1

STRUCTURAL ASPECTS

Stepping away, we

Maybe simplicity of the model makes stuff wrong. Tune to Perin distance profile.

Let

In their study, Perin et al. (2011) heavily rely on a distance-dependent

Here we introduce anisotropic networks tuned to reflect a given distance-dependent connection profile $C(x)$. We are faced with the following problem: Given $C(x) : [0, \sqrt{2}) \to [0, 1]$, find $w : [0, \sqrt{2}) \to [0, \infty)$ such that the probability to have a connection from v_1 to v_2 for arbitrary vertices $v_1 \neq v_2$ in an anisotropic graph $G(n, w)$ with distance $\mathrm{d}(v_1, v_2) = x$ is $C(x)$. The problem is in general highly complex, when nothing can be assumed about $C(x)$. We find an approximate solution to the problem regarding the following geometric relation:

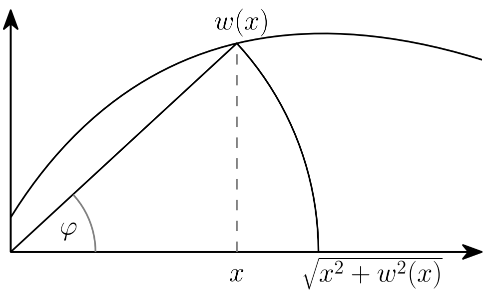

Figure 1.1: Computing connection probability $C(x)$ from non-constant $w(x)$

From Figure 1.1 we have the relation

$$C\left(\sqrt{x^2 + w^2(x)}\right) = \frac{1}{\pi} \arctan \frac{w(x)}{x}. \tag{1.1}$$

In order to solve for $w(x)$ we first consider a linear approximation, expanding

$$C\left(\sqrt{x^2 + w^2(x)}\right) \approx C(x) + \left(\sqrt{x^2 + w^2(x)} - x\right) C'(x).$$

The resulting transcendental equation

$$C(x) + \left(\sqrt{x^2 + w^2(x)} - x\right) C'(x) = \frac{1}{\pi} \arctan \frac{w(x)}{x}$$

is however still too complex in the context of this work. Instead we propose the approximation $\sqrt{x^2 + w^2(x)} \approx x$, which inserting into 1.1 yiels

$$C(x) \approx \frac{1}{\pi} \arctan \frac{w(x)}{x}. \tag{1.2}$$

Under the assumption that $C(x) < \frac{1}{2}$ for all x we obtain the identity

$$w(x) = x \tan\left(\pi\, C(x)\right), \tag{1.3}$$

being aware that it only holds as well as approximation 1.2 does.

Here we use relation 1.3 to generate anisotropic networks reflecting the distance-dependent connectivity profile as found by Perin et al. (2011). For this we finally need to adjust the before arbitrarily determined side length of the network's surface. Perin et al. mapped connectivity in layer 5 of the rat's somatosensory cortex up to a distance of 300 µm. Using this reported distance connectivity to generate anisotropic networks via 1.3, the chosen side length s determines the networks overall connectivity (Figure 1.2 A). We determine $s = 296$ µm to match the overall connection probability of $p = 0.116$ as used before and reported by Song et al. (Figure 1.2 B). The obtained value for s is consistent with the slice thickness of 300 µm used in Perin et al.'s experiment.

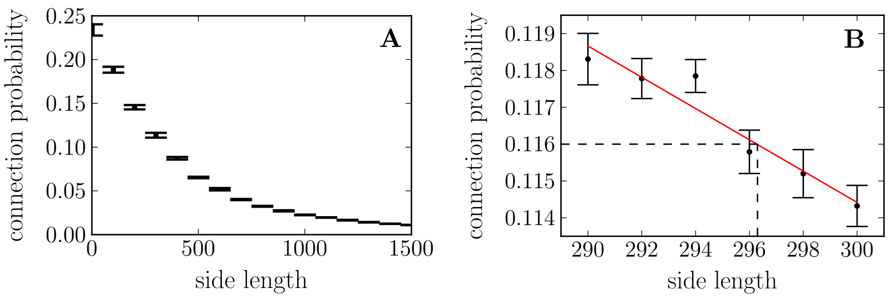

Figure 1.2: Network side length adjusted to match overall connection probability A-B) Side length of the network's surface determines the connection probability, in B) length s is matched to $p = 0.116$, as reported by Song et al. (2005). **C)** Resulting axon width function $w(x)$ from tuning to distance-dependent connection profile as reported by Perin et al. (2011), see also Figure 1.4. Note that $x \gg w(x)$ for most x, justifying the approximation 1.2. **D)** Showing for a single neuron (star) connected (red) and unconnected (gray) neurons in the tuned anisotropic network, revealing characteristic new axon shape.

