3.2 Divided Differences

1.c

Choosing the node close to the , so is in between the nodes.

For the polynomial of degree one interpolates x2 and x3:

> x = c (2.9, 3.0, 3.1, 3.2, 3.4)

> y = c (-4.827866,-4.240058,-3.496909,-2.596792,-0.3330587)

> A = cbind(x, y)

> nidd(A[3:4, ], pi)

$table

[,1] [,2]

[1,] -3.496909 NA

[2,] -2.596792 9.00117

$coef

[1] -3.496909 9.001170

$interp

[1] -3.122526

For degree 2, we choose nodes x1, x2, x3.

> A[2:4 ]

[1] 3.0 3.1 3.2

> nidd(A[2:4, ], pi)

$table

[,1] [,2] [,3]

[1,] -4.240058 NA NA

[2,] -3.496909 7.43149 NA

[3,] -2.596792 9.00117 7.8484

$coef

[1] -4.240058 7.431490 7.848400

$interp

[1] -3.141593

For degree 3, we choose nodes x1, x2, x3, x4.

> A [2:5]

[1] 3.0 3.1 3.2 3.4

> nidd(A[2:5, ], pi)

$table

[,1] [,2] [,3] [,4]

[1,] -4.2400580 NA NA NA

[2,] -3.4969090 7.43149 NA NA

[3,] -2.5967920 9.00117 7.848400 NA

[4,] -0.3330587 11.31867 7.724988 -0.3085292

$coef

[1] -4.2400580 7.4314900 7.8484000 -0.3085292

$interp

[1] -3.141487

For degree 4, we have to use all the nodes

> nidd(A, pi)

$table

[,1] [,2] [,3] [,4] [,5]

[1,] -4.8278660 NA NA NA NA

[2,] -4.2400580 5.87808 NA NA NA

[3,] -3.4969090 7.43149 7.767050 NA NA

[4,] -2.5967920 9.00117 7.848400 0.2711667 NA

[5,] -0.3330587 11.31867 7.724988 -0.3085292 -1.159392

$coef

[1] -4.8278660 5.8780800 7.7670500 0.2711667 -1.1593917

$interp

[1] -3.14159

2.b

> x = (0:4) /10

> y = c (-1, -0.62049958, -0.28398668, 0.00660095, 0.24842440)

> A = cbind(x, y)

> nfdf(A, 0.25)

$coef

[1] -1.00000000 0.37950042 -0.04298752 -0.00293775 0.00009884

$iterp

[1] -0.1327725

It follows that the polynomials that interpolate are, where

4.a

> A = matrix(c(0, -6,

+ + 0.1, -5.89483,

+ + 0.3,-5.65014,

+ + 0.6,-5.17788,

+ + 1.0,-4.28172), ncol=2, byrow= true)

> nidd(A, 0)

$table

[,1] [,2] [,3] [,4] [,5]

[1,] -6.00000 NA NA NA NA

[2,] -5.89483 1.05170 NA NA NA

[3,] -5.65014 1.22345 0.5725000 NA NA

[4,] -5.17788 1.57420 0.7015000 0.2150000 NA

[5,] -4.28172 2.24040 0.9517143 0.2780159 0.06301587

$coef

[1] -6.00000000 1.05170000 0.57250000 0.21500000 0.06301587

$interp

[1] -6

It follows that the interpolating polynomial of degree four is:

P4(x) = −6 + 1.0517x + 0.5725x(x − 0.1) + 0.215x(x − 0.1)(x − 0.3) + .06301587302x(x − 0.1)(x − 0.3)(x − 0.6)

5.a

> x = seq(0, 0.8, 0.2)

> y = c(1, 1.22140, 1.49182, 1.82212, 2.2255)

> A = cbind(x, y)

> nfdf(A, 0.05)

$coef

[1] 1.00000 0.22140 0.04902 0.01086 0.00234

$iterp

[1] 1.05126

It follows that f(0.05) = 1. 05126

5.b

> x = seq(0, 0.8, 0.2)

> y = c(1, 1.22140, 1.49182, 1.82212, 2.2255)

> A = cbind(x, y)

> nbdf(A, 0.65)

$coef

[1] 2.22550 -0.40338 0.07308 -0.01320 0.00234

$iterp

[1] 1.915547

Or use nfdf reverse:

> nfdf(apply(A, 2, rev), 0.65)

$coef

[1] 2.22550 -0.40338 0.07308 -0.01320 0.00234

$iterp

[1] 1.915547

The interpolated value is f(0.65) = 1.915550518.