1 NETWORK MODEL

Referring to anisotropic characteristics in local cortical circuits of the rat's brain, a network model implementing anisotropic tissue geometry is developed. The introduction of a rewiring algorithm and qualitative anisotropy measure lay the foundation for the analysis of structural aspects of this model in Chapter ??.

1.1 REWIRING

eliminate anisotropy through rewiring

It is in our highest interest to compare results to. To this end we introduce an algorithm that preserves distance-dependent connectivity as found in Proposition **??**, but eliminates anisotropy in network connectivity by consecutively rewiring existing connections to new suitable targets.

Rewiring as presented here, will provide the transition from anistropic connectivity to networks isotropic in connectivity, closely resembling purely distance-dependent networks. Applying this process only partially will then allow us to analyse structural features as they change with a varying degree of isotropy, asserting the importance of this process to our study.

In designing the specific rewiring algorithm we identify two requirements that our implementation should satisfy:

1. elimination of anisotropy in connectivity
2. preservation of distance-dependent connectivity

The second point is especially important to us, as we want to impose isotropy on the network at "minimal cost", that is by changing as little as possible about the other characteristics of the network's connectivity. The following process respects both of the points above: For every edge between vertices v and v' with inter-vertex distance x, identify neurons with distance to v in the range of $(x-\varepsilon, x+\varepsilon)$ as potential new targets. Then pick at random one of these vertices, including v', as a new target for the current edge, if such an edge doesn't already exist. In the graph theoretic context we formally define rewiring as follows:

Definition 1.1. Let G be an anisotropic geometric graph with $|V(G)| = n$. Then we define a rewiring of G to be probability space over G^n_Φ, induced by the following process: For every edge $e \in E(G)$ uniformly at random pick a potential new target $t'(e)$ from the set $M_e = T_e \setminus K_e$, where T_e is the set of all vertices that differ in their distance to $s(e)$ less than ε from the distance of $s(e)$ to $t(e)$,

$$T_e = \{v \in V(G) \setminus s(e) \mid |\mathrm{d}(s(e), v) - \mathrm{d}(s(e), t(e))| < \varepsilon\}$$

and K_e the set vertices that already are connected to $s(e)$ by another edge,

$$K_e = \{v \in V(G) \mid \exists\, e' \in E(G) : s(e') = s(e), t(e') = v\}\,.$$

Let $\tilde{C}(x)$ be the distance-dependent connectivity profile of a rewiring R_ε of an anisotropic graph $G_{n,w}$. Denote with $C(x)$ the distance-dependent connection probability of the $G_{n,w}$. We can estimate the

$$\begin{aligned}
\mathbf{E}[\tilde{C}(x)] - C(X) &= \int_{x-\varepsilon}^{x+\varepsilon} f(x')C(x')\,dx - C(x) \\
&= \frac{1}{2\varepsilon}\int_{x-\varepsilon}^{x+\varepsilon} C(x') - C(x)\,dx \\
&= \frac{1}{2\varepsilon}\left\{\int_{x-\varepsilon}^{x} C(x') - C(x)\,dx - \int_{x}^{x+\varepsilon} C(x') - C(x)\,dx\right\} \\
&= \frac{1}{2\varepsilon}
\end{aligned}$$

Remark. Partial rewiring. $R_{\varepsilon,\eta}$

Here we choose $\varepsilon =$??.

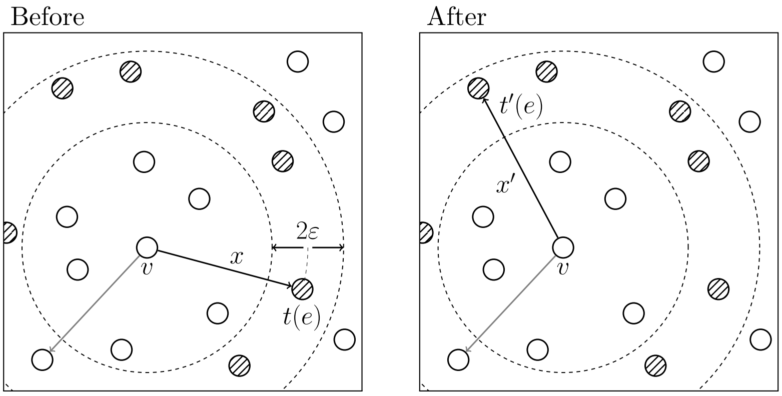

Figure 1.1: Rewiring transforms anisotropic geometric graphs to networks with isotropic connectivity For a given edge e with a distance x from its source vertex v to its target vertex $t(e)$, potential new targets (striped) are found in within a distance $(x-\varepsilon, x+\varepsilon)$ of v. The rewired edge then projects from v to a new target $t'(e)$, randomly chosen from the set of vertices within in this range. Inter-vertex distance between v and $t'(e)$ differs by less than ε from x, ensuring that for small ε the original distance-dependent connectivity is preserved.

1.2 SUMMARY AND DISCUSSION