Hacktopia  $Univ \varepsilon rsity$ 



Hacktopia University

Institute for Open Source Shenanigans and Kernel Comedy (IOSSKC)

Faculty 0101 - Faculty of Recursive Computing Delights

# Unleashing the Power of GNU/Emacs and Org-Mode

# A Cache of Wisdom for Thesis Writing

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Second reviewer: Dr. ing. Bob

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15th January 2024

#### **Abstract**

The fusion of Doom/Emacs, Org Mode, and ETEX presents a potent platform for crafting sophisticated thesis papers. This article provides a detailed guide on how to seamlessly integrate these tools, facilitating a smooth transition between generating HTML and ETEX documents, as well as PDFs, all from a single Org Mode file. Furthermore, a Nix shell, accompanied by a shell.nix file, is introduced to ensure efficient management of the development environment.

#### Zusammenfassung

Die Vereinigung von Doom/Emacs, Org Mode und ETEX bietet eine leistungsstarke Plattform zur Verfassung eleganter Abschlussarbeiten. Dieser Artikel erläutert detailliert, wie man diese Werkzeuge optimal kombiniert, um nahtlos zwischen der Erstellung von HTML- und ETEX-Dokumenten sowie PDFs aus einer einzigen Org-Mode-Datei zu wechseln. Zusätzlich wird eine Nix-Shell mit einer begleitenden shell.nix-Datei vorgestellt, um eine effiziente Verwaltung der Entwicklungs- und Arbeitsumgebung zu gewährleisten.

# **Contents**

1	Introduction / Motivation:	11
	1.1 Why bother writing Org-Mode when I can use 上下X instead?	11
	1.1.1 The possibilities with Org-Mode are endless!	11
	1.2 summery	11
	1.2.1 motivating videos	11
2	(Doom) GNU/Emacs setup and configuration	13
	2.1 packages ~/.config/doom/package.el	13
	2.2 config ~/.config/doom/config.el	13
3	nix shell environment	17
4	start messing around	19
5	convert python 2 latex	21
6	Function composition	23
7	Some random Proof	25
8	plotting functions and their root function / derivation	27
	8.1 python	27
	8.2 tikzpicture	29
	8.3 org plot	29
	8.4 gnu plot	31
9	logic resolution proof	35
	9.1 forward direction ( $ ightarrow$ )	35
	9.2 back direction (←)	36
	9.3 both directions $(\leftrightarrow)$	36

# **List of Listings**

1	nix-shell config for generating a build environment	 17
2	code block containing prime checking functions	 2

# **List of Tables**

5.1	table of library's v	with corresponding version	 21
J. I	table of library 5 v	in corresponding version	 _

# **List of Figures**

function composition and identity	
Composition of Transformations	

# 1 Introduction / Motivation:

Embarking on the journey of using GNU/Emacs with Org-Mode for thesis writing may seem like a daunting task, especially if you're unfamiliar with the intricate workings of GNU/Emacs itself. Some may question the need to delve into Org-Mode when alternatives like ETEX are readily available and widely embraced. However, dismissing the potential of GNU/Emacs and Org-Mode prematurely might mean overlooking a transformative experience that can redefine your approach to document creation.

### 1.1 Why bother writing Org-Mode when I can use 上 instead?

While MEX is a formidable typesetting system, Org-Mode complements it by offering a more versatile and flexible platform for document creation. Org-Mode's simplicity and ease of use, coupled with the ability to seamlessly integrate MEX, provide a unique writing experience that combines the best of both worlds.

### 1.1.1 The possibilities with Org-Mode are endless!

- 1. Include and evaluate code:
  - Integrate code seamlessly within your document, making it a dynamic and interactive platform for technical content.
- 2. Write LeteX in a more relaxed surrounding:
  - Org-Mode simplifies the MEX integration, providing a more user-friendly and intuitive environment for crafting your documents.

### 1.2 summery

For this tutorial, we will be using a distribution of GNU/Emacs called DOOM Emacs. Additionally, on our GNU/Linux platform, we'll leverage a package manager known as Nix to generate an environment for building both PDF and HTML versions of your thesis. This powerful combination of tools will enhance your writing experience and streamline the process of creating, editing, and compiling your thesis documents.