Having determined the neotwork's side length s, we're extending the quiver of generated sample networks for the numerical analysis once more by the "tuned anisotropic graphs" , in which the axon width $w(x)$ was determined such that the networks reflect Perin's connectivity profile. Analyzing the obtained axon width function we note that $x \gg w(x)$ holds for most x, justifying the approximation

$$\sqrt{x^2 + w^2(x)} \approx x$$

a posteriori (Figure 1.3). From the 25 generated networks overall connection probability is extracted as $p = 0.1160 \pm 0.0006$ (SEM), as expected from the choice of s (`f11dca65`).

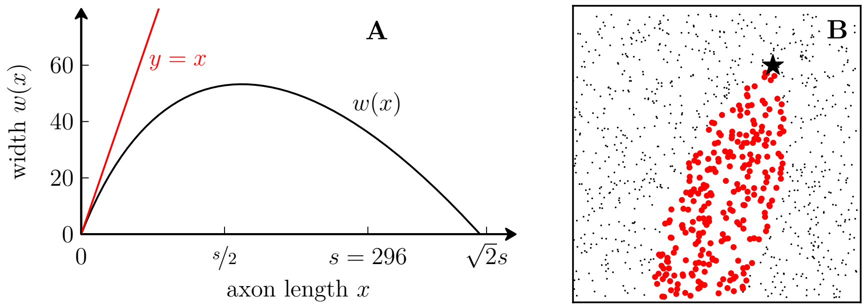

Figure 1.3: Anisotropic network model with tuned axon width w(x) **A-B)** Side length of the network's surface determines the connection probability, in B) length s is matched to $p = 0.116$, as reported by Song et al. (2005). **C)** Resulting axon width function $w(x)$ from tuning to distance-dependent connection profile as reported by Perin et al. (2011), see also Figure 1.4. Note that $x \gg w(x)$ for most x, justifying the approximation 1.2. **D)** Showing for a single neuron (star) connected (red) and unconnected (gray) neurons in the tuned anisotropic network, revealing characteristic new axon shape.

Taking the distance-dependent connection profile $C(x)$ in the anisotropic network model (cf. Theorem **??**), we find that x is strictly (**??**)

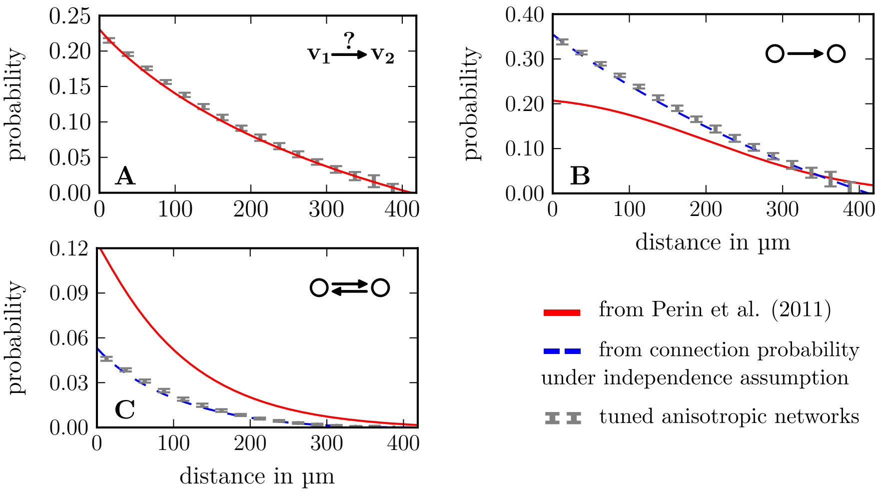

Figure 1.4: Overrepresentation of reciprocal connections independent of Comparison of occurrences of one- and bidirectionally connected neuron pairs in (gray) with profiles found by Perin et al. (2011) (red), shows that overrepresentation of bidirectional pairs is distance-independent and not connected to anisotropy. **A)** Overall connection probability in the adapted anisotropic networks was successfully tuned to reflect connection probability found by Perin et al. **B)-C)** Probabilities for a random neuron pair to display , (875505b0)