UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO
FACULTAD DE INGENIERÍA
CIRCUITOS ELÉCTRICOS
SEMESTRE 2020 - 2

Tarea 3: Ejercicio y apuntes

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TAREA 3

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Friday, 27 March 2020 6:46 PM

// Previo

$$V = RI \; ; \; V = \frac{I}{j\omega C}$$

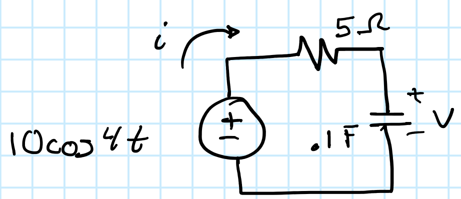
// Alternativa

$$\frac{V}{I} = R \; ; \; \frac{V}{I} = j\omega L \; ; \; \frac{V}{I} = \frac{1}{j\omega C}$$

// Para los 3 casos

$$Z = \frac{V}{I} \quad \Big| \quad Y = \frac{1}{Z} \; ; \; Y = \frac{1}{R} \; ; \; Y = \frac{1}{j\omega L} \; ; \; Y = j\omega C$$

→ Problema | Encontrar $v(t)$ e $i(t)$



// De la fuente

$$\omega = 4 \; \frac{rad}{s}$$

$$V_s = 10 \angle 0^\circ \; [V]$$

// Impedancia

$$Z = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{4(j)(0.1)} = 5 - 2.5j \, [\Omega]$$

// Corriente

$$I = \frac{V_s}{Z} = \frac{10 \angle 0^\circ}{5 - 2.5j} = \frac{10(5 + 2.5j)}{5^2 + 2.5^2} \approx 1.6 + .8j = 1.78 \angle 26.5^\circ$$

[A]

// Voltaje en Capacitor

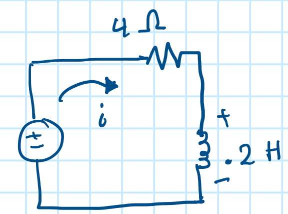
$$V = I Z_c = \frac{I}{j\omega C} = \frac{1.78 \angle 26.5^\circ}{(4j)(.1)} = 4.47 \angle -63.48^\circ \, [V]$$

• Convirtiendo corriente I y voltaje V al dominio del tiempo

$\therefore\ i(t) = 1.789 \cos(4t + 26.5°)$ [A]

$\therefore\ v(t) = 4.47 \cos(4t - 63.43°)$ [V]

Problema | Determine $v(t)$ y $i(t)$



$V_s =$ $5 \sin 10t$

$V_s = 5\sin(10t) = 5\cos(10t - 90)$

$= 5\angle{-90}$ $\quad \omega = 10\ \frac{rad}{s}$

// Impedancia

$Z = 4 + j\omega L = 4 + j(10)(.2) = 4.472\angle{26.57°}$

// Corriente

$$I = \frac{V_s}{Z} = \frac{5\angle{-90}}{4.472\angle{26.57}} = 1.118\angle{-116.57°}\ [A]$$

// Voltaje en Inductor

$$V = I(j\omega L) = (1.118\angle{-116.57})(.2\angle{90}) = 2.236\angle{-26.57°}\ [V]$$

// Dominio Tiempo

$i(t) = 1.118 \cos(10t - 116.57)$ [A]

$v(t) = 2.236 \cos(10t - 26.57°)$ [V]

$\therefore$ El voltaje se adelanta a la corriente

→ Leyes de Kirchoff en el dominio del Fasor

• Para LVK sean $V_1, V_2, \ldots V_n$ los voltajes alrededor de lazo cerrado, entonces

Para LVK sean $V_1, V_2, \dots V_n$ los voltajes alrededor de lazo cerrado, entonces

$$V_1 + V_2 + \dots + V_n = 0$$

- En estado estacionario sinusoidal, cada voltaje pueden ser escrito en forma coseno

$$V_{m_1}\cos(\omega t + \theta_1) + V_{m_2}\cos(\omega t + \theta_2) + \dots +$$
$$V_{m_n}\cos(\omega t + \theta_n) = 0$$

