

10_Stationarity.R

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```
# Course: Time series analysis
# Exercise: 10th / Stationarity
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```

```
require(astsa)
```

```
## Loading required package: astsa
```

```
require(tseries)
```

```
## Loading required package: tseries
```

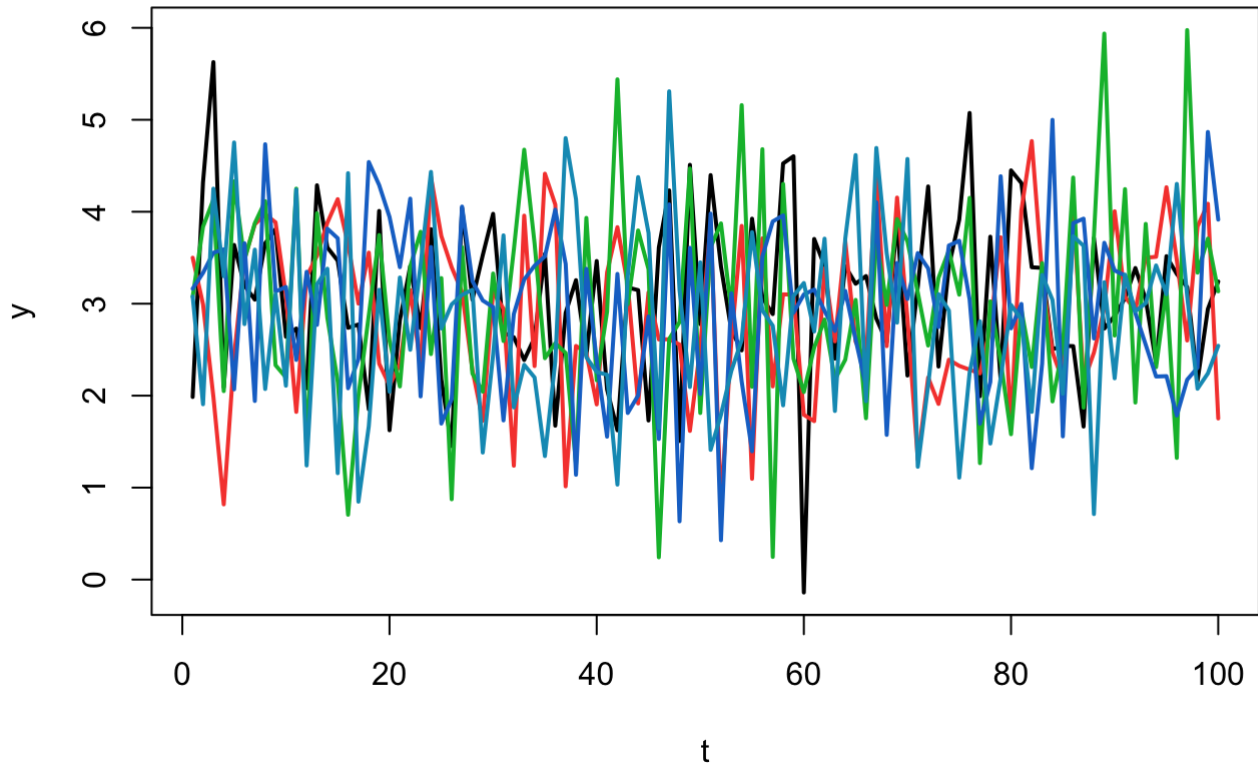
```
## Registered S3 method overwritten by 'quantmod':
##   method             from
##   as.zoo.data.frame zoo
```

```
T <- 100
n <- 5

par(mfrow=c(1,1))

# yt = 3 + et
y1 <- 3 + matrix(rnorm(n*T), ncol=n)
matplot(y1,type="l", lty=1,xlab="t",ylab="y", col=1:n, main="yt = 3 + et",lwd=2)
```

