1 NETWORK MODEL

Referring to anisotropic characteristics in local cortical circuits of the rat's brain, a network model implementing anisotropic tissue geometry is developed. The introduction of a rewiring algorithm and qualitative anisotropy measure lay the foundation for the analysis of structural aspects of this model in Chapter **??**.

1.1 NUMERICAL IMPLEMENTATION

Numerical implementation of the anisotropic random graph model was achieved in Python[1]. Relying on NumPy as part of the scientific Python library SciPy[2] for the more complex mathematical computations, the implementation also uses graph-tool[3], to ensure convenient and efficient handling of the created networks.

The algorithm for the generation of anisotropic networks closely resembles Definition **??**. After randomly distributing N neurons on the square of side-length s, for every neuron a random axon horientation $a \in [0, 2\pi)$ is chosen and an affine transformation, such that the current neuron is located at the origin and its axon projection aligns with the positive x-axis, secures a straightforward implementation of connectivity, using the the inequalities in Definition **??** as a rule for establishing connections.

parameter set chosen to resemble cortical circuits

To harness the numerical implemenation to generate networks, a set of parameters needs to be chosen. The network size N strongly influences the needed computational efforts in calculations based on the generated graphs and has thus been set to $N = 1000$. Choosing the surface side-length arbitrarily as $s = 100$, the axon width w determines connectivity in the network, the relation between width w and overall connection probability p being shown in Figure 1.1. In their analysis of connectivity of thick-tufted layer V pyramidal cells in neonatal rats (day 14), Song et al. (2005) report an overall connection probability of $p = 0.116$, consistent with prior reports of a cortical connection probability of $p \approx 0.1$. Choosing w to be constant, we determine the axon width such that overall connectivity matches the value report by Song et al. and obtain $w/2 = 12.6$ (Figure 1.1).

sample graphs as reference for structural analysis

Having determined a suitable set of parameters as $N = 1000$, $s = 100$ and $w = 25.2$, we generate 25 graphs with this parameter set (label: `N1000w_ax126-flat_graph0-24`). This set of sample graphs will serve as a reference for the following structural analysis. Extending the set by the (partially) rewired sample graphs (see Section **??**) we obtain a resourceful reference for the analysis of structural features of anisotropic geometric graphs, that we will frequently employ to obtain quantitative and qualitative results.

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1 Python Software Foundation. Python Language Reference, version 2.7. Available at `http://www.python.org`

2 Eric Jones, Travis Oliphant, Pearu Peterson and others. NumPy version 1.6.1. Available at `http://www.scipy.org`

3 Tiago P. Peixoto. Efficient network analysis. Version 2.2.18. Available at `http://graph-tool.skewed.de/`

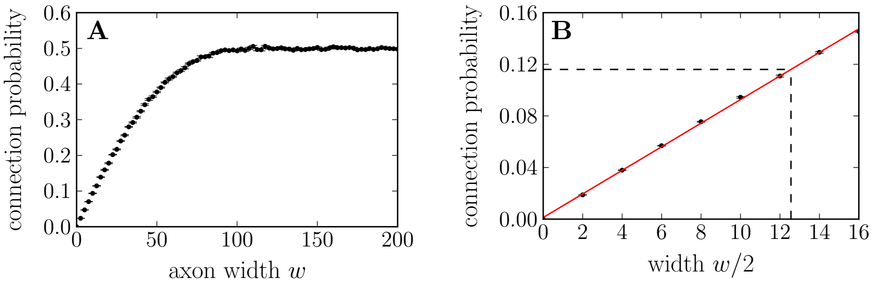

Figure 1.1: Axon width dependent connection probability determines parameter for numerical analysis Generating anisotropic networks with different axon widths w and extracting probability p of directed connection between two random nodes, demonstrates the dependency of p on the width parameter w. **A)** At an axon width of over $w = 100$, exceeding the square's side length, the connection probability saturates at $p = 0.5$, as axon bands are essentially "cutting" the square in a connected and unconnected half (`c5b64f3e`). **B)** For small w the connection probability is a linear function of w, allowing the width $w_S/2$ at which $p(w_S) = 11.6$ to be determined by a linear fit as $w_S/2 = 12.6$ (`585a946f`).

1.2 ANISOTROPY MEASURE

In the last section a method to rewire an anisotropic geometric graph, such that was introduced. From an . In this chapter we introduce.. capturing ..

The $G_{n,\Phi}$ be a geometric graph. Then, for every is the *preferred direction* and its length is

Mardia and Jupp (2000)

Figure 1.2: illustrate varying levels of anisotropy

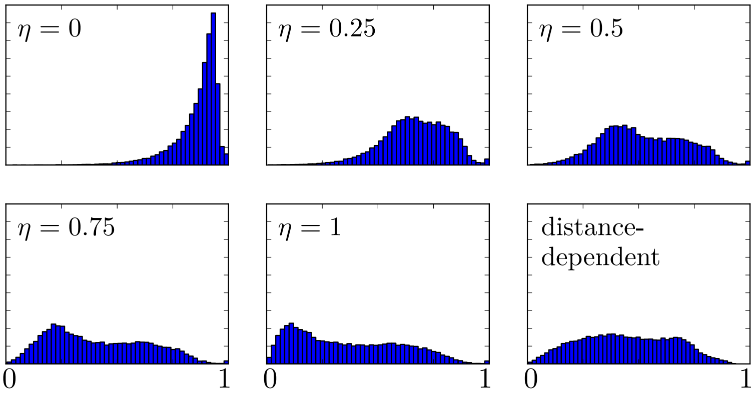

Figure 1.3: Rewiring significantly reduces anisotropy In data taken from the 25 sample graphs (Section 1.1), vertex isotropy degree distribution is shown for the original set of graphs ($\eta = 0$) The characteristic highly anisotropic profile found in the original is already significantly reduced by partial rewiring; anisotropy degree distribution in the fully rewired graphs resemble degree distribution of equivalent purely distance-dependent networks.

... suggesting that fully rewired anisotropic networks do not . There is however one difference in out-degree as an artifact of boundary confinment (Section **??**).

1.3 SUMMARY AND DISCUSSION

BIBLIOGRAPHY

Mardia, Kanti V. and Peter E. Jupp (2000). *Directional Statistics*. 2nd ed. John Wiley & Sons Ltd.

Song, Sen, Per Jesper Sjöström, Markus Reigl, Sacha Nelson, and Dmitri B Chklovskii (2005). Highly Nonrandom Features of Synaptic Connectivity in Local Cortical Circuits. *PLoS Biol* 3.3, pp. 507–519. DOI: `10.1371/journal.pbio.0030068`.