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Temporal Dynamics of the Approximate Number System: Implications for Numerical Cognition and Dyscalculia

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1 Introduction

First conceptualised by Swiss developmental psychologist Jean Piaget, in 1952, the approximate number system (ANS) is an innate cognitive mechanism for evaluating quantities without direct enumeration. This system is a cornerstone of our intuitive grasp of quantity, and whilst seemingly natural, it represents a definitive achievement in the context of successful human evolution. To take one example, this ability to intuitively approximate quantities serves an important role in survival, playing into decisions related to fight or flight and assessing potential chances against distinct quantities of predators.

It is important to note however, that the approximate number system is distinct from the formal system of mathematics that humans supplement this ability with. This is demonstrated in cultures lacking formal mathematical systems, such as newly hatched chicks, who display quantitative intuition when searching for food within collections of objects (Odic and Starr 2018).

Despite this distinction, recent research suggests that when undergoing arithmetic tasks, adults experience similar parietal lobe activation to that incurred during intuitive number approximations (Bonny and Lourenco, 2013). This similar cerebral environment for both the ANS system and its formal mathematical counterpart, provides potential reasoning as to why children with less accurate ANS scores, correlate with those experiencing hindered mathematical ability. It follows that providing an accurate model which best approximates ANS ability across a population, could provide a baseline framework for diagnosis and detection of dyscalculia and other related numerical cognition disorders.

Traditionally, studies have failed to control for response time in ANS experiments, neglecting the potential variation induced by changes in accuracy resulting from the user's desire to answer questions more quickly (Inglis and Gilmore, 2014). While a recent study incorporated this element, demonstrating reduced variation (Park and Starns, 2015), no studies have thus far attempted to ascertain the effect of varying time on ANS ability. In this investigation, therefore, we attempt to model users guess distributions as a function of both innate ANS ability and delay time. We hypothesise that varying the time in which the dots are displayed to the user, will have a resulting effect on the dot guess distribution, where modelling change in variance over time will provide our population fitted temporal distribution model.

2 Methodology

The methodology for our ANS test, was based off a similar system to that applied by Park et al. (2015). Primarily the test consisted of two distinct dot arrays, residing in individual ovals and of two separate colours (Figure 1). Upon initiating the game, the user is randomly assigned a delay time between 0 and 4 seconds, which will persist across the entire duration of 150 rounds and the dots will appear for this selected time duration. The precise number of dots displayed in each round adheres randomly to one of four distinct ratios: 4:3, 7:6, 9:8, or 10:9 (as well as their counterparts). Upon completion of the delay time, the user is prompted to select the class (left or right) with the greatest perceived number of dots and is assigned a period of 3.0 seconds to do so. If the user selects a choice, or 3.0 seconds persists with no user input, the user must manually proceed to the next round, where this process repeats. It must however be noted that due to the variation in assigned delay times, the total duration of the test will vary between participants. We hoped that by manually allowing the user to control the advancement to the next round, this would help to somewhat mitigate the greater tiring effects for those with a longer delay time and subsequently longer test duration.

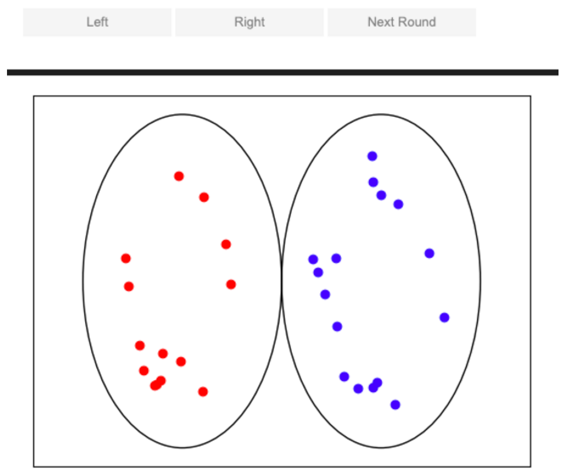


Figure 1: Approximate Number Series (ANS) test used in our methodology. The number of dots displayed adheres to a specific class of dot ratios. Here, the left and right options remain greyed out to ensure the entire displayed period is surpassed before a user choice is made.

Our statistical analysis assumes that approximation of quantities in each group forms a Gaussian Distribution on a continuous line of number estimates. Individual test performance can be quantified by obtaining w the Weber Fraction, which is defined as the test ratio required for an individual to incur a 25% chance of error. Given that any individual naturally incurs a 50% chance of a correct result, the Weber fraction is found when half the potential improvement in guess estimates are made.