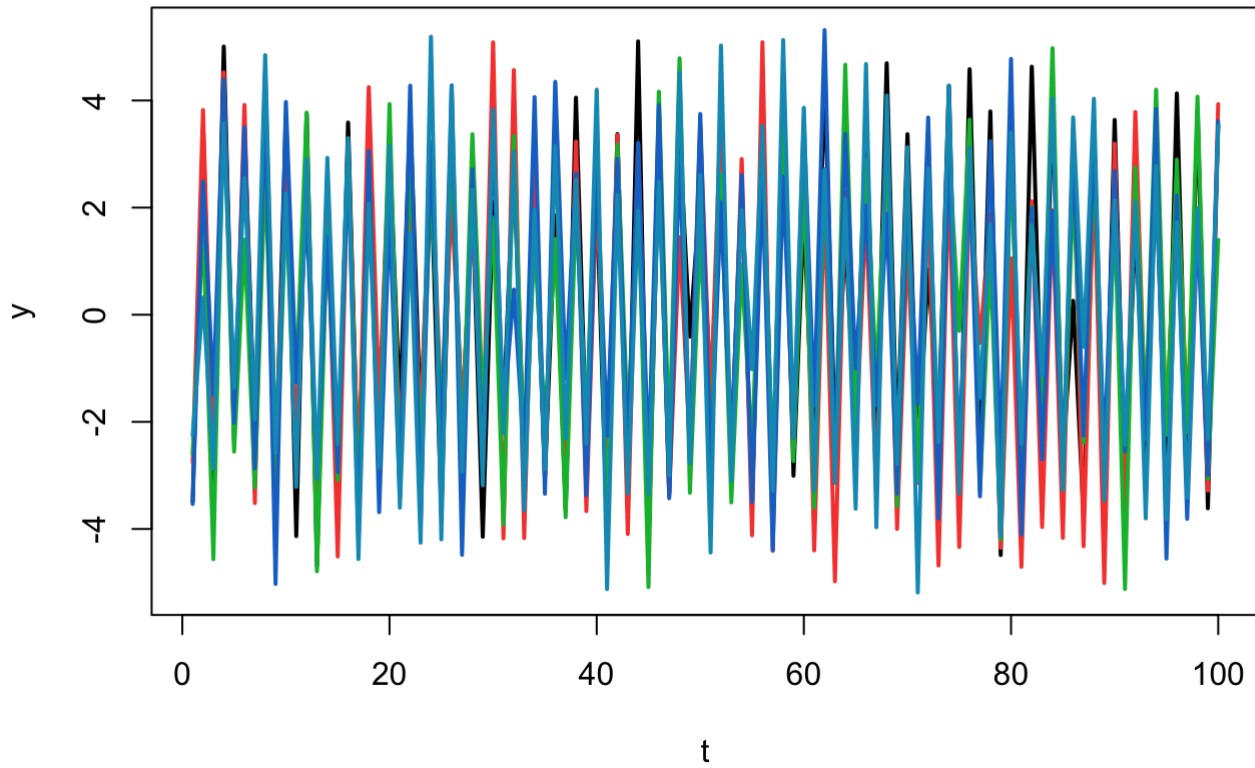
$$y_t = 3 + e_t$$



```
# y_t = 3(-1)^t + e_t
y2 <- matrix(rnorm(n*T),ncol=n)

for(t in 1:nrow(y2)) {
  for(observed in 1:ncol(y2)) {
    y2[t, observed] <- 3 * (-1)^t + y2[t, observed]
  }
}
matplot(y2,type="l", lty=1,xlab="t",ylab="y", col=1:n, main="y_t = 3(-1)^t + e_t",lwd=2
)
```

