Econ 706 Recitation 8 R Code

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In this document, we estimate an AR(1) model (without intercept) using Bayesian techniques with unknown variance. The model is

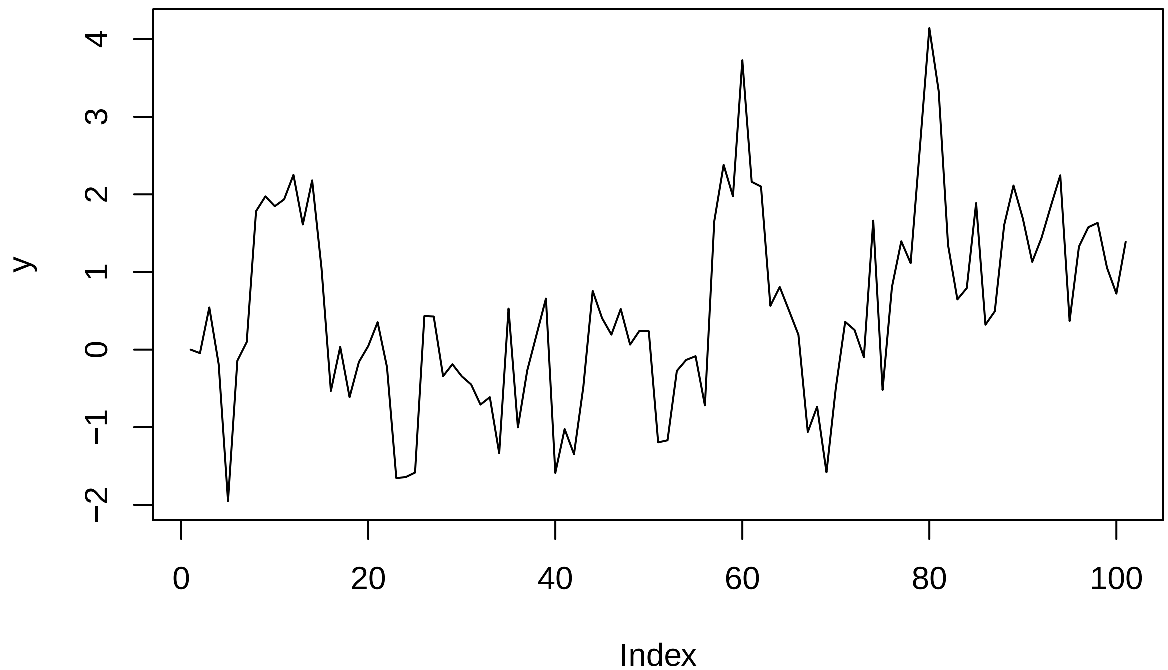
$$y_t = \phi_1 y_{t-1} + u_t \;\; u_t \sim \mathcal{N}(0, \sigma^2)$$

Let's simulate a dataset first.

```
y0=0
sigma=1
phi=0.8
T=100
y=matrix(0,nrow=T+1)
y[1]=y0
ut=rnorm(T,mean=0,sd=sigma^2) #Simulate the Shocks
for (t in 2:(T+1)){
  y[t]=phi*y[t-1]+ut[t-1]
}
```

This is how our data looks like:

```
plot(y, type="l")
```



Define Y to be vector $(y_1, ..., y_T)$ and X to be the vector $(y_0, ..., y_{T-1})$. We will further define prior distributions for both ϕ and γ:

$$\sigma^2 \sim \mathcal{IG}(\underline{\nu}, \underline{s}^2) \;\; \phi|\sigma^2 \sim \mathcal{N}(\underline{\phi}, \sigma^2 \underline{V}_\phi)$$

Then, in a linear Gaussian regression model, posterior takes the form:

$$\bar{\phi} = (X'X + \underline{V}_\phi^{-1})^{-1}(X'Y + \underline{V}_\phi^{-1}\underline{\phi})$$
$$\bar{V}_\phi = (X'X + \underline{V}_\phi^{-1})^{-1}$$
$$\bar{s}^2 = \underline{s}^2 + Y'Y + \underline{\phi}'\underline{V}_\phi^{-1}\underline{\phi} - \bar{\phi}'\bar{V}_\phi^{-1}\bar{\phi}$$
$$\bar{\nu} = \underline{\nu} + T$$

Refer to lecture notes for the derivations of these.

