



Bachelorarbeit

Fachbereich 3 - Mathematik und Informatik

zum Thema

Unleashing the Power of GNU/Emacs and Org-Mode A Cache of Wisdom for Thesis Writing (DRAFT)

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19th August 2024

Abstract

The fusion of Doom/Emacs, Org Mode, and \LaTeX presents a potent platform for crafting sophisticated thesis papers. This article provides a detailed guide on how to seamlessly integrate these tools, facilitating a smooth transition between generating HTML and \LaTeX documents, as well as PDFs, all from a single Org Mode file. Furthermore, a Nix shell, accompanied by a shell.nix file, is introduced to ensure efficient management of the development environment.

Zusammenfassung

Die Vereinigung von Doom/Emacs, Org Mode und \LaTeX bietet eine leistungsstarke Plattform zur Verfassung eleganter Abschlussarbeiten. Dieser Artikel erläutert detailliert, wie man diese Werkzeuge optimal kombiniert, um nahtlos zwischen der Erstellung von HTML- und \LaTeX -Dokumenten sowie PDFs aus einer einzigen Org-Mode-Datei zu wechseln. Zusätzlich wird eine Nix-Shell mit einer begleitenden shell.nix-Datei vorgestellt, um eine effiziente Verwaltung der Entwicklungs- und Arbeitsumgebung zu gewährleisten.

Contents

1	Introduction / Motivation:	11
1.1	Why bother writing Org-Mode when I can use \LaTeX instead?	11
1.1.1	The possibilities with Org-Mode are endless!	11
1.2	summery	11
1.2.1	motivating videos	11
2	(Doom) GNU/Emacs setup and configuration	13
2.1	packages ~/.config/doom/package.el	13
2.2	config ~/.config/doom/config.el	14
3	nix shell environment	17
4	start messing around	19
5	convert python 2 latex	21
6	Function composition	23
7	Some random Proof	25
8	plotting functions and their root function / derivation	27
8.1	python	27
8.2	tikzpicture	28
8.3	org plot	28
8.4	gnu plot	29
9	logic resolution proof	33
9.1	forward direction (\rightarrow)	33
9.2	back direction (\leftarrow)	34
9.3	both directions (\leftrightarrow)	34

List of Listings

1	<code>nix-shell</code> config for generating a build environment	17
2	code block containing prime checking functions	21

List of Tables

5.1	table of library's with corresponding version	21
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List of Figures

6.1	function composition and identity	23
6.2	Graph	24
7.1	\mathbb{R}^3 with linearer transformation A^1	25

1 Introduction / Motivation:

Embarking on the journey of using GNU/Emacs with Org-Mode for thesis writing may seem like a daunting task, especially if you're unfamiliar with the intricate workings of GNU/Emacs itself. Some may question the need to delve into Org-Mode when alternatives like \LaTeX are readily available and widely embraced. However, dismissing the potential of GNU/Emacs and Org-Mode prematurely might mean overlooking a transformative experience that can redefine your approach to document creation.

1.1 Why bother writing Org-Mode when I can use \LaTeX instead?

While \LaTeX is a formidable typesetting system, Org-Mode complements it by offering a more versatile and flexible platform for document creation. Org-Mode's simplicity and ease of use, coupled with the ability to seamlessly integrate \LaTeX , provide a unique writing experience that combines the best of both worlds.

1.1.1 The possibilities with Org-Mode are endless!

1. Include and evaluate code:
Integrate code seamlessly within your document, making it a dynamic and interactive platform for technical content.
2. Write \LaTeX in a more relaxed surrounding:
Org-Mode simplifies the \LaTeX integration, providing a more user-friendly and intuitive environment for crafting your documents.

1.2 summery

For this tutorial, we will be using a distribution of GNU/Emacs called DOOM Emacs. Additionally, on our GNU/Linux platform, we'll leverage a package manager known as Nix to generate an environment for building both PDF and HTML versions of your thesis. This powerful combination of tools will enhance your writing experience and streamline the process of creating, editing, and compiling your thesis documents.