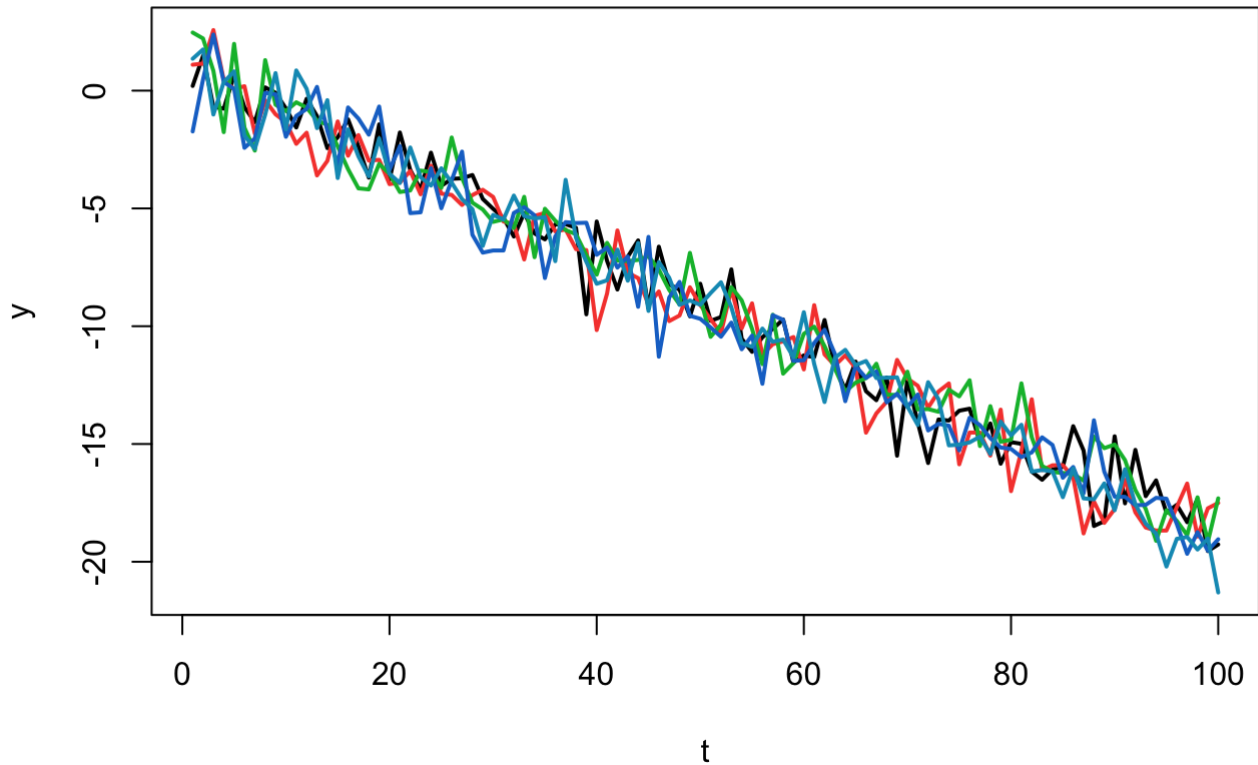
$$y_t = 3(-1)^t + e_t$$



```
# y_t = 1 - 0.2t + e_t
y3 <- matrix(rnorm(n*T), ncol=n)

for(t in 1:nrow(y3)) {
  for(observed in 1:ncol(y3)) {
    y3[t, observed] <- 1 - 0.2*t + y3[t, observed]
  }
}
matplot(y3,type="l", lty=1,xlab="t",ylab="y", col=1:n, main="y_t = 1 - 0.2t + e_t",lwd=
2)
```

