

11_AR_process.R

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```
# Course: Time series analysis
# Exercise: 11th / Auto-regressive process
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```

```
require(astsa)
```

```
## Loading required package: astsa
```

```
require(tseries)
```

```
## Loading required package: tseries
```

```
## Registered S3 method overwritten by 'quantmod':
##   method             from
##   as.zoo.data.frame zoo
```

```
# 1.
#  $y_t = \phi y_{t-1} + \varepsilon_t$        $t = 1, \dots, T$  where  $\varepsilon_t$  is white noise and  $y_0 = 0$  and  $|\phi| < 1$ .

# theoretical ACF  $\lim_{T \rightarrow \infty}$ 
#  $\lim_{T \rightarrow \infty} \rho(\tau) = \gamma(\tau)/\gamma(0) = \text{Cov}(y_t, y_{t+\tau})/\text{Var}(y_t)$ 
#  $\lim_{T \rightarrow \infty} \rho(\tau) = \lim_{T \rightarrow \infty} \hat{\phi}^\tau = 0$ 

#  $y_t = \phi y_{t-1} + \varepsilon_t$       / *  $y_{t-h}$ 
#  $y_{t-h} * y_t = \phi y_{t-h} y_{t-1} + y_{t-h} \varepsilon_t$ 
#  $E(y_{t-h} y_t) = E(\phi y_{t-h} y_{t-1}) + E(y_{t-h} \varepsilon_t)$ 
#  $E(y_{t-h} y_t) = E(\phi y_{t-h} y_{t-1})$ 

ARMAacf(ar = c(0.5), lag.max = 10)
```

```
##           0           1           2           3           4           5
## 1.0000000000 0.5000000000 0.2500000000 0.1250000000 0.0625000000 0.0312500000
##           6           7           8           9          10
## 0.0156250000 0.0078125000 0.0039062500 0.0019531250 0.0009765625
```

```
ARMAacf(ar = c(0.5), lag.max = 100)
```

##	0	1	2	3	4	5
##	1.000000e+00	5.000000e-01	2.500000e-01	1.250000e-01	6.250000e-02	3.125000e-02
##	6	7	8	9	10	11
##	1.562500e-02	7.812500e-03	3.906250e-03	1.953125e-03	9.765625e-04	4.882812e-04
##	12	13	14	15	16	17
##	2.441406e-04	1.220703e-04	6.103516e-05	3.051758e-05	1.525879e-05	7.629395e-06
##	18	19	20	21	22	23
##	3.814697e-06	1.907349e-06	9.536743e-07	4.768372e-07	2.384186e-07	1.192093e-07
##	24	25	26	27	28	29
##	5.960464e-08	2.980232e-08	1.490116e-08	7.450581e-09	3.725290e-09	1.862645e-09
##	30	31	32	33	34	35
##	9.313226e-10	4.656613e-10	2.328306e-10	1.164153e-10	5.820766e-11	2.910383e-11
##	36	37	38	39	40	41
##	1.455192e-11	7.275958e-12	3.637979e-12	1.818989e-12	9.094947e-13	4.547474e-13
##	42	43	44	45	46	47
##	2.273737e-13	1.136868e-13	5.684342e-14	2.842171e-14	1.421085e-14	7.105427e-15
##	48	49	50	51	52	53
##	3.552714e-15	1.776357e-15	8.881784e-16	4.440892e-16	2.220446e-16	1.110223e-16
##	54	55	56	57	58	59
##	5.551115e-17	2.775558e-17	1.387779e-17	6.938894e-18	3.469447e-18	1.734723e-18
##	60	61	62	63	64	65
##	8.673617e-19	4.336809e-19	2.168404e-19	1.084202e-19	5.421011e-20	2.710505e-20
##	66	67	68	69	70	71
##	1.355253e-20	6.776264e-21	3.388132e-21	1.694066e-21	8.470329e-22	4.235165e-22
##	72	73	74	75	76	77
##	2.117582e-22	1.058791e-22	5.293956e-23	2.646978e-23	1.323489e-23	6.617445e-24
##	78	79	80	81	82	83
##	3.308722e-24	1.654361e-24	8.271806e-25	4.135903e-25	2.067952e-25	1.033976e-25
##	84	85	86	87	88	89
##	5.169879e-26	2.584939e-26	1.292470e-26	6.462349e-27	3.231174e-27	1.615587e-27
##	90	91	92	93	94	95
##	8.077936e-28	4.038968e-28	2.019484e-28	1.009742e-28	5.048710e-29	2.524355e-29
##	96	97	98	99	100	
##	1.262177e-29	6.310887e-30	3.155444e-30	1.577722e-30	7.888609e-31	

```
# ACF(Yt)  $\lim_{T \rightarrow \infty} = 0$ 
```

```
# 2.
```

```
T <- 100
```

```
phi_c <- c(-0.9,-0.5, 0.1, 0.1, 0.5, 0.9);
```