- Esto puede ser escrito como:

$$Re\left(V_{m_1} e^{j\theta_1} e^{j\omega t}\right) + Re\left(V_{m_2} e^{j\theta_2} e^{j\omega t}\right) + \dots$$
$$+ Re\left(V_{m_n} e^{j\theta_n} e^{j\omega t}\right) = 0$$

si $V_k = V_{mk} e^{j\theta k}$

$$Re\left[(V_1 + V_2 + \dots + V_n) e^{j\omega t}\right] = 0$$

Puesto $e^{j\omega t} \neq 0$

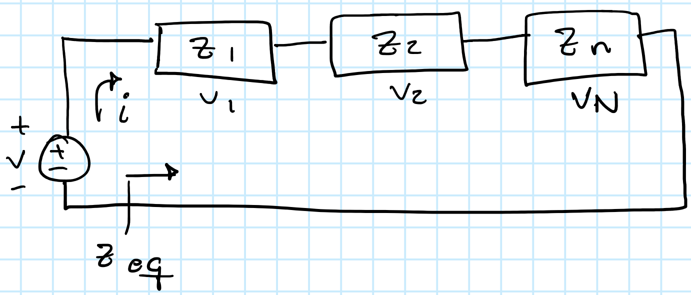
$\therefore V_1 + V_2 + \dots + V_n = 0$

- Demostración que la LVK se mantiene en fasores

$$I_1 + I_2 + \dots + I_n = 0$$

"Combinación de impedancias"

- Considerando N impedancias conectadas en serie y aplicando LVK alrededor del lazo



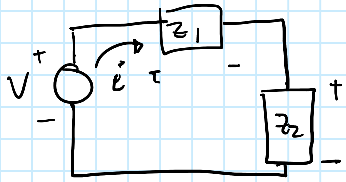
Impedancia terminal

$$Z_{eq} = \frac{V}{I} = Z_1 + Z_2 \dots + Z_n$$

if $N = 2$

$$I = \frac{V}{Z_1 + Z_2}$$

if $N=2$



$$I = \frac{V}{Z_1+Z_2}$$

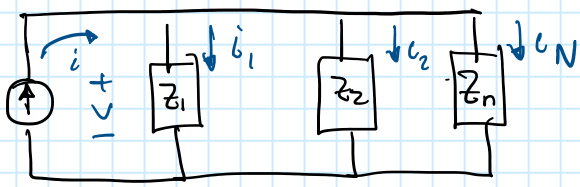
// Puesto que

$V_1 = Z_1 I$ y $V_2 = Z_2 I$

$$\therefore V_1 = \frac{Z_1}{Z_1+Z_2} V \; ; \; V_2 = \frac{Z_2}{Z_1+Z_2} V$$

// Divisor de voltaje

- De la misma manera, se puede encontrar impedancia equivalente



$$I = I_1 + I_2 + \ldots + I_N =$$

$$= V\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n}\right)$$

$$\frac{1}{Z_{eq}} = \frac{I}{V} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n}$$

$Y_{eq} = Y_1 + Y_2 + \ldots + Y_N$ cuando $N=2$

$$Z_{eq} = \frac{1}{Y_{eq}} = \frac{1}{Y_1+Y_2} = \frac{1}{\frac{1}{Z_1}+\frac{1}{Z_2}} = \frac{Z_1 Z_2}{Z_1+Z_2}$$