Let's set an arbitrary prior with the conjugate form.

```
Y=y[2:101]
X=y[1:100]

phiPRIOR=0
VphiPRIOR=2

nuPRIOR=10
ssqPRIOR=1
```

Then we compute posterior parameters as:

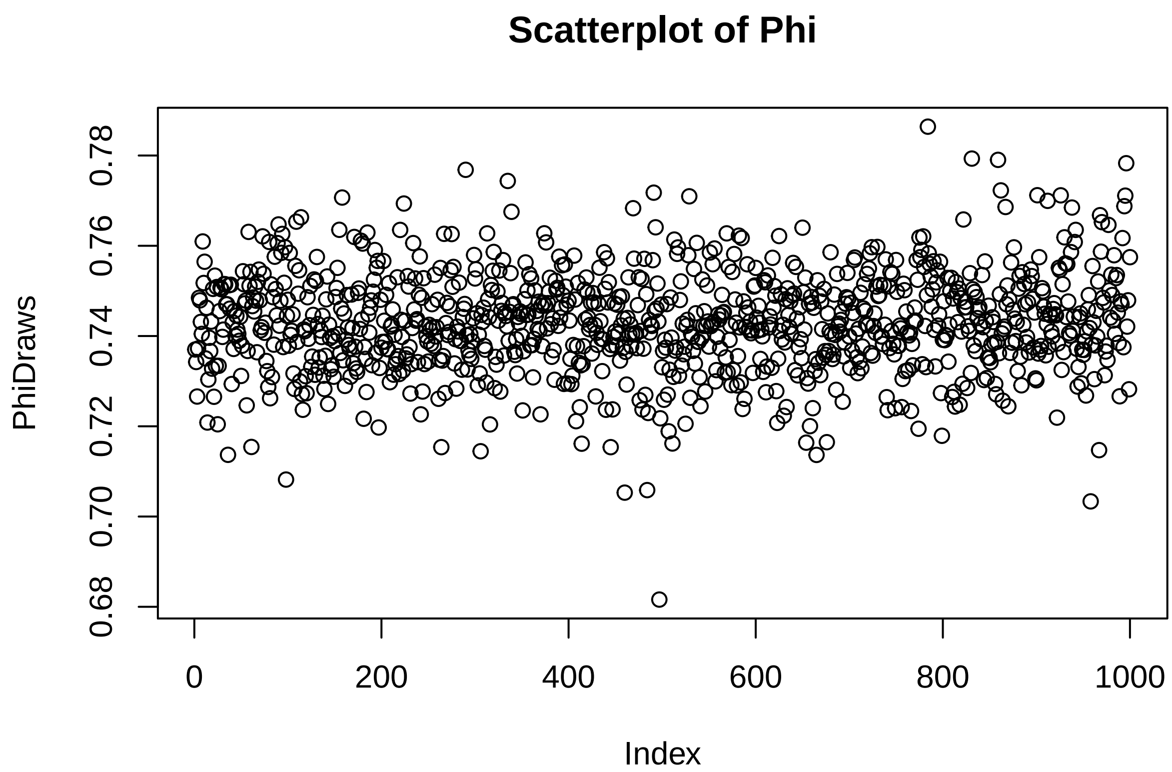
```
phiPOSTERIOR=(t(X) %*% X + VphiPRIOR^-1)^-1 %*% (t(X) %*% Y+(VphiPRIOR^-1  %*%  phiPRIOR))
VphiPOSTERIOR=(t(X) %*% X + VphiPRIOR^-1)^-1
ssqPOSTERIOR=ssqPRIOR+t(Y) %*% Y +phiPRIOR  %*% VphiPRIOR^-1  %*%  phiPRIOR -
                                    phiPOSTERIOR  %*% VphiPOSTERIOR^-1  %*%  phiPOSTERIOR
nuPOSTERIOR=nuPRIOR+T
```

Given that we have the posterior distributions, we can sample from them. We can use direct sampling here. Realize that we are using a different sigma draw for each phi draw. This provides independence across phi draws.