1.2.1 motivating videos

- Literate Documentation with Emacs and Org Mode

1 Introduction / Motivation:

- Consistent Technical Documents Using Emacs and Org Mode
- EmacsConf 2023: Authoring and presenting university courses with Emacs and a full libre software ...
- Emacs Tips - How to Give Presentations with Org Mode
- EmacsConf 2022: Health data journaling and visualization with Org Mode and GNUplot - David O'Toole

2 (Doom) GNU/Emacs setup and configuration

Start by installing Doom Emacs.

<https://youtu.be/37H7bD-G7nE?feature=shared>

2.1 packages ~/.config/doom/package.el

After installing Doom Emacs - follow up by installing useful packages. If you are not familiar with Doom Emacs's `evil` mode (which is a vim style set of key bindings in Emacs), it might be a good idea to install `evil-tutor` and check out the tutorial for `evil` mode.

The `pdf-tools` (optional for this tutorial) will give you additional functionality for working with PDF documents in Doom Emacs, the most important being the support for viewing the PDF document in a Emacs buffer.

Finlay install `org-special-block-extras` for additional `org-mode` blocks for math and other usefull stuff.

Note

If you want to use the special blocks in you current org mode session - do not forget to enable it with: `M-x org-special-block-extras-mode`

```
1 grep "(package" ~/.config/doom/packages.el | sed 's/).*//; s/$/)/'
```

```
1 (package! evil-tutor)
2 (package! pdf-tools)
3 (package! org-special-block-extras)
4 (package! gnuplot)
5 (package! gnuplot-mode)
6 (package! smex)
7 (package! fireplace)
```

Note

Do not forget to `doom sync` after adding new packages to your `package.el` file and reload Emacs.

2 (Doom) GNU/Emacs setup and configuration

2.2 config ~/.config/doom/config.el

To complete the Doom Emacs setup / configuration you only have to adapt your config.el file. We will now add the koma latex document classes and add support for code highlighting.

```
1 grep -v '^\\s*;' ~/.config/doom/config.el | awk 'NF {print $0; blank=0} !NF &&
   ↪ !blank {print ""; blank=1}'
2 # sed '/^$/N;/^\\n$/D;P;D'
```

```
1
2 (global-set-key (kbd "C-x <up>") 'windmove-up)
3 (global-set-key (kbd "C-x <down>") 'windmove-down)
4 (global-set-key (kbd "C-x <left>") 'windmove-left)
5 (global-set-key (kbd "C-x <right>") 'windmove-right)
6
7 (ido-mode 1)
8 (ido-everywhere 1)
9
10 (setq tramp-auto-save-directory "/tmp")
11
12 (after! org
13   (setq org-ascii-text-width 120)
14
15   (setq org-latex-listings 'minted
16     org-latex-packages-alist '((" " "minted"))
17     org-latex-pdf-process
18     '("pdflatex -shell-escape -interaction nonstopmode -output-directory %o
19       ↪ %f"
20       "pdflatex -shell-escape -interaction nonstopmode -output-directory %o
21       ↪ %f"))
22
23   (setq org-latex-minted-options '(("frame" "lines")
24     ("linenos" "true")
25     ("fontsize" "\\small")
26     ("breaklines" "true")
27     ("breakanywhere" "true")
28     ("tabsize" "4")))
29
30   (setq org-latex-classes
31     '(("article" "\\documentclass[11pt]{article}"
32       ("\\section{%s}" . "\\section*{%s}")
33       ("\\subsection{%s}" . "\\subsection*{%s}")
34       ("\\subsubsection{%s}" . "\\subsubsection*{%s}")
35       ("\\paragraph{%s}" . "\\paragraph*{%s}")
36       ("\\subparagraph{%s}" . "\\subparagraph*{%s}")))))
37
38
```

