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International portfolio of stock indices with spatiotemporal correlations: Can investors still benefit from portfolio, when and where?

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ABSTRACT

This paper revisits the topic of international portfolio of stock indices under spatiotemporal correlations, to help people to get better portfolio performance in international stock markets. We firstly develop a mean-*VaR* framework as well as a mean-*CVaR* one, where both of which are with spatiotemporal correlation and other constraints. Then we apply these two frameworks to investigate whether investors can gain in international stock markets or not under various constraints. Our empirical results find that 1) Investors can still benefit from the international portfolio with spatiotemporal correlations, either in the view of avoiding risk or pursuing the profit; 2) the spatiotemporal correlation and exchange rate contribute to the performance of portfolio significantly, and transaction cost and fixed income rarely have effect on the portfolio, both in crisis and calm periods. Additionally, in the period of calm, the skewness of each single return series has some significant impact on the portfolio performance; 3) the portfolio with lowest spatiotemporal correlation with other markets is the optimal choice. In addition, in the calm period, another suitable area can be the one with positive mean and negative skewness of returns, such as in the U.K. market; 4) the mean-*CVaR* framework outperforms the mean-*VaR* one in financial calm period, but equals to the latter in crisis time. Our results demonstrate that the proposed mean-*CVaR* programming framework with spatiotemporal correlation provides a more flexible and effective decision support tool for international portfolio.

1. Introduction

The potential benefits of international diversification were recognized early; see, for example, Balli, Basher, and Jean Louis (2013), Balli, Pericoli, and Pierucci (2014), Berger, Pukthuanthong, and Jimmy Yang (2011), Narayan, Sriananthakumar, and Islam (2014), Solnik (1974) and more recently Balcılar, Demirer, and Hammoudeh (2015).They came to a common conclusion that due to comparably low correlations of international securities, improvements in reward-to-risk performance can be achieved by holding internationally diversified portfolios. Despite a general increase in the correlation of international markets in recent years, which lead to financial globalization and market integration, international investments continue to provide a wider scope for portfolio diversification than that is available in domestic market.

However, over the past few decades, diminishing benefits resulting from international diversification has been an imperative topic

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for academic researchers and international investors (see, e.g., Balcılar et al., 2015; Balli et al., 2013; Berger et al., 2011; Narayan et al., 2014). This phenomenon is first observed and documented within stock markets in developed countries, but is quickly spread to emerging countries (see, e.g., Christoffersen, Errunza, Jacobs, & Jin, 2014; Christoffersen, Errunza, Jacobs, & Langlois, 2012; Lahrech & Sylwester, 2011). In sum, these studies present that if under the same risk, international diversification yields higher returns than that in domestic portfolio, but the benefit is declining. That is to say, from the view of risk adjusted returns, the performance of international portfolio is better than the one purely of domestic diversification. However, the conclusion deducing from above are all from the perspective of time series correlation. Meanwhile, more and more research notices that taking spatiotemporal correlations of indices into account is very necessity (see for example the studies by Tam, 2014; Asgharian, Hess, & Liu, 2013; Fernández-Avilés, Montero, & Orlov, 2012; Gong & Weng, 2016; Weng & Gong, 2016; Ouyang, Zheng, & Jiang, 2014; Cai, Cui, Huang, & Sun, 2017). In this case, one major issue arises, that is whether or not investors can still benefit from the international stock portfolio under spatiotemporal correlation?

In addition, previous literature has brought out many risk factors contributing to international investment. For example, Eun and Resnick (1988) pointed out that international diversification decisions should take currency risk into account. Longin and Solnik (1995) identified that international portfolio of stock indices was influence by many factors, including financial liberalization, international trade, lower market restrictions, reduced transaction costs and advances in communication technologies. Meanwhile, fixed income was also something important to a portfolio (see, e.g., Chen & Wang, 2017; Bessler, Leonhardt, & Wolff, 2016). Besides, the co-movement of stock market was a key topic in finance portfolios, and it was generally believed that the increased market liberalization and integration should lead to increased correlations among stock markets. For instance, Berger et al. (2011) showed that the frontier markets offered significant diversification benefits due to their low integration with the world market. Also, Bergin and Pyun (2016), Eun and Lee (2010); Lahrech and Sylwester (2011) addressed that the correlation and convergence of various markets contribute a lot to a portfolio.

Given the importance of influent factors especially co-movement in determining the magnitude of gains, the second issue arises. That is, which factors are the drivers of international portfolio of stock indices?

Besides, in the aspect of research methods for international investment, some literature already existed, see, for example, De Santis and Gérard (2009) applied gravity methodology to access the optimality of portfolios. At the same time, Rua and Nunes (2009) employed the wavelet analysis to measure the co-movement of assets in the time–frequency space. Then Făt and Dezsi (2012) adopted Principal Component Analysis (PCA) and the Maximum Likelihood (ML) method to study the patterns of relationship of the stock markets. And then Rahim and Masih (2016) applied MGARCH-DCC and wavelet models to figure out whether the Islamic investors can benefit from the portfolio of assets or not. Whereas, the methods mentioned above are not reasonable due to the fact that the investment of assets are only with correlation of time series. Recently, though Asgharian et al. (2013) employed the spatial Durbin model to investigate to what extent that the economic and geographical relations of countries affected the co-movement of their stock markets, it does not account for serially correlation of the residuals of stock returns. However, numerous researchers have presented evidence that the residual of stock returns are serially correlated and in recent years investors have been able to exploit this dependence to obtain abnormally positive expected returns (see, e.g., Lee & Swaminathan, 2000; DeMiguel, Nogales, & Uppal, 2014; Hvidkjaer, 2006; Jegadeesh & Titman, 1993; Lee, Huang, Kuo, & Lee, 2012). Thus, we take serially correlation of the residuals in spatiotemporal model to analyze the international investment of stock indices in our paper.

The objective of our paper are three-fold: 1) from the perspective of investors, to test whether or not they can benefit (or risk sharing) in international investment of stock indices with spatiotemporal correlation; 2) analyzes and testifies the friction factors in international portfolio of stock indices with spatiotemporal correlation; 3) proposes a general and flexible strategy to support the decisions for international portfolio of stock indices, comparing to the problem-dependent strategies in previous literature.

The rest of this paper is organized as follows. Section 2 describes the spatial econometric model that can characterize the spatiotemporal correlation of stock indices and provides a brief overview of the VaR and CVaR measures. Then, Section 3 presents the study design, including the research framework, the empirical data and portfolio management based on the econometric model. And then, Section 4 analyzes the empirical results to investigate the research objective. Finally, the last section discuss about the conclusions and implications.

2. Study models and methods

Before the study, some related models and methods are firstly introduced. In our paper, we will use spatiotemporal-AR model (Gong & Weng, 2016) as the representative of spatial econometric model to characterize the spatiotemporal correlation of stock indices. In addition, we construct the mean-VaR and mean-CVaR frameworks to investigate the performance of international portfolio.

2.1. Spatiotemporal-AR model

Gong and Weng (2016) proposed a spatiotemporal-AR model with three types of spatial dependence including global, industrial branch and regional dependences. They assumed that the residuals of the regression model were serially correlated. Specifically, the spatiotemporal-AR model is expressed as

$$\mathbf{Y}_{N,t} = \lambda_1 \boldsymbol{W}_{1,t}\mathbf{Y}_{N,t} + \lambda_2 \boldsymbol{W}_{2,t}\mathbf{Y}_{N,t} + \lambda_3 \boldsymbol{W}_{3,t}\mathbf{Y}_{N,t} + \boldsymbol{X}_{N,t}\beta + \varepsilon_{N,t},\ \boldsymbol{t} = 1, 2, \cdots, \boldsymbol{T}, \tag{1}$$

$$\varepsilon_{N,t} = \rho\varepsilon_{N,t-1} + e_{N,t}, \tag{2}$$

where$\mathbf{Y}_{N,t}$ denotes an $N \times 1$vector of returns of N stock indices at time t and is used as dependent variable, that is, $\mathbf{Y}_{N,t} = (y_{1,t}, y_{2,t}, \cdots, y_{N,t})'$.$\lambda_1$ is the general dependence parameter; λ_2 represents the parameter of dependence inside industrial branches; λ_3 denotes the regional dependence parameter. Correspondingly, $\boldsymbol{W}_{1,t}$, $\boldsymbol{W}_{2,t}$, $\boldsymbol{W}_{3,t}$ are the spatial weight matrices that reflect dependence in global, industrial branch and regional at time $\boldsymbol{t}$ respectively.$\mathbf{X}_{N,t}$ is an $N \times k$matrix of explanatory variables including *GDP, Re, R, Trade* and R^e. And in these variables, *GDP* stands for Gross Domestic Product growth rate; *CPI* is the inflation rate; *M2* represents broad money growth rate; *Re* is noted as the international reserve growth rate; *R* stands for short-term lending rates; while Trade is import and export trade level and R^e refers to exchange rate. Lastly, ρ denotes the serial correlation coefficient of residual $\varepsilon_{N,t-1}$ and $\boldsymbol{e}_{N,t}$ is an $N \times 1$ vector of the *i. i. d*errors term with $\boldsymbol{e}_{N,t}$ $\boldsymbol{N}(0, \sigma^2\boldsymbol{I}_N)$.