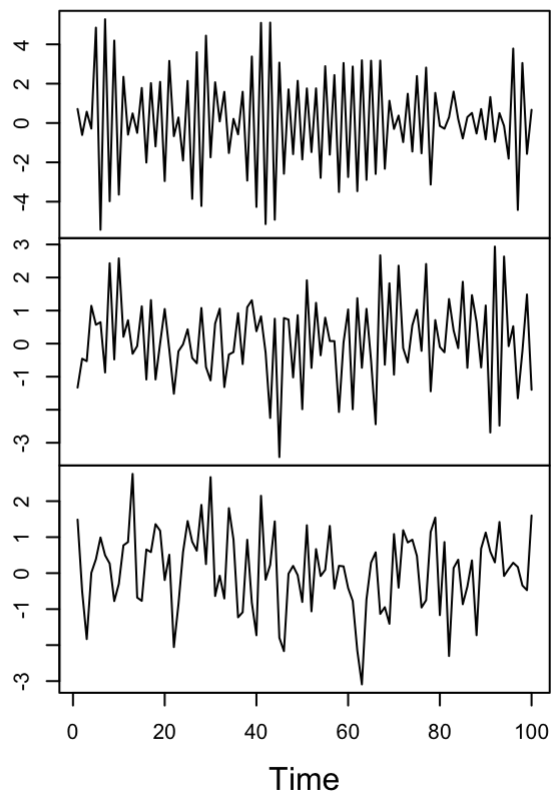
```
AR_models = matrix()
```

```
first_iteration <- TRUE
```

```
for (phi in phi_c) {
  if (first_iteration) {
    first_iteration <- FALSE
    AR_models <- arima.sim(model = list(ar = phi), n = T)
  }
  else {
    AR_models <- cbind(AR_models, arima.sim(model = list(ar = phi), n = T))
  }
}
```

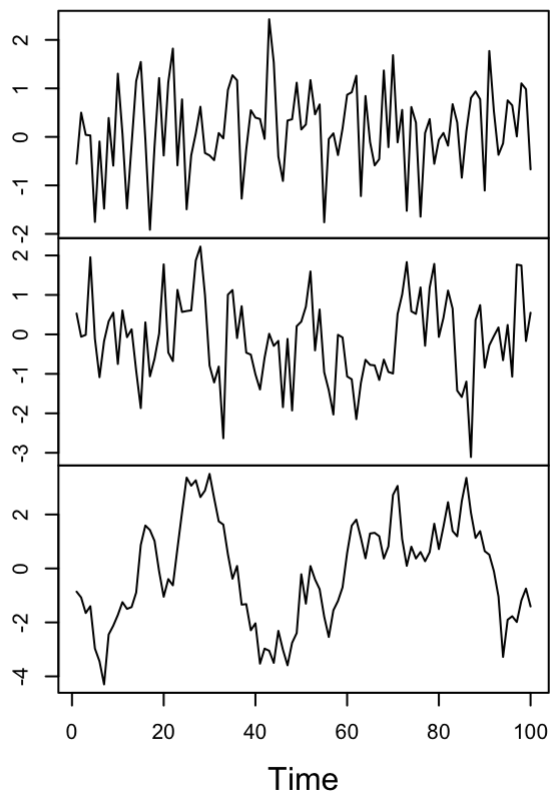
```
plot(AR_models[,])
```

AR_models[1,1]