2.2 config ~/.config/doom/config.el

```
36 (add-to-list 'org-latex-classes
37             ' ("koma-book" "\\documentclass{scrbook}"
38                 ("\\chapter{%s}" . "\\chapter*{%s}")
39                 ("\\section{%s}" . "\\section*{%s}")
40                 ("\\subsection{%s}" . "\\subsection*{%s}")
41                 ("\\subsubsection{%s}" . "\\subsubsection*{%s}")
42                 ("\\paragraph{%s}" . "\\paragraph*{%s}")
43                 ("\\subparagraph{%s}" . "\\subparagraph*{%s}")))
44
45 (add-to-list 'org-latex-classes
46             ' ("koma-article"
47                 "\\documentclass{scrartcl}"
48                 ("\\section{%s}" . "\\section*{%s}")
49                 ("\\subsection{%s}" . "\\subsection*{%s}")
50                 ("\\subsubsection{%s}" . "\\subsubsection*{%s}")
51                 ("\\paragraph{%s}" . "\\paragraph*{%s}")
52                 ("\\subparagraph{%s}" . "\\subparagraph*{%s}")))
53 )
54
55 (setq doom-theme 'doom-one)
56
57 (setq display-line-numbers-type 'relative)
58
59 (setq org-directory "~/org/")
60
```

Note

Before running SPC-h-r-r (M-x doom/reload) - either wait for the nix env setup or install pygments yourself via pip3 install pygments.

3 nix shell environment

Now that Emacs is ready - we can proceed by declaring an `shell.nix` environment file, in which we will specify all the packages we will use for compiling the `org-mode` file and all library's and other stuff used in the thesis (e.g. `numpy` `python` `haskell`).

We will use `gnuplot` for plotting stuff, `graphics` for generating nice graphs in the `dot` language. In addition we need `texlive` and therefore can use `tikz` to generate nice plots/graphs. Last but not least, we install the `pygments` lib we are using for highlighting the code blocks in addition to the `python` packages used in the source code in the thesis.

If you want to install `haskell` or `R` for your thesis and haven't done so system wide - this might be a good opportunity to add the software.

Install Nix (the package manager)

```
sh <(curl -L https://nixos.org/nix/install) --daemon
```

<https://nixos.org/download>

```
1 cat shell.nix
```

Listing 1: nix-shell config for generating a build environment

```
1 #+begin_src nix
2 { pkgs ? import <nixpkgs> {} }:
3
4 pkgs.mkShell {
5   buildInputs = with pkgs; [
6     (python310.withPackages(ps: with ps; [
7       numpy           # for math
8       latexify-py     # for converting python funcs to latex
9       scipy           # for math and physics
10      pygments         # for syntax highlighting
11      matplotlib       # for plotting
12      seaborn          # for plotting
13      scikit-learn     # for ml stuff
14    ]))
15  ]
16 }
```

3 nix shell environment

```
15     texlive.combined.scheme-full
16     graphviz
17     gnuplot
18 ];
19
20 shellHook = ''
21     echo "entering dev environment"
22 '';
23 }
```

Run `nix-shell` to enter the now defined development environment.
After entering our specified environment - you can run `make doom` to start Doom Emacs.

4 start messing around

Since everything should be ready by now - please feel free to mess around with the `thesis.org` file within Emacs.

1. start the nix development environment

```
1 nix-shell # should open a nix shell with all the packages defined in the  
   ↪ local shell.nix config
```

2. open doom Emacs form within the development environment

```
1 make doom # should open a new doom emacs window (from within the nix  
   ↪ shell)
```

3. open `thesis.org`

in doom Emacs, you could either open a file with `M-x dired` or by using `SPC-`.

4. mess around

Finlay adapt the `thesis.org` file for your needs or just explore the file from within Emacs.

Export to HTML and PDF

You can export your derivative with the key binding: `C-c C-e`

5 convert python 2 latex

```
1 import math
2 import numpy as np
3 import scipy
4 import latexify
```

Table 5.1: table of library's with corresponding version

With the imported library's, we can now convert a python function to latex

```
1 def recHelper(x: int, d: int = 3) -> int:
2     # forall x in [3, infty)
3     if x % 2 == 0:
4         return 0
5     elif d > math.floor(math.sqrt(x)):
6         return 1
7     elif x % d == 0:
8         return 0
9     else:
10        return recHelper(x, d + 2)
11
12 def isPrime(x: int) -> int:
13     if x <= 1: # forall x in (infty, 1] : 1
14         return 0
15     elif x == 2:
16         return 1
17     else: # forall x in (2, infty)
18         return recHelper(x, 3)
```