### 1.2.1 motivating videos

- Literate Documentation with Emacs and Org Mode
- Consistent Technical Documents Using Emacs and Org Mode

### 1 Introduction / Motivation:

- EmacsConf 2023: Authoring and presenting university courses with Emacs and a full libre software ...
- Emacs Tips How to Give Presentations with Org Mode
- EmacsConf 2022: Health data journaling and visualization with Org Mode and GNUplot David O'Toole

# 2 (Doom) GNU/Emacs setup and configuration

Start by installing Doom Emacs.

https://youtu.be/37H7bD-G7nE?feature=shared

### **2.1 packages** ~/.config/doom/package.el

After installing Doom Emacs - follow up by installing useful packages. If you are not familiar with Doom Emacs's evil mode (which is a vim style set of key bindings in Emacs), it might be a good idea to install evil-tutor and check out the tutorial for evil mode.

The pdf-tools (optional for this tutorial) will give you additional functionality for working with PDF documents in Doom Emacs, the most important being the support for viewing the PDF document in a Emacs buffer.

Finlay install org-special-block-extras for additional org-mode blocks for math and other usefull stuff.

#### Note

If you want to use the special blocks in you current org mode session - do not forget to enable it with: M-x org-special-block-extras-mode

```
(package! evil-tutor)
(package! pdf-tools)
(package! org-special-block-extras) ;; (use-package org-special-block-extras :ensure t)
(package! gnuplot)
(package! gnuplot-mode)
```

#### Note

Do not forget to doom sync after adding new packages to your package.el file and reload Emacs.

### **2.2 config** ~/.config/doom/config.el

To complete the Doom Emacs setup / configuration you only have to addapt your config.el file. We will now add the koma latex document classes and add support for code highlighting.

#### 2 (Doom) GNU/Emacs setup and configuration

```
;;; $DOOMDIR/config.el -*- lexical-binding: t; -*-
;; Place your private configuration here! Remember, you do not need to run 'doom
;; sync' after modifying this file!
;; ...
;; add latex classes for org-mocde export to latex (pdf)
(after! org
  (setq org-latex-classes
        '(("article" "\\documentclass[11pt]{article}"
           ("\\section{%s}" . "\\section*{%s}")
           ("\\subsection{%s\" . "\\subsection*{%s\")
           ("\\subsubsection{%s}" . "\\subsubsection*{%s}")
           ("\\paragraph{\%s}\" . \\paragraph*{\%s}\")
           ("\subparagraph{%s}" . "\subparagraph*{%s}")))
  (add-to-list 'org-latex-classes
               '("koma-book" "\\documentclass{scrbook}"
                 ("\\chapter{\%s}" . "\\chapter*{\%s}")
                 ("\section{%s}" . "\section*{%s}")
                 ("\\subsection{%s\" . "\\subsection*{%s\")
                 ("\\subsubsection{%s\" . "\\subsubsection*{%s\")
                 ("\\paragraph{\%s}\" . \\paragraph*{\%s}\")
                 ("\subparagraph{%s}" . "\subparagraph*{%s}"))
  (add-to-list 'org-latex-classes
               '("koma-article"
                 "\\documentclass{scrartcl}"
                 ("\section{%s}" . "\section*{%s}")
                 ("\\subsection{%s\" . "\\subsection*{%s\")
                 ("\\subsubsection{%s}" . "\\subsubsection*{%s}")
                 ("\\paragraph\%s\\" . "\\paragraph\*\\\\s\\")
                 ("\\subparagraph{\%s}\" . \\subparagraph*{\%s}\")))
  )
;; for syntax highlighting in org-mode
;; (setq org-latex-src-block-background 'minted
(setq org-latex-listings 'minted
     org-latex-packages-alist '(("" "minted"))
      org-latex-pdf-process
      '("pdflatex -shell-escape -interaction nonstopmode -output-directory %0 %f"
        "pdflatex -shell-escape -interaction nonstopmode -output-directory %0 %f"
        "pdflatex -shell-escape -interaction nonstopmode -output-directory %0 %f"))
```

;; -----

### Note

Before running SPC-h-r-r (M-x doom/reload) - either wait for the nix env setup or install pygments yourself via pip3 install pygments.

## 3 nix shell environment

Now that Emacs is ready - we can procued by declaring an shell.nix environment file, in which we will specify all the packages we will use for compiling the org-mode file and all library's and other stuff used in the thesis (e.g. numpy python haskell).