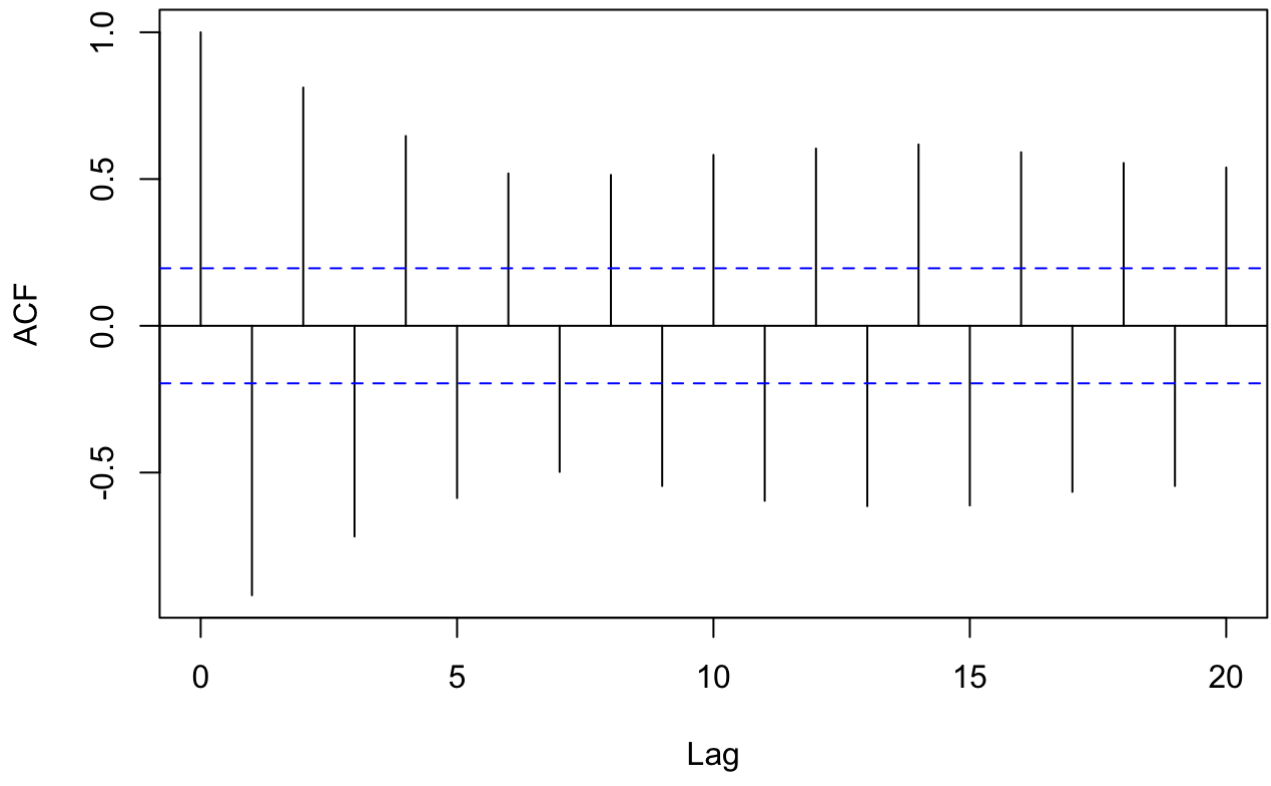
AR_models[, 1]

arima.sim(model=AR_models[,1])



