4.4 Composite Numerical Integration

1.a

Using Trapezoid rule with n = 4 yields h =(b−a)/4=0.25. This gives the following as the partition:



R code:

f = function(x){x\*log(x)}

trapezoid (1,2, f, n=4)

[1] 0.639900477688

Exact answer is

R code:

f = function(x){x\*log(x)}

integrate(f,1,2)

0.63629436112 with absolute error < 7.1e-15

which gives an absolute error of

|0.63990047768799 − 0.63629436111989| = 0.0036061165680954

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

2.a

Using Simpson’s rule with n = 4 yields h =(b−a)/4=0.25. This gives the following as the partition:



R code:

f = function(x){x\*log(x)}

simpsons (1, 2, f, n=4)

[1] 0.636309829797

Exact answer is

which gives an absolute error of

|0.63630982979695 − 0.63629436111989| = 0.000015468677058594

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

3.a

Using Midpoint rule with n = 4 yields , This gives the following as the partition:



R code:

f = function(x){x\*log(x)}

midpt (1, 2, f, n=4)

[1] 0.633096365058

Exact answer is

which gives an absolute error of

|0.63309636505765 − 0.63629436111989| = 0.0031979960622374

(the error is slightly smaller than Trapezoid Rule)!

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The exact answer (according to Wolfram Alpha) is



4. a

Using Composite Trapezoid rule with n = 8 yields , This gives the following as the partition:



R code:

f4 = function(x) {x^2 \* exp(-x^2)}

trapezoid (0, 2, f4, n=8)

[1] 0.421582037198

The absolute error is

|0.4215820371981 − 0.42272505649248| = 0.0011430192943747

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

4. b

Using Composite Trapezoid rule with n = 8 yields , This gives the following as the partition:



R code:

f4 = function(x) {x^2 \* exp(-x^2)}

simpsons (0, 2, f4, n=8)

[1] 0.422716187934

The absolute error is

|0.42271618793398 − 0.42272505649248| = 0.0000088685585001858

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

4. c

Using Composite Midpoint rule with n = 8 yields , This gives the following as the partition:



*4241792084996*

*The absolute error is*

*|0.42417920849963−0.42272505649248|= 0.0014541520071525*

*(the error is slightly smaller than Trapezoid Rule)!*