We will use <code>gnuplot</code> for plotting stuff, <code>graphics</code> for generating nice graphs in the dot language. In addition we need <code>texlive</code> and therefore can use <code>tikz</code> to generate nice plots/graphs. Last but not least, we install the <code>pygments</code> lib we are using for highlighting the code blocks in addition to the <code>python</code> packages used in the source code in the thesis.

If you want to install haskell or R for your thesis and haven't done so system wide - this might be a good opportunity to add the software.

```
Install Nix (the package manager)

sh <(curl -L https://nixos.org/nix/install) --daemon

https://nixos.org/download
```

```
cat shell.nix
```

Listing 1: nix-shell config for generating a build environment

```
{ pkgs ? import <nixpkgs> {} }:
pkgs.mkShell {
 buildInputs = with pkgs; [
    (python310.withPackages(ps: with ps; [
             # for math
latexify-py # for converting python funcs to latex
             # for math and physics
scipy
            # for syntax highlighting
pygments
matplotlib # for plotting
seaborn
             # for plotting
scikit-learn # for ml stuff
   1))
    texlive.combined.scheme-full
   graphviz
```

### 3 nix shell environment

```
gnuplot
];
shellHook = ''
   echo "entering dev environment"
   '';
}
```

Run nix-shell to enter the now defined development environment. After entering our specified environment - you can run make doom to start Doom Emacs.

# 4 start messing around

Since everything should be ready by now - please feel free to mess around with the README.org file within Emacs.

1. start the nix development environment

```
nix-shell # schould open a nix shell with all the packages defined in the local shell.
```

2. open doom Emacs form within the development environment

```
make doom # should open a new doom emacs window (from within the nix shell)
```

- 3. open README.org in doom Emacs, you could either open a file with M-x dired or by using SPC-.
- mess around
   Finlay adapt the README.org file for your needs or just explore the file from within Emacs.

#### **Export to HTML and PDF**

You can export your derivative with the key binding: C-c C-e

# 5 convert python 2 latex

```
import math
import numpy as np
import scipy
import latexify
```

Table 5.1: table of library's with corresponding version

```
latexify.__version___ '0.0.0a0'
np.__version___ '1.24.2'
scipy.__version___ '1.10.1'
```

With the imported library's, we can now convert a python function to latex

```
def recHelper(x: int, d: int = 3) -> int:
1
        # forall x in [3, infty)
2
        if x % 2 == 0:
3
            return 0
        elif d > math.floor(math.sqrt(x)):
            return 1
        elif x \% d == 0:
            return 0
8
        else:
9
            return recHelper(x, d + 2)
10
11
    def isPrime(x: int) -> int:
12
        if x <= 1: # forall x in (infty, 1] : 1
13
            return 0
14
        elif x == 2:
15
            return 1
16
        else: # forall x in (2, infty)
17
            return recHelper(x, 3)
18
```

Listing 2: code block containing prime checking functions

In line 12 is the main definition of the recursive basecase. Line 1 starts by declaring a recursive helper for the main prime checking function.

### 5 convert python 2 latex

$$isPrime(x) = \begin{cases} 0, & \text{if } x \leq 1\\ 1, & \text{if } x = 2\\ recHelper(x, 3), & \text{otherwise} \end{cases}$$

$$\operatorname{recHelper}(x,d) = \left\{ \begin{array}{ll} 0, & \text{if } x \% \ 2 = 0 \\ 1, & \text{if } d > \lfloor \sqrt{x} \rfloor \\ 0, & \text{if } x \% \ d = 0 \\ \operatorname{recHelper}\left(x,d+2\right), & \text{otherwise} \end{array} \right.$$

Now we can call the isPrime function with the parameter 5 and 6:

- is 5 a prime number? result of isPrime function: True
- is 6 a prime number? result of isPrime function: False

# **6 Function composition**

Graph using the dot language

```
digraph G {
   node [shape=circle, fontname="Courier", fontsize=16]
   edge [fontname="Courier", fontsize=16]
   A:nw -> A [label="id"]
   A -> B [label="f"]
   B -> C [label="g"]
   A -> C [xlabel="g . f "]
   {rank=same; A B}
}
```

