Felix Carter – MATLAB Language Dynamics

Introduction

The MATLAB file language_dynamics.m contains the function language_dynamics which takes a vector x containing the values for B, I and P (representing the proportion of the population which are classified as basic, independent/intermediate or proficient speakers of the language respectively) and calculates the rates of change of B, I and P with time using global parameters. The parameters used in this question are from the model of Welsh speakers in the research paper Barrett-Walker et al., 2020 and can be found defined in the file q1_params.m. These are set as global parameters. The equations for rates of change are from this research paper and given by:

$$\frac{dB}{dt} = \dot{B} = r(B + I + (1 - \alpha)P) - \beta_{BI}BP - rB$$

$$\frac{dI}{dt} = \dot{I} = \beta_{BI}BP - \beta_{IP}IP - rI$$

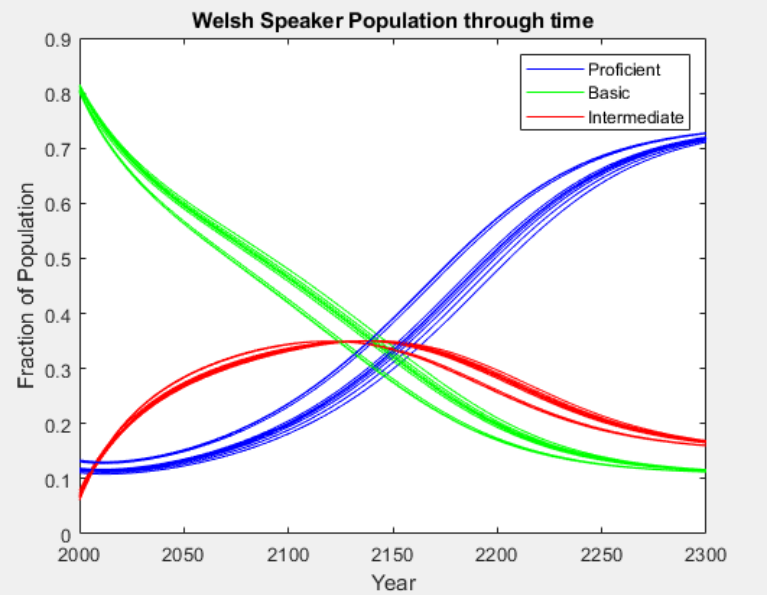
$$\frac{dP}{dt} = \dot{P} = \alpha rP + \beta_{IP}IP - rP$$

The file q1_evaluate.m evaluates the system based on the initial values given for B, I and P population proportions (based on data from Wales in 1991 in the study and defined locally as the vector x) and outputs the values of dB/dt, dI/dt and dP/dt respectively based on these initial conditions, and the global parameters set in q1_params.m. This is done by running the function language_dynamics.

Welsh Speaking Population

The file q3a_run.m outputs a graph in MATLAB of the population levels for each year, from running ode45. The function q3_dynamics (from q3_dynamics.m) is used within this. A for loop is used so that ode45 is ran 10 times, each with slightly different random initial conditions B, I, P corresponding to the speaker population proportions. (these are very similar to the 1991 Welsh initial conditions from the study, just with some slight random variation).

The function q3_dynamics is ran using ode45, and then the values of t against B, I and P respectively are plotted on a graph between 2000 and 2300 for the ten iterations. (Basic is in green, Intermediate is in red, and Proficient is in blue, as in the Fig. 3a graph in the research paper):



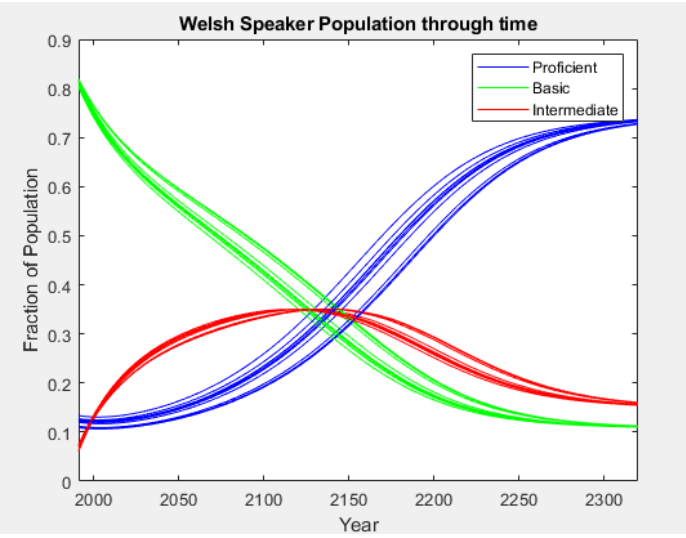
Each iteration produces slightly different results due to the random initial conditions, however we can see that the results for these conditions seem to converge to fixed points similar to the solution B=0.1076, I=0.1485, P=0.7440.