In our paper, the three types of spatial dependence are cut into one, the reason for that is the research object of Gong and Weng (2016) is confined to Chinese stock market and its research data is chosen from individual stocks, which can characterize the spatial effect in the global, industrial branch and regional wide. However, the research object of our paper is global stock markets, and composite indices are used as proxy variables. This means that the composite indices have reflected the price movements and trends of overall individual stock market in every country. Thus, it is hard and unnecessary to tell the influence of industrial and regional dependence apart in the international composite indices. In addition, the values of λ_2 and λ_3 are far smaller than the one of λ_1, see in Gong and Weng (2016) for more details. For these two reasons, we only take the spatial dependence of stock markets in global wide into account. Thus, we can transform Eq. (1) into the following form

$$\mathbf{Y}_{N,t} = \lambda_1 \boldsymbol{W}_{1,t}\mathbf{Y}_{N,t} + \boldsymbol{X}_{N,t}\boldsymbol{\beta} + \varepsilon_{N,t}, \quad \boldsymbol{t} = 1, 2, \cdots, \boldsymbol{T}. \tag{1}$$

In addition, the spatial weights are defined as the proximity of development speed of GDP, which implies that the closer speed of development is, the greater spatial interactions between the two indices are. The reason for that is more and more researches consider that the proximity of culture and institution is the cause of spatial interaction between two economies (Ahmad & Hall, 2017; Brennan & Martin, 2012), and the spatial weights matrix must extend the notion of proximity from geographical proximity to economic proximity, technological proximity, social proximity, etc. (Liu & Prucha, 2018). Therefore, we can set up a spatial weights matrix based on the proximity of development speed of GDP which represents a kind of economic institution.

2.2. VaR and CVaR

As a tool of measuring and managing risk, *VaR* has been widely applied by investment institutions, which originally proposed by JPMorgan Bank to fill the gap in measures of market risk. Generally, we adopt the definition of *VaR* defined by Jorion (1991) that *VaR* is the maximum expected loss within a certain target range under a certain confidence interval. Mathematically, the *VaR* can be expressed by

$$\mathbf{P}(\boldsymbol{R_t} \le \mathbf{VaR}_t^{\alpha}|\Omega_{t-1}) = \boldsymbol{\alpha}, \tag{3}$$

where $\boldsymbol{R_t}(\mathbf{t} = 1, 2, \cdots, \boldsymbol{T})$ stands for the time series of portfolio returns of assets, $\boldsymbol{\alpha}$ is the probability level and Ω_{t-1} denotes the information set at time $\boldsymbol{t} - 1$. In other words, the one-step-ahead *VaR* at α probability level can be computed as

$$\boldsymbol{VaR_t^a} = \hat{\boldsymbol{\mu}} + \varnothing^{-1}(\boldsymbol{\alpha}) * \sqrt[2]{\hat{\boldsymbol{\sigma}}_{port}} \tag{4}$$

where $\hat{\boldsymbol{\mu}}$ denotes the predicted portfolio return, $\varnothing^{-1}(\boldsymbol{\alpha})$ is the $\boldsymbol{\alpha}$-quantile of the standard normal distribution and $\hat{\boldsymbol{\sigma}}_{port}$is the predicted portfolio variance for additional information(Gong & Weng, 2016).

However, using *VaR* as a measure of portfolio risk is a very difficult problem. That is, if the distribution of portfolio returns is not subject to normal or log-normal, the VaR will present its non-convex and non-subadditive nature (Artzner et al., 1999; Mausser & Rosen, 1998). It implies that if portfolio returns is a non-normal distribution, the corresponding *VaR* will generate multiple local extremes that efficient optimization techniques cannot be applied. In view of this, scholars are forced to search for a similar percentile risk measure. Thus, *CVaR* is a perfect candidate for conducting a *VaR*-style portfolio management.

Theorem:. *For a portfolio return variable $R_t(t = 1, 2, \cdots, T)$ with distribution function F, the CVaR of R_t at the $\alpha \in (0, 1)$ quantile level, which proposed by* Rockafeller and Uryasev (2000) *, can be defined as*

$$\mathrm{CVaR}_\alpha = E[-R_t\,|-R_t \geqslant VaR_t^\alpha] = E[f(x,y)|f(x,y) \geqslant VaR_t^\alpha] \; = VaR_t^\alpha + 1\Big/T(1-\alpha)\sum_{t=1}^{T}[f(x,y)_t\_VaR_t^\alpha, 0] \tag{5}$$

where E[.] is the conditional expectation on the condition of $\{-R_t \ge VaR_t^\alpha\}$. That is to say, the threshold meets

$$\Pr[-R_t \ge VaR_t^\alpha] = \alpha \text{ or } VaR_t^\alpha = -F_t^{-1}(\alpha) \tag{6}$$

where $\Pr[.]$ is the probability of an event and $F^{-1}(.)$ is the inverse function of the distribution.

3. Study design

In this section, we outline how to apply the spatiotemporal-AR model to manage the international portfolio. We describe the research framework in Section 3.1 and data-set in Section 3.2. Next, in Section 3.3, we construct the programming framework to

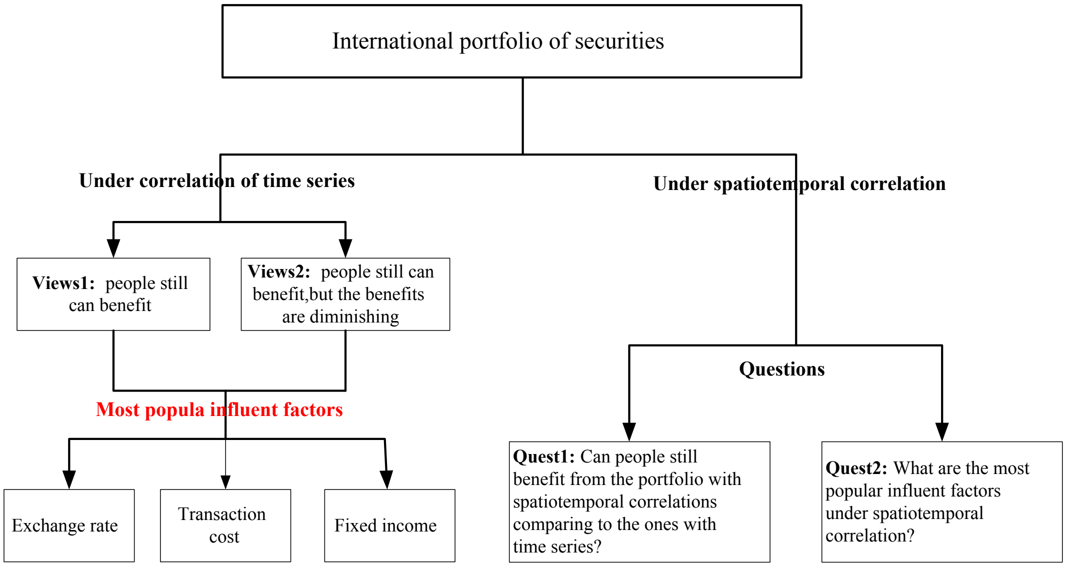


Fig. 1. Research framework of this paper.

evaluate the performance of the portfolio based on the spatiotemporal-AR model.

3.1. Research framework

Our research rout is: By comparing the performance of the portfolio under two kinds of correlations, including correlation of time series and the one of spatiotemporal, to figure out some questions. To find out more information, we can refer to Fig. 1.

From Fig. 1, we can see that this figure is divided into two parts, the left one and the right one. Where, the left one is under correlation of time series and describes the views on the international portfolio of stock indices as well as the influent factors, which are all drawn from the literature mentioned in the introduction. As to the right part under spatiotemporal correlation, there are two questions need to be solved, which is exactly the objective of our study.

3.2. Data

There are several notes to be illustrated about the empirical data: 1). our data consists of six stock indices, including China (SSE), U.S. (DJI), Japan (N225), Germany (DAX), France (FCHI) and U.K. (ISEQ). The data series chosen here are from Dec 1, 2006 to May 31, 2015, and the returns of each series are in daily logarithmic form, which are defined as $Y_t = lnP_t^s - lnP_{t-1}^s$, where P_t^s is the price of stock index at time t. The data selection is based on the following two reasons: (1) they are typically to be our study sample, since the six stock indices represent the major financial markets, which ranging from the developed countries to developing states. (2) The time range selection is also reasonable. Specifically, choosing Dec 1, 2006 as the starting point is the reason that financial market is still in a calm period before the time. Later on, there are turmoil financial crisis and post crisis. So the sample data covers the calm and crisis eras and is very representative. 2). The entire sample is divided into five sub-samples. The reason for that is Asgharian et al. (2013) found the spatial impacts change over time, so it is necessary to divide the whole research sample into several subsamples. Also, in Gong and Weng (2016), they break the entire sample into three subsamples according to the timeline of the global financial crisis (GFC), in view of the spatial impacts perform differently between two GFCs. , using the dating method suggested by Refs. (Lleo & Ziemba, 2012; Tamakoshi & Hamori, 2014; Zhang, Podobnik, Kenett, & Eugene Stanley, 2014). Whereas other works identify financial crises based on the authors' judgment or chronological events (Hlaing & Kakinaka, 2017; Laeven & Valencia, 2008). Recent studies on international financial flows identify episodes of sudden stop and flight as a sign of the deterioration of economic conditions (Waelti, 2015). Referring to these researches, in our paper, we divide the full sample into five sub-sample periods. Then, the partitioned result is summarized as follows: Subprime mortgage crisis is from Dec 1, 2006 to Mar 31, 2008; Global financial tsunami period is the period during Apr 1, 2008 to Nov 28, 2008; Real economy crisis/secondary disaster starts on Dec 1, 2008 and ends on Nov 30, 2009; while European debt crisis period captures the period from Dec 1, 2009 to Feb 29, 2012 and financial and economic recovery period refers to Mar 1, 2012 to May 31, 2015. In each period, we cut them into two parts once again: one is used as a rolling window of trading days to estimate spatiotemporal dependence parameters; the other one is applied to predict the VaR and CVaR of out-of-samples. 3). Other data collected to be used as the friction factors in the programming frameworks, such as exchange rate, deposit rate and so on, are all changed into daily logarithmic form.

