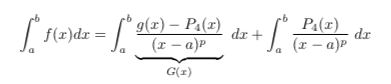
4.9 Improper Integrals

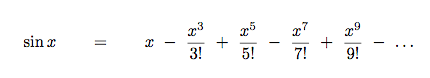
+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

1.a

Using the formula:



Taylor Polynomial of sin(x):



For this problem g(x) = sin x, a = 0, b = 1,

Now, integrating using Simpson’s Rule and combining the result with the above yields:

G = function(x) {

z = (sin(x) - x + x ^ 3 / 6) / x ^ (1 / 4)

z[is.na(z)] = 0

z

}

options (digits = 12)

simp = simpsons (0, 1, G, n = 4, plot = true)

simp

[1] 0.001432199

simp + 166/315

[1] 0.528416325711

Thus,

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

2.a

For this problem, a change of variable can make the problem easier (it switches the singularity from the right endpoint to the left):

Let u =1-x; du=-dx; -x = u-1

Thus, apply this method to the last integral (and multiply later by later.

It follows that

And

Now, integrating , using Simpson’s Rule and combining the result with the above yields:

G2 = function(x){

z = (exp(x) - (1+x +(x^2)/2 +(x^3)/6 +(x^4)/24) ) / sqrt(x)

z[x == 0] = 0;

z

}

simp = simpsons(0, 1, G2, n = 6, plot = true)

simp

[1] 0.00176061533325

exp(-1)\*(simp + 11051/3780)

[1] 1.07615978529

Thus,

+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

3.a

First convert the improper integral into one that has a left endpoint singularity at zero, in this form:

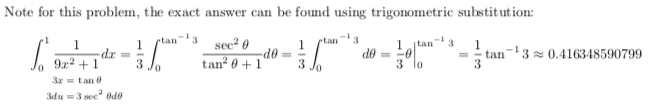
Let

Using Simpson’s with n = 4 yields:

f = function(x) { 1/(9\* x^2 +1) }

simpsons(0, 1, f, 4, plot=TRUE)

[1] 0.411264869151



+\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*\*+

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