

## NETWORK MODEL

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Referring to anisotropic characteristics in local cortical circuits of the rat's brain, a network model implementing anisotropic tissue geometry is developed. The introduction of a rewiring algorithm and qualitative anisotropy measure lay the foundation for the analysis of structural aspects of this model in Chapter ??.

## 1.1 REWIRING

*eliminate  
anisotropy  
through rewiring*

It is in our highest interest to compare results to. To this end we introduce an algorithm that preserves distance-dependent connectivity as found in Proposition ??, but eliminates anisotropy in network connectivity by consecutively rewiring existing connections to new suitable targets.

Rewiring as presented here, will provide the transition from anisotropic connectivity to networks isotropic in connectivity, closely resembling purely distance-dependent networks. Applying this process only partially will then allow us to analyse structural features as they change with a varying degree of isotropy, asserting the importance of this process to our study.

In designing the specific rewiring algorithm we identify two requirements that our implementation should satisfy:

1. elimination of anisotropy in connectivity
2. preservation of distance-dependent connectivity

The second point is especially important to us, as we want to impose isotropy on the network at “minimal cost”, that is by changing as little as possible about the other characteristics of the network’s connectivity. The following process respects both of the points above: For every edge between vertices  $v$  and  $v'$  with inter-vertex distance  $x$ , identify neurons with distance to  $v$  in the range of  $(x - \varepsilon, x + \varepsilon)$  as potential new targets. Then pick at random one of these vertices, including  $v'$ , as a new target for the current edge, if such an edge doesn’t already exist (Figure 1.1). In the graph theoretic context we formally define a rewiring as follows:

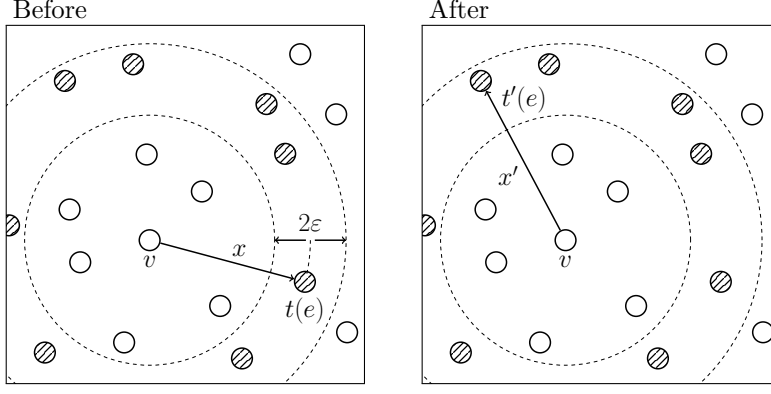
**Definition 1.1.** Let  $G$  be an anisotropic geometric graph with  $|V(G)| = n$ . Then we define a *rewiring* of  $G$  to be probability space over  $G_{\Phi}^n$ , induced by the following process: For every edge  $e \in E(G)$  uniformly at random pick a potential new target  $t'(e)$  from the set  $M_e = T_e \setminus K_e$ , where  $T_e$  is the set of all vertices that differ in their distance to  $s(e)$  less than  $\varepsilon$  from the distance of  $s(e)$  to  $t(e)$ ,

$$T_e = \{v \in V(G) \setminus s(e) \mid |d(s(e), v) - d(s(e), t(e))| < \varepsilon\}$$

and  $K_e$  the set vertices that already are connected to  $s(e)$  by another rewired edge,

$$K_e = \{v \in V(G) \mid \exists e' \in E'(G) : s(e') = s(e), t(e') = v\},$$

where  $E'(G)$  is the set of all edges that have been rewired already.



**Figure 1.1: Rewiring transforms anisotropic geometric graphs to networks with isotropic connectivity** For a given edge  $e$  with a distance  $x$  from its source vertex  $v$  to its target vertex  $t(e)$ , potential new targets (striped) are found in within a distance  $(x - \varepsilon, x + \varepsilon)$  of  $v$ . The rewired edge then projects from  $v$  to a new target  $t'(e)$ , randomly chosen from the set of vertices within in this range. Inter-vertex distance between  $v$  and  $t'(e)$  differs by less than  $\varepsilon$  from  $x$ , ensuring that for small  $\varepsilon$  the original distance-dependent connectivity is preserved. (Note that all targets within range are eligible for rewiring as no other edges exist. In general this is not the case.)

*Remark.* In the way Definition 1.1 is formulated, it is possible for  $M_e$  to be empty for an edge  $e$ . In this case no new edge is realized and the resulting, rewired network has  $|E(G)| - 1$  edges. In practice this happens less than

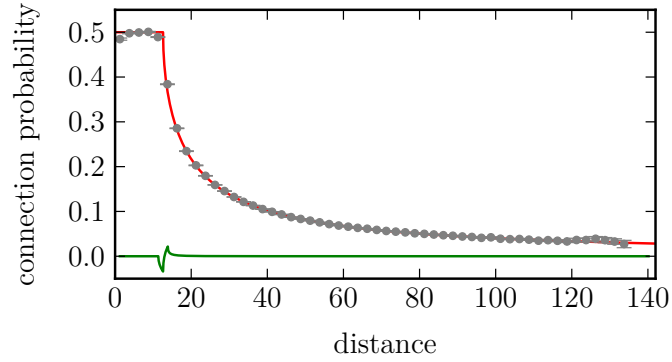
Let  $\tilde{C}(x)$  be the distance-dependent connectivity profile of a rewiring  $R_\varepsilon$  of an anisotropic graph  $G_{n,w}$ . Denote with  $C(x)$  the distance-dependent connection probability of the  $G_{n,w}$ . We can estimate the

$$\begin{aligned}
 \mathbf{E}[\tilde{C}(x)] - C(x) &= \int_{x-\varepsilon}^{x+\varepsilon} f(x')C(x') dx - C(x) \\
 &= \frac{1}{2\varepsilon} \int_{x-\varepsilon}^{x+\varepsilon} C(x') - C(x) dx \\
 &= \frac{1}{2\varepsilon} \left\{ \int_{x-\varepsilon}^x C(x') - C(x) dx - \int_x^{x+\varepsilon} C(x') - C(x) dx \right\} \\
 &= \frac{1}{2\varepsilon}
 \end{aligned}$$

As a generalization of Definition 1.1, we define a partial rewiring  $R_{\varepsilon,\eta}$ , finding new targets only for a fraction  $\eta$  of all edges:

**Definition 1.2.** Let  $\varepsilon > 0$  and  $0 \leq \eta \leq 1$ . A *partial rewiring*  $R_{\varepsilon,\eta}$  of an anisotropic geometric graph  $G_{n,w}$  is then a rewiring  $R_\varepsilon$  of  $G_{n,w}$ , in which every edge is rewired with a probability of  $\eta$ , otherwise it remains. To avoid the occurrence of multiple edges,  $K_e$  is then extended to include the targets of all edges originating from  $s(e)$  that will not be rewired.

Here we choose  $\varepsilon = ??$ .



**Figure 1.2: Predicted distance-dependent connection probability profile is matched by numerical results** Averaging distance-dependent connection probabilities over the 25 sample graphs, we find the expected profile calculated in Theorem ?? is matched perfectly by the numerical results. (4f4dfcf1)

## 1.2 SUMMARY AND DISCUSSION