Listing 2: code block containing prime checking functions

In line 12 is the main definition of the recursive basecase. Line 1 starts by declaring a recursive helper for the main prime checking function.

$$\text{isPrime}(x) = \begin{cases} 0, & \text{if } x \leq 1 \\ 1, & \text{if } x = 2 \\ \text{recHelper}(x, 3), & \text{otherwise} \end{cases}$$

5 convert python 2 latex

$$\text{recHelper}(x, d) = \begin{cases} 0, & \text{if } x \% 2 = 0 \\ 1, & \text{if } d > \lfloor \sqrt{x} \rfloor \\ 0, & \text{if } x \% d = 0 \\ \text{recHelper}(x, d + 2), & \text{otherwise} \end{cases}$$

Now we can call the `isPrime` function with the parameter 5 and 6:

- is 5 a prime number? result of `isPrime` function: True
- is 6 a prime number? result of `isPrime` function: False

6 Function composition

Graph using the dot language

```
1 digraph G {  
2   node [shape=circle, fontname="Courier", fontsize=16]  
3   edge [fontname="Courier", fontsize=16]  
4   A:nw -> A [label="id"]  
5   A -> B [label="f"]  
6   B -> C [label="g"]  
7   A -> C [xlabel="g . f"]  
8   {rank=same; A B}  
9 }
```

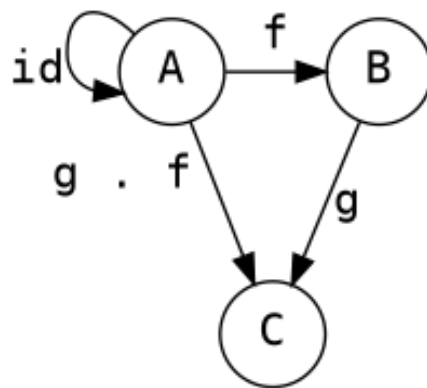


Figure 6.1: function composition and identity

Graph using latex tikz picture

6 Function composition

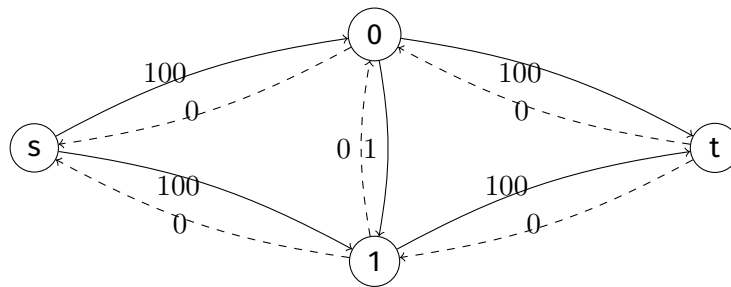


Figure 6.2: Some random Graph

7 Some random Proof

Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \in \mathbb{R}^{3 \times 3}.$$

Proof that the following applies to all $\beta \in \mathbb{N}_{>0}$:

$$A^\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix}.$$

‘Matrix multiplication as composition | Chapter 4, Essence of linear algebra’ footnote <https://youtu.be/XkY2DOUCWMU> The Matrix A can also be interpreted as a linear transformation in \mathbb{R}^3 .

This interpretation makes it easier to understand the theorem to be proven, and makes it seem almost trivial.

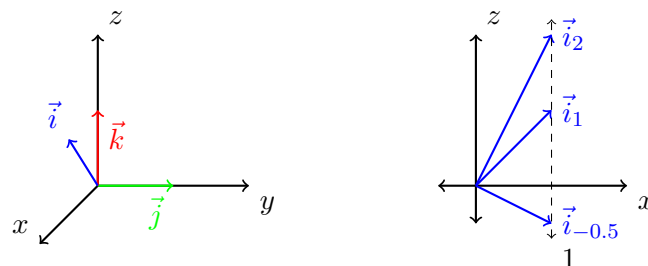


Figure 7.1: \mathbb{R}^3 with linearer transformation A^1

With $\vec{i} = (1, 0, \beta)^T$ and $\vec{j} = (0, 1, 0)^T$ and $\vec{k} = (0, 0, 1)^T$.

