UNIVERSIDAD NACIONAL
AUTÓNOMA DE MÉXICO
FACULTAD DE INGENIERÍA
CIRCUITOS ELÉCTRICOS
SEMESTRE 2020 - 2

Tarea y Apuntes 4

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TAREA_4

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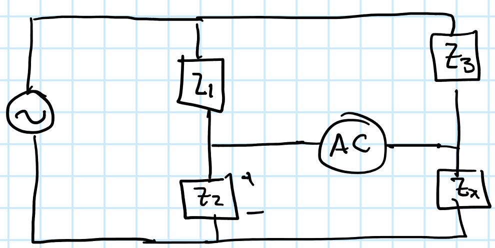
- Un circuito puente en A.C es utilizado en la medición de la Inductancia L de un inductor o la capacitancia de un capacitor



Tal puente es un circuito balanceado, cuando no fluye ninguna corriente a través del medidor en A.C.

// Divisor de voltaje

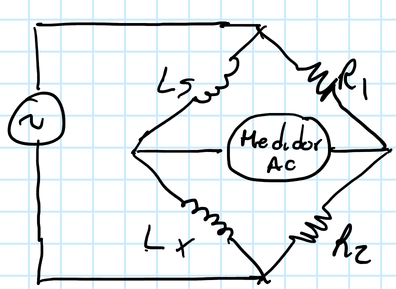
$$V_1 = \frac{Z_2}{Z_1 + Z_2} V_S = V_2 = \frac{Z_x}{Z_3 + Z_x} V_S$$

// Reduciendo

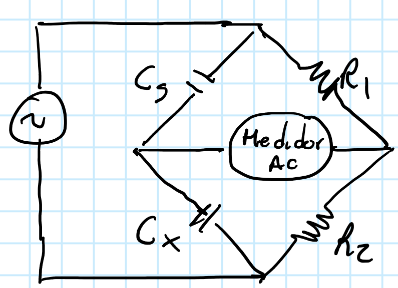
$$\frac{Z_2}{Z_1 + Z_2} = \frac{Z_x}{Z_3 + Z_x} ;$$

$$Z_3 Z_2 = Z_x Z_1$$

$$\therefore Z_x = \frac{Z_3}{Z_1} Z_2$$ ← Ecuación Balanceada para un puente en A.C.



$$L_x = \frac{R_2}{R_1} L_S$$



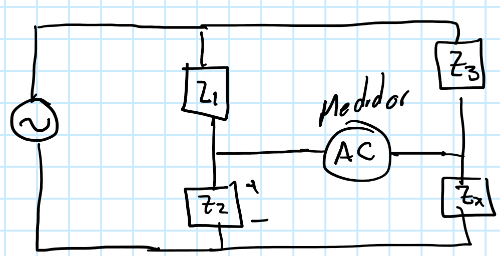
$$C_x = \frac{R_2}{R_1} C_S$$

- L_x y C_x son desconocidos mientras L_s y C_s si lo son, en estos casos R_1 y R_2 son variados hasta que el medidor tenga una lectura nula.

→ Ejemplo

El siguiente circuito está balanceada cuando Z_1 es un resistor de $1[k\Omega]$ y Z_2 de $4.2[k\Omega]$, mientras que Z_3 es una combinación paralelo de un resistor de $1.5[M\Omega]$ y un capacitor de $12[pF]$ y una $f = 2[kHz]$

a) Los componentes en serie que hagan Z_x
b) Los componentes en paralelo que hagan Z_x



Circuito B

$$Z_x = \frac{Z_3}{Z_1} Z_2$$

$$Z_x = R_x + j\,X_x$$

$$Z_1 = 1000[\Omega]$$
$$Z_2 = 4200[\Omega]$$

$$Z_3 = R_3 \,||\, \frac{1}{j\omega C_3} =$$

$$= \frac{\frac{R_3}{j\omega C_3}}{R_3 + \frac{1}{j\omega C_3}} = \frac{R_3}{1 + j\omega R_3 C_3}$$

