1 NETWORK MODEL

Referring to anisotropic characteristics in local cortical circuits of the rat's brain, a network model implementing anisotropic tissue geometry is developed. The introduction of a rewiring algorithm and qualitative anisotropy measure lay the foundation for the analysis of structural aspects of this model in Chapter ??.

1.1 ANISOTROPIC GEOMETRIC NETWORK MODEL

Taking up the concept of anisotropy in neural connectivity introduced in the last section, we propose here, as basis for this study, a simple geometric network model featuring anisotropic connectivity. Constructing such a model, we're challenged with resembling the anisotropic aspects outlined last section as closely as possible, while at the same time making the model as simple and as abstract as possible, to allow for an abstract and analytical study of such anisotropic networks.

With this in mind, we propose the following model: On a square surface of side length s, a number of N point neurons are randomly, uniformly distributed. Connected neighbors are then calculated for each neuron separately and independently, by determining the randomly, uniformly distributed direction of the neuron's single axon. In this direction the axon traverses over the surface describing a straight path, terminating only when an edge of the surface is reached. Directed contacts are made with every neuron that is within a width $w(x)$ of the axon's trajectory, where in general w depends on the axon length x at this point (Figure 1.1).

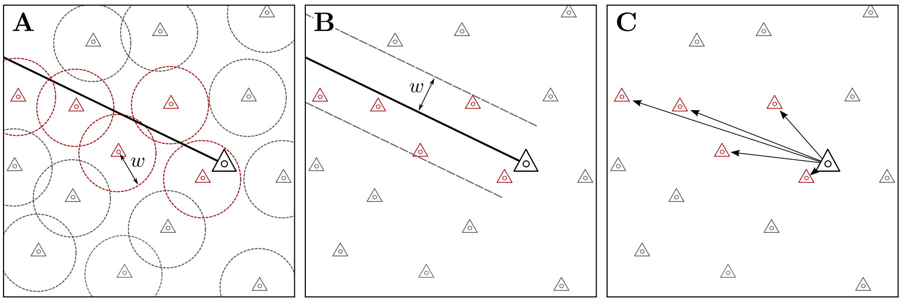

Figure 1.1: Anisotropic geometric network model and interpretations of width parameter w Illustrating the process of finding connections for one neuron (large triangle, black), the axon describes a linear trajectory in an arbitrary direction and until terminating on the surface's edge. Target neurons (red) are encountered along the path within a (here constant) distance w, which is in **A)** interpreted as a dendritic radius or, equivalently, in **B)** as a "bandwith" of the axon. Connections to the encountered targets are then established as projections in **C)**, consistent with the directed nature of synapses in biological networks (cf. Chapter **??**).

random axonal orientation yields relevant connectivity

The implementation of arbitrary axonal orientation is crucial to the model. Although cortical axons are described as consistently projecting downwards (Braitenberg and Schüz 1998, cf. Section **??**), combining exclusively vertically aligned axons with the simplified axonal and dendritic morphological profiles would result in a "vertically staggered connectivity" - neurons could then only project to targets located below them. It is in fact not a vertical alignment of axon orientation, but the anisotropy in neural connectivity - the observation of neuronal targets aligning with the axonal projection - that we try to capture and analyze in this model.

We will refer to the model as the *anisotropic geometric network model*. Trying to provide a simple, abstract model isolating anisotropy in connectivity, in most of this study the width $w(x)$ is assumed to be constant, $w(x) = w$, a notable exception being the exploration in ??. In the graph theoretic context the anisotropic network model is a random graph model, in which a realization of the random process results in a geometric directed graph with a special mode of connectivity. We can formally define such realization as:

Definition 1.1 (Anisotropic geometric graph)**.** Let $n \in \mathbb{N}$ and $w \in (0, \infty)$. An *anisotropic geometric graph* $G_{n,w}$ then consists of a tuple (G, Φ, a), of a simple directed graph G with $|V(G)| = n$ vertices and the maps $\Phi : V(G) \to [0,1]^2$ and $a : V(G) \to [0, 2\pi)$, such that for every vertex pair $v, v' \in V(G)$ and edge $e \in E(G)$ with $s(e) = v$ and $t(e) = v'$ exists if and only if the inequalities

$$R_{-a(v)} \left(\Phi(v') - \Phi(v)\right) \hat{e}_x \geq 0 \quad \text{and} \quad \left| R_{-a(v)} \left(\Phi(v') - \Phi(v)\right) \hat{e}_y \right| \leq \frac{w}{2}$$

hold. Here R_φ is the rotation matrix of angle φ in the Cartesian plane and $\hat{e}_x, \hat{e}_y$ are the standard unit vectors.

The anisotropic random graph model then is then giving the probability distribution over the set of anisotropic random graphs by describing a random process generating such graph.

Definition 1.2 (Anisotropic random graph model)**.** Let $n \in \mathbb{N}$ and $w > 0$. The *anisotropic random graph model* $G(n, w)$ is a probability space over the set of anisotropic geometric graphs with a probability distribution induced by the following process: Let G be an empty graph with n vertices. Assign randomly and uniformly to every vertex $v \in V(G)$ a position $\Phi(v) \in [0,1]^2$ and axonal orientation $0 \leq a(v) < 2\pi$. Then add edges such that (G, Φ, a) is an anisotropic geometric graph $G_{n,w}$.

As with every geometric graph model introduced, we restrict the surface to be the unit square. This does not limit the model, as only the relative width of the axon band in regard to the surface's side length is determining connectivity statistics - the expected number of connections is easily obtained by the quotient of the area covered by the axon and the surface area, making connectivity statistics in the anisotropic random graph model scale-free.

anisotropic model is scale-free

The following maybe interpreted as a study of anisotropic geometric graphs in the light of a neuroscientific context. To enable such an analysis, a few more concepts are needed. The introduction of those concepts composes the rest of the chapter. A first important step is the numerical implementation of the anisotropic network model.

1.2 SUMMARY AND DISCUSSION