For the convenience of subsequent sections, the returns and skewness of each series of stock index are shown in Table 1. From this table, we can see that though the mean of each series of return is very small, the distribution is severely skewed. And most the skewness of the stock indices is negative but the mean values are positive, the reason for that may be the returns are negative in most

Table 1
Summary statistics of the returns of stock indices in different stock markets.

Country	Entire period		Subprime mortgage crisis		Global financial tsunami	
	Mean	Skewness(%)	Mean	Skewness(%)	Mean	Skewness(%)
U.S.	0.0002	−26.8617	0.0002	−22.5537	−0.0022	−5.2209
Germany	0.0003	−13.8539	0.0003	−51.0760	−0.0019	24.5336
France	−0.0001	−2.3835	0.0000	−41.8284	−0.0022	20.6332
U.K.	−0.0002	56.6421	−0.0004	4.5053	−0.0051	21.8636
Japan	0.0000	−66.3705	−0.0004	−23.6771	−0.0024	−50.7498
China	0.0005	−40.2096	0.0019	−91.3100	−0.0030	43.1794
Country	The real economy crisis/secondary disaster		European debt crisis period		Financial and economic recovery period	
	Mean	Skewness(%)	Mean	Skewness(%)	Mean	Skewness(%)
U.S.	0.0008	21.0753	0.0004	−33.2999	0.0004	−16.4200
Germany	0.0008	−16.2331	0.0004	−11.4847	0.0005	−36.9007
France	0.0006	−4.7428	−0.0001	15.5492	0.0003	−22.7297
U.K.	0.0005	−30.0940	0.0002	−7.5819	0.0007	57.9177
Japan	0.0006	−13.3804	0.0000	−110.7531	0.0008	−30.6370
China	0.0019	−63.3990	−0.0005	−38.9296	0.0004	−4.6234

Notes: Returns refer to the stock-index log returns. The St. Dev. mentioned above is in percentage. Subprime mortgage crisis refers to the period Dec 1, 2006-Mar 31, 2008; Global financial tsunami is the period during Apr 1, 2008-Nov 28, 2008; Real economy crisis/secondary disaster is from Dec 1, 2008 to Nov 30, 2009; while European debt crisis period is Dec 1, 2009–Feb 29, 2012 and financial and economic recovery period refers to Mar 1, 2012-May 31, 2015.
Source: All data is from the Wind Financial Database of China.

time but a few positive values are strong enough to influence the negative ones. As a result, the distribution of the return of each stock index is left offset and has a positive mean value. On the contrary, the skewness of some indices are positive and the mean values are negative, the reason for that is the returns are positive in most time but a few negative values are strong enough to influence the positive ones, which result as negative values. In addition, there are several indices, e.g. the index of U.S. in the real economy crisis/ secondary disaster and the index of the U.K. in financial and economic recovery period, which have both positive skewness and mean values. The reason for that is the sum of few positive values is overweight the most one of negative values. Thus, we can deduce that the data is not in a normal distribution, and the series is not independence. Furthermore, we can believe that a spatiotemporal model is sufficient to deliver the white noise residuals needed to estimate the conditional correlations.

3.3. *Portfolio management using spatiotemporal-AR model*

For commercial banks and individual investors, one of the major concerns is to minimize the risk of investment portfolio. To address this concern, we compute the *VaR* and *CVaR* metrics based on the spatiotemporal-AR model, and set minimal values of the metrics as the objective of programming framework to help investors make a portfolio management. In addition, we select three popular factors shown in Fig. 1, including exchange rate, transaction cost and fixed income, as the constraints of portfolio. As for the reason of choosing the three factors as constraints, we try to test that whether the most popular three factors under correlation of time series still contribute to the portfolio under spatiotemporal correlation. Thus, we have construct the mean-*VaR* and mean-*CVaR* programming frameworks. More details can be seen in Section 4.1.

3.3.1. *Mean-VaR and mean-CVaR programming frameworks*

In our two programming frameworks, we choose the macro variables to be the explanatory variables in the regression Eq.(1)', and other factors such as fixed return and transaction cost are included only in the simulation framework. Reasons for that are 1) Generally speaking, we choose the macro variables such as changes in exchange rate, unexpected inflation, GDP growth and changes in interest rate to be the explanatory variables in international investment, see in Asgharian et al. (2013); 2) The objective functions and constraints can be transformed into each other. Thus, transaction costs and other factors relegated to the constraints is equivalent to the one in the target regression model. More specific can be seen as follows:

In the mean-*VaR* framework, the objective function is to minimize the *VaR* which is based on the spatiotemporal-AR model. X_i, $i = 1, 2, \cdots, n$, denotes the weight of individual asset, where $0 \le X_i \le 1$ and $x_1 + x_2 + \cdots + x_n = 1$. R_t is the portfolio returns at time t and it is computed from formula (c). R_0 denotes the expected portfolio returns of deposit rate with equal weighted. C_i refers to transaction cost , which applies the form of V-Shaped cost here for its convexity and easy computation. C_i^r is the ratio of transaction cost and is commonly within the range of $0.001 \sim 0.003$. Y_i and R^e are the log form returns of stock indices and exchange rate, respectively. Thus, we can figure out some features of the constraints from formula (a) ∼ (d). That is, the formula (a) can ensure the investor get a guaranteed of minimum income and formula (b) is the constraint of transaction cost. While for formula (c), it is the portfolio revenue and is composed of the returns from stock indices and the currency. The formula (d) is the short sales constraint, since some emerging markets are subject to such kind of securities regulator measures, e.g. China market.

Mean-*VaR* framework

$$\mathbf{minVaR} \tag{7}$$

$$st.\begin{cases} \boldsymbol{R_t} \geqslant \boldsymbol{R}_0 & (a) \\ \boldsymbol{C_t} = \sum_{\boldsymbol{i}=1}^{\boldsymbol{n}} \boldsymbol{C_t^i}\ |\mathrm{x}_{\boldsymbol{i}}^1 - \mathrm{x}_{\boldsymbol{i}}^0| & (b) \\ \boldsymbol{R_t} = \sum_{\boldsymbol{i}=1}^{\boldsymbol{n}} \mathrm{x}_i^t \boldsymbol{Y_i^t} + \sum_{\boldsymbol{i}=1}^{\boldsymbol{n}} \mathrm{x}_i^t \boldsymbol{R_i^e} - \boldsymbol{C_t} & (c) \\ \sum_{\boldsymbol{i}=1}^{\boldsymbol{n}} \mathrm{x}_i^t = 1,\ 0 \leqslant \mathrm{x}_i^t \leqslant 1,\ \ \boldsymbol{t} = 1,\ 2, \ldots, 5 & (d) \end{cases}$$

Mean-*CVaR* framework

$$\boldsymbol{minF}_\alpha(\mathrm{x},\eta_\alpha) = \eta_\alpha + 1\Big/(1-\alpha)T\sum_{t=1}^{T} Z^t \tag{8}$$

$$st.\begin{cases} \boldsymbol{R_t} \geqslant \boldsymbol{R}_0 & (e) \\ Z^t \geqslant -\boldsymbol{R_t} - \eta_\alpha & (f) \\ Z^t \geqslant 0 & (g) \\ \eta_\alpha = VaR_\alpha & (h) \\ \sum_{i=1}^{n} \mathrm{x}_i^t = 1,\ 0 \leqslant \mathrm{x}_i^t \leqslant 1, & (i) \\ \boldsymbol{C_t} = \sum_{i=1}^{n} \boldsymbol{C_t^i}\ |\mathrm{x}_{\boldsymbol{i}}^1 - \mathrm{x}_{\boldsymbol{i}}^0| & (j) \\ \boldsymbol{R_t} = \sum_{i=1}^{n} \mathrm{x}_i^t \boldsymbol{Y_i^t} + \sum_{i=1}^{n} \mathrm{x}_i^t \boldsymbol{R_i^e} - \boldsymbol{C_t} & (k) \end{cases}$$

While in the mean-*CVaR* framework, the objective function is to minimize the *CVaR* measure. Referring to formula (5), the objective function can be rewritten in a compact form for easy computation. That is, divide the *CVaR* into two parts, one is in formula (8), the other one is shown as in formula (f). Other variables and inequalities in formulas from (e) to (k) are with the same meanings as in the mean-VaR framework.

As a footnote, the criterion to evaluate the power of mean-VaR and mean-CVaR methods is the severity of excess loss. If CVaR is greater than VaR, it means that VaR cannot capture some tail risk, according to the definitions and properties of VaR and CVaR. Though in some sub-period in Section 4, the CVaR is slightly larger than VaR, it still can verify the fact that the mean-CVaR is superior to the mean-VaR.

3.3.2. Simulation algorithm

Since two kind of programming frameworks have been built, our further work is to propose the simulation algorithm for the frameworks.