// Puesto que

$$I_1 = \frac{Z_2}{Z_1+Z_2} I \quad I_2 = \frac{Z_1}{Z_1+Z_2} I$$

Divisor de corriente

TAREA 3

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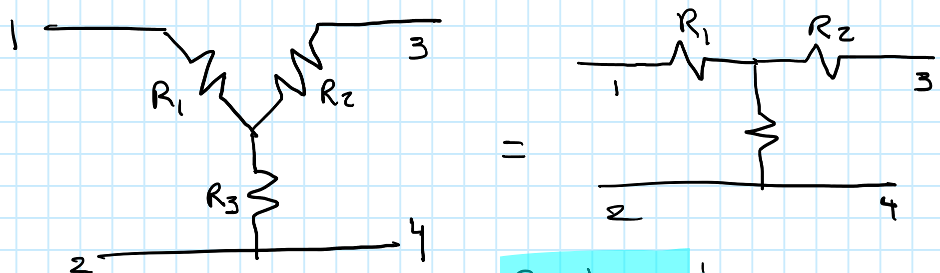
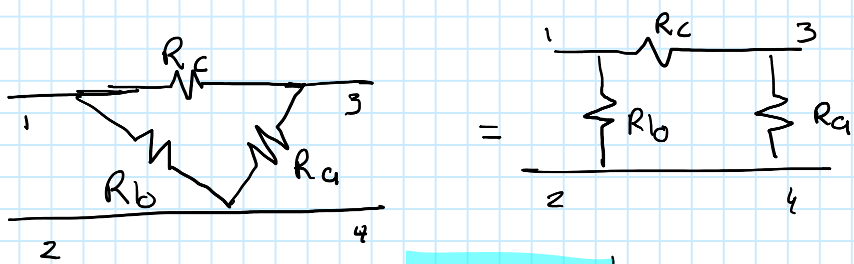
TRANSFORMACIÓN DE IMPEDANCIAS

- Considerando los circuitos idénticos, en estrella



Se tiene que $R_{12}(Y) = R_1 + R_3$

- Considerando los circuitos idénticos, en Delta



Se tiene que $R_{12}(\Delta) = R_b \parallel (R_c + R_a)$

- Partiendo de:

$$R_{12}(Y) = R_{12}(\Delta)$$

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \quad (1)$$

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \quad (2)$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad (3)$$

// Restando (3) en (1)

$$R_1 + R_3 - R_2 - R_3 = R_1 - R_2 = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} - \frac{R_a(R_b+R_c)}{R_a+R_b+R_c}$$

$$\therefore R_1 - R_2 = \frac{R_c(R_b-R_a)}{R_a+R_b+R_c} \quad (4)$$

//Sumando (2) y (4)

$$\therefore R_1 = \frac{1}{2}\left[\frac{R_aR_c + R_bR_c + R_bR_c - R_aR_c}{R_a+R_b+R_c}\right] =$$

$$= \frac{R_bR_c}{R_a+R_b+R_c} \quad (5)$$

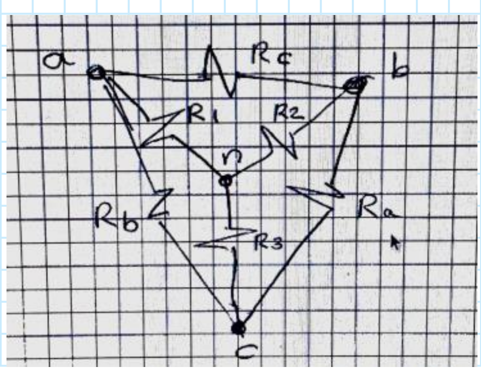
//Restando (4) de (2)

$$2R_2 = \frac{R_aR_c + R_bR_c - R_bR_c + R_aR_c}{R_a+R_b+R_c} \; ; R_2 = \frac{R_aR_c}{R_a+R_b+R_c} \quad (6)$$

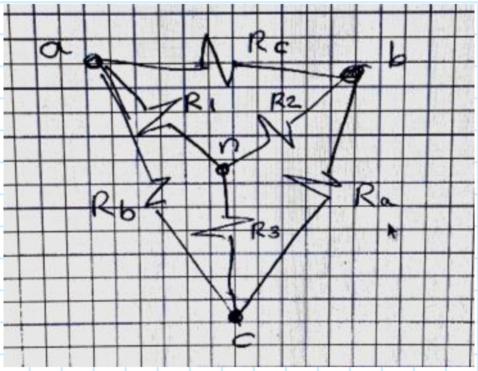
//Restando (5) de (1)

$$R_3 = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} - \frac{R_bR_c}{R_a+R_b+R_c} = \frac{R_aR_b}{R_a+R_b+R_c} \quad (7)$$