$$y_t = 1 - 0.2t + e_t$$

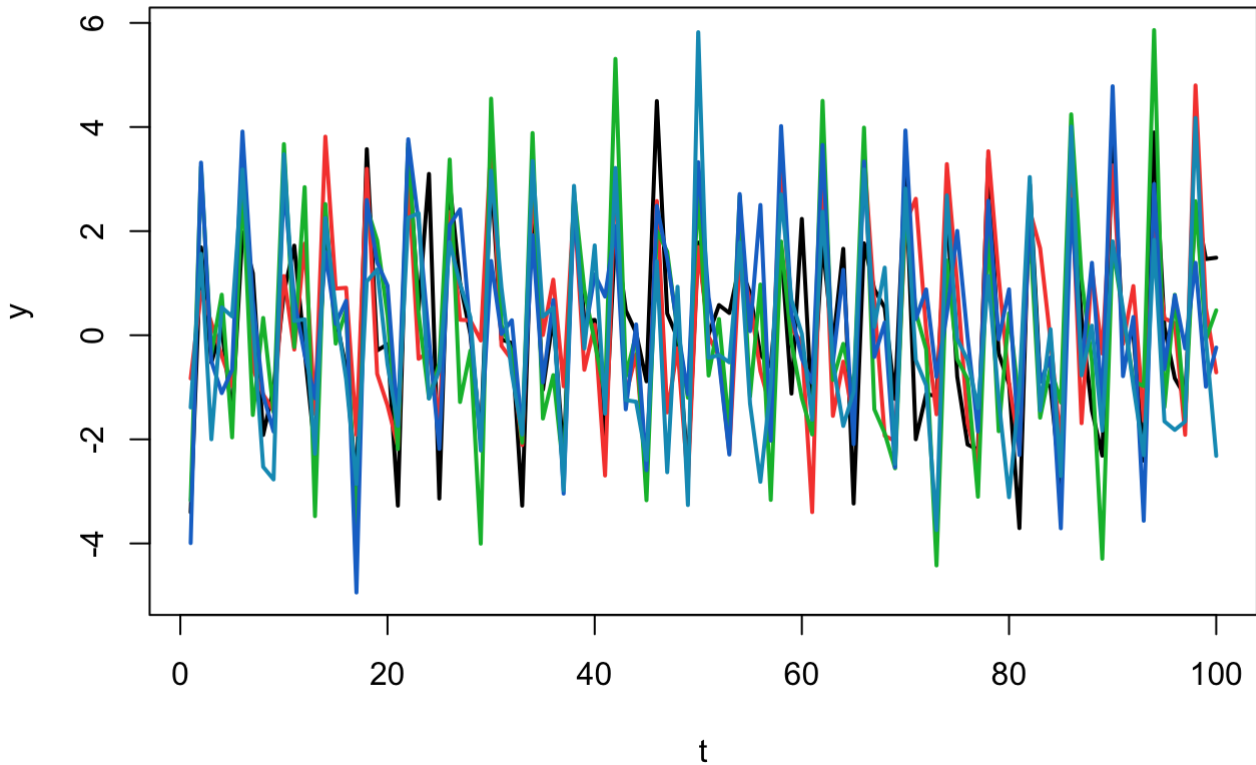


```
# y_t = -2+e_t / t %% 4 = 1, 3+e_t / t %% 4 = 2, e_t / t %% 4 = 3, -1 + e_t / t %% 4 = 0
y4 <- matrix(rnorm(n*T), ncol=n)

for(t in 1:nrow(y4)) {
  for(observed in 1:ncol(y4)) {
    if (t %% 4 == 1) y4[t, observed] <- -2 + y4[t, observed]
    else if (t %% 4 == 2) y4[t, observed] <- 3 + y4[t, observed]
    else if (t %% 4 == 4) y4[t, observed] <- -1 + y4[t, observed]

  }
}
matplot(y4,type="l", lty=1,xlab="t",ylab="y", col=1:n, main="stochastic y 4",lwd=2)
```

