3]

> f = function(x) {x^3 - 25}

> bisection (2, 3, f)

[1] 2.924018

> bisection (2, 3, f, iter=true)

Midpt (p) LeftB (a) RightB (b) err (b-a)

1 2.500000 2.000000 3.000000 5.000000e-01

2 2.750000 2.500000 3.000000 2.500000e-01

3 2.875000 2.750000 3.000000 1.250000e-01

4 2.937500 2.875000 3.000000 6.250000e-02

5 2.906250 2.875000 2.937500 3.125000e-02

6 2.921875 2.906250 2.937500 1.562500e-02

7 2.929688 2.921875 2.937500 7.812500e-03

8 2.925781 2.921875 2.929688 3.906250e-03

9 2.923828 2.921875 2.925781 1.953125e-03

10 2.924805 2.923828 2.925781 9.765625e-04

11 2.924316 2.923828 2.924805 4.882812e-04

12 2.924072 2.923828 2.924316 2.441406e-04

13 2.923950 2.923828 2.924072 1.220703e-04

14 2.924011 2.923950 2.924072 6.103516e-05

15 2.924042 2.924011 2.924072 3.051758e-05

16 2.924026 2.924011 2.924042 1.525879e-05

17 2.924019 2.924011 2.924026 7.629395e-06

18 2.924015 2.924011 2.924019 3.814697e-06

19 2.924017 2.924015 2.924019 1.907349e-06

20 2.924018 2.924017 2.924019 9.536743e-07

21 2.924017 2.924017 2.924018 4.768372e-07

22 2.924018 2.924017 2.924018 2.384186e-07

23 2.924018 2.924018 2.924018 1.192093e-07

24 2.924018 2.924018 2.924018 5.960464e-08

The third root of 25 is approximately P14 = 2.92401

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**Question 9**

|pn - p|

, that is 1x

Simplify and solve for n:

-n

n = 13.287712

> f = function(x) {x^3 - x - 1}

> bisection (1, 2, f)

[1] 1.324718

> bisection (1, 2, f, iter=true)

Midpt (p) LeftB (a) RightB (b) err (b-a)

1 1.500000 1.000000 2.000000 5.000000e-01

2 1.250000 1.000000 1.500000 2.500000e-01

3 1.375000 1.250000 1.500000 1.250000e-01

4 1.312500 1.250000 1.375000 6.250000e-02

5 1.343750 1.312500 1.375000 3.125000e-02

6 1.328125 1.312500 1.343750 1.562500e-02

7 1.320312 1.312500 1.328125 7.812500e-03

8 1.324219 1.320312 1.328125 3.906250e-03

9 1.326172 1.324219 1.328125 1.953125e-03

10 1.325195 1.324219 1.326172 9.765625e-04

11 1.324707 1.324219 1.325195 4.882812e-04

12 1.324951 1.324707 1.325195 2.441406e-04

13 1.324829 1.324707 1.324951 1.220703e-04

14 1.324768 1.324707 1.324829 6.103516e-05

15 1.324738 1.324707 1.324768 3.051758e-05

16 1.324722 1.324707 1.324738 1.525879e-05

17 1.324715 1.324707 1.324722 7.629395e-06

18 1.324718 1.324715 1.324722 3.814697e-06

19 1.324717 1.324715 1.324718 1.907349e-06

20 1.324718 1.324717 1.324718 9.536743e-07

21 1.324718 1.324718 1.324718 4.768372e-07

22 1.324718 1.324718 1.324718 2.384186e-07

23 1.324718 1.324718 1.324718 1.192093e-07

24 1.324718 1.324718 1.324718 5.960464e-08

P14 = 1.324829

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**Question 13**

Define the sequence {Pn}. **Show** that , but that {Pn} diverges.

Given {Pn},

Then Pn = , and Pn-1 =

So, Pn – Pn-1 =

Thus,

By the definition of harmonic series, we can recognize that {Pn} is a harmonic series. By the definition of harmonic series, we also it will always diverge.

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