Conclusión: Cada resistor en la red. Y es el producto de los resistores en las 2 ramas adyacentes Δ, dividido por la suma de 3 resistores



| de 3 resistores



→ Conversión Y a Δ de ⑤, ⑥ y ⑦ se tiene que

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_b R_c (R_c R_a) + R_c R_a (R_a R_b) + R_a R_b (R_b R_c)}{(R_a + R_b + R_c)^2}$$

$$= \frac{R_a R_b R_c}{R_a + R_b + R_c} \quad (8)$$

// Dividiendo ⑧ entre ⑤, ⑥ y ⑦

$$\frac{\dfrac{R_a R_b R_c}{R_a + R_b + R_c}}{\dfrac{R_b R_c}{R_a + R_b + R_c}} = R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \quad (9)$$

2° Caso

$$\frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{\dfrac{R_a R_b R_c}{R_a + R_b + R_c}}{\dfrac{R_c R_a}{R_a + R_b + R_c}} = R_b \quad (10)$$

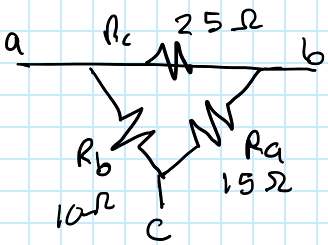
Por último,

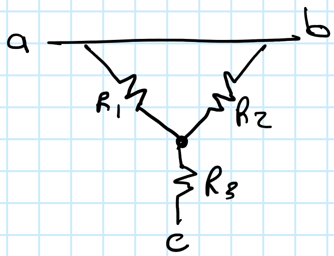
$$\frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{\dfrac{R_a R_b R_c}{R_a + R_b + R_c}}{R_a R_b} = R_c \quad (11)$$

$$\frac{R_1R_2+R_2R_3+R_3R_1}{R_3} = \frac{\overline{R_a+R_b+R_c}}{\dfrac{R_aR_b}{R_a+R_b+R_c}} = R_c \quad (11)$$

- De (9), (10) y (11) la regla de conversión Y − Δ es "Cada Resistor en la red Δ es la suma de todos los posibles productos de resistores Y tomados 2 a la vez, dividido por el resistor Y puesto"

Problema: Convierta la red Δ a su equivalente Y



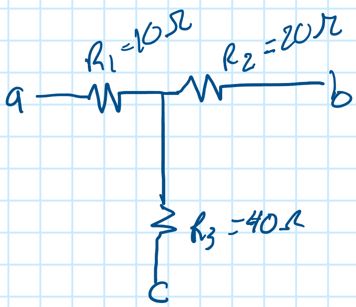


$$R_1 = \frac{R_bR_c}{R_a+R_b+R_c} = \frac{25\times10}{25+10+15} = \frac{250}{50} = 5[\Omega]$$

$$R_2 = \frac{R_cR_a}{R_a+R_b+R_c} = \frac{25\times15}{50} = 7.5[\Omega]$$

$$R_3 = \frac{R_aR_b}{R_a+R_b+R_c} = \frac{15\times10}{50} = 3[\Omega]$$

→ Problema: Transforma la red Y a Δ

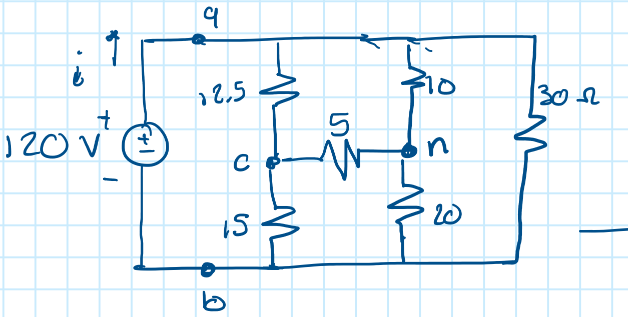


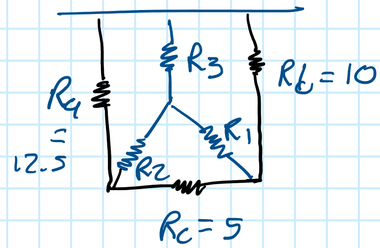
$$R_a = \frac{200+800+400}{10} = 140\,[\Omega]$$