(1) Simulation algorithm for the mean-*VaR* framework

Step 1: For Eqs. (1)' and (2), we adopt unconditional Maximum Likelihood (ML) to estimate the parameters using the in-sample data. In the process of parameter estimation, it may encounter some problems that affect the fitting goodness of the model. Firstly, singular matrix may be occur when inverse operation on the spatial matrix, which would affect the accurate results of the parameter estimation. Secondly, the applied model is a liner model, which may be another reason to lower the fitting goodness of the procedure used. In the international wide, investment of stock indices would face more complicated and confused environment, which may cause the applied liner model lose the ability to characterize this complex environment so well. Finally, the data used is daily data, and it may result the sample not large enough, which would affect the accurate of the estimation result (Bell & Bockstael, 2000; Carrión-Flores, Flores-Lagunes, & Guci, 2018). To sum up, these three reasons will affect the fitting goodness of the model, and will thus lead the R^2 of the model in Table 2 is not very high. However, in the future research, we will try to find a more reasonable method to set the spatial weights matrix, as well as to build a more practical spatiotemporal model, to heighten the fitness of the estimated model. More detail information can be referred to Section 2.3 in Gong and Weng (2016) and the result of parameter estimation is shown in Table 2.

Step 2: Use the estimated parameters to predict the returns of stock indices Y_t in formulas (1)' and (2) first. Then, compute the portfolio returns R_t and covariance CoV_t, according to Section 4.2.2 in Gong and Weng (2016). Thus, we get the predicted portfolio returns and covariance to compute the *VaR* by resorting to formulas (3) and (4).

Step 3: Set the *VaR* as the initial value of the 'fmincon' routine, which is a constrained minimum of a scalar function that subjected to the conditions from (a) to (d).

Step 4: Set initial values for other parameters, including confidence level, the daily deposit rate, the ratio of transaction cost and portfolio weighted. Taking portfolio weighted setting for example, we assign the weight with equal values. That is,

$$X= [x_1, x_2, \ldots, x_i] = 1/n[1,\ 1, \ldots, 1]',\ i = 1,\ 2,\ \ldots, n.$$

Step 5: Run the 'fmincon' function and simulate it for M times (M ≥ 10000), then we can get the vector of portfolio returns R_t, the

Table 2
The results of parameter estimation based on the spatiotemporal-AR model.

Corresponding variables	The real economy crisis/secondary disaster.		The financial and economic recovery period	
	Estimate	Std.	Estimate	Std.
GDP	0.610	0.036	0.565^{**}	0.008
CPI	0.110^{***}	0.001	-0.066^{***}	0.001
M_2	0.200^{***}	0.013	0.238^{***}	0.007
Re	0.230^{**}	0.042	0.186^{***}	0.009
R	0.150^{***}	0.041	0.143^{***}	0.022
Trade	0.111^{**}	0.002	0.092^{***}	0.003
R^e	0.100^{***}	0.065	0.079^{**}	0.006
$Y_{N,t}$	0.651^{*}	0.069	0.645^{*}	0.009
$\varepsilon_{N,t-1}$	0.210	0.109	0.207	0.074
LL	−643.3975		−420.7215	
R^2	0.3475		0.3564	

Notes: 1.*Std.* represents standard errors of the estimated parameters. *LL* represents the log-likelihood and R^2 is the goodness of fit.
2. *** means statistical significance at the level of 1%, **means statistical significance at the level of 5% and *means statistical significance at the level of 10%.

VaR metric as VaR_t and investment weight x_i^t in the mean-*VaR* framework, where$i = 1, 2, \cdots, n$; $t = 1, 2, \cdots, T$.

(2) Simulation algorithm for the mean-*CVaR* framework

Step 1: Based on the result of *VaR* from Step3 in the mean-*VaR* framework, we can get the initial value of *CVaR* metric by resorting to formula (5), which is a scalar function that subjected to the conditions from (e) to (k).

Step 2: Similar to the process from Steps 3–5 in the mean-*VaR* framework, we use 'fmincon' routine to simulate it for M times (M ≥ 10000) on the base of all well initial-setting variables. Then we get the vector of portfolio returns, the *CVaR* metric and optimal portfolio weights in the mean-*CVaR* framework. Next, we will go to analyze the simulation results.

4. Analysis results

4.1. What are the most contributing factors?

In this section, our analysis is based on the performance of the efficient frontiers based on the two frameworks. For lack of space, we take two periods including crisis and calm time for example, namely the real economy crisis/secondary disaster and the financial and economic recovery period, to figure out the contributing factors under spatiotemporal correlation.

4.1.1. Analysis in the real economy crisis/secondary disaster

In this subsection, we impose one single, two, or three of the friction factors (namely fixed-return, exchange rate and transaction cost) on to the programming framework at a time, to figure out which constraint significantly contribute to the performance of our portfolio on earth. As a result, the efficient frontiers under various constraints are almost the same. For limited space, we just show the efficient frontiers without any other constraints but for spatiotemporal correlation in Fig. 2, while figures with other constraints are shown in the appendix.

From the results of Fig. 2 and the other ones in the appendix, they imply that the traditional friction factors have little effect on the optimal portfolio in the real economy crisis/secondary disaster. But what are the important contributing factors? Whether the spatial correlation of stock indices affects the portfolio most? Thus, we compare the performance of efficient frontiers that are just with time series correlation with the one only with spatiotemporal correlation. Specific details are shown in Fig. 3.

From Figs. 2 and 3, it is obvious that the portfolio revenue and risk is different between the two correlations. Take the results at 99% percent for example, the max revenue with spatiotemporal correlation is up to 0.003 in Fig. 2 and the one with time series correlation is near 0.002 in Fig. 3 (we only focus on the line at the top part of the graphs and beginning with the turning points, and the risk analysis in the following is the same), while the minimize revenue with spatiotemporal correlation is 0.0005 but the one with time series correlation is near 0.0012. These results mean that the fluctuation range of portfolio revenue with spatiotemporal correlation is larger. On the opposite, from the view of risk, the max VaR/CVaR with spatiotemporal correlation is up to 0.03 and the one with time series correlation is near 0.053, while the minimize VaR/CVaR with spatiotemporal correlation is 0.0005 but the one with time series correlation is near 0.027. This result means that the risk with time series correlation is larger than the one with spatiotemporal correlation. Thus, it can be deduce that the difference of portfolio revenue and risk between the two correlations must result in different performances of efficient frontiers. Therefore, spatiotemporal correlation is an important influent factor. But one thing we cannot neglect is that in the Eq. (1)', exchange rate is an explanatory variable and is significant in the equation (see the coefficient of 'R^e' in Table 2). That is to say, exchange rate is playing an important role in the spatiotemporal correlation. Therefore, we can deduce that spatiotemporal correlation and exchange rate are the main influent factors in the international portfolio, while

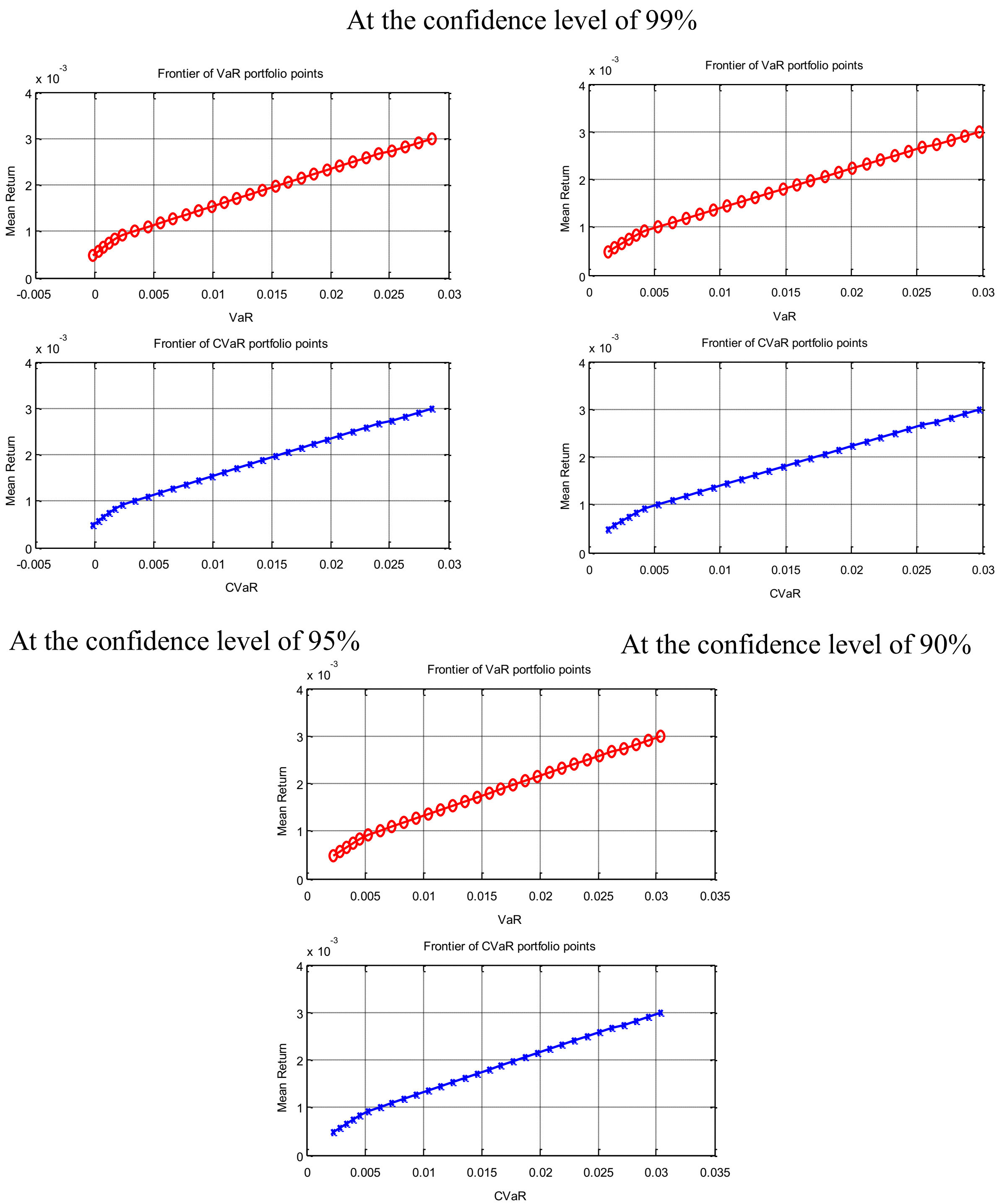
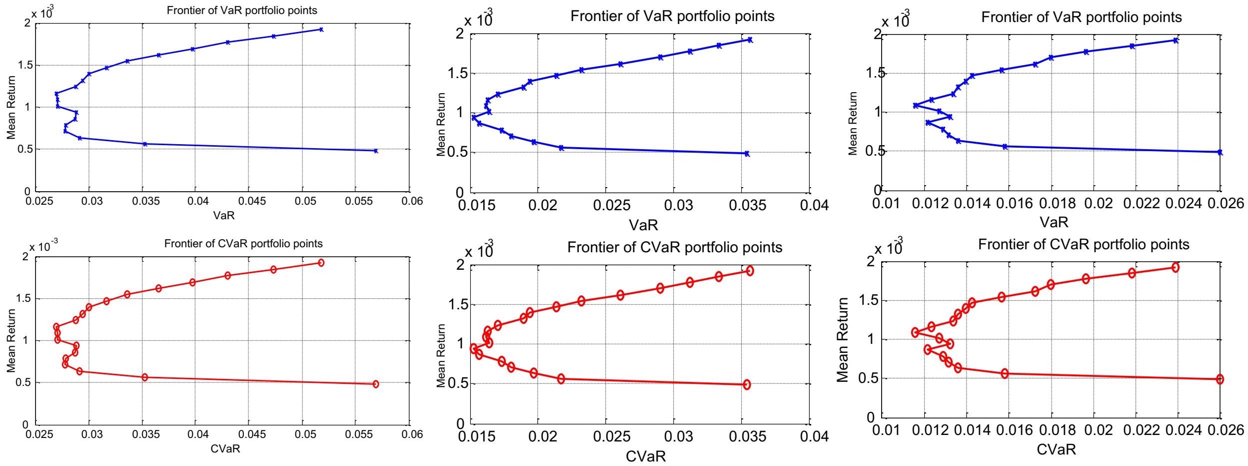


