4.6 Romberg Integration

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1. a

Using R yields the following

f=function(x){x^2\*log(x)}

rm = romberg (1, 1.5, f, 3);

> rm

$R

[,1] [,2] [,3]

[1,] 0.2280741 NA NA

[2,] 0.2012025 0.1922453 NA

[3,] 0.1944945 0.1922585 0.1922593

$intg

[1] 0.1922593

> exact = 1/9\*(1.5^3\*(3\*log (1.5)-1) +1); exact

[1] 0.1922593577328

> abs (exact - rm$intg)

[1] 2.04183521757e-08

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

1. a

R code:

f=function(x) { x^2\*log(x)}

rm=romberg(1,1.5,f,4)

rm

$R

[,1] [,2] [,3] [,4]

[1,] 0.2280741 NA NA NA

[2,] 0.2012025 0.1922453 NA NA

[3,] 0.1944945 0.1922585 0.1922593 NA

[4,] 0.1928181 0.1922593 0.1922594 0.1922594

$intg

[1] 0.1922594

> exact = 1/9\*(1.5^3\*(3\*log (1.5)-1) +1); exact

[1] 0.1922593577328

> abs (exact - rm$intg)

[1] 2.04183521757e-08

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5. a, b, c, d, e

f=function(x) {sqrt(1+cos(x)^2)}

rm=romberg(0,48,f,10);rm

$R

[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]

[1,] 62.43737 NA NA NA NA NA NA NA NA NA

[2,] 57.28856 55.57229 NA NA NA NA NA NA NA NA

[3,] 56.44375 56.16215 56.20147 NA NA NA NA NA NA NA

[4,] 56.26305 56.20282 56.20553 56.20560 NA NA NA NA NA NA

[5,] 56.21877 56.20401 56.20409 56.20407 56.20406 NA NA NA NA NA

[6,] 58.36268 59.07732 59.26887 59.31752 59.32973 59.33279 NA NA NA NA

[7,] 58.45044 58.47970 58.43985 58.42669 58.42320 58.42232 58.42209 NA NA NA

[8,] 58.46557 58.47061 58.47001 58.47048 58.47066 58.47070 58.47071 58.47072 NA NA

[9,] 58.46925 58.47048 58.47047 58.47048 58.47048 58.47048 58.47048 58.47048 58.47048 NA

[10,] 58.47017 58.47047 58.47047 58.47047 58.47047 58.47047 58.47047 58.47047 58.47047 58.47047

$intg

[1] 58.47047

the predictions wildly vary:

* R1,1 62.437371400655
* R2,2 55.572291687125
* R3,3 56.201470719242
* R4,4 56.205598853742
* R5,5 56.204062386108
* R6,6 59.332787007054
* R7,7 58.422092969344
* R8,8 58.470717384470
* R9,9 58.470479083106
* R10,10 58.470469052880

\*

It looks like it’s converging to 56.20, but changes a lot at R6,6. This comes because of the large interval that we are using, and that f is periodic. The problem at the sixth level is that the h is nearly a multiple of π/2, which gives many values at the maximum of f. This exaggerates the estimate of the integral.  
If we take advantage of the period of f, then we can get a better estimate with fewer iterations. For example, the curve is a periodic function with over 15 periods. If we find the value from 0 to π/2 and then multiply by 30, and integrate the last partial period, we should get a better answer with smaller n. We are able to get all digits of accuracy with only n = 6 instead.

rm = romberg(0, pi/2, f, 6);rm

$R

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 1.896119 NA NA NA NA NA

[2,] 1.909972 1.914589 NA NA NA NA

[3,] 1.910099 1.910141 1.909845 NA NA NA

[4,] 1.910099 1.910099 1.910096 1.910100 NA NA

[5,] 1.910099 1.910099 1.910099 1.910099 1.910099 NA

[6,] 1.910099 1.910099 1.910099 1.910099 1.910099 1.910099

$intg

[1] 1.910099

rm2=romberg(30\*pi/2, 48, f, 6);rm2

$R

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 1.139625 NA NA NA NA NA

[2,] 1.160793 1.167849 NA NA NA NA

[3,] 1.165841 1.167524 1.167502 NA NA NA

[4,] 1.167088 1.167504 1.167502 1.167502 NA NA

[5,] 1.167399 1.167502 1.167502 1.167502 1.167502 NA

[6,] 1.167476 1.167502 1.167502 1.167502 1.167502 1.167502

$intg

[1] 1.167502

> 30\*rm$intg+rm2$intg

[1] 58.47047

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We can compare to the exact answer, which is can be found in R as it is related to the distribution function of the standard normal distribution:

erf (1) = 2\*Φ() − 1 = 0.84270079294971

> 2\*pnorm (sqrt (2))-1

[1] 0.8427008

erf = function(x) { 2/sqrt(pi)\*exp(-x^2)}

options (digits = 12)

$R

[,1] [,2] [,3] [,4] [,5] [,6] [,7]

[1,] 0.771743332258 NA NA NA NA NA NA

[2,] 0.825262955597 0.843102830043 NA NA NA NA NA

[3,] 0.838367777441 0.842736051389 0.842711599479 NA NA NA NA

[4,] 0.841619221245 0.842703035846 0.842700834810 0.842700663942 NA NA NA

[5,] 0.842430505490 0.842700933572 0.842700793420 0.842700792763 0.842700793269 NA NA

[6,] 0.842633227681 0.842700801745 0.842700792956 0.842700792949 0.842700792950 0.84270079295 NA

[7,] 0.842683902045 0.842700793500 0.842700792950 0.842700792950 0.842700792950 0.84270079295 0.84270079295

$intg

[1] 0.84270079295

It agrees with the answer at the R7,7 spot.