Looking at the graph produced, it seems that the proportions of Welsh speakers are not quite stable in 2300. The graph from Fig. 3a in the research paper begins in 1991 and ends in approximately 2320, so by changing the ICs to 1991 and extending the time until 2320 in the file q3a_run2.m, the results outputted are more similar to the fixed point we expect, for example

```
fixed_point =

    0.1102    0.1547    0.7351
```

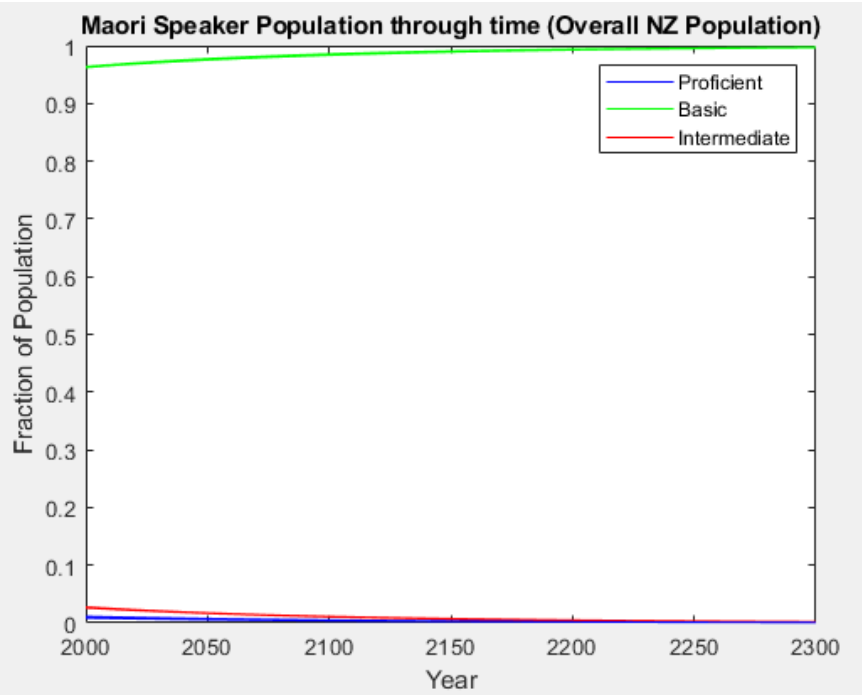
Rounded to 2d.p. gives B=0.11, I=0.15, P=0.74, matching our fixed to 2 decimal places.



The graph suggests that it will take until approximately the mid-2300s for the population to fully stabilise.

Māori Speaking Population

The function q3_dynamics is used with ode45 in order to simulate the population dynamics between the years 2000 and 2300 for these initial conditions, and a graph is produced to simulate the population dynamics:



This shows that for these initial conditions and parameters, the proportion of the population who are categorised as intermediate or proficient Māori speakers decreases towards zero, and the proportion of the population classified as basic approaches 100%. As the paper explains, if the entire population is classified as basic, the language is classified as extinct, so this graph suggests that the Māori language will go extinct in New Zealand based on the current learning rates and population proportions. Indeed, in the year 2300 the language proportions are predicted to be approximately:

```
fixed_point =


    0.9971    0.0020    0.0009
```

This means that well over 99% of the population are classified into the basic category in 2300. With such a small proportion of the population being able to speak the language (approximately 0.09% in the proficient category), it would effectively be extinct and the graph shows that with these parameters and initial conditions, the proportion of the population who can speak Māori would only decrease with time and would never increase.

This is a stark contrast to the predictions for the Welsh language, where the proportion of the population who can speak the language proficiently is expected to be approximately 74% by 2300 – despite the vast majority of Wales being classified into the basic speaker category for the initial conditions, and both countries having similar intergenerational transmission rates α. However, the

initial percentage of the population classified as basic is much greater for the Māori language is much greater than for the Welsh language, and the learning rate parameters β_{IP}=0.0016 and β_{BI}=0.0138 for Māori are far lower than the learning rate parameters for Welsh (β_{IP}=0.0510 and β_{BI}=0.0969).