$$R_b = \frac{200+800+400}{20} = 70\,[\Omega]$$

$$R_c = \frac{200+800+400}{40} = 35\,[\Omega]$$

Problemas: Encuentre la resistencia equivalente R_{ab} y calcule la corriente i



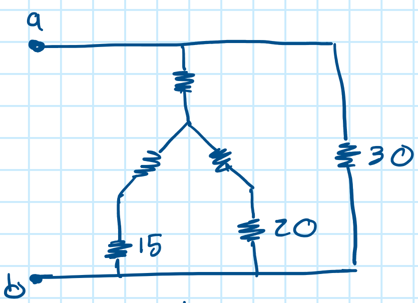


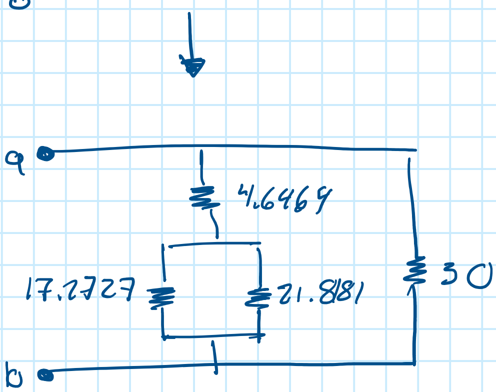
Circuito Equivalente

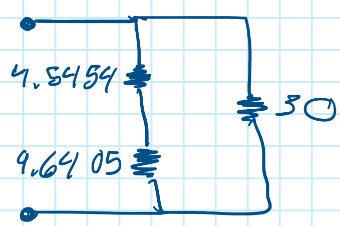
$$R_1 = \frac{50}{27.5} = 1.8181 \qquad R_3 = \frac{125}{27.5} = 4.5454$$

$$R_2 = \frac{62.8}{27.5} = 2.2727$$

// Resistencia Equivalente







$$R_{eq} = \frac{1}{\frac{1}{4.5454 + 9.6405} + \frac{1}{30}} =$$

$$= \boxed{9.6315\ [\Omega]}$$

$$\therefore \frac{V}{I} = R \ ; \ I = \frac{V}{R} = \frac{120}{9.6315} =$$

$$I = \boxed{12.4591\ [A]}$$

TAREA 3

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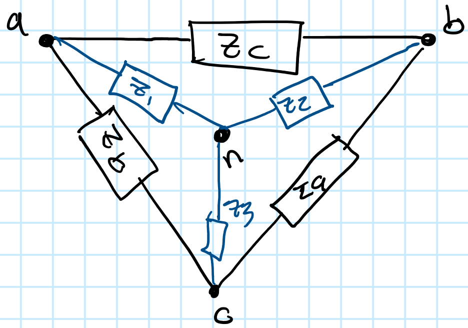
Conversiones en un circuito con Impedancia



Conversion Y - Δ

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 + Z_3 + Z_3 Z_1}{Z_3}$$

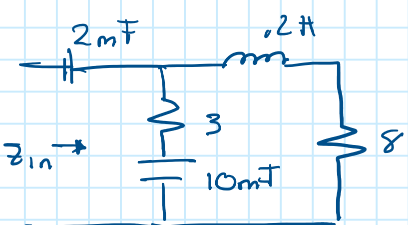
Conversion Δ - Y

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} \qquad Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

→ Problemas

1) Encuentre Z_{in}, asumir $\omega = 50 \frac{rad}{s}$



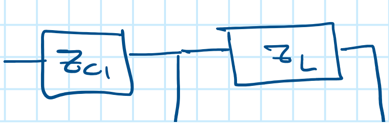
$$Z_{C_1} = \frac{-j}{50(2\times10^{-3})} = -10j\,[\Omega]$$