Given that the Gaussian Distribution is given by

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{1}$$

The probability of error in an ANS test can be given as

$$P_e = \frac{1}{2}\text{erfc}\left(\frac{1-r}{w\sqrt{2(1+r^2)}}\right) \tag{2}$$

Where erfc is the complementary error function, r is the ratio of dots in each set, and w is defined as $w = 1 - r_w$, where r_w is the ratio at which a 25% error incurs for a fixed time delay.

Using this equation, we fitted the model to the individual users' error rates at each given ratio. This was performed by iterating through all values of w within a specified range and selecting that value which minimised the sum of the squared residuals (SSR).

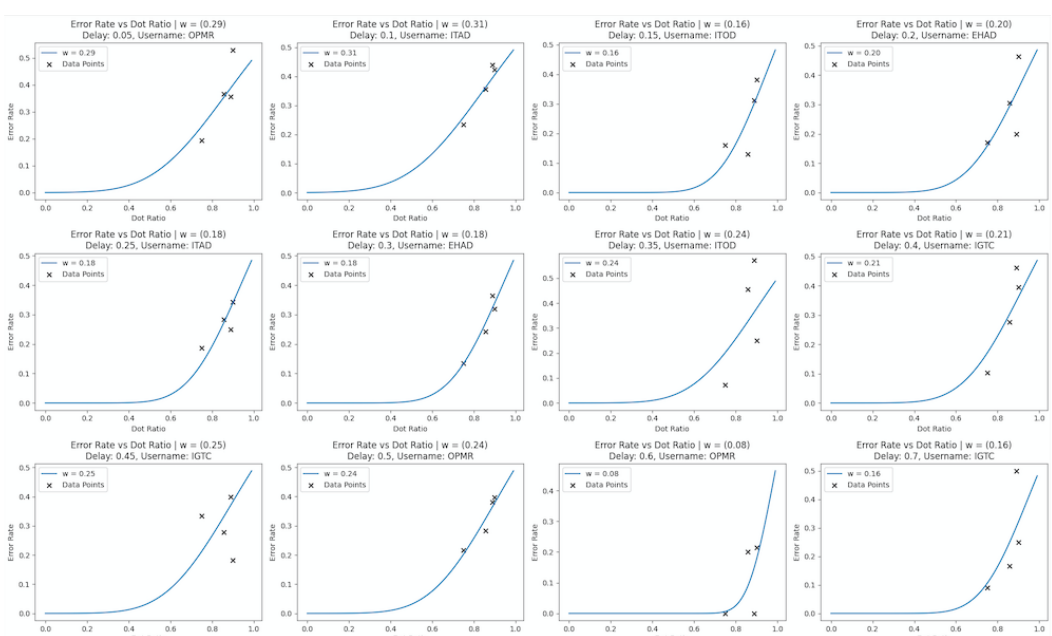
As such, w provides an individual measure of ANS performance, and can be used to calculate the accompanying expected standard deviation in an individual's guess distribution. This was performed by finding the minimum of two Gaussian distribution probability density functions, representing the value to be integrated over. We then applied numerical integration between the upper and lower bound, effectively calculating the area under the curve within the overlapping region. This procedure occurred iteratively, over a range of sigma values, such that the correct value was obtained when the total area under the overlapping region equalled 0.5 (If a guess is made in the range of overlap, there remains a 50% chance this guess will be correct and thus total error probability becomes 0.5^2 – our desired 25%.

In order to statistically define the extent of relationship between ANS performance and time, we fitted an ordinary least squares (OLS) regression on logarithmically transformed Weber fractions, and the time variable to which they related to. A power curve was chosen to fit this relationship given the likely parabolic relationship between increasing time and ANS error, wherein a time point is reached in which no further improvements can be made. This approach enabled us to model any potential multiplicative effects between variables, testing the hypothesis that that a relationship exists and if this is more than merely additive. The statistical significance of the generated model coefficients was determined using t-tests of significance level 0.05. The F-statistic was calculated to define the overall fit of the model.

As a final stage, we fitted a 3-Dimensional model from the w-calculated distribution functions, as a collective function of delay time. This was used to calculate the expected error rate, as half the area under the distribution overlaps, at any selected delay time and dot ratio.

