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APPENIDX

1.1 REPRODUCIBILITY OF COMPUTATIONAL RESULTS

Computational implementations of network models and numerical analysis of their features are an integral part of this study. In order to ensure the reproducibility of this part of the analysis, *Sumatra*[1], a software for provenance capture of computational projects, was used. As a "lab notebook for computational researchers", the software combines version control of code with the capture of inputs and outputs, as well as the used parameter sets, for each computational process (Davison 2012). Providing a database and various interfaces, the tool allows not only direct management of simulations and data but provides a platform to make the computational reproducible and accessible to others.

Sumatra labels

In this study, for every computational result referenced in the text, a label was cited, referring to the record of the process in Sumatra's database. Code and produced data are understood as part of the work of this thesis; giving references to computational processes in which the results were obtained, allows the reader at any point to see the exact implementation of the described process or to look up a parameter not explicitly mentioned in the text (Figure 1.1). Through this, the work presented here displays reproducibility and transparency of the described methods and results, meeting this important requirement in all scientific work.

use of free, open source software

Further contributing to the reproducibility of this work is the exclusive use of free and open source software. With the single exception of *Mathematica*[2], all tools used to create this work are freely available and can be modified as their source code is openly accessible. The list of software used in this thesis includes, additionally to the programs mentioned throughout the text, *GNU Emacs*[3], *Inkscape*[4], *GNU Image*

1 Sumatra: Automated tracking of scientific computations. Available at `http://neuralensemble.org/sumatra/`
2 Wolfram Mathematica Version 7.0.1.0. See `http://www.wolfram.com/mathematica/`
3 GNU Emacs Version 24.3. Free Software Foundation. Available at `http://www.gnu.org/software/emacs/`
4 Inkscape Version 0.48.3.1. Available at `http://www.inkscape.org`

Manipulation Program[5] and *TeX Live*[6]. The document's layout is based on an adapted version of *Classicthesis*[7].

```
------------------------------------------------------------------------
Label            : com_inp_N500-trials5-step010
Timestamp        : 2014-06-06 17:29:39.663398
Reason           : common input variance
Outcome          :
Duration         : 1087.83452988
Repository       : GitRepository at /users/hoffmann/research
Main_File        : comp/common_input_variance.py
Version          : 10690e97199851f08955ed7944f7eadfe71b4bf2
Script_Arguments : com_inp_N500-trials5-step010 <parameters>
Executable       : Python (version: 2.7.3) at /usr/bin/python
Parameters       : rew_frac_min = 0.1
                 : n_trials = 5
                 : rew_margin = 1.25
                 : Torus = False
                 : l_ax = 1000
                 : rew_frac_max = 1.0
                 : N = 500
                 : ed_l = 296
                 : rew_frac_step = 0.1
                 : save_the_graph = True
                 : comp_label = "common_input_variance/"
                 : self_loops_allowed = False
                 : parallel_edges_allowed = False
Input_Data       : []
Launch_Mode      : serial
Output_Data      :[common_input_variance/com_inp_N500-trials5-step010.p
                 : (1aacdd2202ba8ce6a419b6e8cf5d241c2ef37454)]
User             : Felix Hoffmann <Felix11H@github.nomail>
Tags             : clustering
------------------------------------------------------------------------
```

Figure 1.1: Example Sumatra record entry Showing the label, parameter set and outputs, Sumatra record entries display the full information required to reproduce referenced results. The version number is here of critical importance; it refers to a snapshot of the code base at the time of simulation. Through it the exact code at this point can be recovered and used to reproduce results.

5 GNU Image Manipulation Program Version 2.8.8. Available at `http://www.gimp.org/`

6 Tex Live 2012. D.E. Knuth. Available at `https://www.tug.org/texlive/`

7 Classicthesis 4.1. André Miede. Available at `http://www.ctan.org/pkg/classicthesis`

```
In[1]:= f[d_] = Piecewise[{{1 / (s * (d) ^ (1 / 2)) - 1 / (s^2), 0 < d < s^2}, {0, d > s^2}}]
```

Out[1]= $\begin{cases} -\frac{1}{s^2} + \frac{1}{\sqrt{d}\, s} & 0 < d < s^2 \\ 0 & \text{True} \end{cases}$

```
In[2]:= g[x_] := Convolve[f[d], f[d], d, x, Assumptions → {d ∈ Reals, x ∈ Reals}]
        Simplify[g[x], {s > 0, x ∈ Reals}]
```

Out[3]= $\begin{cases} \frac{\pi s^2 - 4 s \sqrt{x} + x}{s^4} & x > 0 \,\&\&\, s^2 \ge x \\ -\frac{2 s^2 + x + \frac{4 s^3}{\sqrt{-s^2+x}} - \frac{4 s x}{\sqrt{-s^2+x}} - 2 s^2 \operatorname{ArcTan}\left[\frac{s}{\sqrt{-s^2+x}}\right] + i s^2 \operatorname{Log}\left[s - i \sqrt{-s^2+x}\right] - i s^2 \operatorname{Log}\left[s + i \sqrt{-s^2+x}\right]}{s^4} & s^2 < x \,\&\&\, 2 s^2 > x \\ 0 & \text{True} \end{cases}$

```
In[4]:= h[x_] := g[x^2] * 2 * x
In[5]:= Simplify[h[x], {s > 0, x ∈ Reals, x > 0}]
```