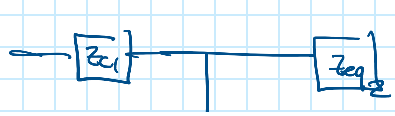
$$Z_{C_2} = \frac{-j}{50(10\times10^{-3})} = -2j\,[\Omega]$$

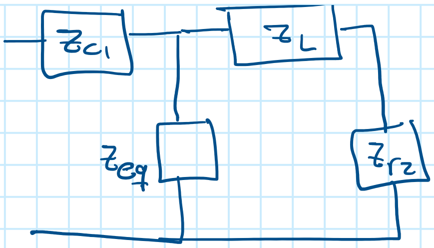
$Z_{r_1} = 3\Omega$

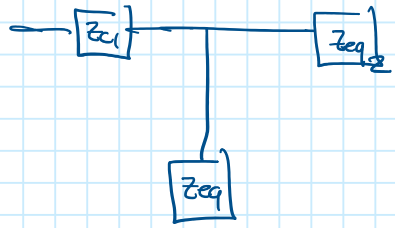
$Z_{r_2} = 8\Omega$

$Z_L = (j)(50)(.2) = 10j\,[\Omega]$









$Z_{eq} = Z_{r_1} + Z_{C_2} = 3 - 2j\ [\Omega]$ | $Z_{eq_2} = Z_L + Z_{r_2} = 8 + 10j\ [\Omega]$

//Obteniendo equivalente total

$$Z_{aux} = Z_{eq} + Z_{eq_2} = \left(\frac{1}{Z_{eq}} + \frac{1}{Z_{eq_2}}\right)^{2} = \left(\frac{1}{3-2j} + \frac{1}{8+10j}\right)^{2} =$$

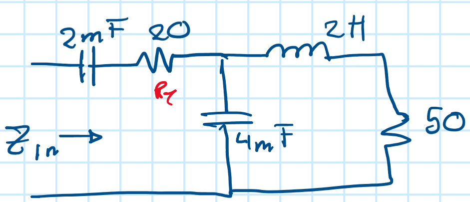
$$= \left(\frac{3+2j}{13} + \frac{8-10j}{164}\right)^{-1} \cong \left(\frac{149}{533} + \frac{99}{1066}j\right)^{-1} =$$

$$\cong \underbrace{\frac{596}{185} - \frac{198}{185}j}\ [\Omega]$$

$$\therefore Z_{eq_{Total}} = Z_{aux} + Z_{C_1} = \ldots + (-10j)$$

$$\cong \frac{596}{185} - \frac{2048}{185}j\ [\Omega] \cong 3.22 - 11.07j$$

(2) Determine Z_{in} cuando $\omega = 10\ \frac{rad}{s}$



$Z_L = 20j$

//Impedancias

$$Z_{C_1} = -\frac{1}{\omega C_1}j = -\frac{1}{(10)(2)(10^{-3})}j$$

$$Z_{C_1} = -30j$$

$$Z_{C_2} = \frac{1}{10\,(4\times10^{-3})}j = -25j$$

//Reduciendo elementos ← Sin esquemas (Directo)

L y R_2 = Serie | $Z_{eq} = 30 + 20j$

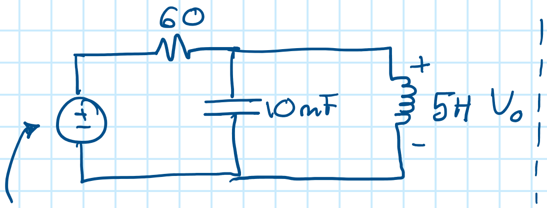
Z_{eq} y C_2 = Paralelo | $Z_{eq_C} = \dfrac{1}{\frac{1}{30+20j} + \frac{1}{-25j}} \cong \underbrace{12.37 - 23.76j}$

Z_{eq} y Z_2 = Paralelo | $Z_{eqc} = \dfrac{1}{\frac{1}{30+20j} + \frac{1}{-25j}} = \underbrace{12.37 - 23.76j}$