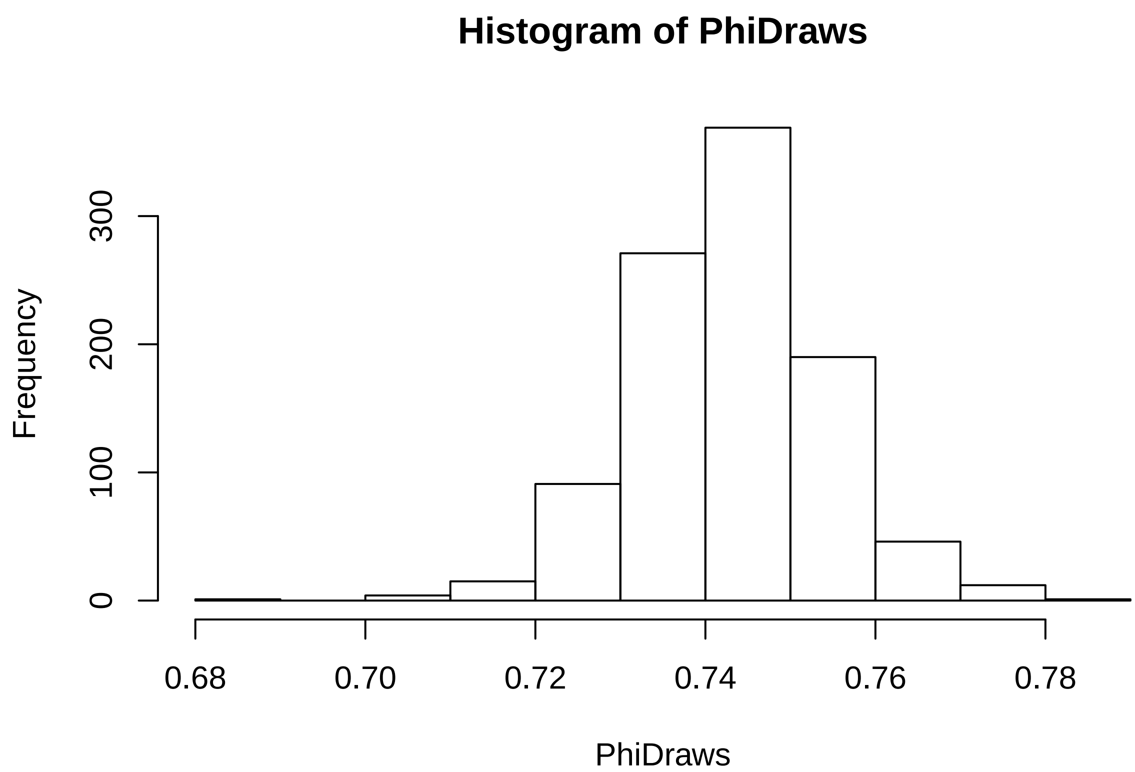
```
N=1000 #Sample size

SigmaSQDraws=matrix(0,1000,1)
PhiDraws=matrix(0,1000,1)

SigmaSQDraws=rinvgamma(N,ssqPOSTERIOR,nuPOSTERIOR)
for (ii in 1:N){
PhiDraws[ii]=rnorm(1,phiPOSTERIOR,(SigmaSQDraws[ii]^2)*VphiPOSTERIOR)
}
```

Now, we can plot our samples and compute basic descriptive statistics.

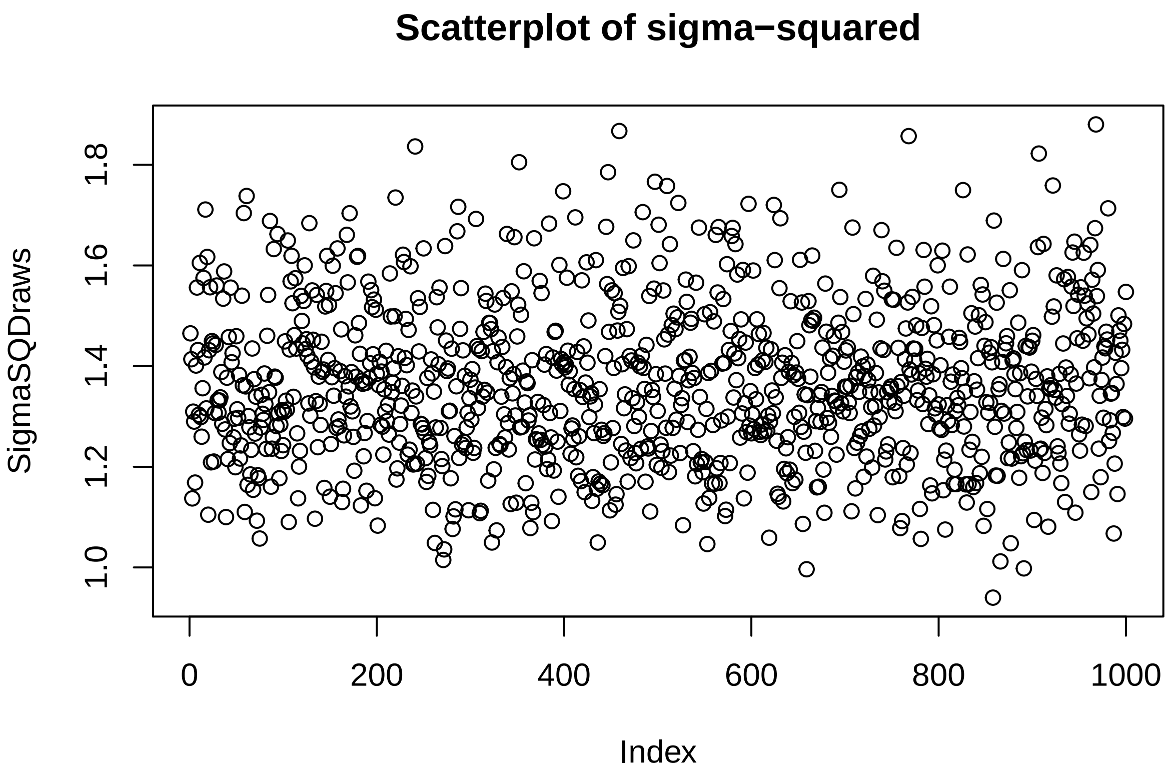
```
plot(PhiDraws,main="Scatterplot of Phi")
```