stochastic y 4



```
# Given  $\sigma^2(et) = \sigma^2$ 
# determine (weakly) stationary y and compute  $E(yt)$  and  $Var(yt)$ , if (strictly) stationary compute the auto-correlation-function

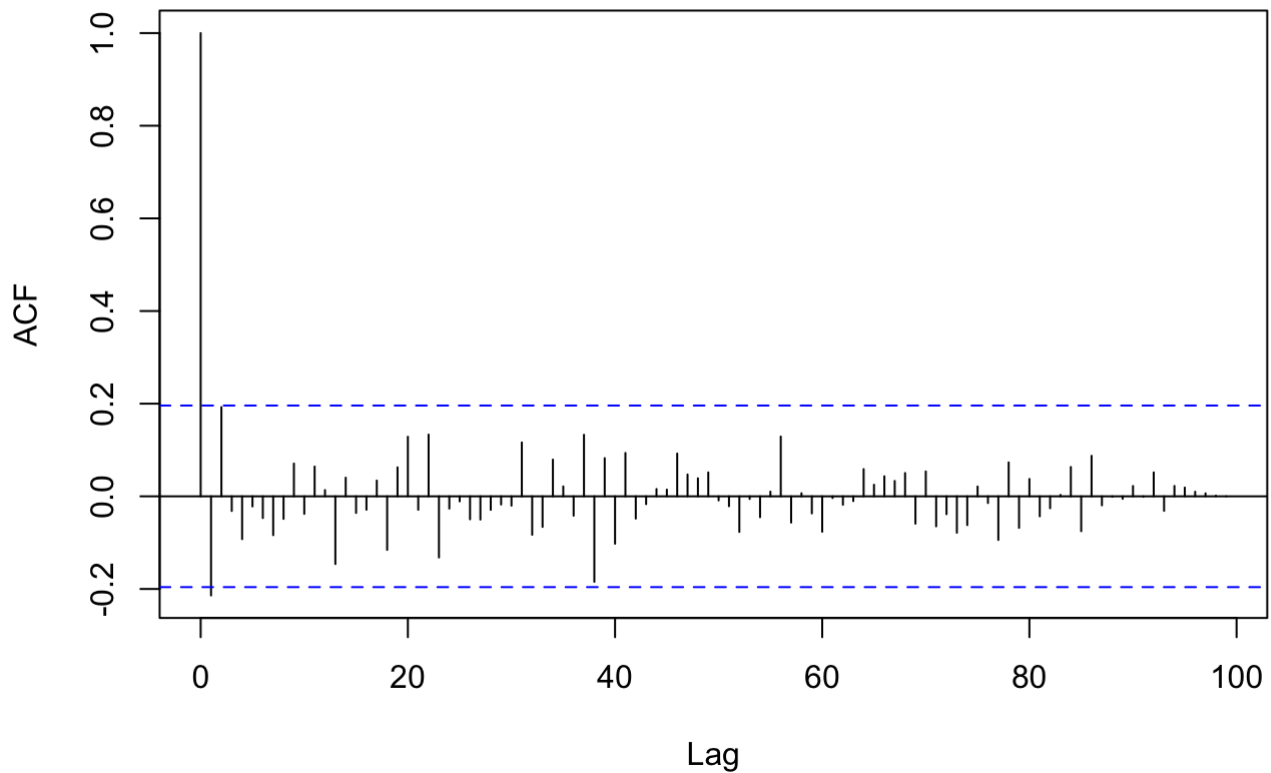
# def (weakly) stationary y:
# I:  $\mu(t) = \mu$ 
# II:  $\sigma^2(t) = \sigma^2$ 
# III:  $\gamma(t, s) = \gamma(t - s)$ 

# 1.)
#  $y_t = 3 + e_t$ 

# I.  $\mu(t) = 3$  const. meets 1st criteria
# II.  $\sigma^2(t) = V(e_t)$  meets 2nd criteria
# III.  $\gamma(t, s) = \gamma(t - s)$ :
#  $Cov(y_t, y_{t+h})$ 
#  $Cov(3 + e_t, 3 + e_{t+h}) = Cov(e_t, e_{t+h})$  ...equal covariance for all t. meets 3rd criteria
# Therefore (weakly) stationary.

acf(y1[,3], lag.max = T)
```