Puesto que

$R_3 = 1.5\ M\Omega$ y $C_3 = 12\ pF$

$$\therefore\ Z_3 = \frac{1.5\times10^6}{1+(j)(2\pi)(2\times10^3)(1.5\times10^6)(12\times10^{-12})} \cong 1.427 - .3228j\ [M\Omega]$$

a) Asumiendo que Z_x está compuesta de elementos en serie

$$R_x + j\,X_x = \frac{4200}{1000}\left(1.427 - j(.3228)\right)\times10^6 = 5.993 - 1.356j\,[M\Omega]$$

Agrupando

$R_x = 5.993\,[M\Omega]$

$X_x = \frac{1}{\omega C} = 1.35\times10^6$

$$C = \frac{1}{\omega X_x} = \frac{1}{(2\pi)(2\times10^3)(1.356\times10^6)}$$

Agrupando $R_x = 5.995 [M\Omega]$ $\quad C = \frac{1}{\omega X_x} = \frac{1}{(2\pi)(2\times10^3)(1.356\times10^6)}$

$X_x = \frac{1}{\omega C} = 1.35\times10^6$

$C = 58.69 [pF]$

b) Si Z_x está configurada de componentes en paralelo

$$Z_x = \frac{4200}{1000} R_3 || \frac{1}{j\omega C_3} = 4.2 Z_3$$

Puesto que Z_3 consta de R_3 y $X_3 = \frac{1}{\omega C_3}$ existen varias formas de obtener $4.2 Z_3$

- Suponiendo que

$R_x = 1.4 R_3 = 2.1 [M\Omega]$

$X_x = \frac{1}{\omega C_x} = 3 X_3 = \frac{3}{\omega C_3}$; $C_x = \frac{1}{3} C_3 = 4 [pF]$

Problema

En el circuito B, suponer que el balance se consigue cuando Z_1 es un resistor de $4.8 [k\Omega]$, $Z_2 = 10\,\Omega$ en serie con un inductor de $.25\,\mu H$, Z_3 es un resistor de $12 [k\Omega]$ y $f = 6 [MHz]$. Determinar los componentes en serie que hacen Z_x

Resultado $25\,\Omega$, en serie un inductor de $.625\,\mu H$

$Z_1 = 4.8 [k\Omega]$

$Z_2 = 10 + (.25\times10^{-6})(2\pi)(6\times10^6) j = 10 + 3\pi j$

$Z_3 = 12 [k\Omega]$

$V_1 = \frac{Z_1}{Z_1 + Z_2} \quad V_2 = \frac{Z_x}{Z_3 + Z_x}$ $\quad \frac{Z_1}{Z_1 + Z_2} = \frac{Z_x}{Z_3 + Z_x}$

$$\therefore \quad Z_x = \frac{Z_2}{Z_1} Z_3 = \frac{10 + 3\pi j}{4800}(12000) = \left(\frac{5}{2}\right)(10 + 3\pi j)$$

$$Z_x = 25 + \frac{15\pi}{2} j \, [\Omega]$$

$\therefore R_x = 25\ [\Omega]$ //

$X_x = \frac{15\pi}{2} = \omega L \quad ; \quad L = \frac{15\pi}{2\omega} \cong 6.25 \times 10^{-7}$

$\omega = (2\pi)(6\times10^{6})$

$\therefore L \cong 0.625\ [\mu H]$ //

TAREA_4

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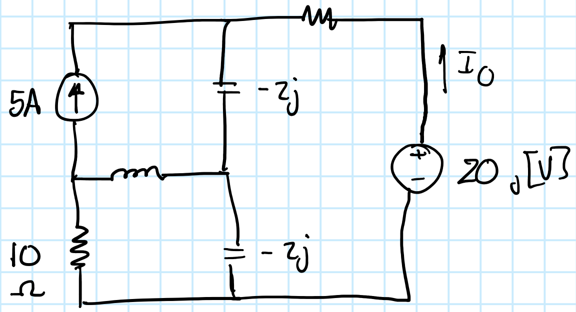
Teorema de Superposición

Puesto que el teorema aplica a Circuitos D.C, de la misma manera aplica a Circuito A.C.