Z_{eqc} y Z_b = Serie | $Z_{eq_T} = 20 - 50j + \ldots =$

$\therefore Z_{eq_T} = 32.37 - 73.78j$ //

3 Determine V_o



$20 \cos(4t - 15°)$

$20 \cos(4t - 15°) = 20 \angle -15$

$\omega = 4 \ \frac{rad}{s}$

$Z_L = 5 \cdot 4j = 20j = 20 \angle 90°$

$Z_C = -\dfrac{1}{(4)(10 \times 10^{-3})} j = -25j =$

$= 25 \angle -90$

//Reducción

$Z_{LC} = \dfrac{1}{\frac{1}{20j} + \frac{1}{-25j}} = 100j = 100 \angle 90$

NOTA

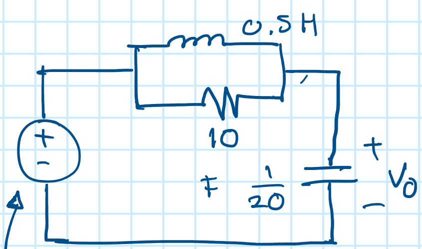
L y C tienen mismo voltaje

$\therefore V_o = \dfrac{100j}{60 + 100j} \ v \cong .73 + .44j \, [V] \cong .857 \angle 31° \ [V]$

$V_o = (.857 \angle 31°)(20 \angle -15) \cong 17.14 \angle 16°$ //

Si tomamos mayor precisión serían $\angle 15.967°$

4 Calcule V_o



$10 \cos(10t + 75°)$

$\omega = 10 \ \frac{rad}{s}$

$10 \angle 75°$

$Z_L = 5j$

$Z_C = -\dfrac{1}{\frac{10}{20}} j = -2j$

// Reducciendo

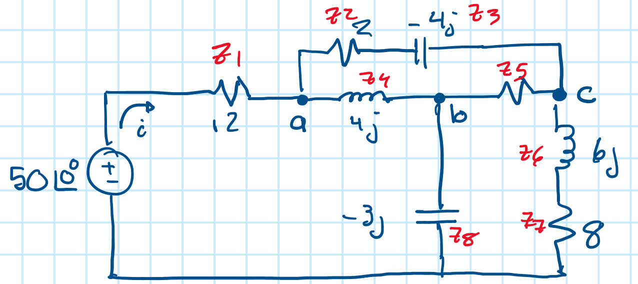
$$Z_{LR} = \frac{1}{\frac{1}{5j} + \frac{1}{10}} = 2 + 4j$$

$$V_0 = \frac{Z_L}{Z_C + Z_{LR}} [V] =$$

$$= \frac{-2j}{2 + 2j} = -.5 - .5j \, [V]$$

$$\therefore V_0 = (.7071 \angle -135^\circ)(10 \angle 75^\circ) \cong 17.071 \angle -60^\circ$$

