

APPENDIX

1.1 REPRODUCIBILITY OF COMPUTATIONAL RESULTS

Computational implementations of network models and numerical analysis of their features are an integral part of this study. In order to ensure the reproducibility of these results, *Sumatra*, a software for provenance capture of computational projects, was used (Davison 2012). As a “lab notebook for computational researchers”, the software combines version control of code with the capture of inputs and outputs, as well as the used parameter sets, for each computational process. Providing a database and various interfaces, the tool allows not only direct management of simulations and data but provides a platform to make the computational reproducible and accessible to others.

In this study, for every computational result referenced in the text, a label was cited, referring to the record of the process in Sumatra’s database. As an integral part . Thus ensuring accessibility and reproducibility of the work presented here.

*on the use of
Sumatra labels*

The

1.2 MATHEMATICA

```

In[1]:= f[d_] = Piecewise[{{1 / (s * (d) ^ (1 / 2)) - 1 / (s^2), 0 < d < s^2}, {0, d > s^2}}]

Out[1]= 
$$\begin{cases} -\frac{1}{s^2} + \frac{1}{\sqrt{d} s} & 0 < d < s^2 \\ 0 & \text{True} \end{cases}$$


In[2]:= g[x_] := Convolve[f[d], f[d], d, x, Assumptions -> {d ∈ Reals, x ∈ Reals}]
Simplify[g[x], {s > 0, x ∈ Reals}]

Out[3]= 
$$\begin{cases} \frac{\pi s^2 - 4 s \sqrt{x} + x}{s^4} & x > 0 \ \&\& \ s^2 \geq x \\ -\frac{2 s^2 + x + \frac{4 s^3}{\sqrt{-s^2+x}} - \frac{4 s x}{\sqrt{s^2+x}} - 2 s^2 \text{ArcTan}\left[\frac{s}{\sqrt{-s^2+x}}\right] + i s^2 \text{Log}\left[s - i \sqrt{-s^2+x}\right] - i s^2 \text{Log}\left[s + i \sqrt{-s^2+x}\right]}{s^4} & s^2 < x \ \&\& \ 2 s^2 > x \\ 0 & \text{True} \end{cases}$$


In[4]:= h[x_] := g[x^2] * 2 * x

In[5]:= Simplify[h[x], {s > 0, x ∈ Reals, x > 0}]

Out[5]= 
$$2 x \begin{cases} \frac{\pi s^2 - 4 s x + x^2}{s^4} & s \geq x \\ -\frac{2 s^2 + x^2 + \frac{4 s^3}{\sqrt{-s^2+x^2}} - \frac{4 s x^2}{\sqrt{s^2+x^2}} - 2 s^2 \text{ArcTan}\left[\frac{s}{\sqrt{-s^2+x^2}}\right] + 2 s^2 \text{ArcTan}\left[\frac{\sqrt{-s^2+x^2}}{s}\right]}{s^4} & s < x \ \&\& \ \sqrt{2} s > x \\ 0 & \text{True} \end{cases}$$


In[6]:= (*For s == 1, h becomes*)

In[7]:= Simplify[h[x], {s == 1, x ∈ Reals, x > 0}]

Out[7]= 
$$2 x \begin{cases} \pi + (-4 + x) x & x \leq 1 \\ -2 - x^2 + 4 \sqrt{-1 + x^2} - 2 \text{ArcCot}\left[\frac{1}{\sqrt{-1+x^2}}\right] + 2 \text{ArcTan}\left[\frac{1}{\sqrt{-1+x^2}}\right] & 1 < x < \sqrt{2} \\ 0 & \text{True} \end{cases}$$


In[8]:= (*Expected Value*)
s := 1.
Integrate[x * h[x], {x, 0, Sqrt[2]}]

Out[9]= 0.521405