Series y1[, 3]



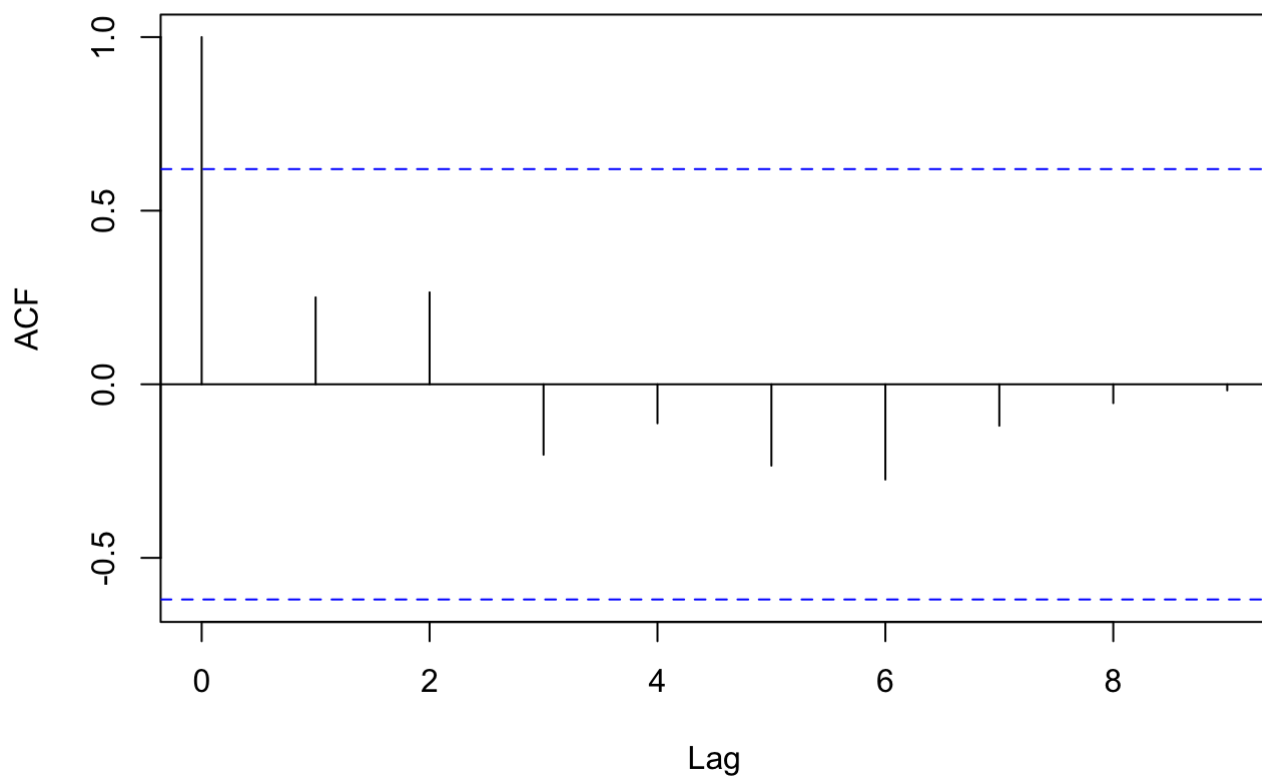
```
# 2.)
#
#  $y_t = 3(-1)^t + \epsilon_t$ 
# I.  $\mu(t) = 3(-1)^t$  ... is dependent on  $t$  and therefore does not meet criteria I.
# Therefore No stationarity.

# 3.)
#  $Y_t = 1 - 0.2t + \epsilon$ 
# I.  $\mu(t) = -0.2t$  ... is dependent on  $t$  and therefore does not meet criteria I.
# Therefore No stationarity.

# 4.)
#  $y_t = -2 + \epsilon_t \mid t \% 4 = 1, 3 + \epsilon_t \mid t \% 4 = 