Fig. 2. Efficient frontiers are just with spatiotemporal correlations in the real economy crisis/secondary disaster.

fixed-return and transaction cost have little impact on the portfolio. This conclusion is contrary to the verdicts in the existing research literature (see, e.g., Chen & Wang, 2017; Bessler et al., 2016; Eun & Resnick, 1988; Longin & Solnik, 1995). But the result is consistent with the research that spatial dependences existing in the financial market (see, e.g., Cai et al., 2017; Asgharian et al., 2013; Fernández-Avilés et al., 2012; Ouyang et al., 2014; Tam, 2014). As a matter of fact, many countries are going to have stronger and stronger spatial relationship with each other, especially the ones sharing with the same culture including the social legal system, the political system and the norms of interpersonal relationships, or economic policy including fiscal policy, monetary policy and income policy and so on (Abate, 2016; Zhang, Mo, Liu, & Liu, 2018). Thus, we can explain why exchange rate which is an economic policy of



At the confidence level of 99% At the confidence level of 95% At the confidence level of 90%

Fig. 3. Efficient frontiers are just with time series correlation in the real economy crisis/secondary disaster.

one country and spatiotemporal correlation are the main influent factors. For space limited, we take the results of efficient frontier based on the spatiotemporal-AR model at 95% confidence level as representative to display in Fig. 2. From this figure, it is easy to find that the shape of efficient frontier is almost close to a straight line in each subfigure. This result means that the more profit, the more risk, which completely complies with the portfolio theory and proves that the calculate results are valid.

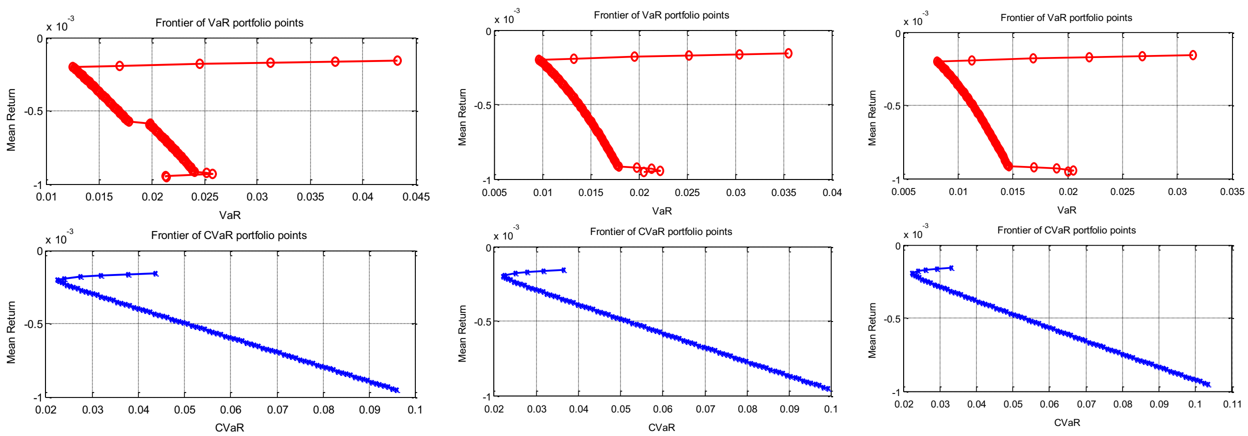
In short, in the real economy crisis/secondary disaster, the traditional factors such as transaction cost and fixed return have rarely effect on the performance. The reason for that maybe the performance of international portfolio is mainly affected by spatiotemporal correlation and exchange rate. Next, we will see if the same conclusion can be drawn in the financial and economic recovery period.

4.1.2. Analysis in the financial and economic recovery period

In this section, we still impose one single, two, or three of the friction factors, namely fixed-return, exchange rate and transaction cost, on to the programming framework at a time, to figure out which constraint contributes to the international portfolio on earth. The results still show that transaction cost and fixed return have little effect on the efficient frontiers. Now, we still take the efficient frontiers at 95% confidence level as an example, to show more specific in Fig. 4.

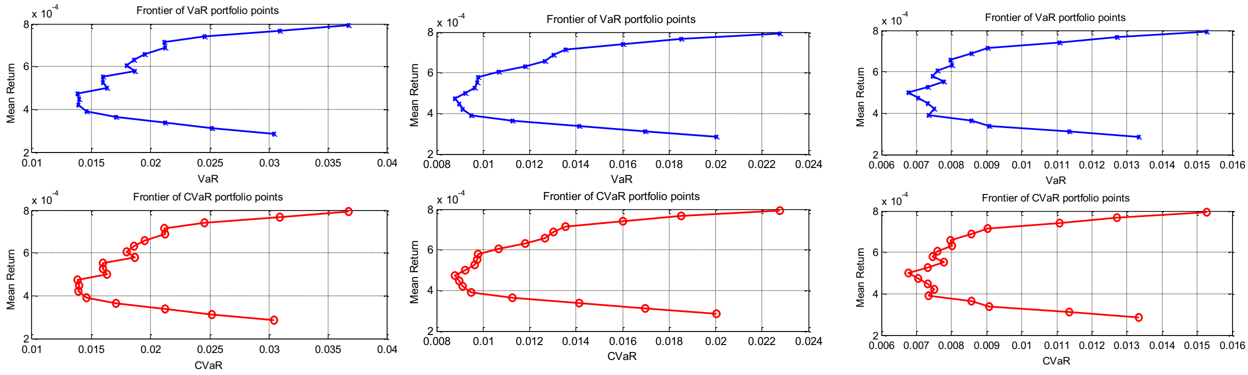
From Fig. 4, we can find that the figures of efficient frontier, which are the lines at the top part of the graphs and beginning with the turning points, are almost the same. As the analysis above, we take the efficient frontiers with spatiotemporal correlation and time series dependence for comparison, to inspect whether or not the spatial correlation has impact on the performance of the portfolio in the financial and economic recovery period. Specific can be referred to Figs. 4 and 5.

It is easy to find that the portfolio revenues under two correlations are subtle different, but the risk with spatiotemporal correlation is larger. It proves again that the international portfolios are insensitive to transaction cost and fixed return in the financial and



At 99% confidence level At 95% confidence level At 90% confidence level

Fig. 4. Efficient frontiers are just with spatiotemporal correlations in the financial and economic recovery period.



At 99% confidence level At 95% confidence level At 90% confidence level

Fig. 5. Efficient frontiers are just with time series correlations in the financial and economic recovery period.

economic recovery period, but the spatiotemporal correlation of every single market. In addition, similarity to the reason from the above, exchange rate is playing an important role in the spatiotemporal correlation and thus it is a significant factor to the performance of portfolio.