This means that only the x coordinate is transformed. No matter how often you exponentiate this transformation, the y and z axes will not change, but the x axis will increasingly approach the y axis.

\mathbb{Z}

$$\forall \beta \in \mathbb{N}^* \mid \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix}$$

7 Some random Proof

*Proof of important theorem. **Beweis** (durch vollständige Induktion nach β):*
 Induktionsvoraussetzung (\star):

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix}$$

Bemerkung 1.

Induktionsbeginn ($\beta = 0$): is also a valid starting point ...

Induktionsbeginn ($\beta = 1$):

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Induktionsschritt ($\beta \rightsquigarrow \beta + 1$):

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{\beta+1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (\beta+1) & 0 & 1 \end{pmatrix} \quad (7.1)$$

$$\Leftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{\beta} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (\beta+1) & 0 & 1 \end{pmatrix} \quad (7.2)$$

$$\Leftrightarrow^{\star} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (\beta+1) & 0 & 1 \end{pmatrix} \quad (7.3)$$

$$\Leftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta+1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta+1 & 0 & 1 \end{pmatrix} \quad (7.4)$$

□

8 plotting functions and their root function / derivation

8.1 python

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 import seaborn as sns
4
5 # Set seed for reproducibility
6 np.random.seed(42)
7
8 # Generate random 5x5 covariance matrix
9 cov_matrix = np.random.rand(5, 5)
10 cov_matrix = np.dot(cov_matrix, cov_matrix.T)
11
12 # Generate random data with the specified covariance matrix
13 data = np.random.multivariate_normal(mean=[0, 0, 0, 0, 0], cov=cov_matrix,
14 ↪ size=100)
15
16 # Create scatter plot
17 plt.figure(figsize=(12, 6))
18
19 # Scatter plot
20 plt.subplot(1, 3, 1)
21 sns.scatterplot(x=data[:, 0], y=data[:, 1])
22 plt.title('Scatter Plot')
23
24 # Box plot
25 plt.subplot(1, 3, 2)
26 sns.boxplot(data=data)
27 plt.title('Box Plot')
28
29 # Violin plot
30 plt.subplot(1, 3, 3)
31 sns.violinplot(data=data, inner='quartile', palette='muted')
32 plt.title('Violin Plot')
33
34 plt.tight_layout()
35 # plt.show()
```

8 plotting functions and their root function / derivation

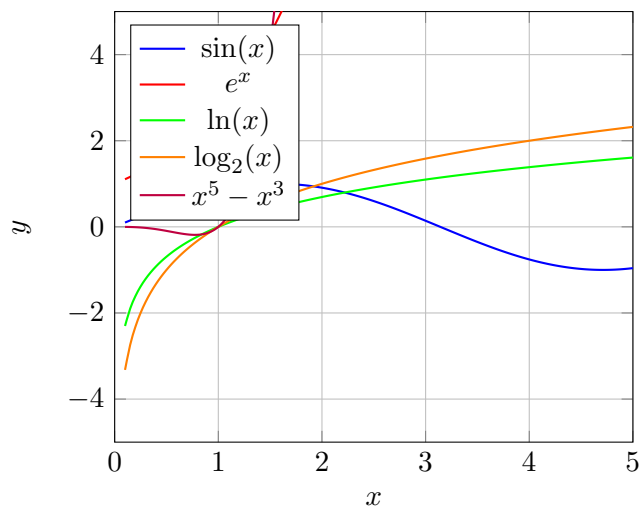
```
35 plt.savefig("random-py-plot.png")