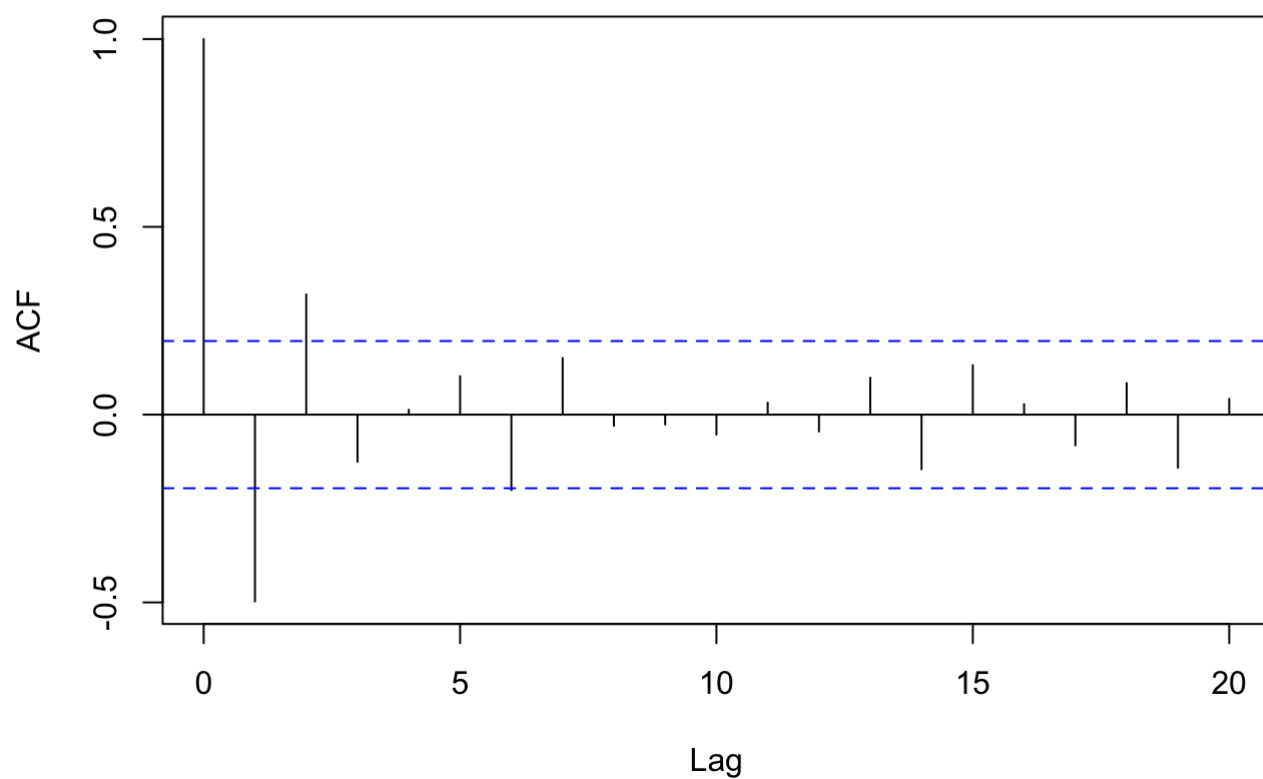
```
plot(acf(AR_models[,1]))
```

Series AR_models[, 1]