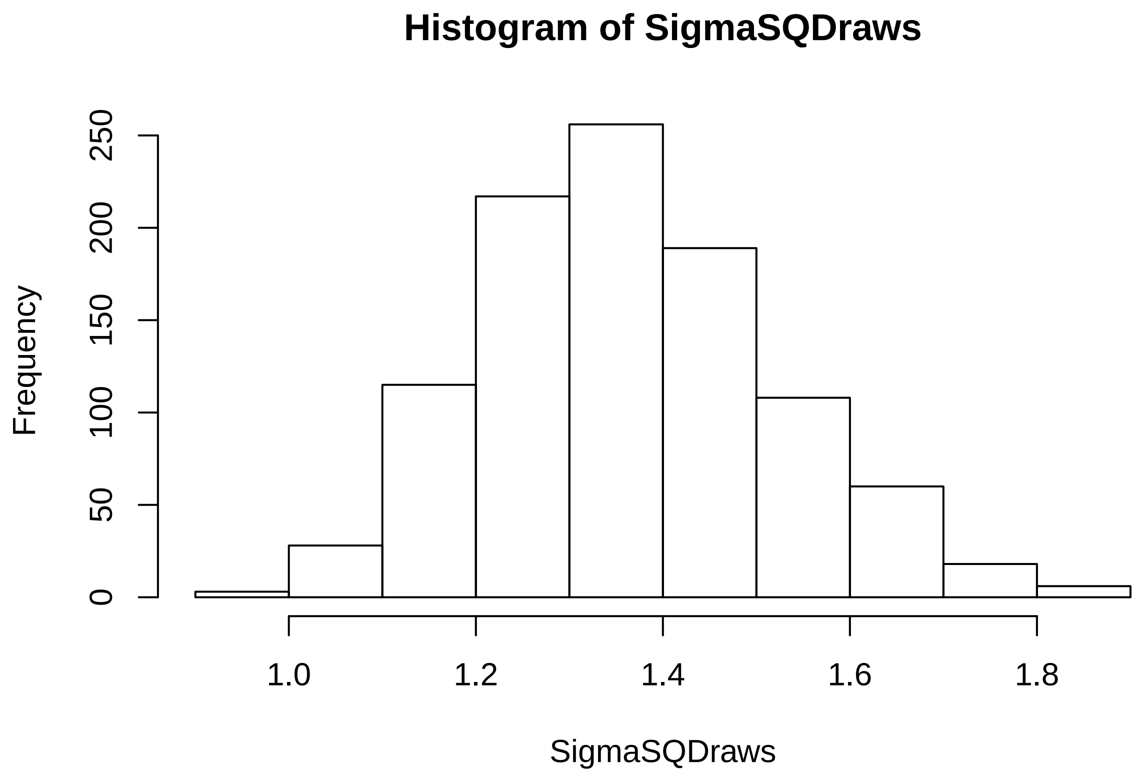
```r
hist(PhiDraws)
```




```r
plot(SigmaSQDraws,main="Scatterplot of sigma-squared")
```




```r
hist(SigmaSQDraws)
```




```
phiPOSTERIORMEAN=mean(PhiDraws)
phiPOSTERIORMEDIAN=median(PhiDraws)
```

Posterior mean of phi becomes 0.742961 and posterior median of phi becomes 0.7425722. Lastly, we construct an equial tail probability (not equal tail length!) credible set:

```
phiCredibleSetUB=quantile(PhiDraws,probs=.975)
phiCredibleSetLB=quantile(PhiDraws,probs=.025)

SigmaSQCredibleSetUB=quantile(SigmaSQDraws,probs=.975)
SigmaSQCredibleSetLB=quantile(SigmaSQDraws,probs=.025)
```

The credible set for phi becomes [0.7208311,0.7647422] and for sigma-squared becomes [1.0901578,1.6937331].