1

STRUCTURAL ASPECTS

1.1 DEGREE DISTRIBUTION

The in- and out-degree of vertex in a directed graph describes the number of incoming and outgoing connection from and to other vertices (cf. Definition ??). As a fundamental concept in graph and network theory, the degree distribution is integral in the categorization of networks and allows for the estimation of graph properties.

Degree distribution was shown to have strong impact on the dynamics of neuronal networks models commonly used in computational neuroscience research (Roxin 2011). Increasing in-degree variance for example could be connected to the appearance of oscillations in the network. Extracting degree distributions from biological networks however, remains a challenge as many neurons need to be tracked simultaneously to obtain enough data to confidently estimate degree distributions.

Here we analyze in- and out-degrees in the anisotropic network model. First we find that compared to the binomial in-degree distribution of a Gilbert random graph model, in-degrees of vertices in anisotropic networks display higher variance and their distribution is skewed to the left (Figure 1.1). However, this specific in-degree profile is not an intrinsic property of anisotropy, as the distribution remains stable under manipulation of the anisotropy degree and closely matches the profile of a purely distance-dependent network (Figure 1.2). This result agrees with findings of Perin et al. (2011, Fig. S3), who were able to recreate degree distributions from their experiment with layer V thick-tufted pyramidal cells in neonatal rats from the extracted distance-dependent connection profiles alone.

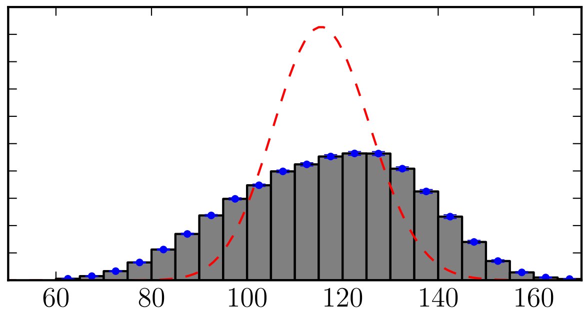

Figure 1.1: In-degree distribution in anisotropic networks shows comparably high variance and is skewed to the left From 250 anisotropic networks in-degree distributions were extracted and are shown in a normed histogram plot, errorbars SEM. Comparison with the binomial degree distribution (red) of a Gilbert random graph model with matching parameter set ($N = 1000$, $p = 0.116$) shows higher variance of in-degrees in anisotropic networks (sample variance = 344.54, variance of binomial distribution $Np(1-p) = 102.44$.) Skewness to the left of the sample is -0.1763. (`9326138e`)

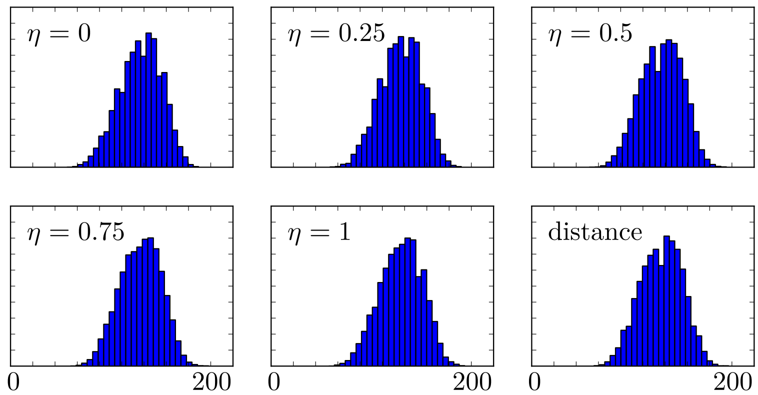

Figure 1.2: In-degree distribution not affected by varying degrees of anisotropy In-degree distributions from the 25 sample graphs (ref ??) and their rewiring stages are plotted in normed histograms and listed from rewiring factor $\eta = 0$ (original anisotropic) to $\eta = 1$ (completely rewired, maximal isotropy). Comparison shows that varying degrees of anisotropy do not influence the degree distribution, in fact in-degree distributions match with the degree distribution of an equivalent distance-dependent network shown bottom-right (`77995b6b`).