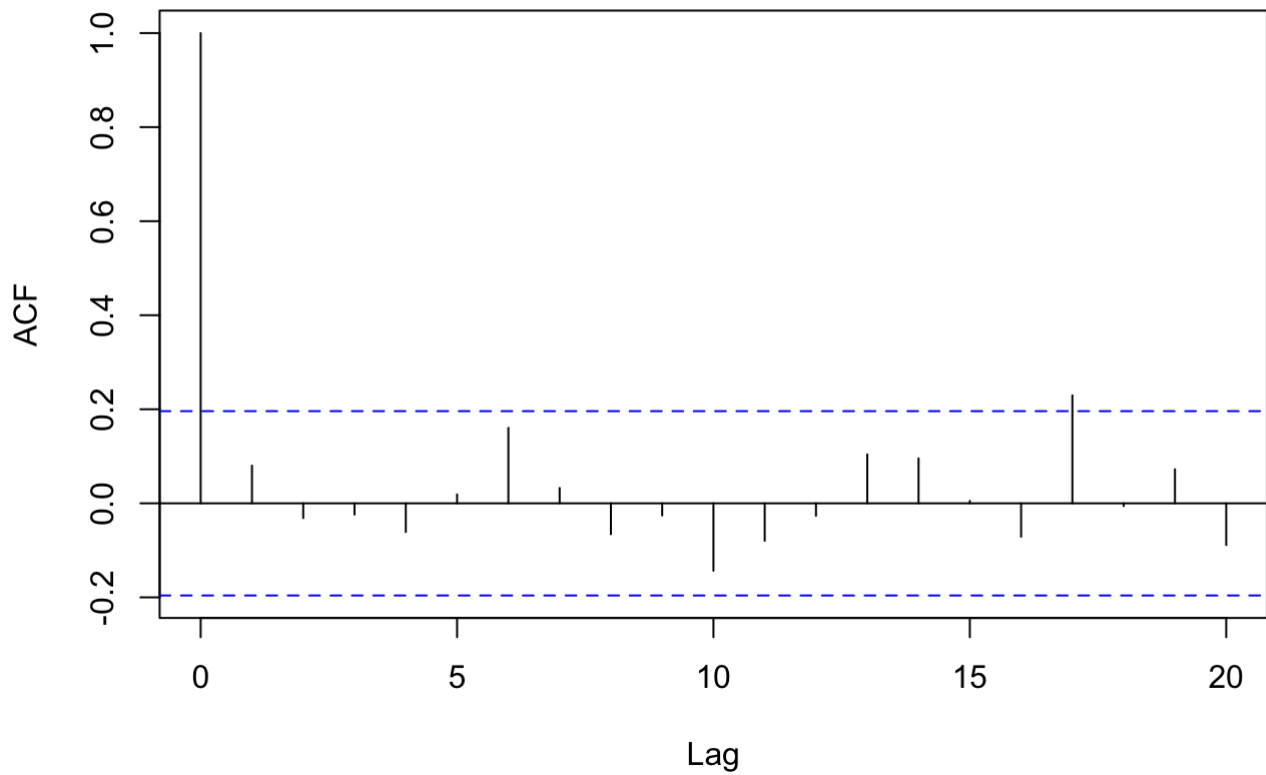
```
plot(acf(AR_models[,2]))
```

Series AR_models[, 2]



