

In[1]:= **f[d_]** = **Piecewise**[{**{1 / (s * (d) ^ (1 / 2)) - 1 / (s^2), 0 < d < s^2}, {0, d > s^2}}**}]

$$\text{Out[1]} = \begin{cases} -\frac{1}{s^2} + \frac{1}{\sqrt{d} s} & 0 < d < s^2 \\ 0 & \text{True} \end{cases}$$

In[2]:= **g[x_]** := **Convolve**[**f[d], f[d], d, x, Assumptions -> {d ∈ Reals, x ∈ Reals}**]
Simplify[**g[x], {s > 0, x ∈ Reals}**]

$$\text{Out[3]} = \begin{cases} \frac{\pi s^2 - 4 s \sqrt{x} + x}{s^4} & x > 0 \ \&\& \ s^2 \geq x \\ -\frac{2 s^2 + x + \frac{4 s^3}{\sqrt{-s^2 + x}} - \frac{4 s x}{\sqrt{-s^2 + x}} - 2 s^2 \text{ArcTan}\left[\frac{s}{\sqrt{-s^2 + x}}\right] + i s^2 \text{Log}\left[s - i \sqrt{-s^2 + x}\right] - i s^2 \text{Log}\left[s + i \sqrt{-s^2 + x}\right]}{s^4} & s^2 < x \ \&\& \ 2 s^2 > x \\ 0 & \text{True} \end{cases}$$

In[4]:= **h[x_]** := **g[x^2] * 2 * x**

In[5]:= **Simplify**[**h[x], {s > 0, x ∈ Reals, x > 0}**]

$$\text{Out[5]} = 2 x \begin{pmatrix} \frac{\pi s^2 - 4 s x + x^2}{s^4} & s \geq x \\ -\frac{2 s^2 + x^2 + \frac{4 s^3}{\sqrt{-s^2 + x^2}} - \frac{4 s x^2}{\sqrt{-s^2 + x^2}} - 2 s^2 \text{ArcTan}\left[\frac{s}{\sqrt{-s^2 + x^2}}\right] + 2 s^2 \text{ArcTan}\left[\frac{\sqrt{-s^2 + x^2}}{s}\right]}{s^4} & s < x \ \&\& \ \sqrt{2} s > x \\ 0 & \text{True} \end{pmatrix}$$

In[6]:= **(*For s == 1, h becomes*)**

In[7]:= **Simplify**[**h[x], {s == 1, x ∈ Reals, x > 0}**]

$$\text{Out[7]} = 2 x \begin{pmatrix} \pi + (-4 + x) x & x \leq 1 \\ -2 - x^2 + 4 \sqrt{-1 + x^2} - 2 \text{ArcCot}\left[\frac{1}{\sqrt{-1 + x^2}}\right] + 2 \text{ArcTan}\left[\frac{1}{\sqrt{-1 + x^2}}\right] & 1 < x < \sqrt{2} \\ 0 & \text{True} \end{pmatrix}$$

In[8]:= **(*Expected Value*)**

s := 1.

Integrate[**x * h[x], {x, 0, Sqrt[2]}**]

Out[9]= 0.521405