⑤ Encuentre la corriente I

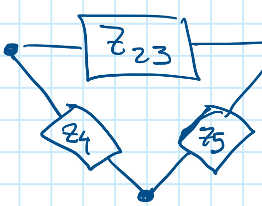


// Elementos en serie

Z_2 y Z_3 = $2 - 4j$

Z_6 y Z_7 = $8 + 6j$

// Elementos en Delta



$$Z_a = \frac{(2-4j)(4j)}{10} = 1.6 + .8j$$

$$Z_b = \frac{(2-4j)(8j)}{10} = 1.6 - 3.2j$$

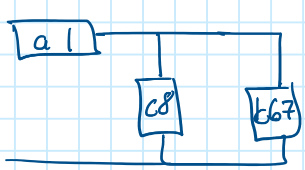
$$Z_c = \frac{32j}{10} = 3.2j$$

// Al reducir podremos considerar nuevos elementos en Serie

Z_b y Z_{67} = $9.6 + 2.8j$

Z_c y Z_8 = $.2j$

Z_a y Z_1 = $13.6 + .8j$



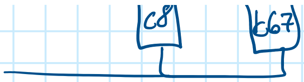
// Reducción de elemento en Paralelo

$$Z_{c8\,b67} = Z_{eq} = \frac{1}{\frac{1}{.2j} + \frac{1}{9.6 + 2.8j}} =$$

$$Z_{eq} = .0038 + .1988j$$

// Equivalente Total

Z_{eq} y Z_{a1} = Z_{eq} = $13.6038 + .1988j$

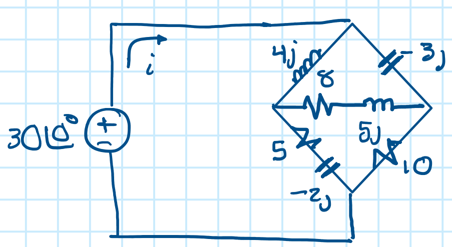


$$Z_{eq_7} \text{ y } Z_{q1} = Z_{eq_{Total}} = 13.6038 + .1988j$$

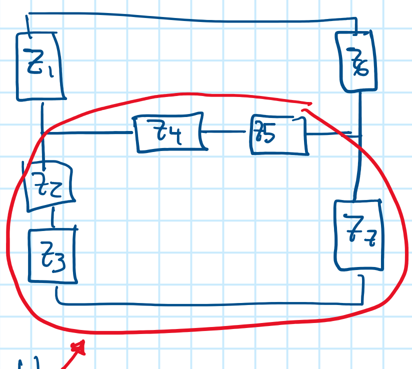
$$Z_{eq_{Total}} \approx 13.6 + .2j$$

$$\therefore I = \frac{V}{Z} = \frac{50\angle 0^\circ}{13.6\angle 4^\circ} \approx 3.66\angle -4^\circ$$

6 Encuentre I



//Equivalente



//Reduciendo

$$Z_2 \text{ y } Z_3 = 5 - 2j$$

$$Z_4 \text{ y } Z_5 = 8 + 5j$$

//El delta Z_{45}, Z_{23} y Z_7

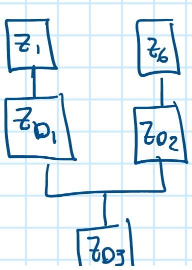
$$Z_{D_1} = \frac{(8+5j)(5-2j)}{8+5j+5-2j} = \frac{18+25j}{23+3j}$$

$$= \frac{1172}{538} + \frac{57}{538}j$$

$$Z_{D_2} = \frac{(Z_{45})(Z_7)}{Z_{45}+Z_{23}+Z_7} = \frac{(8+5j)(10)}{23+3j} \approx \frac{495}{269} + \frac{455}{269}j$$

$$Z_{D_3} = \frac{(Z_{23})(Z_7)}{\cdots} = \frac{(5-2j)(10)}{23+3j} = \frac{545}{269} - \frac{305}{269}j$$

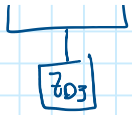
//Reducción Parcial



//Reduciendo en Serie

$$Z_1 \text{ y } Z_{D_1} = 4j + \left[\frac{1172}{538} + \frac{57}{538}j\right]$$

$$Z_6 + Z_{D_2} = -3j + [\quad]$$



$Z_6 + Z_{D2} = -3j + [\ \dots\ ]$

$\therefore Z_{parcial_1} = Z_1 + Z_{D1} \overset{\sim}{=} 2.1877 + 4.105j$

$\therefore Z_{parcial_2} = Z_6 + Z_{D2} \overset{\sim}{=} 3.6988 - 1.30j$

$$Z_{Paralelo} = \frac{1}{\frac{1}{2.18 + 4.10j} + \frac{1}{3.6988 - 1.30j}} \overset{\sim}{=} 2.67 + .8212j$$

// Equivalente Total

$$Z_{eq} = (2.67 + .8212j) + Z_{D3} = 4.696 - 3.3j$$

$$\therefore Z_{eq} = \frac{V}{I} \; ; \quad I = \frac{V}{Z_{eq}} = \frac{30}{4.696 - 3.3j} \overset{\sim}{=}$$

$$I \overset{\sim}{=} 6.351 + .453j \; [A]$$