4.2 Richardson Extrapolation

# (1.b)

f=function(x) {x + exp(x)}

# Three Point Central Difference (Eq. 4.5)

df=function(x,h) { (f(x+h) - f(x-h))/(2\*h)}

Richardson = function (x0, h, n) {

tabl=matrix (, n, n)

tabl[1,1] =df(x0,h)

h=h/2

for (i in 2: n) {

tabl[i,1] = df(x0, h)

for (j in 2:i)

tabl[i,j]=tabl[i,j-1]+(tabl[i,j-1]-tabl[i-1,j-1])/(4^(j-1)-1)

h=h/2

}

return(tabl)

}

options(digits=12)

richardson(0,.4,3)

This gives the following table:

# [, 1] [,2] [,3]

# [1,] 2.02688081451 NA NA

# [2,] 2.00668001271 1.9999464121 NA

# [3,] 2.00166750020 1.9999966627 2.00000001274

Thus, N3(0.4) = 2.000000012735509. The exact answer is 2.

# \*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*

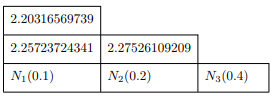
# (1.C)

Example 1 uses the formula (4.5) from section 4.1. Thus, the N1 function is

It follows that

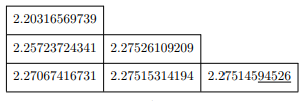
This leads to the first iteration of Richardson’s formula for the derivative:

The table is now:



Note that we fill the table in a row at a time. The best approximation for the derivative so far is N2(0.4) = 2.27526109209. We now proceed with the last row:

which completes the table:



Thus, N3(h) = 2.27514594526 is the best approximation for f 0 (1.05) so far.

The actual answer:

f’ (1.05) = 21.05(cos (1.05) + sin (1.05) ln 2) = 2.27514584172

it agrees to six decimal places!