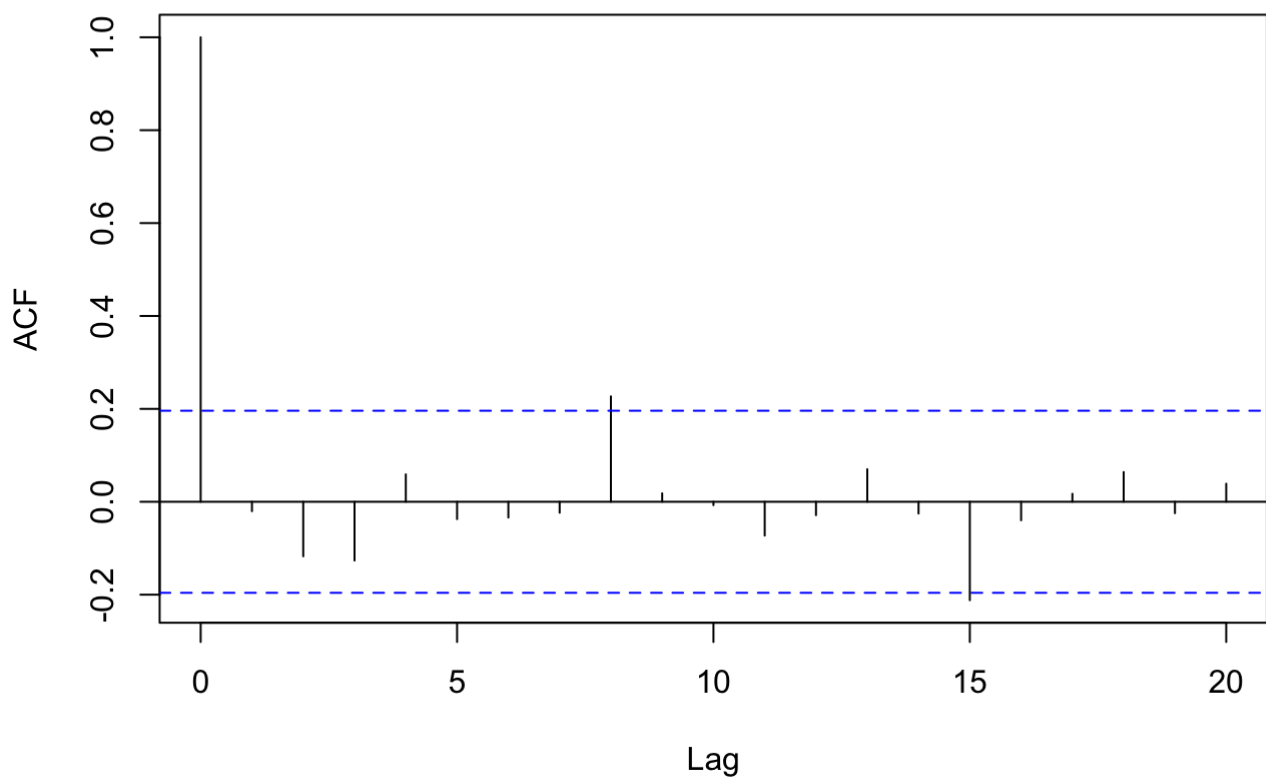
```
plot(acf(AR_models[,3]))
```

Series AR_models[, 3]



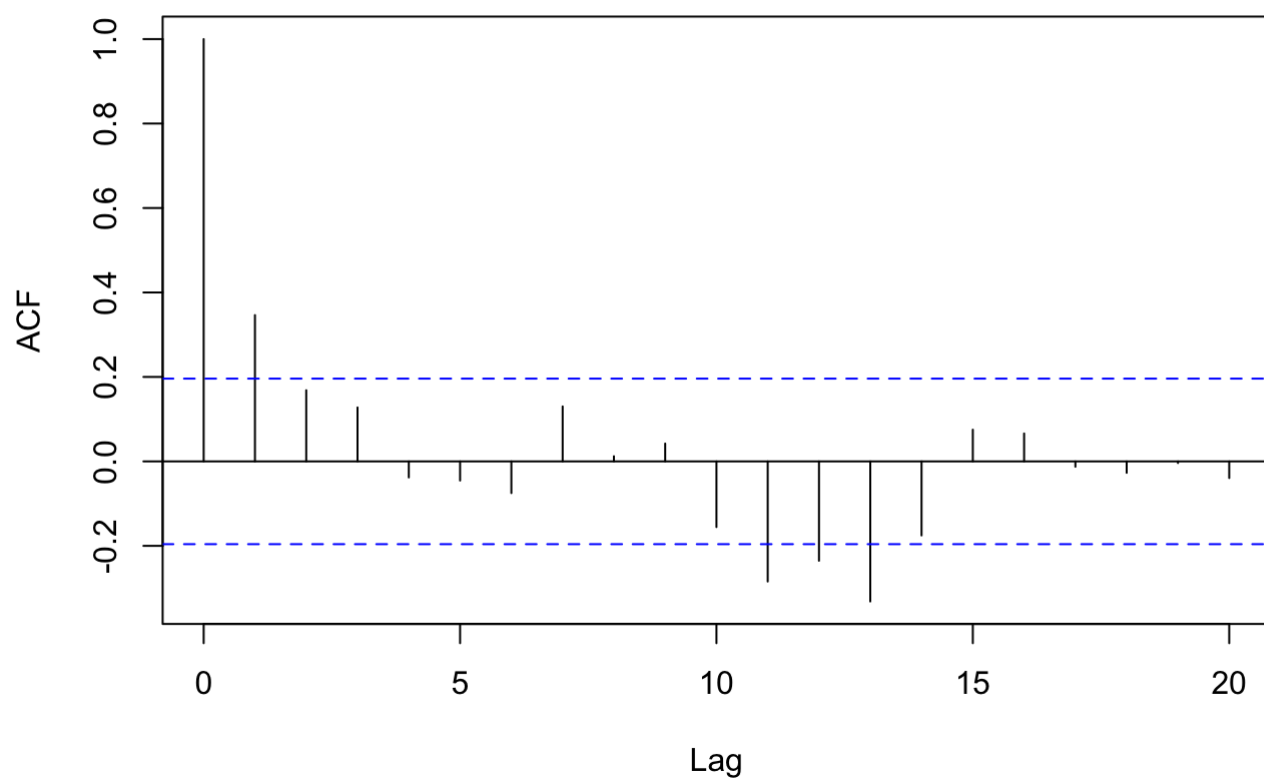
```
plot(acf(AR_models[,4]))
```

Series AR_models[, 4]



```
plot(acf(AR_models[,5]))
```

Series AR_models[, 5]



```
plot(acf(AR_models[,6]))
```

Series AR_models[, 6]