While the out-degree distribution of vertices in the anisotropic network also shows itself stable under rewiring, its distribution is drastically different from the out-degree distribution in a comparable distance-dependent network (Figure 1.3). The asymmetric, long-tailed distribution is identified as an artifact of the anisotropic network's spatial confinement; a neuron, closely located near a surface edge, might have an axon projection out of the square causing minimal out-degree or, projecting through the entire length of the surface, may have maximal out-degree. Approximating the expected number of outgoing connections for a vertex in an anisotropic network of size N, side-length s and axon width w as

$$N\frac{wl}{s^2},$$

with parameters $N = 1000$ and $\frac{w}{s} = 0.252$, we obtain an upper bound for the expected out-degree,

$$N\frac{wl}{s^2} \leq N\frac{w}{s}\sqrt{2} \approx 350.$$

If $f(l)$ is the probability density function to find axon length l for a random node in the anisotropic network model, the out-degree distribution is then approximated by

$$\mathbf{Pr}[d_{\text{out}} = N\frac{wl}{s^2}] = f(l),$$

see also Figure 1.4.

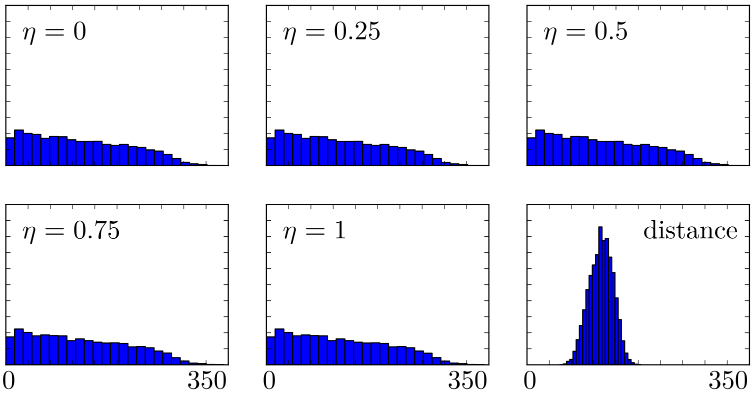

Figure 1.3: Out-degree distribution not affected by varying anisotropy but highly different from distance-dependent networks Out-degree distributions from the 25 sample graphs (ref ??) and their rewiring stages are plotted in normed histograms and listed from rewiring factor $\eta = 0$ (original anisotropic) to $\eta = 1$ (completely rewired, maximal isotropy). While varying degrees of anisotropy do not influence the degree distribution, the characteristic out-degree profile is drastically different from the distribution found in equivalent distance-dependent networks (`77995b6b`).

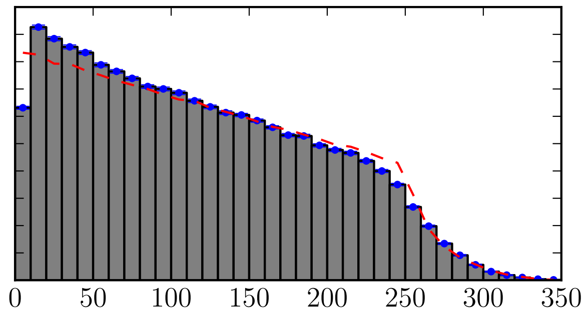

Figure 1.4: Characteristic out-degree distribution as an artifact of network's boundaries The incline at the beginning is only barely visible due to the choice of bin size. (cf. **??**) (`019555b0`)

1.2 SMALL WORLD PROPERTIES

Sporns papers

1.3 MOTIFS

BIBLIOGRAPHY

Perin, Rodrigo, Thomas K. Berger, and Henry Markram (2011). A synaptic organizing principle for cortical neuronal groups. *Proceedings of the National Academy of Sciences* 108.13, pp. 5419–5424. DOI: 10.1073/pnas.1016051108.

Roxin, Alex (2011). The Role of Degree Distribution in Shaping the Dynamics in Networks of Sparsely Connected Spiking Neurons. *Frontiers in Computational Neuroscience* 5. DOI: 10.3389/fncom.2011.00008.

Symbol	**Description**
L	Length
Ma	Mach number
p	Pressure