```

Mathematica 1.1: Computation of probability density function for distance between two random points in square of side length s as supplement to proof of Theorem ?? . Note that form of final result Out[7] differs from solution given in ?? for $1 < x < \sqrt{2}$. While proof of equivalence could not be achieved analytically, expressions given are numerically equivalent.

```

In[200]:= (*Expected distance between reciprocally connected pairs*)
r = NIntegrate[x * w[x] * c[x]^2 / 0.02451, {x, 0, Sqrt[2]}]

Out[200]= 0.20001

In[201]:= (*Probability of connection in third pair when other two are reciprocally connected*)
p = c[4 / Pi * r]

Out[201]= 0.164749

In[202]:= (*Probabilities for motif 9, 15 and 16 are then:*)
(1 - p)^2
2 * p * (1 - p)
p^2

Out[202]= 0.697643

Out[203]= 0.275214

Out[204]= 0.0271424

```

Mathematica 1.2: Computation of three motifs for Section ?? . Function $c[x]$ is the distance-dependent probability distribution from Theorem and $w[x]$ the probability density function for distance between two random points in a box (cf. Mathematica ??, Moltchanov 2012).

1.3 SUPPLEMENTARY FIGURES

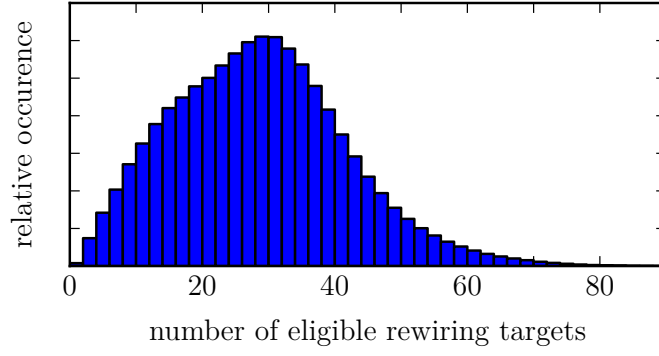


Figure 1.1: (4afc2727)

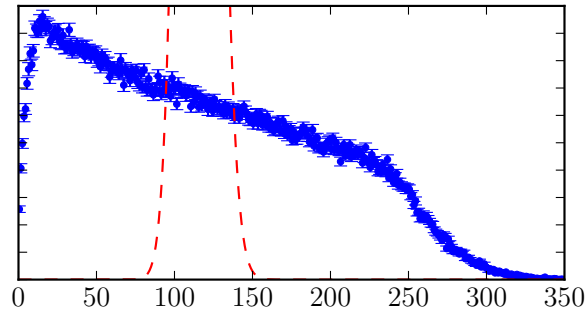


Figure 1.2: (c7ee86d7)

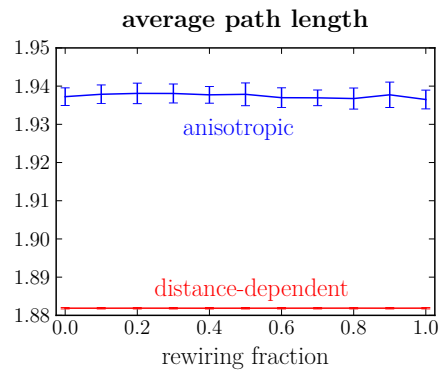


Figure 1.3: Average path length for anisotropic and distance-dependent networks, $N = 1000$. (064f9b10)

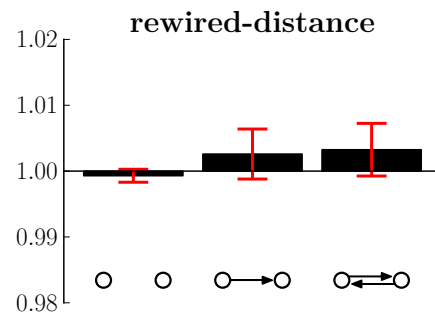


Figure 1.4: Probabilities for connections in neuron pairs are identical in distance-dependent and rewired anisotropic networks. (c5f1462b)