AR(p) with ϕ_0 :

$$\tilde{Y}_t = \phi_0 + \phi_1 \tilde{Y}_{t-1} + \dots + \phi_p \tilde{Y}_{t-p} + \varepsilon_t \qquad \varepsilon_t \sim iid(0, \sigma^2)$$

I: const. mean?

$$E[\tilde{Y}_t] = E(\phi_0) + E(\phi_1 \tilde{Y}_{t-1}) + \dots + E(\phi_p \tilde{Y}_{t-p}) + \cancel{E(\varepsilon_t)}$$

$$E(\tilde{Y}_t) = \phi_0 + \sum_{i=1}^{p} \phi_i \, E(\tilde{Y}_{t-i}) + 0$$

$$\mu = \phi_0 + \sum_{i=1}^{p} \phi_i \, \mu$$

$$\underline{\underline{\mu = \frac{\phi_0}{1 - \sum_{i=0}^{p} \phi_i}}}$$

II: const Var.

$$Var(\tilde{Y}_t) = Var(\phi_0) + Var(\phi_1 Y_{t-1}) + \dots + Var(\phi_p Y_{t-p}) + Var(\varepsilon_t)$$

$$Var(\tilde{Y}_t) = 0 + \phi_1^2 Var(Y_{t-1}) + \dots + \phi_p^2 Var(Y_{t-p}) + \sigma^2$$

$$Var(Y_{t-p}) = Var(\tilde{Y}_t) = const.$$

$$Var(\tilde{Y}_t) = \phi_1^2 Var(\tilde{Y}_t) + \phi_p^2 Var(Y_{t-p}) + \sigma^2$$

$$Var(\tilde{Y}_t) = \frac{\sigma^2}{1 - \phi_1^2 + \dots \phi_p^2} = const. \checkmark$$

III Const. Covariance ?

$$Cov(\tilde{Y}_t, \tilde{Y}_{t-p}) = Cov(Y_t, \cancel{\phi_0} + \phi_1 \tilde{Y}_{t-1} + \dots + \phi_p \tilde{Y}_{t-p} + \cancel{\varepsilon_t})$$

$\downarrow$ iid!

$$= E(\tilde{Y}_t \tilde{Y}_{t-p}) = \phi_1 E(Y_t Y_{t-1}) + \dots + \phi_p E(Y_t Y_{t-p})$$

$$= \left(\sum_{i=1}^{p} \phi_i\right) \cdot Var(Y_{t-p}) = Cov(\tilde{Y}_t, \tilde{Y}_{t-p})$$

$$= \frac{(\phi_1 + \dots + \phi_p)\sigma^2}{1 - \phi_1 + \dots + \phi_p} = \text{const.} \checkmark$$