Puesto que las impedancias dependen de la frecuencia entonces se debe tener un circuito en el dominio de la frecuencia para cada una frecuencia.

La respuesta total debe ser obtenido sumando los respuestas individuales en el dominio del tiempo.

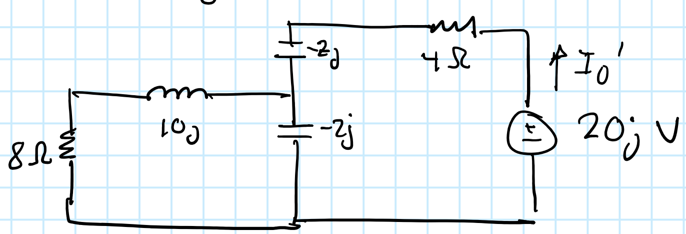
Ejemplo



$I_0 = I_0' + I_0''$

donde I_0' y I_0'' son debido a las fuentes de voltaje y corriente

// Para I_0' consideramos

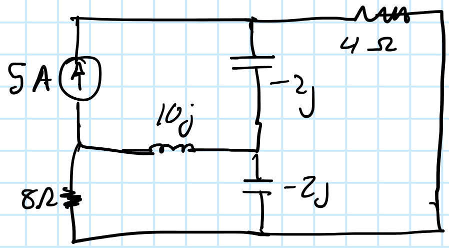


$$Z = \frac{-2j\,(8+10j)}{-2+8+10j} = .25 - 2.25j$$

$$\therefore I_0' = \frac{20j}{(4-2j)+Z} = \frac{20j}{4.25-4.25j} \cong -2.353 + 2.353j$$

// Para I_0'' Consideramos el circuito siguiente

$(8+8j)I - 10j\,I_3 + 2j\,I_2 = 0$



$(8+8j)I - 10j\,I_3 + 2j\,I_2 = 0$

$(4-4j)I_2 + 2j\,I_1 + 2j\,I_3 = 0$

$I_3 = 5A$

/ Sustituyendo

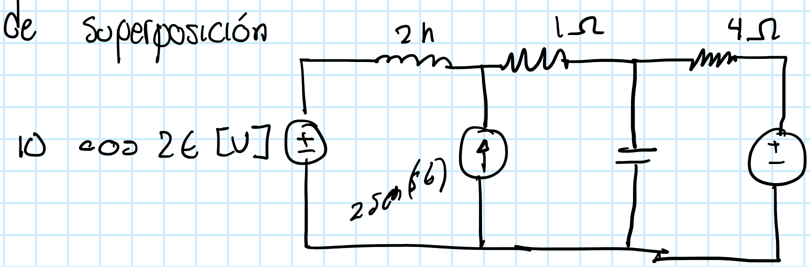
$I_1 = (2+2j)I_2 - 5$

$I_2 = \dfrac{90-40j}{34} \approx 2.647 - 1.176j = \underline{-I_0''}$

$\therefore I_0 = I_0' + I_0'' = -5 + 3.529j = 6.12\angle 144.78° \; [A]$

Problema 1   * Fue una aproximación realmente cercana

Encuentre V_0 en el siguiente circuito mediante teorema de superposición

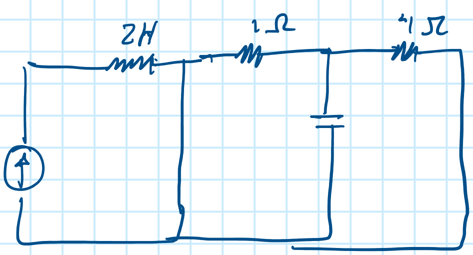


$V_0 = V_0' + V_0''$

V_0'

