**2.4 Error Analysis for Iterative Methods**

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**Question 1**

f = function(x){ exp(6\*x) + 3\*exp(2\*x)\*(log(2))^2 - exp(4\*x)\*log(8 )- (log(2))^3 }

fp = function(x){6\* exp(6\*x)+6\* exp(2\*x)\*(log(2))^2 – 4\* exp(4\*x)\*log(8 ) }

Using Newton’s method and the Modified Newton’s method results (using an initial guess of p0 = −.5)

f = function(x) { (exp(2\*x)-log(2))^3 }

fp = function(x) { 6\*(exp(2\*x)-log(2))^2\*exp(2\*x) }

> newtons(-0.5, f, fp, iter = true)

P\_value abs (P - P0)

0 -0.5000000 NA

1 -0.3526384 1.473616e-01

2 -0.2854364 6.720201e-02

3 -0.2476465 3.778993e-02

4 -0.2247398 2.290665e-02

5 -0.2103222 1.441761e-02

6 -0.2010516 9.270572e-03

7 -0.1950131 6.038549e-03

8 -0.1910478 3.965316e-03

9 -0.1884303 2.617448e-03

10 -0.1866968 1.733579e-03

11 -0.1855460 1.150720e-03

12 -0.1847811 7.649423e-04

13 -0.1842721 5.089871e-04

14 -0.1839332 3.388932e-04

15 -0.1837075 2.257374e-04

16 -0.1835571 1.504067e-04

17 -0.1834568 1.002334e-04

18 -0.1833900 6.680556e-05

19 -0.1833455 4.452960e-05

20 -0.1833158 2.968309e-05

21 -0.1832960 1.978726e-05

22 -0.1832828 1.319086e-05

23 -0.1832740 8.793613e-06

24 -0.1832682 5.862280e-06

25 -0.1832643 3.908129e-06

26 -0.1832617 2.605394e-06

27 -0.1832599 1.736918e-06

28 -0.1832588 1.157940e-06

29 -0.1832580 7.719580e-07

30 -0.1832575 5.146377e-07

31 -0.1832571 3.430914e-07

32 -0.1832569 2.287274e-07

33 -0.1832568 1.524848e-07

34 -0.1832567 1.016565e-07

35 -0.1832566 6.777099e-08

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**Question 2**

Using the Modified Newton’s method results (using an initial guess of p0 = −.5)

> f = function(x) {(exp(2\*x)-log(2))^3 }

> fp = function(x) {6\*(exp(2\*x)-log(2)) ^2\*exp(2\*x) }

> fp2 = function(x) {12\*exp(2\*x)\*(exp(2\*x)-log(2))\*(3\*exp(2\*x)-log(2)) }

> mnewtons(-0.5, iter = true)

p\_n err (b-a)

1 -0.2653689 2.346311e-01

2 -0.1896445 7.572443e-02

3 -0.1832971 6.347399e-03

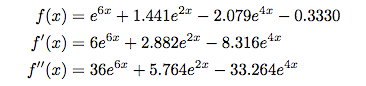
4 -0.1832565 4.063208e-05

5 -0.1832565 1.651055e-09

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**Question 3**

The first and second derivatives of



Using the same initial guess as above gives:

f = function(x){exp(6\*x) + 1.441\*exp(2\*x) - 2.079\*exp(4\*x) - 0.3330}

fp = function(x){6\*exp(6\*x) + 2.882\*exp(2\*x) - 8.316\*exp(4\*x)}

fp2 = function(x){36\*exp(6\*x) + 5.764\*exp(2\*x) - 33.264\*exp(4\*x)}

**newtons** (-0.5, f, fp, 1e-15, 100, iter=TRUE)

P\_value abs (P - P0)

0 -0.5000000 NA

1 -0.3524184 1.475816e-01

2 -0.2849803 6.743813e-02

3 -0.2468135 3.816674e-02

4 -0.2232329 2.358067e-02

5 -0.2075095 1.572340e-02

6 -0.1954008 1.210863e-02

7 -0.1810352 1.436565e-02

8 -0.1525931 2.844209e-02

9 -0.1620145 9.421450e-03

10 -0.1673611 5.346572e-03

11 -0.1693411 1.980020e-03

**mnewtons** (-0.5, f, fp, fp2, eps=1e-5, iter = true)

p\_n err (b-a)

1 -0.2648968 2.351032e-01

2 -0.1855788 7.931810e-02

3 -0.2027591 1.718031e-02

4 -0.4421827 2.394236e-01

5 -0.2394289 2.027538e-01

6 -0.1787358 6.069315e-02

7 -0.1736578 5.078018e-03

8 -0.1706224 3.035346e-03

9 -0.1696706 9.518626e-04

10 -0.1696068 6.376037e-05

11 -0.1696065 2.507491e-07

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**Question 6**

1. show that sequence defined by converges quadratically to zero.

Since , then converges quadratically to zero.

1. Show that sequence defined by does not converges quadratically to zero, regardless of the size of the exponent k > 1.

Since K>1, and

Thus,

By definition, the asymptotic error constant is infinite, which means that it is impossible for the sequence to be quadratically converge to zero for K>1

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**Question 8**

Suppose ***p*** is a root of multiplicity m of is continuous on an open interval containing p.

Show the following fixed-point method has

Given p is a root of multiplicity of m, then it follows that

Taking the derivative of the function

We want to show that p is a zero of

Given:

Let , then

Let’s simplify the function

Taking the derivative of the function

when the function evaluate at ***p***, a lot of the terms will disappear,

Thus

So

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**Question 11**

Rewrite the equation

Let ,

It follows that

Taking the derivative of the function

Since then

So

Taking the derivative of the function

thus

Taking the 2nd derivative of the function

Since

Taking the 2nd derivative of the function

Evaluate at p, yields

So

Thus the 2nd derivative of the function evaluated at ***p*** is zero.

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