From the analysis, we can sum up that the international portfolio selection are little effect by transaction cost and fixed return, but the exchange rate and spatiotemporal correlation. Next, we will go to the second questions that about the portfolio revenue.

4.2. Can investors still benefit from the portfolio, when and where?

In this section, we still only take two periods as examples for space limitations, including the real economy crisis/secondary disaster and the financial and economic recovery time, to testify whether or not the investors can still benefit from the portfolio under spatiotemporal correlation.

4.2.1. Analysis in the real economy crisis/secondary disaster

Though, we can dig out something about portfolio revenue from Fig. 2, it cannot tell us the portfolio weights under a certain level of risk and the trend of weights changing or something else. Thus, to get more details, we will analyze and compare the statistics from Table 3.

As a footnote, there are something needed to be explained for the Table 3. That is, there exist so many points on the efficient frontier (see Fig. 2, all the points on the lines are belong to efficient frontiers) that it is hard to list all of them. For brevity, the paper

Table 3
Comparison of mean-VaR with mean-CVaR under spatiotemporal correlation in the real economy crisis/secondary disaster.

	at 90% confidence level					at 95% confidence level					at 99% confidence level				
rpv/VaR	0.1742	0.0241	0.0147	0.0117	0.0105	0.0331	0.0172	0.0127	0.0109	0.0101	0.0209	0.0149	0.0119	0.0105	0.0099
VaR	0.0033	0.0454	0.1100	0.1971	0.2856	0.0146	0.0636	0.1268	0.2116	0.2974	0.0231	0.0732	0.1356	0.2193	0.3038
rpv	0.0006	0.0011	0.0016	0.0023	0.0030	0.0005	0.0011	0.0016	0.0023	0.0030	0.0005	0.0011	0.0016	0.0023	0.0030
U.S.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6872	0.3486	0.0000	0.0000	0.0000	0.6872	0.7622	0.3811	0.3500
German	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
France	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
U.K.	0.2089	0.9086	0.6608	0.3304	0.1105	0.0900	0.3128	0.3586	0.3304	0.1174	0.0800	0.3128	0.0000	0.0000	0.0000
Japan	0.7911	0.0000	0.0000	0.0000	0.0000	0.9100	0.0000	0.0000	0.0000	0.0000	0.9200	0.0000	0.0000	0.0000	0.0000
China	0.0000	0.0914	0.3392	0.6696	0.8895	0.0000	0.0000	0.2928	0.6696	0.8826	0.0000	0.0000	0.2378	0.6189	0.6500
rpv/CVaR	0.1742	0.0241	0.0147	0.0117	0.0105	0.0003	0.0002	0.0001	0.0001	0.0001	0.0209	0.0149	0.0119	0.0105	0.0099
CVaR	0.0033	0.0454	0.1100	0.1971	0.2856	0.0001	0.0006	0.0013	0.0021	0.0030	0.0231	0.0732	0.1356	0.2193	0.3038
rpd	0.0006	0.0011	0.0016	0.0023	0.0030	0.0000	0.0000	0.0000	0.0000	0.0000	0.0005	0.0011	0.0016	0.0023	0.0030
U.S.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6872	0.3486	0.0000	0.0000	0.0000	0.6872	0.7622	0.3811	0.3500
German	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
France	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
U.K.	0.2089	0.9086	0.6608	0.3304	0.1105	0.0900	0.3128	0.3586	0.3304	0.1170	0.0800	0.3128	0.0000	0.0000	0.0000
Japan	0.7911	0.0000	0.0000	0.0000	0.0000	0.9100	0.0000	0.0000	0.0000	0.0000	0.9200	0.0000	0.0000	0.0000	0.0000
China	0.0000	0.0914	0.3392	0.6696	0.8895	0.0000	0.0000	0.2928	0.6696	0.8830	0.0000	0.0000	0.2378	0.6189	0.6500

Notes: There are three groups of confidence level to be presented. In each group, the top-half are the results of the mean-VaR and the bottom-half are the results of the mean-CVaR frameworks.
Each number represents the mean values of out-of-sample.

only selects five columns of data to present. Take the columns at 90% confidence level for example, the first column is the minimum portfolio return and the final is the maximum one, while the three middle columns are selected by the same step length. Then, the same criterion is applied to other levels of confidence and more information can be referred to Table 3.

1) *Portfolio revenue analysis*. From Table 3, it is obvious that all of the portfolio still can get higher returns even with the short selling constraint in this period, which draws coincide with Guidi and Ugur (2014) that portfolio diversification can benefit during the real economy crisis/secondary disaster.
2) *Portfolio weights analysis.* As for the portfolio weights, they are almost the same at different confidence levels, especially at 95% and 99% levels of confidence. Taking the confidence level at 95% for example, the investment concentrates on Japan's stock market at first, and then it gradually focuses on the U.S., the U.K. and China markets, finally it focus on China and the U.K. markets both in the mean-*VaR* and mean-*CVaR* frameworks. As to 99% level of confidence, almost the same case happens as at 95% level of confidence. These results cannot be explained only from the view of correlation of time series and spatial correlation, but also from the skewness of each stock returns series. More specifically, we describe the choosing strategy at 95% and 99% levels of confidence. In the real economy crisis/secondary disaster, all of the six markets are of highly spatiotemporal correlations, which could be seen from the regression coefficient of $Y_{N,t}$ in Table 2. From this Table, we can see that the coefficient is 0.651 and the value is positive, which means that the stock markets are affected by each other and could cause huge risk. Therefore, two points can be drawn: (1) in this period, it is wise to invest in Japan and the U. K markets, if investors want to get revenue at the smallest risk. And among of these two markets, the Japan market should be chosen priority since its weight is far higher than the one in the U. K. market. In addition, it is interesting to find that even Chinese market and the U.S. one are with larger positive mean values of returns, they are not be chosen. The reason for that maybe Japan has the lowest spatiotemporal correlations with other markets in this period (Gong & Weng, 2016). (2) If to pursuit the highest portfolio revenue at the expense of the most serious risk, the investors can put most of their money in China market. There may be two reasons for that: One is that China market is in a rapid development speed that can generate a higher profit, see Table 1, the mean value of its returns is 0.0019, which is over higher than the ones in other markets; the other one is that China market emerges serious left deviation, which reach to −63.399%, this may cause the portfolio face the serious risk.

While at 90% confidence level, investors can invest in Japan and the U.K. markets, but still keep Japan market as the absolutely main component (See Table 3, the weight is near 0.80 in Japan and only 0.20 in the U.K. market) for lower risk. If continue to pursuit more profit, the investors can put their money into the U.K. and China markets. Similar to confidence levels at 95% and 99%, the strategy of getting highest revenue is putting most of their money into Chinese stock market.

The results analyzed above are greatly different from the one according to the unconditional time series correlation. In other words, China market will be the first choice at the lowest risk with unconditional time series correlation (Li, Qiu, Chen, Zhong, & Wu, 2017). This conclusion is obviously not fit for the realistic since China market is an emerging market and it always faces the relatively larger fluctuations than the other markets. Thus, it cannot be the market with lowest risk to be chosen.

3) *Portfolio risk analysis.* As we all known, the greater the portfolio revenue, the greater the risk, and thus we have to master some information about the portfolio risk. Here we can conclude something from Table 3 that about the *VaR* and *CVaR* metrics: the two risk measures are almost with the same values if at the same confidence level, this suggests that the *VaR* metric based on the spatiotemporal-AR model has a good ability to forecast the risk in these period. In another word, there is no expected shortfall if the *VaR* metric computing from spatiotemporal-AR model.
4) *Sharpe Ratio analysis.* As we all known, each portfolio can calculate a Sharpe Ratio, and the higher the ratio, the better the portfolio. So we can deduce from the value of ratio that invest in Japan and the U.K. markets is the best portfolio for the risk-averse investors, see the value of rpv/*VaR* and rpv/*CVaR* in Table 3.

In sum, investors still can benefit from the international portfolio in the real economy crisis/secondary disaster. If pursuing lower risk, they can invest in Japan and the U.K. markets. On the contrary, if pursuit higher revenue, they can invest mainly in China market. Next, we go to analyze the result of investment income in the financial and economic recovery period.

4.2.2. Analysis in the financial and economic recovery period

Before analyzing the investment income situation in the financial and economic recovery period, we should note that there are only a few points on the efficient frontier and the line of efficient frontier is nearly close to a horizontal line, comparing with large invalid points and a steep diagonal line on the lower part of broken lines (see Fig. 4). This suggests that the portfolio returns are very insensitive to risk. More specific are presented in Table 4.