3 Results

In total 37 participants completed the study, who each met the minimum threshold of 75 completed rounds for responses to be recorded. The time delays recorded ranged from 0.05 seconds to 4 seconds, covering a full distribution of delay intervals. Fitting of the error function to data from each of the 37 participants, yielded respective Weber fractions in the range of 0.069 to 0.31.



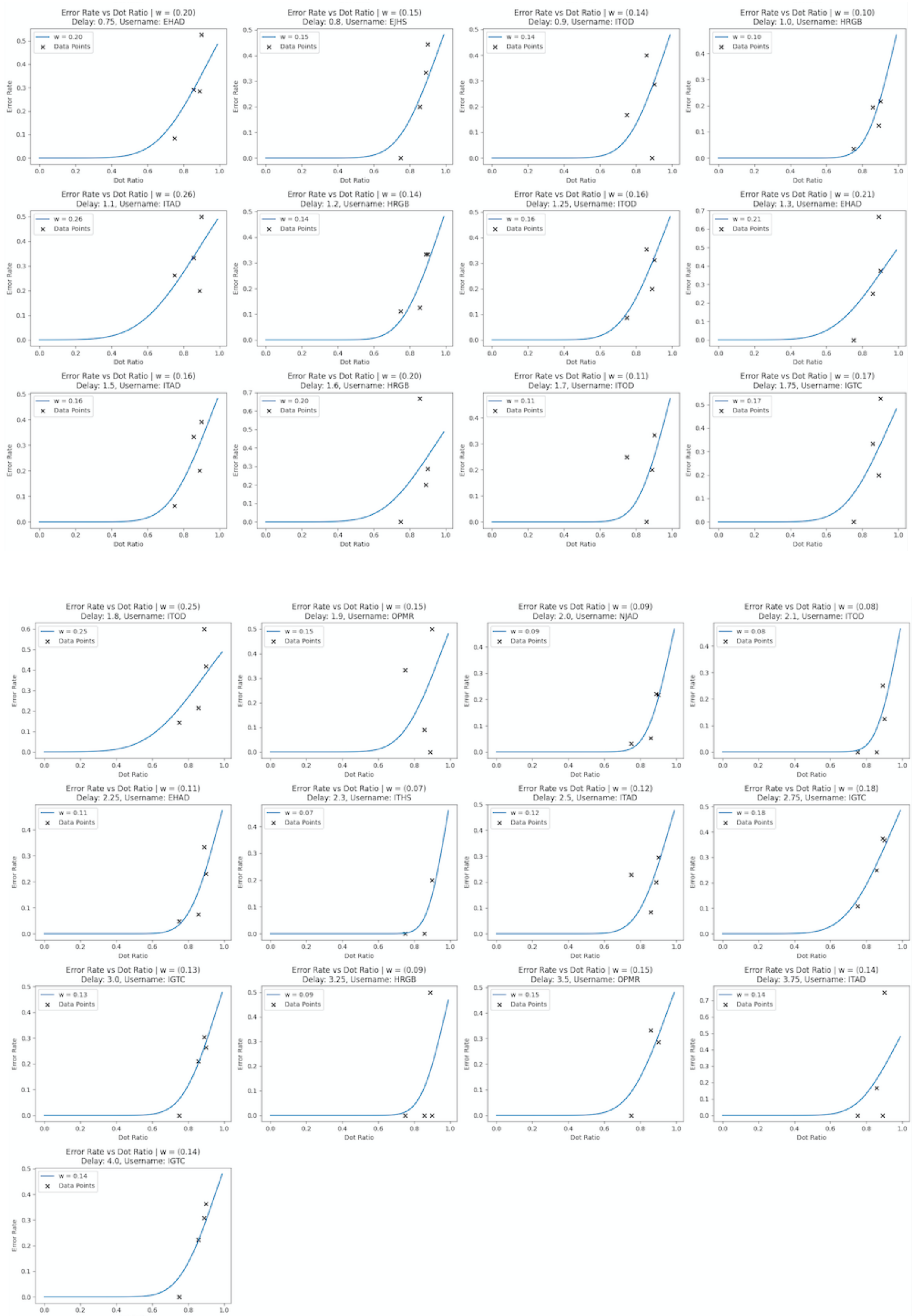


Figure 2: Fitted error functions, for each of 37 individuals with recorded responses, given the average user error rates at each dot ratio and a fixed delay value. Models were fitted by iterating through w values in the range of 0.01 to 1 and finding that which minimised SSR.