$10\angle 0$

$w = 2$



$Z_L = Lwj = 4j$

$Z_C = \dfrac{1}{2(.1)j} = -5j$

$Z_{CR_1} = \left(\dfrac{1}{4} + \dfrac{1}{-5j}\right)^{-1} = \dfrac{\frac{1}{4} - \frac{j}{5}}{\frac{1}{4}^2 + \frac{1}{5}^2} \approx \dfrac{100}{41} - \dfrac{80}{41}j$

$Z_{CR_1R_2} = 1 + (Z_{CR_1}) \approx \underbrace{\dfrac{141}{41} - \dfrac{80}{41}j}$

$\therefore Z = (\ldots) + 4j = \dfrac{141}{41} + \dfrac{84}{41}j$

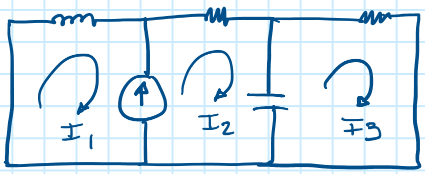
$I = \dfrac{10\angle 0°}{Z}$

$\approx 2.146 - 1.278j$

$\therefore Z = (\ldots) + 4j = \frac{141}{41} + \frac{84}{41} j \quad = 2.146 - 1.278j$

$\therefore V = (2.146 - 1.278j)(1) = 2.146 - 1.278j \cong 2.49 \angle -30.78^\circ$

// Definiendo sentidos y sistema de ecuaciones en Mallas



$I_1 10j + I_2 - 2j(I_2 - I_3) = 0$

$\hookrightarrow 10j\, I_1 + (1 - 2j) I_2 + 2j\, I_3 = 0$

$4 I_3 - 2j(I_3 - I_2) = 0$

$\hookrightarrow 2j\, I_2 + (4 - 2j) I_3 = 0$

Extra: $-I_1 + I_2 = 2 \cos(5t - 90^\circ)$

S.E

Forma Matricial

$$\begin{bmatrix} 0 & 2j & 4-2j \\ 10 & 1-2j & 2j \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2\cos(5t-90) \end{bmatrix}$$

$\det(A) = -44 + 30j$

$$\therefore \Delta i_1 = \frac{1}{\det A} \begin{bmatrix} 0 & 2j & 4-2j \\ 0 & 1-2j & 2j \\ 2\cos(5t-90) & 1 & 0 \end{bmatrix} \approx .2256 + .3356j$$

$$\Delta i_2 = \frac{1}{\det A} \begin{bmatrix} 0 & 0 & 4-2 \\ 10 & 0 & 2j \\ -1 & 2\cos(5t-90) & 0 \end{bmatrix} \approx .2256 - 1.619j$$

$$\Delta i_3 = \frac{1}{\det A} \begin{bmatrix} 0 & 2 & 0 \\ 10 & 1-2j & 0 \\ -1 & 1 & 2\cos(5t-90) \end{bmatrix} \approx -.62059 - .4231j$$

$$\det A \begin{bmatrix} \cdots & 1+2j & \cdots \\ -1 & 1 & 2\cos(5t-90) \end{bmatrix} = -.4231j$$

$$\therefore V_0 = I_2 Z = (.2256 - 1.6693j)(1) \cong 1.679\cos(5t - 82.8^\circ)$$

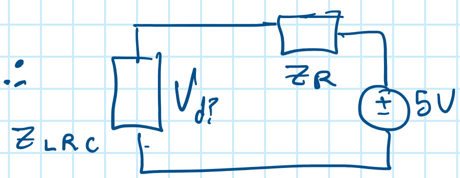
V_0''

- $Z_L = 2j$
- $Z_C = \dfrac{-j}{(.1)(1)} = -10j$

$Z_{LR} = 1 + 2j$

$Z_{LRC} = \dfrac{1}{\frac{1}{1+2j} - \frac{1}{-10j}} \cong \dfrac{20}{13} + \dfrac{30}{13}j\ \Omega$