2, \epsilon_t \mid t \% 4 = 3, -1 + \epsilon_t \mid t \% 4 = 0$ 
# if  $t \% 4 = 1$  #  $\mu(t) = -2, \sigma^2(t) = V(\epsilon_t)$ 
# if  $t \% 4 = 2$  #  $\mu(t) = +3, \sigma^2(t) = V(\epsilon_t)$ 
# if  $t \% 4 = 3$  #  $\mu(t) = 0, \sigma^2(t) = V(\epsilon_t)$ 
# if  $t \% 4 = 0$  #  $\mu(t) = -1, \sigma^2(t) = V(\epsilon_t)$ 

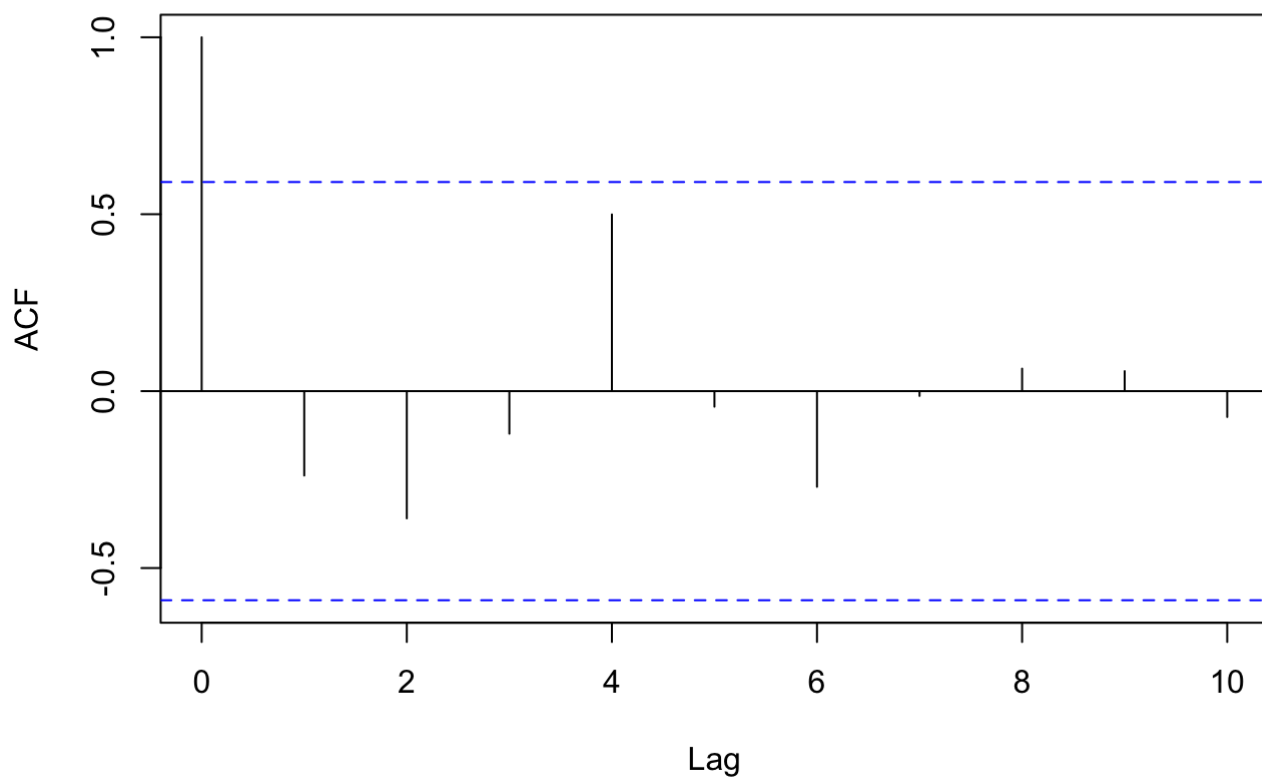
row_select <- c(0,4,8,12,16,20,24,28,32,36,40);
acf(y4[row_select,3], lag.max = T)
```