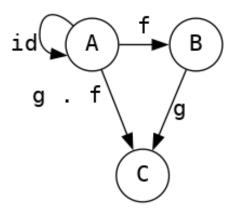


Figure 6.1: function composition and identity

Graph using latex tikz picture

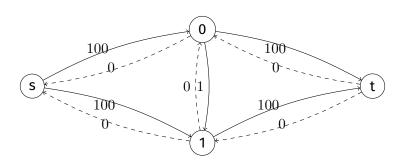


Figure 6.2: Some random Graph

### **7 Some random Proof**

Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \in \mathbb{R}^{3 \times 3}.$$

Proof that the following applies to all  $\beta \in \mathbb{N}_{>0}$ :

$$A^{\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix}.$$

'Matrix multiplication as composition | Chapter 4, Essence of linear algebra' footnote https://youtu.be/XkY2DOUCWMU The Matrix A can also be interpreted as a linear transformation in  $\mathbb{R}^3$ .

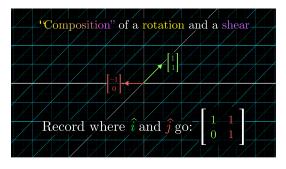


Figure 7.1: Composition of Transformations
Source: https://www.3blue1brown.com/lessons/matrix-multiplication

This interpretation makes it easier to understand the theorem to be proven, and makes it seem almost trivial.

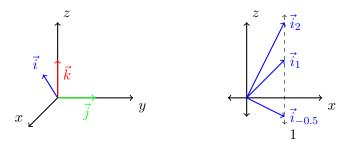


Figure 7.2:  $\mathbb{R}^3$  with linearer transformation  $A^1$ 

With 
$$\vec{i} = (1, 0, \beta)^T$$
 and  $\vec{j} = (0, 1, 0)^T$  and  $\vec{k} = (0, 0, 1)^T$ .

This means that only the x coordinate is transformed. No matter how often you exponentiate this transformation, the y and z axes will not change, but the x axis will increasingly approach the y axis.

72

$$\forall \beta \in \mathbb{N}^* \mid \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix}$$

*Proof of important theorem.* **Beweis** (durch vollständige Induktion nach  $\beta$ ): Induktionsvoraussetzung (\*):

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix}$$

### Bemerkung 1.

Induktionsbeginn ( $\beta = 0$ ): is also a valid starting point ...

Induktionsbeginn ( $\beta = 1$ ):

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Induktionsschritt ( $\beta \leadsto \beta + 1$ ):

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{\beta+1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (\beta+1) & 0 & 1 \end{pmatrix}$$
 (7.1)

$$\Leftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}^{\beta} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (\beta + 1) & 0 & 1 \end{pmatrix}$$
 (7.2)

$$\stackrel{\star}{\iff} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (\beta + 1) & 0 & 1 \end{pmatrix}$$
 (7.3)

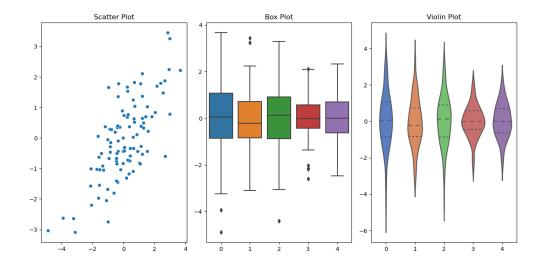
$$\Leftrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta + 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta + 1 & 0 & 1 \end{pmatrix}$$
 (7.4)

# 8 plotting functions and their root function / derivation

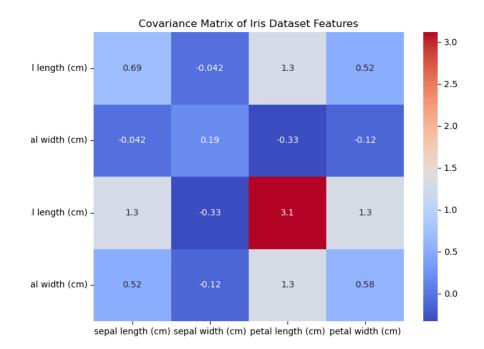
### 8.1 python

```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
# Set seed for reproducibility
np.random.seed(42)
# Generate random 5x5 covariance matrix
cov_matrix = np.random.rand(5, 5)
cov_matrix = np.dot(cov_matrix, cov_matrix.T)
# Generate random data with the specified covariance matrix
data = np.random.multivariate_normal(mean=[0, 0, 0, 0, 0], cov=cov_matrix, size=100)
# Create scatter plot
plt.figure(figsize=(12, 6))
# Scatter plot
plt.subplot(1, 3, 1)
sns.scatterplot(x=data[:, 0], y=data[:, 1])
plt.title('Scatter Plot')
# Box plot
plt.subplot(1, 3, 2)
sns.boxplot(data=data)
plt.title('Box Plot')
# Violin plot
plt.subplot(1, 3, 3)
sns.violinplot(data=data, inner='quartile', palette='muted')
plt.title('Violin Plot')
plt.tight_layout()
# plt.show()
plt.savefig("random-py-plot.png")
```