1) *Portfolio revenue analysis.* It is interesting to see from Table 4 that the portfolio revenues are all negative at all levels. The reason for that may be the index in the U.K. market is serious right deviation that reaching to 57.9177%, which means that most of investment returns in the U.K. market are negative. Furthermore, the negative values have positive spatiotemporal correlations (see Table 2, the regression coefficient of $Y_{N,t}$ is 0.645) with other indices, which resulting a negative portfolio value in the whole of financial and economic recovery period. Interestingly, it seems to be got some gains in this period, since the mean values of all markets are positive. Fortunately, the negative results are all very small, which are approximately equal to −0.0002. As a foot, since the value of return is so close to identify in the whole financial and economic recovery period, the values listed in Table 4 are

Table 4
Comparison of mean-VaR with mean-CVaR under spatiotemporal correlations in the financial and economic recovery period.

	at 90% confidence level					at 95% confidence level					at 99% confidence level				
rpv/VaR	−0.02467	−0.01569	−0.00931	−0.00652	−0.00491	−0.02068	−0.01335	−0.00807	−0.00572	−0.00434	−0.01591	−0.01043	−0.00645	−0.00465	−0.00357
VaR	0.00812	0.01203	0.01905	0.02542	0.03144	0.00969	0.01414	0.02198	0.02898	0.03554	0.01260	0.01810	0.02748	0.03568	0.04325
rpv	−0.00020	−0.00019	−0.00018	−0.00017	−0.00015	−0.00020	−0.00019	−0.00018	−0.00017	−0.00015	−0.00020	−0.00019	−0.00018	−0.00017	−0.00015
U.S.	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000	0.0000	0.0000	0.0000	0.0000
Germany	0.00000	0.00000	0.00000	0.00000	0.00000	0.00775	0.00000	0.00000	0.00000	0.00000	0.0077	0.0000	0.0000	0.0000	0.0000
France	0.00814	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000	0.0000	0.0000	0.0000	0.0000
U.K.	0.99186	0.85901	0.57267	0.28634	0.10000	0.99225	0.85901	0.57267	0.28634	0.10000	0.9923	0.8590	0.5727	0.2863	0.10000
Japan	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000	0.0000	0.0000	0.0000	0.00000
China	0.00000	0.14099	0.42733	0.71366	0.90000	0.00000	0.14099	0.42733	0.71366	0.90000	0.0000	0.1410	0.4273	0.7137	0.90000
rpv/CVaR	−0.00882	−0.00850	−0.00755	−0.00658	−0.00466	−0.00899	−0.00843	−0.00725	−0.00619	−0.00422	−0.00884	−0.00793	−0.00659	−0.00538	−0.00354
CVaR	0.02256	0.02236	0.02399	0.02617	0.03309	0.02213	0.02255	0.02498	0.02979	0.03651	0.02252	0.02396	0.02749	0.03698	0.04360
rpd	−0.00020	−0.00019	−0.00018	−0.00017	−0.00015	−0.00020	−0.00019	−0.00018	−0.00017	−0.00015	−0.00020	−0.00019	−0.00018	−0.00017	−0.00015
U.S.	0.02139	0.00000	0.00000	0.00000	0.00000	0.02139	0.00000	0.00000	0.00000	0.00000	0.02139	0.00000	0.00000	0.00000	0.00000
Germany	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
France	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
U.K.	0.97861	0.88811	0.66608	0.44405	0.20000	0.97861	0.88811	0.66608	0.44405	0.20000	0.97861	0.88811	0.66608	0.44405	0.20000
Japan	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
China	0.00000	0.11189	0.33392	0.55595	0.80000	0.00000	0.11189	0.33392	0.55595	0.80000	0.00000	0.11189	0.33392	0.55595	0.80000

Notes: There are three groups of confidence level to be presented. In each group, the top-half one are the results of the mean-VaR framework and the bottom-half one are the results of the mean-CVaR framework. Each number represents the mean values of out-of-sample

kept five decimal places.

2) *Portfolio weights analysis.* Through comparing the results at different confident levels in Table 4, it is easy to find that the portfolio weights are almost the same at different levels. Taking confidence level at 99% for example, the investment concentrates on the U.K. market at first, then the weight in the U.K. market becomes smaller and smaller while the one in China market grows larger and larger. Finally, the investment focuses on China market, both in mean-*VaR* and mean-*CVaR* frameworks. Reasons for those are: 1) the U.K. market has lower spatiotemporal correlations with other markets in this period (Weng & Gong, 2016). According to the portfolio theory, the U.K. market should be the first candidate to be chosen. 2) Thought, China market is higher spatiotemporal correlations with other markets, its negative skewness is lowest (see Table 1, its skewness is only −4.62%, comparing with the second lower one (−16.42%) in U.S. with the same mean returns). Based on that information, China market can be another investment choice. Again, the results emphasize that spatiotemporal correlations and skewness of the returns are the factors contribute significantly to the portfolio.

   As a foot, the result analyzed above is similar to the one according to the unconditional time series correlation.

3) *Portfolio risk analysis.* It is obvious that the *CVaR* metric is larger than the *VaR* one at any confidence levels. This shows that the constraints in the mean-*CVaR* framework are tighter than the one in *VaR* constraints at the benchmark of the same returns. Put differently, the *VaR* metric cannot predict the risk so well and there is still existing expected shortfall.
4) *Sharpe Ratio analysis.* The Sharpe Ratio shows that the risk is very sensitive to the portfolio revenue in the financial and economic recovery period. Taking Table 4 for example, if an investor gets a portfolio return of −0.00019, he should face a risk of 0.02444. While pursuing for larger profit, such as −0.00015, he should cost higher risk, which can reach 0.04360. This implies that the increasing of risk is significantly exceeds the profit. Thereby, we can deduce that even if the investors cannot gain a lot in the international portfolio, but they still can avoid high risk. This conclusion is consistent with the one in Mimouni, Charfeddine, and Al-Azzam (2016). Thus, we can draw the same conclusion as in the real economy crisis/secondary disaster that investors still can benefit from international portfolio in the calm period.

In sum, though the portfolio revenues are all negative at any level, investors still can benefit from the international portfolio by avoiding the portfolio risk in the financial and economic recovery period. In addition, the portfolio weights are almost the same at different levels, and if the investment focus on the U.K. market, investors can keep the risk of portfolio minimize.

4.3. Robustness test and investment strategies

We will do the robustness test from three aspects:

1) Changing the periods of the sample. We use the mean-*VaR* and mean-*CVaR* frameworks to test the performances of portfolio in

Table 5
Comparison results of the performance of the two frameworks.

	Correlation	In the real economy crisis / secondary disaster	In the financial and economic recovery
Influent factors	Under time series correlation	1) exchange rate 2) transaction cost 3) fixed income	1) exchange rate 2) transaction cost 3) fixed income
	Under spatiotemporal correlation	1)Spatiotemporal correlation 2) exchange rate	Mainly effected by Spatiotemporal correlation and exchange rate; partly by the skewness of each single return series
Benefit or not	Under time series correlation	Investors can get higher positive portfolio returns	The portfolio returns is positive but is declining.
	Under spatiotemporal correlation	Still can get positive portfolio returns.	Portfolio revenues are negative at all levels, but the values are very small.
Performance comparison of the two frameworks	Under time series correlation	The mean-*CVaR* framework outperforms the mean-*VaR* one.	the mean-*CVaR* framework outperforms the mean-*VaR* one
	Under spatiotemporal correlation	The performance of mean-*CVaR* framework is equal to the mean-*VaR* one	the mean-*CVaR* framework outperforms the mean-*VaR* one
Investment strategy	Under time series correlation	China market will be the first choice, if pursuing the revenue at the expense of lower risk.(Li, Qiu, Chen, Zhong, & Wu, 2017).	Still can avoid the portfolio risk, since the portfolio revenues are negative at all levels, but the values are a little small (Bergin, & Pyun, 2016).
	Under spatiotemporal correlation	If pursuing the revenue with lower risk, they can mainly invest in Japan and the U.K. markets. On the contrary, if pursuit higher revenue, they can mainly invest in China and the U.K. markets.	Still can avoid the portfolio risk. (If the investment mainly focus on the U.K. market, investors can keep the risk of portfolio minimize).

other three periods. The results show that (1) the conclusion drawn from subprime mortgage crisis is the same as the one from the financial and economic recovery period; (2) The conclusion drawn from global financial tsunami and European debt crisis is similar to the one from the real economy crisis/secondary disaster.

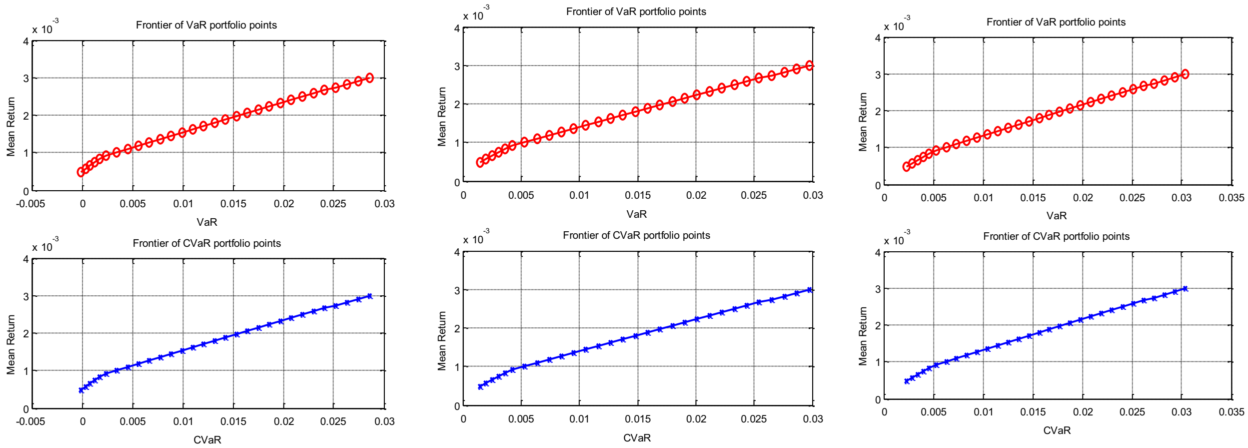
2) Changing the research sample. We adopt other markets as research sample, such as the markets of Australia, Russian, China, Brazil, Canada and U.S. As a result, it also can be drawn that the investor can benefit from the portfolio in the real economy crisis/secondary disaster.
3) Relaxing the constraints of portfolio programming frameworks. That is, we relax the speculating on the stock market by deleting the constraint $0 \le X_i^t \le 1$ that in formula (d) and formula (i), which are contained in the mean-*VaR* framework and the mean-*CVaR* framework, respectively. We do this test is for the reason that short-sales and short purchase constraints do not exist in some countries, such as in the U.S. market. This result is basically consistent with the one that analysis in Section 4.2.

