AR(p) process

$$Y_t = \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + \varepsilon_t \qquad \varepsilon_t \sim iid(0, \sigma^2)$$

$$\rho(i) = \phi_1 \rho(1-i) + \phi_2 \rho(2-i) + \dots + \phi_p \rho(p-i) \qquad i = 1, \dots, T$$

$$Y_t = \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + \varepsilon_t \qquad / \cdot (Y_{t-i})$$

$$Y_{t-i} Y_t = \phi_1 Y_{t-1} Y_{t-i} + \dots + \phi_p Y_{t-p} Y_{t-i} + \varepsilon_t Y_{t-i}$$

$$E[Y_{t-i} Y_t] = \phi_1 E[Y_{t-1} Y_{t-i}] + \dots + \phi_p E[Y_{t-p} Y_{t-i}] + \cancel{E[\varepsilon_t Y_{t-i}]}$$

$$\gamma_i = \phi_1 \gamma_{1-i} + \dots + \phi_p \gamma_{p-i}$$

$$\gamma_i = \sum_{j=1}^{p} \phi_j \gamma_{j-i}$$

$$\rho_i = \sum_{j=1}^{p} \phi_j \rho_{j-i}$$

$$\Rightarrow \rho(i) = \phi_1 \rho(1-i) + \phi_2 \rho(2-i) + \dots + \phi_p (p-i) \qquad i = 1, \dots T$$