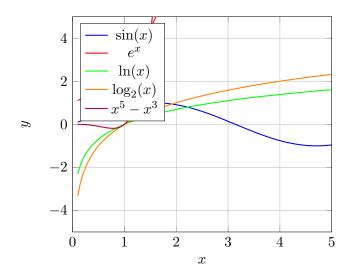
### 8 plotting functions and their root function / derivation



```
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn.datasets import load_iris
import numpy as np
# Load the Iris dataset from scikit-learn
iris = load_iris()
iris_data = iris.data
iris_feature_names = iris.feature_names
# Create a covariance matrix
cov_matrix = np.cov(iris_data, rowvar=False)
# Create a heatmap using Seaborn
plt.figure(figsize=(8, 6))
sns.heatmap(cov_matrix, annot=True, cmap='coolwarm', xticklabels=iris_feature_names, yt
plt.title('Covariance Matrix of Iris Dataset Features')
# plt.show()
plt.savefig("random-covariance-fig.png")
```



## 8.2 tikzpicture

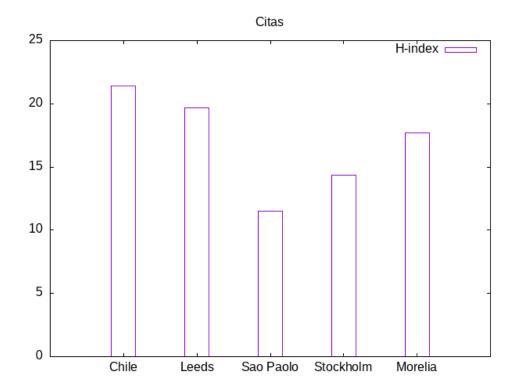


## 8.3 org plot

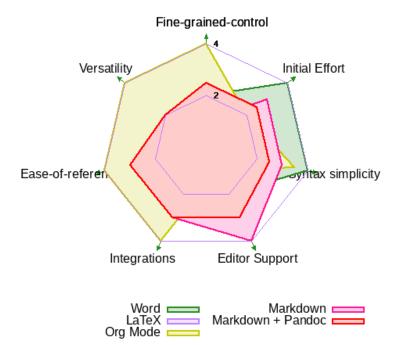
see https://orgmode.org/manual/Org-Plot.html

## 8 plotting functions and their root function / derivation

Sede	Max cites	H-index
Chile	257.72	21.39
Leeds	165.77	19.68
Sao Paolo	71.00	11.50
Stockholm	134.19	14.33
Morelia	257.56	17.67



Format	Fine-grained-control	Initial Effort	Syntax simplicity	Editor Support	Int
Word	2	4	4	2	
£T <sub>E</sub> X	4	1	1	3	
Org Mode	4	2	3.5	1	
Markdown	1	3	3	4	
Markdown + Pandoc	2.5	2.5	2.5	3	



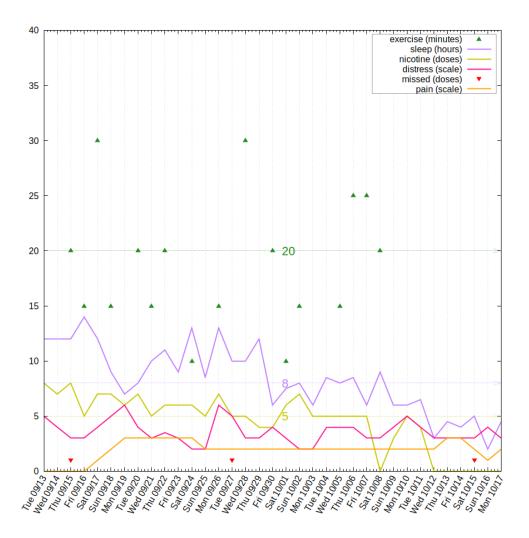
### An evaluation of plaintext document formats