$\therefore$



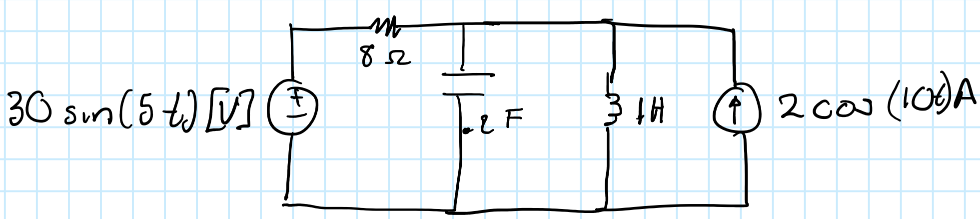
- $Z_R = 4$

$$V = \frac{Z_{LRC}}{Z_{LRC} + Z_R}(5) = \;\cong\; \frac{25}{13} + \frac{50}{39}i$$

$$I = \frac{V_z}{Z_{LR}} = \frac{\frac{25}{13} + \frac{50}{39}j}{1+2j} = \frac{35}{39} - \frac{20}{39}j$$

$$\therefore V_0 = \frac{35}{39} - \frac{20}{39}j \cong .8974 - .512820j = 1.033\angle 29.74\ [V]$$

Problema 2



V_0'

$Z_C = \dfrac{-j}{(.5)(.2)} = -j \qquad Z_L = 5j \qquad \therefore Z = 8 - \dfrac{5}{4}j$

$Z_{CL} = \left(\dfrac{1}{-j} + \dfrac{1}{5j}\right)^{-1} = -\dfrac{5}{4}j$

$$\therefore V_0 = \frac{-\frac{5}{4}j}{8-\frac{5}{4}j}\left(30\angle{-90^\circ}\right) \cong 4.6313\angle{-171.119^\circ}$$

$$= 4.6313\cos(5t - 171.119^\circ)$$

Comprobando

$$V_0 = 4.6313\cos(5t-171.119^\circ) =$$

$$= 4.6313\,\text{sen}(5t - 81.119^\circ)$$

V_0''

$$Z_C = \frac{-j}{(.2)(10)} = -\frac{1}{2}j$$

$$Z_L = 10j$$

$$Z_{RC} = \left(\frac{1}{8} - \frac{1}{\frac{1}{2}j}\right)^{-1} = \frac{8}{257} - \frac{128}{257}j$$

$$Z_{eq} = \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)^{-1}$$

$$= \frac{Z_1 Z_2 Z_3}{Z_2 Z_3 + Z_1 Z_3 + Z_2 Z_1}$$

$$I = \frac{Z_{eq}}{Z_2} \cong \frac{12160}{580} + \frac{800}{5801}j \cong 2.096 + .1379j$$

$$\therefore V_0 = (2.096 + .1379j)\left(-\frac{1}{2}j\right) \cong 1.048\angle{-86.23^\circ}$$

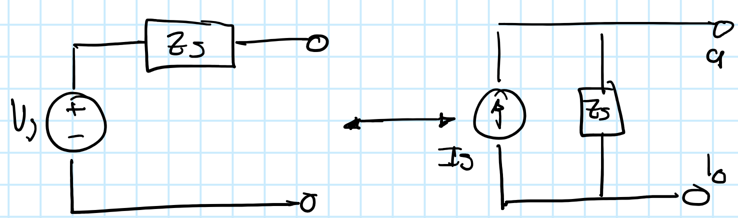
$$= 1.048\cos(10t - 86.23^\circ)$$

Total

$$V_0 = V_0' + V_0'' = 4.6313\,\text{sen}(5t - 81.119^\circ) + 1.048\cos(10t - 86.23)$$