Overall, the empirical result testifies that the conclusions we draw are quite correct. To spell out the analysis and conclusions in the previous sections, we sum up the information, and the result is shown in Table 5.

5. Conclusions and implications

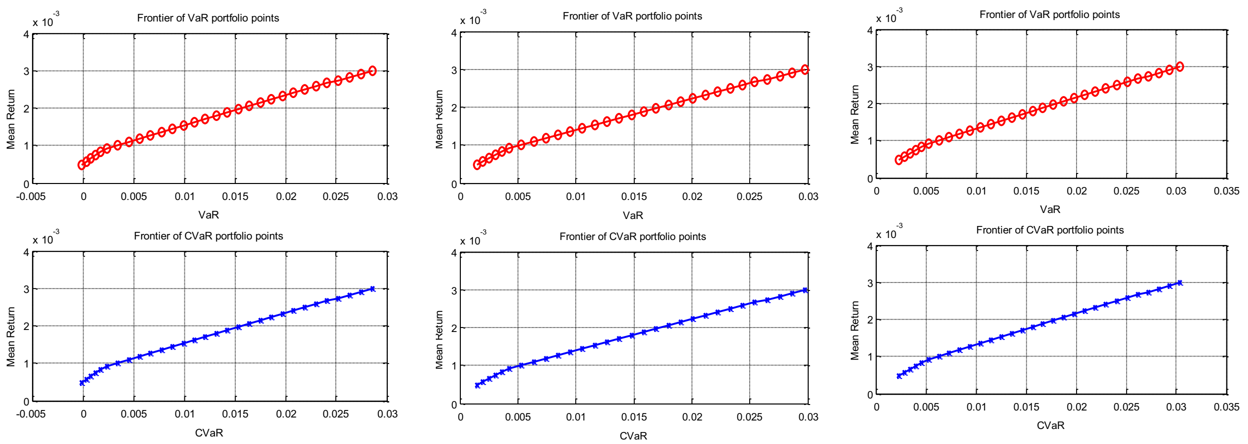
In this paper, we propose the mean-*VaR* and mean-*CVaR* programming frameworks with spatiotemporal correlations, where the objective of the frameworks are to minimize the *VaR*/*CVaR* metrics. Then, we apply these two frameworks to analyze the portfolio strategy of the international stock indices.

We find that: 1) the spatiotemporal correlation and exchange rate contribute to the performance of portfolio significantly, and



At 99% confidence level At 95% confidence level At 90% confidence level

Fig. 6. Efficient frontiers are with spatiotemporal correlation, exchange rate and transaction cost in the real economy crisis/secondary disaster.



At 99% confidence level At 95% confidence level At 90% confidence level

Fig. 7. Efficient frontiers are with spatiotemporal correlation and exchange rate in the real economy crisis/secondary disaster.

transaction cost and fixed income rarely have effect on the portfolio, both in crisis and calm periods. In addition, if the market has the lowest spatiotemporal correlation with other ones, it should be the optimal choice, whether in crisis or calm periods. 2) In the period of calm, the skewness of each single return series has some significant impact on the portfolio performance. In other words, if one market is with the lowest negative skewness, and the positive of mean return, then it should be chosen in the calm period. 3) Investors can still benefit from the international portfolio with spatiotemporal correlations, either in the view of avoiding risk or pursuing the profit; 4) the mean-*CVaR* framework outperforms the mean-*VaR* one with spatiotemporal correlations in financial calm period, but equals to the latter in crisis time.

From the conclusion above, we can get some implications as follows:

1) In different periods, the first primary is to sort the influencing factors first, according to their contributions to the performance of the portfolio, to figure out which market should be chosen. Usually, the most contributing factor is the spatiotemporal correlation and exchange rate, and then is the skewness of each single return series. Thus, investors should get a preliminary forecast of the portfolio risk, such as estimating the spatiotemporal correlations from the spatiotemporal models, and the one with lowest spatiotemporal correlation with other markets. Besides, in the calm period, another suitable area can be the one with positive mean and negative skewness of returns, such as the U.K. market.
2) The correlations of stock indices must contain correlations of both time series and spatial dependence. This implies that, if only considering the correlation of time series, the investors will underestimate the risk of international portfolio and cannot reach their expected returns, even suffered a greater loss in practice. In addition, we should use the mean-*CVaR* framework to help us to get more effective decision in financial calm period.

These findings can help investors better manage the portfolio risk and get some diversification benefits from international stock markets.

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Appendix. Supplementary Figs.

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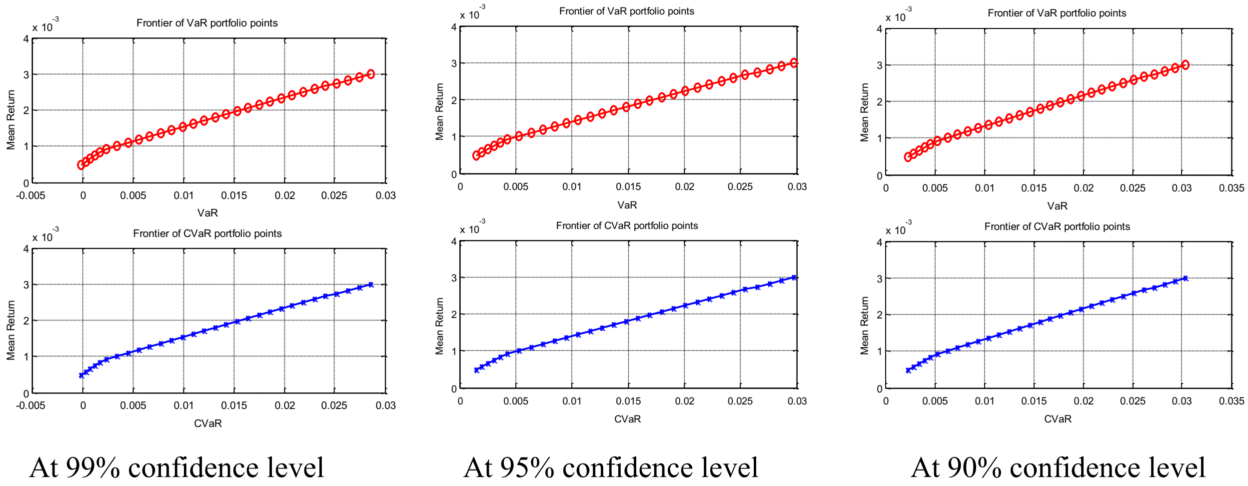

Fig. 8. Efficient frontiers are with spatiotemporal correlation, transaction cost, exchange rate and fixed income in the real economy crisis/secondary disaster.

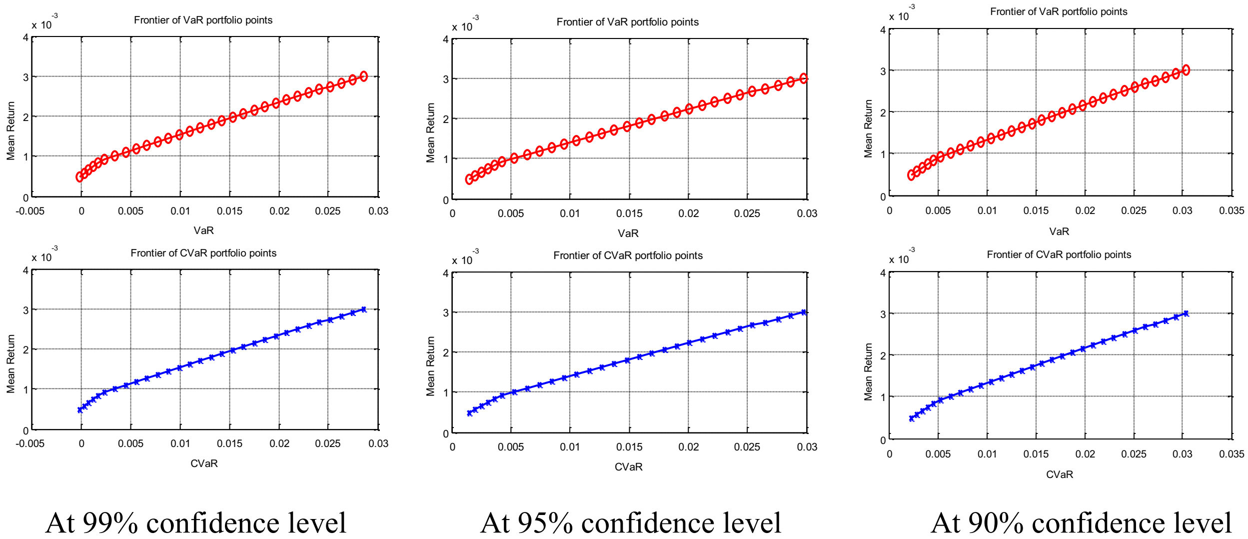

Fig. 9. Efficient frontiers are with spatiotemporal correlation and fixed income in the real economy crisis/secondary disaster.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.najef.2018.12.002.

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