---


1  import seaborn as sns
2  import matplotlib.pyplot as plt
3  from sklearn.datasets import load_iris
4  import numpy as np
5
6  # Load the Iris dataset from scikit-learn
7  iris = load_iris()
8  iris_data = iris.data
9  iris_feature_names = iris.feature_names
10
11 # Create a covariance matrix
12 cov_matrix = np.cov(iris_data, rowvar=False)
13
14 # Create a heatmap using Seaborn
15 plt.figure(figsize=(8, 6))
16 sns.heatmap(cov_matrix, annot=True, cmap='coolwarm',
17             ↳ xticklabels=iris_feature_names, yticklabels=iris_feature_names)
18 plt.title('Covariance Matrix of Iris Dataset Features')
19 # plt.show()
20 plt.savefig("random-covariance-fig.png")
```

8.2 tikzpicture



8.3 org plot

see <https://orgmode.org/manual/Org-Plot.html>

Sede	Max cites	H-index
Chile	257.72	21.39
Leeds	165.77	19.68
Sao Paolo	71.00	11.50
Stockholm	134.19	14.33
Morelia	257.56	17.67

Format	Fine-grained-control	Initial Effort	Syntax simplicity	Editor Support	Integratio
Word	2	4	4	2	
TEX	4	1	1	3	
Org Mode	4	2	3.5	1	
Markdown	1	3	3	4	
Markdown + Pandoc	2.5	2.5	2.5	3	

Sede	Max cites	
Chile	257.72	WWWWWWWWWWWWWW
Leeds	165.77	WWWWWWWWWh
Sao Paolo	71.00	WWW;
Stockholm	134.19	WWWWWW:
Morelia	257.56	WWWWWWWWWWWWWW
Rochefourchat	0.00	
test ...	42.0	WW

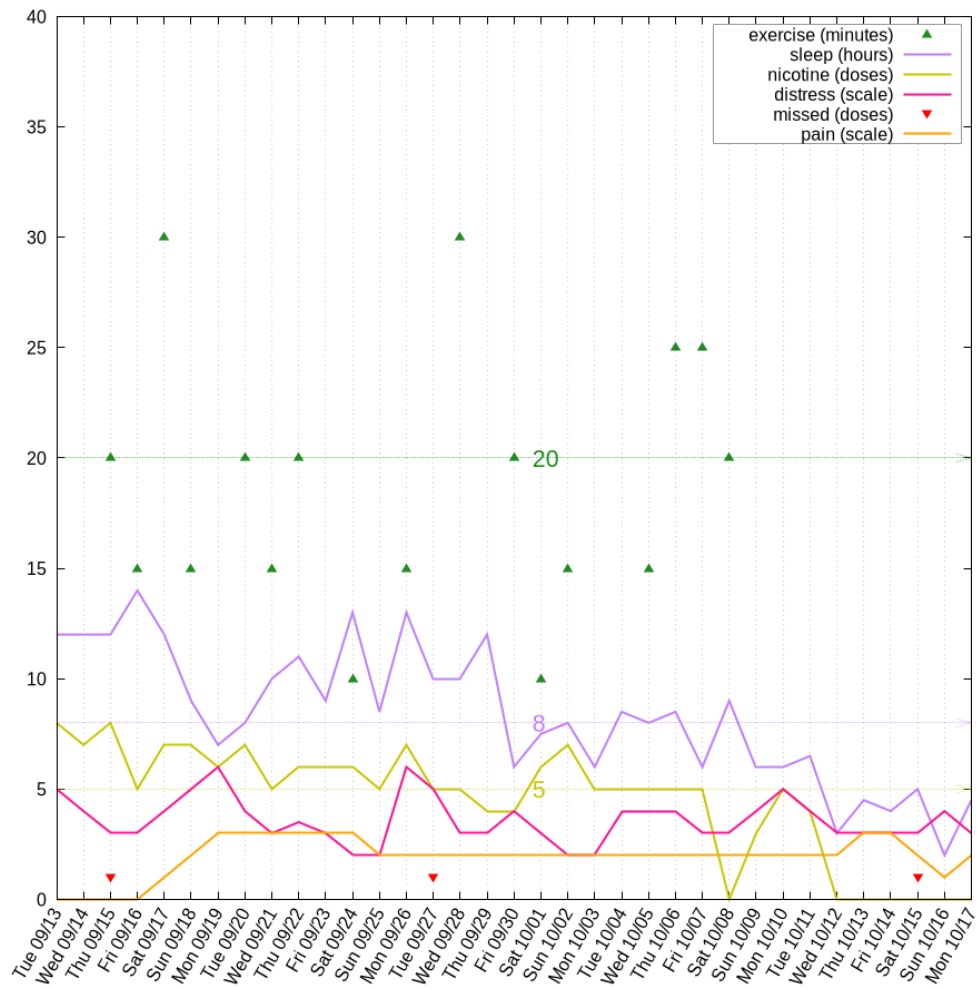
8.4 gnu plot

see https://gitlab.com/dto/health-template/-/tree/main?ref_type=heads by David O'Toole

8 plotting functions and their root function / derivation

Date	exercise minutes	sleep hours	nicotine doses	distress scale	missed dose
[2022-09-13 Tue]	0	12	8	5	
[2022-09-14 Wed]	0	12	7	4	
[2022-09-15 Thu]	20	12	8	3	
[2022-09-16 Fri]	15	14	5	3	
[2022-09-17 Sat]	30	12	7	4	
[2022-09-18 Sun]	15	9	7	5	
[2022-09-19 Mon]	0	7	6	6	
[2022-09-20 Tue]	20	8	7	4	
[2022-09-21 Wed]	15	10	5	3	
[2022-09-22 Thu]	20	11	6	3.5	