Problema 3

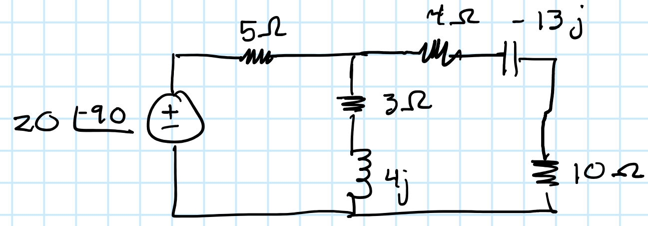
Transformación en fuentes de A.C



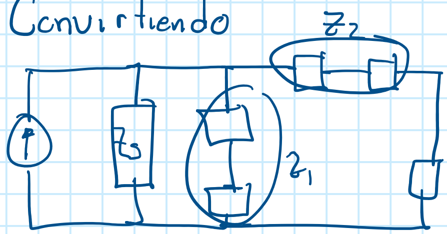
$$V_S = Z_S I_S \qquad I_S = \frac{V_S}{Z_S}$$

Calcular V_x en el

Calcular V_x en el siguiente circuito



Convirtiendo



$I_s = -4j$

$Z_s = 5$

$Z_1 = 3+4j$

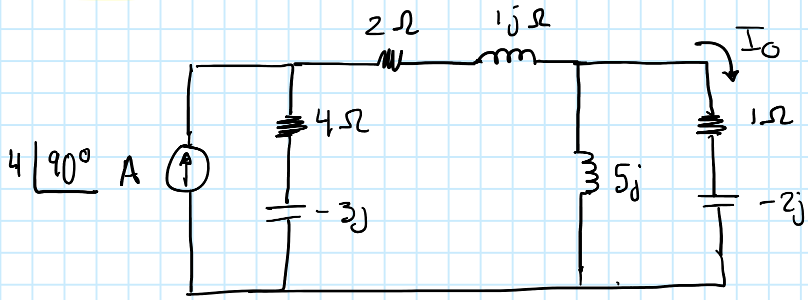
$Z_2 = 14-13j$

$$\therefore Z_{eq} = \left(\frac{1}{5} + \frac{1}{3+4j} + \frac{1}{14-13j}\right)^{-1} = \frac{3270}{1313} + \frac{1135}{1313}j$$

$$I = \frac{Z_{eq}}{Z_2}(I_s) = \left(\frac{(\;.\;.\;.\;)}{14-13j}\right)(-4j) = .4874 + .8644j$$

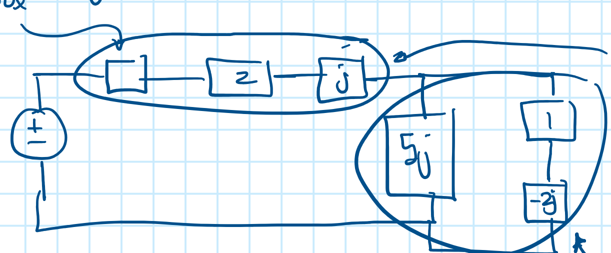
$$V_x = (.4874 + .8644j)(10) = 5.5194 \angle -27.97° [V]$$

Problema 4



//Conversión

$R_{Aux} = 4-3j$

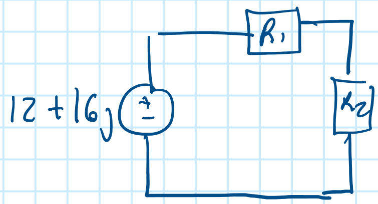


- $R_1 = 4-3j+2+j = 6-2j$
- $R_2 = \left(\frac{1}{5j} + \frac{1}{1-2j}\right)^{-1}$

$= \frac{5}{2} - \frac{5}{2}j$

//Reducido

$$V = (4-3j)(4\angle 90^\circ) = 12+16j$$



$$V = \frac{R_2}{R_1+R_2}V = \frac{\frac{5}{2}-\frac{5}{2}j}{(6-2j)+\left(\frac{5}{2}-\frac{5}{2}j\right)}(12+16j) =$$

$$= \frac{220}{37} + \frac{160}{37}j = 5.9459 + 4.324j$$

$$V = IZ\ ;\ I_0 = \frac{5.9459 + 4.324}{1-2j} \approx -.54054 + 3.2432j$$

$$\therefore I_0 \approx 3.2879\angle 99.46^\circ\ [A]$$