**2.5 Accelerating Convergence**

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**Question 1**

b. P0=0.75, Pn = , n = 1,2,3…

> g=function(x) { sqrt(exp(x)/3) }

> fixpt = rep(.75,7) #fixpt[1]=initial guess; values after will be replaced by next line

> for (i in 2:length(fixpt)) fixpt[i] = g(fixpt[i-1]) #one line fixed point method!

> A=cbind(aitkens(fixpt)) #cbind combines vectors as columns of a matrix

> rownames(A)=1:(length(fixpt)-2) #change the row names to the iteration count

> colnames(A)=c("phat") #call the column name phat!

> A

phat

1 0.9078586

2 0.9095675

3 0.9099169

4 0.9099888

5 0.9100037

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**Question 3**

Choosing initial guess P0= 1.5, using Steffensen method

gp3=function(x) { (x+1)^(1/3) }

> steff(1.5, gp3, iter = true)

P\_0 p\_1 p\_2 abs(p - p0)

0 1.500000 NA NA NA

1 1.500000 1.357209 1.330861 0.1751008176

2 1.324899 1.324752 1.324724 0.0001812249

using fixed-point method

> fixedPoint(1.5, gp3, iter = true)

p\_Old abs (P - p\_Old)

0 1.500000 0.000000e+00

1 1.357209 1.427912e-01

2 1.330861 2.634785e-02

3 1.325884 4.977185e-03

4 1.324939 9.444108e-04

5 1.324760 1.793521e-04

6 1.324726 3.406607e-05

7 1.324719 6.470693e-06

8 1.324718 1.229085e-06

9 1.324718 2.334607e-07

10 1.324718 4.434511e-08

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**Question 4**

gp4=function(x) { 2^(-x) }

> steff(0.5, gp4, iter = true)

current\_p p\_1 p\_2 abs(p - p0)

0 0.5000000 NA NA NA

1 0.5000000 0.7071068 0.6125473 0.142187669

2 0.6421877 0.6407406 0.6413836 0.001001877

using fixed-point method

fixedPoint(.5, gp4, iter = true)

p\_Old abs (P - p\_Old)

0 0.5000000 0.000000e+00

1 0.7071068 2.071068e-01

2 0.6125473 9.455945e-02

3 0.6540409 4.149353e-02

4 0.6354978 1.854301e-02

5 0.6437186 8.220796e-03

6 0.6400610 3.657621e-03

7 0.6416858 1.624786e-03

8 0.6409635 7.222699e-04

9 0.6412845 3.209719e-04

10 0.6411419 1.426576e-04

11 0.6412053 6.340098e-05

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**Question 7**

> p = 1/(1:100) # generates a sequence of 100 iterates

> pn=aitkens(p)

> which(round(pn,10)>=.05)

[1] 1 2 3 4 5 6 7 8 9

> pn[1:10]

[1] 0.25000000 0.16666667 0.12500000 0.10000000 0.08333333 0.07142857 0.06250000 0.05555556

[9] 0.05000000 0.04545455

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**Question 8**

A sequence {Pn} is said to be super linearly convergent to ***p*** if

1. Show that if of order for , then {Pn} is super linearly convergent to p.

The definition of Pn converges to P of order is:

Suppose Pn converges to P of order >1.

Then it follows that

Since Pn converges to P of order , so

Which means that

Thus, Pn converges super linearly to P.