[2022-09-23 Fri]	0	9	6	3	
[2022-09-24 Sat]	10	13	6	2	
[2022-09-25 Sun]	0	8.5	5	2	
[2022-09-26 Mon]	15	13	7	6	
[2022-09-27 Tue]	0	10	5	5	
[2022-09-28 Wed]	30	10	5	3	
[2022-09-29 Thu]	0	12	4	3	
[2022-09-30 Fri]	20	6	4	4	
[2022-10-01 Sat]	10	7.5	6	3	
[2022-10-02 Sun]	15	8	7	2	
[2022-10-03 Mon]	0	6	5	2	
[2022-10-04 Tue]	0	8.5	5	4	
[2022-10-05 Wed]	15	8	5	4	
[2022-10-06 Thu]	25	8.5	5	4	
[2022-10-07 Fri]	25	6	5	3	
[2022-10-08 Sat]	20	9	0	3	
[2022-10-09 Sun]	0	6	3	4	
[2022-10-10 Mon]	0	6	5	5	
[2022-10-11 Tue]	0	6.5	4	4	
[2022-10-12 Wed]	0	3	0	3	
[2022-10-13 Thu]	0	4.5	0	3	
[2022-10-14 Fri]	0	4	0	3	
[2022-10-15 Sat]	0	5	0	3	
[2022-10-16 Sun]	0	2	0	4	
[2022-10-17 Mon]	0	4.5	0	3	

8.4 gnu plot



9 logic resolution proof

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

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9.1 forward direction (\rightarrow)

$$\begin{array}{c}
 \frac{\frac{[A]^3 \quad [A \rightarrow B]^1}{B} [\text{mp}] \quad [\neg B]^2}{\perp} [\neg E] \\
 \frac{\perp}{\neg A} [\neg I^3] \\
 \frac{\neg A}{\neg B \rightarrow \neg A} [\rightarrow I^2] \\
 \frac{\neg B \rightarrow \neg A}{(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)} [\rightarrow I^1]
 \end{array}$$

9.2 back direction (\leftarrow)

$$\begin{array}{c}
 \frac{\frac{\frac{[A]^5 \quad \frac{[\neg B]^6 \quad [\neg B \rightarrow \neg A]^4}{\neg A} [\text{mp}]}{\perp} [\neg E]}{B} [\text{raa}^6]}{A \rightarrow B} [\rightarrow I^5]}{(\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)} [\rightarrow I^4]
 \end{array}$$

9.3 both directions (\leftrightarrow)

$$\frac{(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A) \quad (\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)}{\vdash (A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)} [\leftrightarrow I]$$