Out[5]= $2 x \left(\begin{cases} \frac{\pi s^2 - 4 s x + x^2}{s^4} & s \ge x \\ -\frac{2 s^2 + x^2 + \frac{4 s^3}{\sqrt{-s^2+x^2}} - \frac{4 s x^2}{\sqrt{-s^2+x^2}} - 2 s^2 \operatorname{ArcTan}\left[\frac{s}{\sqrt{-s^2+x^2}}\right] + 2 s^2 \operatorname{ArcTan}\left[\frac{\sqrt{-s^2+x^2}}{s}\right]}{s^4} & s < x \,\&\&\, \sqrt{2}\, s > x \\ 0 & \text{True} \end{cases} \right)$

```
In[6]:= (*For s == 1, h becomes*)
In[7]:= Simplify[h[x], {s == 1, x ∈ Reals, x > 0}]
```

Out[7]= $2 x \left(\begin{cases} \pi + (-4 + x)\, x & x \le 1 \\ -2 - x^2 + 4 \sqrt{-1 + x^2} - 2 \operatorname{ArcCot}\left[\frac{1}{\sqrt{-1+x^2}}\right] + 2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1+x^2}}\right] & 1 < x < \sqrt{2} \\ 0 & \text{True} \end{cases} \right)$

```
In[8]:= (*Expected Value*)
        s := 1.
        Integrate[x * h[x], {x, 0, Sqrt[2]}]
Out[9]= 0.521405
```

Mathematica 1.1: Computation of probability density function for distance between to random points in square of side length s as supplement to proof of Theorem ??. Note that form of final result `Out[7]` differs from solution given in ?? for $1 < x < \sqrt{2}$. While proof of equivalence could not be achieved analytically, expressions given are numerically equivalent.

```
In[200]:= (*Expected distance between reciprocally connected pairs*)
          r = NIntegrate[x * w[x] * c[x]^2 / 0.02451, {x, 0, Sqrt[2]}]
Out[200]= 0.20001
In[201]:= (*Probability of connection in third pair when other two are reciprocally connected*)
          p = c[4 / Pi * r]
Out[201]= 0.164749
In[202]:= (*Probabilities for motif 9, 15 and 16 are then:*)
          (1 - p)^2
          2 * p * (1 - p)
          p^2
Out[202]= 0.697643
Out[203]= 0.275214
Out[204]= 0.0271424
```

Mathematica 1.2: Computation of three motifs for Section ??. Function `c[x]` is the distance-dependent probability distribution from Theorem and `w[x]` the probability density function for distance between two random points in a box (cf. Mathematica 1.1, Moltchanov 2012).

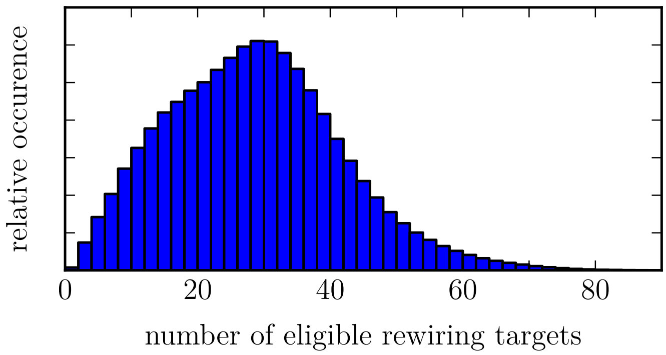

Figure 1.2: (4afc2727)

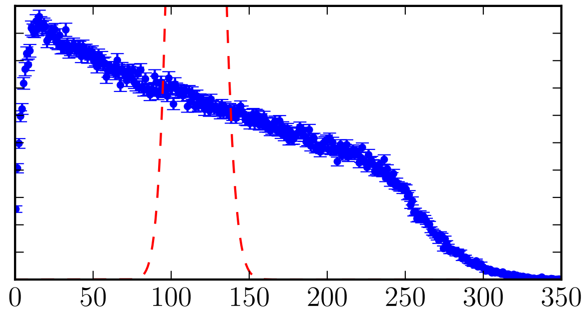

Figure 1.3: (c7ee86d7)

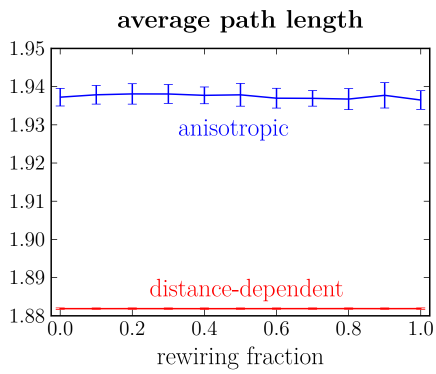

Figure 1.4: Average path length for anisotropic and distance-dependent networks, $N = 1000$. (064f9b10)

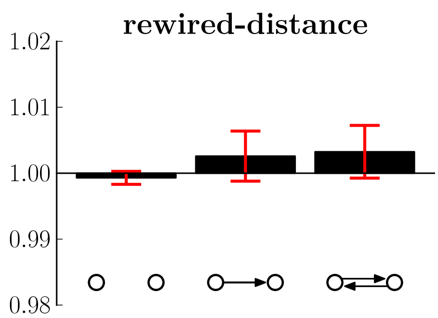

Figure 1.5: Probabilities for connections in neuron pairs are identical in distance-dependent and rewired anisotropic networks. (c5f1462b)