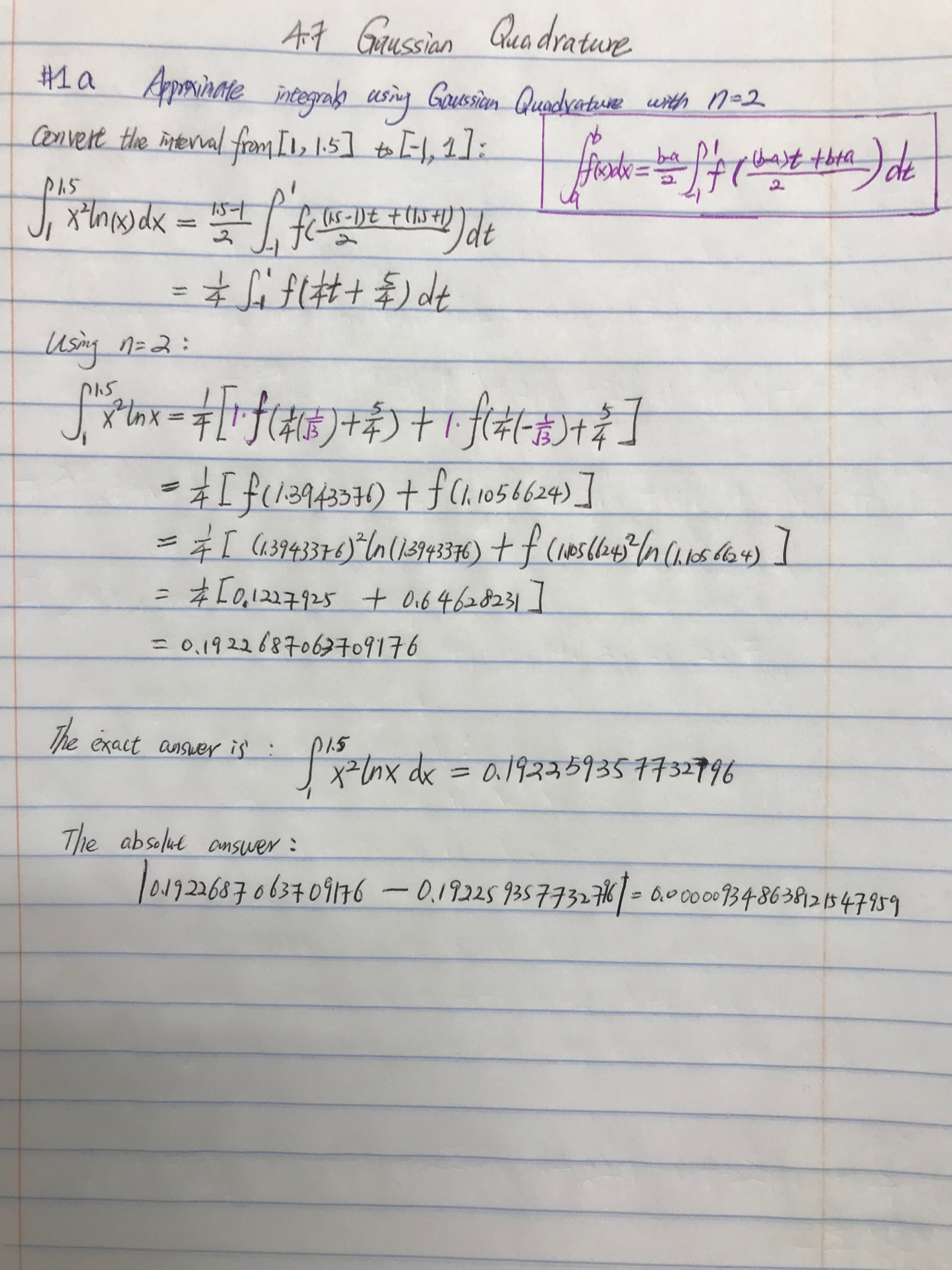
4.7 Gaussian Quadrature

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

1. a



+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

1. a

Using n=3:

1.056350832689629

=0. 192259377256879

The absolute error is

|0.192259377256879−0.192259357732796|= 0.00000001952408298921959

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

1. a

Using n = 4, we have

= 0.25{0.3478548451374538 ***f*** (0.25(−0.8611363115940526) + 1.25)

+ 0.6521451548625461 ***f***(0.25(−0.3399810435848563) +1.25)

+0.6521451548625461 ***f***(0.25( 0.3399810435848563) + 1.25)

+0.3478548451374538 ***f***(0.25( 0.8611363115940526) + 1.25)}

= 0.25 {0.01270976103900079 + 0.13517931157193655 + 0.33580948870208782

+ 0.28533886990642765}

=0.1922593578048632

The absolute error is

|0.1922593578048632−0.192259357732796|= 0.00000000007206715779695116

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

1. a

using R code, with n= 5:

f=function(x){x^2\*log(x)}

gaussianquad (1, 1.5, f, 5)

[1] 0.192259357733

> actual = 1/9\*(1.5^3\*(3\*log(1.5)-1)+1);actual

[1] 0.192259357732796

> abs (gaussianquad (1,1.5, f,5)- actual)

[1] 3.460287612000457e-13

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

1. a

Let’s start with the n = 2 case.

The Legendre polynomial is , which has the roots of and =

The formula for the c constants is found in the preamble of Theorem 4.7 of our book.

Thus

Since =

The answer is exactly the same for c2 as all you have to do is switch indices of the xi’s in the problem.

The answer will finish with

Thus, the nodes and coefficients for the Gaussian Quadrature rule with n = 2 is

and,

Next, for the case of n = 3.

The Legendre polynomial is x(x2−3/5) =0*,* which has the roots of and

Using the formula in the preamble of Theorem 4.7 again (with n = 3) yields:

