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A Simplified Jump Process for Common Stock Returns

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I. Introduction

The specification of a statistical distribution which accurately models the behavior of stock returns continues to be a salient issue in financial economics. With the introduction of arithmetic and geometric Brownian motion models, much attention has recently focused on a Poisson mixture of distributions as an appropriate specification of stock returns. For example, see [12], [3], [8], [10], [5], and [1]. Consistent with empirical evidence, these models yield leptokurtic security return distributions and, furthermore, the specification has much economic intuition. In particular, one may always decompose the total change in stock price into ''normal'' and ''abnormal'' components. The ''normal'' change may be due to variation in capitalization rates, a temporary imbalance between supply and demand, or the receipt of any other information which causes marginal price changes. This component is modelled as a lognormal diffusion process. The ''abnormal'' change is due to the receipt of any information which causes a more than marginal change in the price of the stock and is usually modeled as a Poisson process.

The specification of stock price dynamics plays a fundamental role in the valuation of contingent claims. In their seminal paper, Black and Scholes [2] provided a preference-free option valuation formula assuming the continuity of the underlying common stock price process. There then exists a self-financing dynamic position in stock and riskless bonds which replicates the option's cash flows. The absence of arbitrage opportunities yields an equilibrium option value. Alternatively, assuming stock returns follow a discontinuous process and the risks associated with the jump component are diversifiable, Merton [8] develops an option pricing formula given that the jumps are governed by a Poisson process.

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The Poisson process may be characterized as follows. During a time interval of length h, the probability of an event occurring may be expressed as:

$$\begin{aligned}
&\text{Prob [no events occur in the time interval } (t,\ t+h)] = 1 - \lambda h + 0(h) \\
(1)\ &\text{Prob [one event occurs in the time interval } (t,\ t+h)] = \lambda h + 0(h) \\
&\text{Prob [more than one event occurs in the time interval } (t,\ t+h)] = 0(h)
\end{aligned}$$

where $0(h)$ is the asymptotic order symbol and λ denotes the mean number of information arrivals (events) per unit time. When such an event occurs, there is an instantaneous jump in the stock price of size Y, the random variable Y being assumed independent of the lognormal diffusion process. Formally, we express this model by the following stochastic differential equation:

$$\frac{dS}{S} = \alpha\, dt + \sigma\, dz + dq \tag{2}$$

where

- $S \equiv$ common stock price,
- $\alpha \equiv$ instantaneous expected return on the stock,
- $\sigma^2 \equiv$ instantaneous variance of the stock return, conditional on no arrivals of ''abnormal'' information,
- $z \equiv$ standardized Wiener process,
- $q \equiv$ Poisson process, assumed independent of z,
- $\lambda \equiv$ intensity of the Poisson process (mean number of ''abnormal'' information arrivals per unit time).

The jump size, Y, has posited distribution $\ln Y \sim N(\mu, \delta^2)$.

Although corresponding equilibrium option pricing formulas have been put forward, difficulties arise with the empirical implementation and verification of this financial process. Press [12], *a priori,* constrains the instantaneous expected rate of return on the stock to be zero, $\alpha = 0$. Employing the method of cumulants and a sample of ten NYSE listed common stocks over the period 1926 through 1960, Press frequently obtains negative estimates of the variance parameters σ^2 and δ^2. Beckers [1], in modifying the Press procedure, sets the mean jump size equal to zero, $\mu = 0$, rather than $\alpha = 0$. Again employing the method of cumulants, Beckers often obtains negative estimates of σ^2 and δ^2. Fehr and Rosenfeld [5], alternatively, consider maximum likelihood estimation of the Poisson mixture of distributions under the simplifying assumption that the jump size is a fixed known constant. Unfortunately, their results are not statistically powerful in that the corresponding parameter estimates have standard errors which are generally quite large.

Our aim in this paper is to introduce a new and simpler model for the behavior of stock prices. The model, while retaining economic intuition, is consistent with the empirical evidence on stock returns and becomes amenable to statistical estimation.

II. A Bernoulli Jump Process

A Bernoulli jump process will be put forward as an appropriate model for information arrivals and, as such, stock price jumps. To motivate this specification, we consider the Poisson model in further detail.

A Poisson process possesses stationary independent increments with probability mechanism (1). Let the random variable N denote the number of events that occur in a time interval of length t. Define $h = t/n$ for an arbitrary integer n and subdivide the interval $(0,t)$ into n equal subintervals each of length h. Letting X_i denote the number of events that occur in subinterval i and by the stationary independent increment assumption,

$$N = \sum_{i=1}^{n} X_i$$

is the sum of n independent identically distributed random variables such that

$$\begin{aligned} &\text{Prob}\left[X_i = 0\right] = 1 - \lambda h + 0(h) \\ (3)\qquad &\text{Prob}\left[X_i = 1\right] = \lambda h + 0(h) \qquad \text{for} \quad i = 1, 2, \ldots, n. \\ &\text{Prob}\left[X_i > 1\right] = 0(h) \end{aligned}$$

For n large, each X_i has approximately the Bernoulli distribution with parameter $\lambda h = \lambda t/n$. Consequently, being the sum of n independent identically distributed Bernoulli random variables, N has the binomial distribution, approximately. That is

$$\text{Prob}\ [N = k] \cong \binom{n}{k}\left(\frac{\lambda t}{n}\right)^{k}\left(1 - \frac{\lambda t}{n}\right)^{n-k} \qquad k = 0, 1, 2, \ldots, n.$$

It is well known (for example, [6]) that

$$\lim_{n\to\infty}\binom{n}{k}\left(\frac{\lambda t}{n}\right)^{k}\left(1 - \frac{\lambda t}{n}\right)^{n-k} = \frac{\exp\,(-\,\lambda t)(\lambda t)^{k}}{k!} \qquad k = 0, 1, 2, \ldots .$$

This is a standard construction of the Poisson process (for further details, see [7]). Assume now that t is very small. It follows then that we can satisfactorily approximate N by the Bernoulli variate X defined by

$$\begin{aligned} P\ [X = 0] &= 1 - \lambda t, \\ P\ [X = 1] &= \lambda t. \end{aligned}$$

The distinguishing characteristic of the Bernoulli jump process is that over a fixed period of time, t, either no information impacts upon the stock price or one relevant information arrival occurs with probability λt, where λ is the rate of the process. No further information arrivals over this period of time are allowed.

If jumps in stock prices correspond to the arrival of ''abnormal'' information, by very definition the number of such information arrivals ought not to be very large. For practical considerations, if t corresponds to one trading day, no more than one ''abnormal'' information arrival is to be expected on average. Furthermore, if returns were computed for finer time intervals, the Bernoulli model would converge to the Poisson model.

The advantage of the Bernoulli jump process is that more satisfactory empirical analyses are available. Maximum likelihood estimation of the relevant parameters is practically and economically implementable. Under rather mild regularity conditions [4], satisfied in the present context, the resultant parameter estimates are asymtotically optimal. That is, they are asymptotically unbiased, consistent, and efficient in that the corresponding Cramer-Rao lower bound is attained. Significantly, the statistically most powerful test of the null hypothesis $\lambda = 0$ can be implemented. That is, the presence of a jump component in common stock returns can be empirically ascertained.

III. Statistical Analysis

Following Beckers [1], we *a priori* set the mean jump size equal to zero, $\mu = 0$, guaranteeing a symmetric return distribution. This economically reasonable prespecification simplifies the cumulant estimation methodology (see [1] for further details) but does not alter the generality of the maximum likelihood estimation procedure. Our model then results in a daily security return whose density f is a Bernoulli mixture of Gaussian densities:

$$f(x) = (1 - \lambda)\phi\left(\alpha, \sigma^2\right) + \lambda\phi\left(\alpha, \sigma^2 + \delta^2\right)$$

where

$$\phi\left(\mu, \sigma^2\right) = \left(2\pi\sigma^2\right)^{-1/2} \exp\left(-(x - \mu)^2/2\sigma^2\right).$$

This should be contrasted with the density b derived by Beckers [1] where the daily security return involves a Poisson mixture of Gaussian densities:

$$b(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda}(\lambda)^n}{n!} \phi\left(\alpha, \sigma^2 + n\delta^2\right).$$

Both models allow for discontinuous jumps and yet their daily returns are continuously distributed with bounded density functions. Further, note that $f(x) = b(x) + 0(\lambda^2)$. That is, for small values of λ, $f(x)$ and $b(x)$ are practically indistinguishable.

Let $K_i^{(f)}$ and $K_i^{(b)}$ denote, respectively, the cumulants i associated with the densities f and b for any integer i. The relationship between the cumulants and moments of a distribution is detailed by Kendall and Stuart [9]. Beckers [1] establishes

$$
\begin{aligned}
K_1^{(b)} &= \alpha, \\
K_2^{(b)} &= \sigma^2 + \lambda\delta^2, \\
K_3^{(b)} &= 0, \\
K_4^{(b)} &= 3\delta^4\lambda, \\
K_5^{(b)} &= 0, \\
K_6^{(b)} &= 15\delta^6\lambda.
\end{aligned}
\tag{4}
$$

The method of cumulants sets the sample cumulants ($\overline{K}_i^{(b)}$) equal to the population cumulants and solves for the parameter estimates $\overline{\lambda}$, $\overline{\sigma}^2$, $\overline{\delta}^2$, and $\overline{\alpha}$. Using (4) Beckers [1] shows

$$
\begin{aligned}
\overline{\lambda} &= 25\left(\overline{K}_4^{(b)}\right)^3 \Big/ 3\left(\overline{K}_6^{(b)}\right)^2, \\
\overline{\sigma}^2 &= \overline{K}_2^{(b)} - 5\left(\overline{K}_4^{(b)}\right)^2 \Big/ 3\overline{K}_6^{(b)}, \\
\overline{\delta}^2 &= \overline{K}_6^{(b)} \big/ 5\overline{K}_4^{(b)}, \\
\overline{\alpha} &= \overline{K}_1^{(b)}.
\end{aligned}
\tag{5}
$$

Although this method produces consistent estimates, they are not efficient. Indeed the sign of $\overline{\delta}^2$ depends on the sign of $\overline{K}_6^{(b)}$ for leptokurtic returns. Consequently, erratic behavior of the sixth sample cumulant results in negative variance estimates.

To derive the cumulants for the Bernoulli mixture, the corresponding moment-generating function is employed. Letting R denote the daily return, then

$$
\mathrm{E}\left[e^{sR}\right] = (1-\lambda)\exp\left(\alpha s + \sigma^2 s^2/2\right) + \lambda \exp\left(\alpha s + \left(\sigma^2 + \delta^2\right)s^2/2\right).
$$

The moments of the distribution may consequently be derived and, therefore, the cumulants. It is a straightforward task to show

$$\begin{aligned}
K_1^{(f)} &= \alpha, \\
K_2^{(f)} &= \sigma^2 + \lambda\delta^2, \\
K_3^{(f)} &= 0, \\
K_4^{(f)} &= 3\delta^4\lambda(1 - \lambda), \\
K_5^{(f)} &= 0, \\
K_6^{(f)} &= 15\delta^6\lambda(1 - \lambda)(1 - 2\lambda).
\end{aligned}$$

These results when compared to (4) are found to be similar. Again by equating sample with population cumulants we derive the estimators $\hat{\lambda}$, $\hat{\sigma}^2$, $\hat{\delta}^2$, and $\hat{\alpha}$:

$$\begin{aligned}
\hat{\lambda} &= \left(1 \pm \sqrt{(3K^*/(3K^* + 100))}\right)/2, \quad \text{where } K^* = \left(\overline{K}_6^{(f)} / \overline{K}_4^{(f)}\right)^2, \\
\hat{\sigma}^2 &= \overline{K}_2^{(f)} - \hat{\lambda}\hat{\delta}^2, \\
\hat{\delta}^2 &= \overline{K}_6^{(f)} \Big/ \left(\overline{K}_4^{(f)}(5(1 - 2\hat{\lambda}))\right), \\
\hat{\alpha} &= \overline{K}_1^{(f)}.
\end{aligned}$$

A '+' is taken in $\hat{\lambda}$ if $\overline{K}_6^{(f)} < 0$ and a '−' if $\overline{K}_6^{(f)} > 0$ in order to ensure $\hat{\delta}^2 > 0$. Of course, $0 \leq \hat{\lambda} \leq 1$.

The Bernoulli jump process admits maximum likelihood estimation. Assuming n daily security returns $\underset{\sim}{x} = (x_i,\ i=1, \ldots, n)$ and denoting $\underset{\sim}{\gamma} = (\lambda,\sigma^2,\delta^2,\alpha)$, the logarithm of the likelihood function is

$$\ln L(\underset{\sim}{x};\ \underset{\sim}{\gamma}) = \sum_{i=1}^{n} \ln f\left(x_i;\ \underset{\sim}{\gamma}\right).$$

Necessary conditions for the existence of maximum likelihood estimators $\underset{\sim}{\gamma}^*$ are provided by

$$\frac{\partial \ln L(\underset{\sim}{x};\ \underset{\sim}{\gamma})}{\partial \gamma_i} = 0 \quad i = 1, 2, 3, 4 \tag{6}$$

whereas corresponding sufficient conditions require the positive definiteness of $-H(\underset{\sim}{x};\underset{\sim}{\gamma}^*)$, the matrix $H(\underset{\sim}{x};\underset{\sim}{\gamma})$ being defined by

$$H(\underset{\sim}{x};\ \underset{\sim}{\gamma})_{ij} = \frac{\partial^2 \ln L(\underset{\sim}{x};\ \underset{\sim}{\gamma})}{\partial \gamma_i\, \partial \gamma_j} \quad i,j = 1, 2, 3, 4.$$

The likelihood equations (6) are nonlinear. Hence a numerical technique, the multidimensional Newton-Raphson procedure [11], was applied to solve (6). The eigenvalues of $-H(\underset{\sim}{x};\underset{\sim}{\gamma}^*)$ were determined to be positive ensuring the attainment of a local maximum. Alternative starting values always yielded identical results. Significantly, maximum likelihood estimation allows the construction of approximate confidence intervals for the parameters of interest. Furthermore, these confidence intervals are asymptotically optimal [14] and $\underset{\sim}{\gamma}^*$ is asymptotically multivariate normally distributed with variance-covariance matrix approximated by $-H(\underset{\sim}{x};\underset{\sim}{\gamma}^*)^{-1}$ [13].

IV. Empirical Results

The data consist of 47 NYSE listed stocks each with 500 daily return observations spanning the period September 15, 1975 to September 7, 1977. The data source is the CRSP daily return file wherein dividends are added back in computing returns and weekend returns are treated as overnight returns. The data coincide with that employed in Beckers' [1] analysis. Notice that, by treating weekend returns as daily returns, we may tend to induce jumps even when the underlying common stock price process is continuous. This caveat applies to all such estimation methodologies. The four parameters to be estimated are λ, the intensity of the information arrival; σ^2, the instantaneous variance of the return on the stock; δ^2, the variance of the logarithm of the jump; and α, the instantaneous expected return on the stock. Three sets of estimation procedures are employed: Beckers' method of cumulants; the corresponding method of cumulants for the Bernoulli jump process; and maximum likelihood estimation. The Beckers' technique does assume a different model specification than the other two procedures; yet for small values of λ, the models are very similar and hence comparison of the estimation techniques is deemed appropriate. The results are in Table 1.

For each sampled common stock, the first row presents estimates derived by Beckers' cumulants method, the second row Bernoulli cumulants estimates, while corresponding maximum likelihood estimates are in the third row. Approximate standard errors are also provided for the maximum likelihood estimates; they are not readily available for the method of cumulants procedures.

Under the Beckers' procedure, approximately 60 percent of the sampled stocks yielded negative estimates of σ^2 or δ^2. This occurrence is reduced to approximately 20 percent for the Bernoulli cumulants procedure. However, maximum likelihood estimation produces no negative estimates of variance. It should be noted that when the cumulants procedures do yield positive variance estimates, the resultant parameter estimates are similar to the maximum likelihood estimates. Also, the standard errors of the maximum likelihood estimates are extremely small, confirmimg the preferability of this estimation methodology.

Table 1 also provides the statistically most powerful test for the presence of jumps in the sampled common stock returns. We consider the likelihood ratio statistic

$$\Lambda = -2(\ln L(\underset{\sim}{x}; \underset{\sim}{\gamma}^*) - \ln L(\underset{\sim}{x}; \underset{\sim}{\gamma}^\circ))$$

where $\underset{\sim}{\gamma}^*$ and $\underset{\sim}{\gamma}^\circ$ denote the maximum likelihood estimates of γ in the presence and in the absence of the Bernoulli jump structure, respectively. Under the null hypothesis that security returns are consistent with a lognormal diffusion process without Bernoulli jump structure, Λ is asymptotically distributed χ^2 with two degrees of freedom. Only five stocks did not demonstrate the presence of jumps at the 5 percent significance level. Moreover, over 78 percent of the stocks indicated the presence of jumps at the 1 percent significance level. These results are strongly supportive of the presence of jumps in security returns.

V. Conclusions

This paper has put forth a simplified jump process for common stock returns. A Bernoulli jump process models information arrivals and, as such, stock price jumps. The specification maintains the desirable economic properties associated with movements in common stock returns and is empirically tractable. Both the method of cumulants and maximum likelihood estimation are implemented. Maximum likelihood estimates of the variance parameters are consistently positive, whereas method of cumulants variance estimates are not. Maximum likehood estimation also allows for the statistically most powerful test to detect the presence of jumps in common stock returns. Our empirical analysis confirms the presence of jumps in a majority of the sampled common stock returns.

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TABLE 1
Parameter Estimates[a]

Stock	λ	$\sigma^2 \times 10^4$	$\delta^2 \times 10^4$	$\alpha \times 10^2$	Λ
ASA	11.8607	−8.4649	1.3256	−.0989	
	.4282	3.3076	9.2265	−.0989	
	.4484	3.1889	9.0762	−.1251	18.266**
	(.1573)	(.9000)	(2.0498)	(.1127)	
AT&T	3242.6641	−16.5906	.0053	.0841	
	.4956	.1533	.5985	.0841	
	.3667	.2133	.6456	.0761	23.498**
	(.1413)	(.0500)	(.1681)	(.0285)	
ARC	7.6731	4.4572	−.3564	.0412	
	.5888	.5413	2.0063	.0412	
	.5570	.6183	1.9828	.0357	16.660**
	(.1611)	(.2337)	(.3512)	(.0547)	
Avon	.0708	2.0237	19.8845	.0767	
	.0586	2.1114	22.5262	.0767	
	.0529	2.2288	22.7789	.0451	81.448**
	(.0253)	(.1950)	(10.527)	(.0716)	
Boeing	.9713	5.2207	−2.0137	.1702	
	.7262	.0327	4.4507	.1702	
	.6114	.7062	4.1851	.1862	31.654**
	(.0881)	(.2168)	(.5401)	(.0696)	
Burroughs	.1564	1.7471	4.5300	−.0307	
	.1078	1.8328	5.7755	−.0307	
	.2760	1.4620	3.6032	.0002	16.294**
	(.1567)	(.3147)	(1.297)	(.0666)	
Chase Manhattan	.7984	−.0825	3.2563	.0476	
	.2558	.8113	6.6682	.0476	
	.2286	.9776	6.7336	.0274	91.846**
	(.0623)	(.1493)	(1.6315)	(.0563)	
Citicorp	12.5114	−2.7871	.4213	.0062	
	.4300	1.1900	3.0104	.0062	
	.4907	1.0419	2.9397	.0079	16.260**
	(.1652)	(.3257)	(.6196)	(.0668)	
Coca Cola	15.1537	6.1171	−.2933	.0378	
	.5637	.3756	2.3019	.0378	
	.2638	.9071	2.9041	.0255	26.728**
	(.1056)	(.1458)	(.8881)	(.0532)	
Control Data	.2809	3.6858	3.0633	.0698	
	.1569	3.8459	4.4640	.0698	
	.0838	3.9913	6.6214	.0650	1.650
	(.1564)	(.5673)	(7.5148)	(.0945)	
Deere	.0728	2.0814	−2.4020	.0424	
	.9400	−.6591	2.7293	.0424	
	.7707	.5805	1.7208	.0538	4.024
	(.1779)	(.4118)	(.3176)	(.0604)	
Delta Airlines	10.4024	7.7158	−.5313	.0431	
	.5766	.1887	3.4684	.0431	
	.3455	.8588	3.8600	−.0203	44.934**
	(.0880)	(.1513)	(.8307)	(.0572)	
Digital Equipment	1.0459	4.8219	−1.3726	.0553	
	.7196	1.1374	3.1252	.0553	
	.3257	2.2921	3.3593	.0599	2.155
	(.2630)	(.5518)	(1.5851)	(.0806)	

TABLE 1 (cont.)

Parameter Estimates[a]

Stock	λ	$\sigma^2 \times 10^4$	$\delta^2 \times 10^4$	$\alpha \times 10^2$	Λ
Du Pont	.0122	1.3408	−3.1193	.0058	
	.9882	−1.8544	3.1948	.0058	
	.7360	.5289	1.0514	−.0069	4.238
	(.2066)	(.2623)	(.2392)	(.0508)	
Ford	1.3073	2.5757	−.7274	.1077	
	.7033	.3533	1.8155	.1077	
	.5813	.6156	1.7369	.1289	11.990**
	(.1820)	(.2522)	(.3157)	(.0544)	
GE	540.5481	21.3974	−.0371	.0641	
	.5108	.4671	1.7251	.0641	
	.3244	.7147	1.9531	.0545	19.870**
	(.1388)	(.1480)	(.5742)	(.0487)	
GM	14.0827	−1.7193	.2243	.1044	
	.4340	.7021	1.6981	.1044	
	.3939	.7633	1.7156	.1015	12.916**
	(.2153)	(.2376)	(.5154)	(.0513)	
GTE	.1868	.5011	1.5707	.1058	
	.1217	.5417	2.0761	.1058	
	.1564	.5179	1.7680	.1066	26.936**
	(.1022)	(.0860)	(.7914)	(.0375)	
Homestake	24.7265	−9.1454	.5294	.0338	
	.4500	1.5644	5.2919	.0338	
	.6307	.9044	4.8245	−.0042	25.480**
	(.1093)	(.3583)	(.6348)	(.0789)	
Honeywell	.7998	1.0339	2.8750	.1173	
	.2560	1.8251	5.8915	.1173	
	.4128	1.4245	4.6307	.0658	24.858**
	(.1626)	(.4587)	(1.0922)	(.0759)	
IBM	.4792	1.4621	−.8219	.0929	
	.7928	−.0447	1.4038	.0929	
	.4842	.4601	1.2560	.0821	15.596**
	(.1712)	(.1427)	(.2746)	(.0435)	
Int. Min. & Chem.	.1616	1.0675	8.1833	−.0012	
	.1103	1.2314	10.4999	−.0012	
	.1421	1.1599	8.6580	−.0061	92.864**
	(.0496)	(.1480)	(2.6865)	(.0567)	
ITT	1.1276	2.5162	−.8606	.1237	
	.7130	.1054	2.0201	.1237	
	.3302	.8110	2.2264	.1028	19.259**
	(.1270)	(.1480)	(.6349)	(.0519)	
Kennecott	.0422	2.7170	25.5228	−.0462	
	.0375	2.3137	27.5931	−.0462	
	.1346	1.8628	11.0689	−.1106	65.888**
	(.0631)	(.2736)	(4.1103)	(.0711)	
Loews	3.1952	−.3322	1.1850	.1336	
	.3653	1.8460	4.5931	.1336	
	.3223	2.0143	4.4671	.1005	13.966**
	(.2423)	(.6554)	(1.7414)	(.0811)	
McDonalds	.0985	2.4074	−2.7583	.0172	
	.9235	−.8717	3.2567	.0172	
	.7579	.5248	2.2131	.0204	10.472**
	(.1258)	(.2721)	(.2910)	(.0618)	

TABLE 1 (cont.)

Parameter Estimates[a]

Stock	λ	$\sigma^2 \times 10^4$	$\delta^2 \times 10^4$	$\alpha \times 10^2$	Λ
Merrill Lynch	.0198	6.2813	-13.0903	.0463	
	.9813	-7.3216	13.5980	.0463	
	.7684	1.8333	5.4575	-.0195	3.728
	(.1437)	(.9410)	(.9404)	(.1106)	
Monsanto	146.5012	11.9460	-.0716	-.0056	
	.5206	.5558	1.7344	-.0056	
	.5102	.5967	1.7240	-.0056	18.207**
	(.3524)	(.5316)	(.3701)	(.0515)	
Natl. Semiconductor	.2390	5.1335	18.7370	-.0944	
	.1425	5.8775	26.2042	-.0944	
	.0971	6.6274	30.8437	.0070	35.472**
	(.0620)	(.8152)	(15.1631)	(.1281)	
Northwest Airlines	.1579	4.0792	-3.8034	.0771	
	.8914	-.8523	4.8583	.0771	
	.4907	1.4774	2.3769	.0339	7.076 *
	(.1914)	(.7017)	(2.1054)	(.0796)	
Occidental Petroleum	305.1706	56.9002	-.1731	.0923	
	.5143	.9589	6.0508	.0923	
	.4024	1.5483	6.2703	.0640	36.513**
	(.1217)	(.4054)	(1.3324)	(.0829)	
Pfizer	2.6580	-.1215	.9866	.0338	
	.3534	1.3117	3.3649	.0338	
	.2189	1.6175	4.0361	.0331	16.132**
	(.1514)	(.3186)	(1.7714)	(.0674)	
Polaroid	.0471	4.5996	19.3319	-.0104	
	.0414	4.6389	21.0761	-.0104	
	.0117	4.9862	44.8426	-.0488	15.654**
	(.0131)	(.3511)	(46.2779)	(.1016)	
RCA	.0454	2.7422	8.7400	.1264	
	.0400	2.7588	9.5006	.1264	
	.0105	2.9192	20.8751	.1098	8.775 *
	(.1615)	(.2124)	(26.5610)	(.0776)	
Schlumberger	.7755	.8905	1.3065	6727	
	.2531	1.2339	2.6460	.6727	
	.2865	1.2086	2.4293	.0410	10.771**
	(.2738)	(.3923)	(1.1883)	(.0617)	
Searle	.1932	1.8312	12.6435	-.0095	
	.1245	2.1786	16.8350	-.0095	
	.1352	2.2358	15.1581	-.1100	79.918**
	(.0621)	(.3484)	(5.5312)	(.0810)	
Sears	.1292	1.7714	1.8522	.0196	
	.9060	-.5346	2.2812	.0196	
	.5992	.5984	1.5661	-.0492	8.218 *
	(.1821)	(.2376)	(.3050)	(.0569)	
Skyline	.5785	6.0488	-2.9035	-.0076	
	.7747	.2745	5.2857	-.0076	
	.7144	.9016	4.8537	-.0032	16.109**
	(.1093)	(.4542)	(.5774)	(.0874)	
Sperry	.1877	1.5503	5.0901	.0028	
	.1221	1.6832	6.7351	.0028	
	.3987	1.0575	3.6323	.0154	29.095**
	(.2016)	(.4373)	(.9731)	(.0644)	

TABLE 1 (cont.)

Parameter Estimates[a]

Stock	λ	$\sigma^2 \times 10^4$	$\delta^2 \times 10^4$	$\alpha \times 10^2$	Λ
Syntex	.1402	2.7556	19.7381	−.0737	
	.0998	3.0622	24.6586	−.0737	
	.1239	2.9914	20.4448	−.1151	76.582**
	(.0472)	(.3571)	(6.8773)	(.0891)	
Texas Instruments	.2039	1.6230	4.1775	−.0006	
	.1289	1.7491	5.6289	−.0006	
	.1902	1.6068	4.5629	.0095	20.208**
	(.1272)	(.2919)	(2.0044)	(.0663)	
Tiger	10.4301	19.4991	−1.3026	.0307	
	.5765	1.0053	8.5136	.0307	
	.3095	2.8857	9.8336	−.0941	30.018**
	(.1326)	(.6556)	(2.8574)	(.1001)	
U.S. Steel	1.3691	2.8029	−.7058	−.0362	
	.6965	.5857	1.7962	−.0362	
	.6883	.6354	1.7453	−.0413	6.786 *
	(.2284)	(.4406)	(.3063)	(.0585)	
Upjohn	.1195	1.7321	9.1772	.0215	
	.0887	1.8388	11.1569	.0215	
	.1544	1.6391	7.7041	.0137	47.799**
	(.0728)	(.2354)	(2.8287)	(.0669)	
Western Union	.3880	3.3870	−2.3274	.1241	
	.8130	−.5388	3.7180	.1241	
	.4871	.8225	3.4146	.0805	29.775**
	(.0977)	(.1765)	(.5896)	(.0625)	
Westinghouse	.9798	.2060	4.0148	.0942	
	.2746	1.6948	8.9045	.0942	
	.2216	1.9889	2.7560	−.0107	54.446**
	(.0740)	(.2934)	(2.7182)	(.0786)	
Xerox	121.1467	21.7790	−.1521	.0114	
	.5227	1.5904	3.3534	.0114	
	.2599	2.2711	4.1256	.0037	7.682 *
	(.2060)	(.4833)	(1.9751)	(.0793)	

* Indicates significance at 5% level.
** Indicates significance at 1% level.
[a] Standard errors of maximum likelihood estimates are in parentheses.