Series y4[row_select, 3]



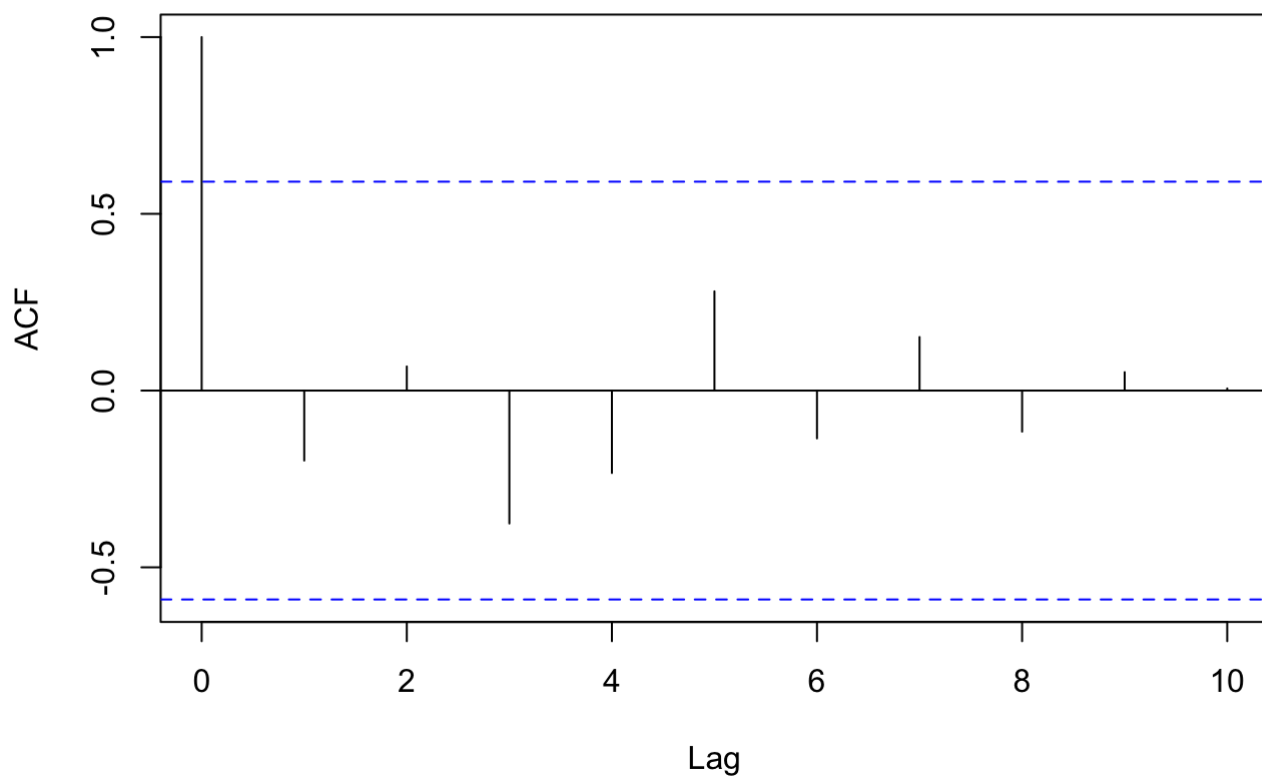
```
row_select <- row_select + 1  
acf(y4[row_select,3], lag.max = T)
```

Series y4[row_select, 3]



```
row_select <- row_select + 2  
acf(y4[row_select,3], lag.max = T)
```

Series y4[row_select, 3]



```
row_select <- row_select + 3  
acf(y4[row_select,3], lag.max = T)
```


Series y4[row_select, 3]