Sede	Max cites	
Chile	257.72	WWWWWWWWWW
Leeds	165.77	WWWWWWh
Sao Paolo	71.00	www;
Stockholm	134.19	wwwww:
Morelia	257.56	wwwwwwwww
Rochefourchat	0.00	
test	42.0	WW

## 8.4 gnu plot

see https://gitlab.com/dto/health-template/-/tree/main?ref\_type=heads by David
O'Toole

Date	exercise minutes	sleep hours	nicotine doses	distress scale	missed doses
		·			
[2022-09-13 Di]	0	12	8	5	0
[2022-09-14 Mi]	0	12	7	4	0
[2022-09-15 Do]	20	12	8	3	1
[2022-09-16 Fr]	15	14	5	3	0
[2022-09-17 Sa]	30	12	7	4	0
[2022-09-18 So]	15	9	7	5	0
[2022-09-19 Mo]	0	7	6	6	0
[2022-09-20 Di]	20	8	7	4	0
[2022-09-21 Mi]	15	10	5	3	0
[2022-09-22 Do]	20	11	6	3.5	0
[2022-09-23 Fr]	0	9	6	3	0
[2022-09-24 Sa]	10	13	6	2	0
[2022-09-25 So]	0	8.5	5	2	0
[2022-09-26 Mo]	15	13	7	6	0
[2022-09-27 Di]	0	10	5	5	1
[2022-09-28 Mi]	30	10	5	3	0
[2022-09-29 Do]	0	12	4	3	0
[2022-09-30 Fr]	20	6	4	4	0
[2022-10-01 Sa]	10	7.5	6	3	0
[2022-10-02 So]	15	8	7	2	0
[2022-10-03 Mo]	0	6	5	2	0
[2022-10-04 Di]	0	8.5	5	4	0
[2022-10-05 Mi]	15	8	5	4	0
[2022-10-06 Do]	25	8.5	5	4	0
[2022-10-07 Fr]	25	6	5	3	0
[2022-10-08 Sa]	20	9	0	3	0
[2022-10-09 So]	0	6	3	4	0
[2022-10-10 Mo]	0	6	5	5	0
[2022-10-11 Di]	0	6.5	4	4	0
[2022-10-12 Mi]	0	3	0	3	0
[2022-10-13 Do]	0	4.5	0	3	0
[2022-10-14 Fr]	0	4	0	3	0
[2022-10-15 Sa]	0	5	0	3	1
[2022-10-16 So]	0	2	0	4	0
[2022-10-17 Mo]	0	4.5	0	3	0



# 9 logic resolution proof

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

### 9.1 forward direction ( $\rightarrow$ )

$$\begin{array}{c|c} & \underline{ \begin{array}{ccc} & \underline{[A]^3 & [A \rightarrow B]^1}[\mathsf{mp}] & [\neg B]^2 \\ & \underline{ & \underline{ } \\ & \underline{ } \\ & \underline{ \begin{array}{ccc} & \underline{ } \\ & \underline{ } \\ & \underline{ } \\ & \underline{ } \\ & \underline{ \end{array} \\ & \underline{ \begin{array}{ccc} & \\ & \underline{ } \\ & \underline{ } \\ & \underline{ } \\ & \underline{ \end{array} \\ & \underline{ \begin{array}{ccc} & \\ & \underline{ } \\ & \underline{ } \\ & \underline{ } \\ & \underline{ \end{array} \\ & \underline{ \begin{array}{ccc} & \\ & \underline{ } \\ & \underline{ } \\ & \underline{ \end{array} \\ & \underline{ \begin{array}{ccc} & \\ & \underline{ } \\ & \underline{ } \\ & \underline{ \end{array} \\ & \underline{ \begin{array}{ccc} & \\ & \underline{ } \\ & \underline{ } \\ & \underline{ \end{array} \\ & \underline{ \end{array} \\ & \underline{ \begin{array}{ccc} & \\ & \underline{ } \\ & \underline{ \end{array} \\ & 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# 9.2 back direction ( $\leftarrow$ )

### 9.3 both directions ( $\leftrightarrow$ )

$$\frac{ (A \to B) \to (\neg B \to \neg A) \quad (\neg B \to \neg A) \to (A \to B)}{\vdash (A \to B) \leftrightarrow (\neg B \to \neg A)} [\leftrightarrow \mathbf{I}]$$