Subsequent OLS fitting to the logarithmically transformed Weber fractions and their corresponding delay values yielded a statistically significant relationship, summarised in the following table, with the resulting power curve visualised in Figure 3.

Table 1: Summary of Key Statistical Findings

Statistic	**Value**	**Interpretation**
R-squared	0.313	31.3% of variance in Y explained by the model.
Prob (F-statistic)	0.000 316	Evidence of a significant relationship.
coef (x_1)	-0.1965	Change in expected log transformed Weber fraction for a unit change in log transformed delay.
$P > \|t\|$ (x_1)	< 0.001	X's coefficient is statistically significant (< 0.05 threshold).

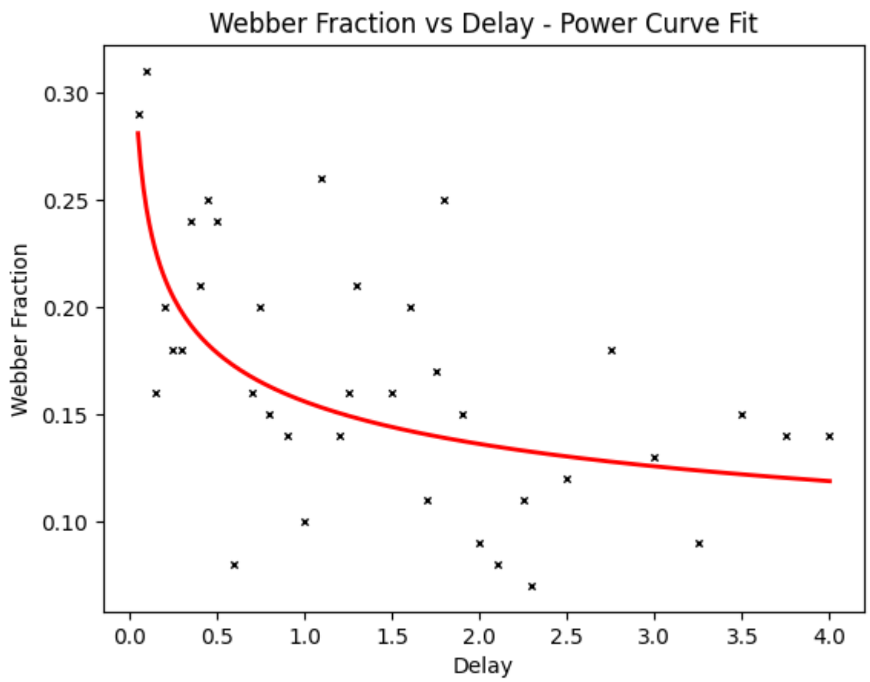
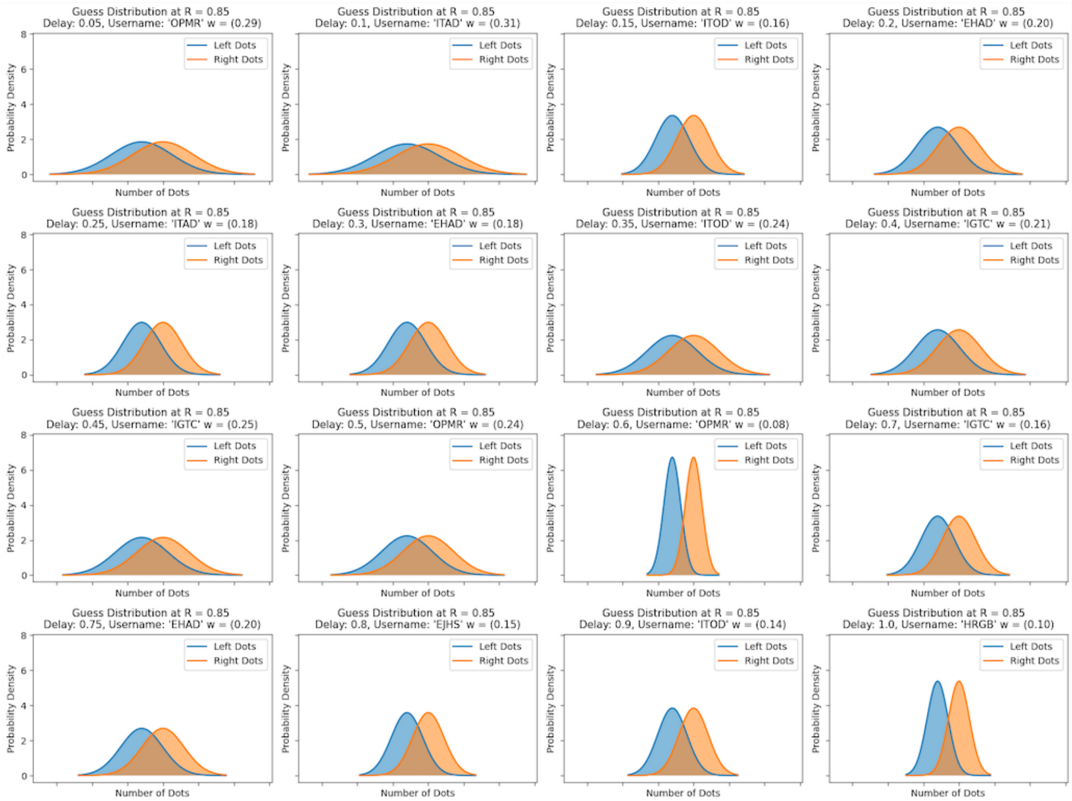
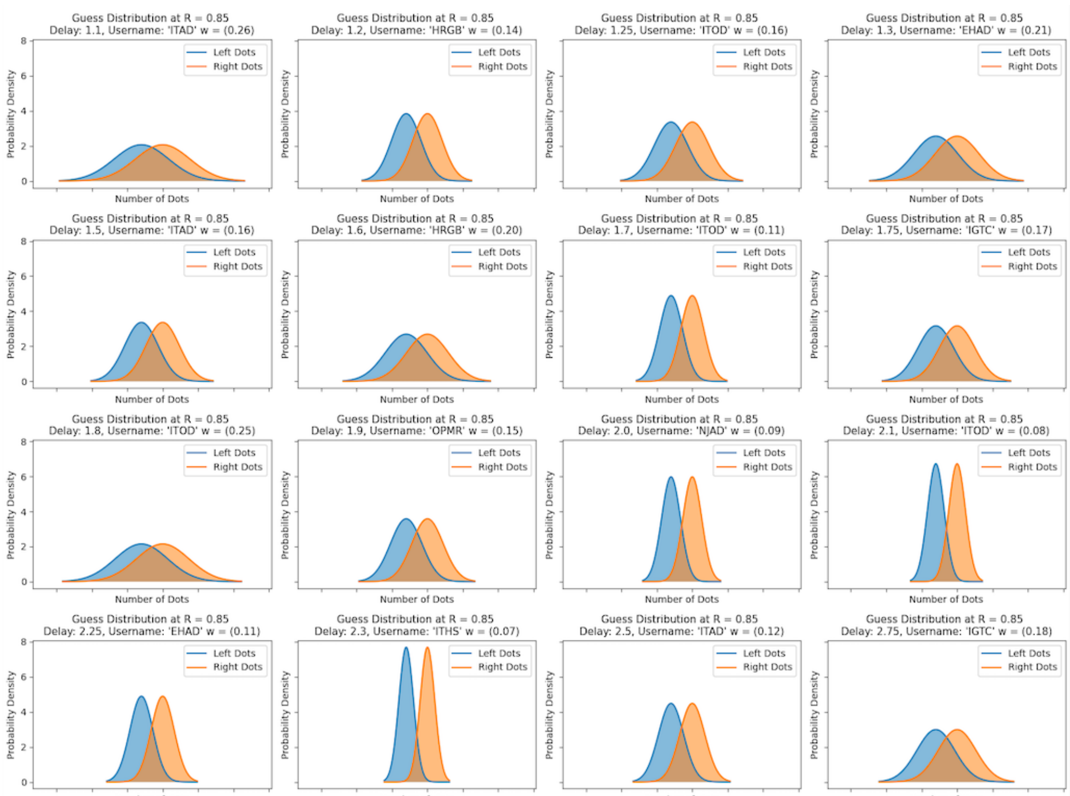


Figure 3: Fitted power curve for calculated Weber fractions vs delay time. The power curve was fitted to minimise the SSR of the logarithmically transformed data points.

Given the existence of a relationship, this was then modelled in both time and distribution space, by obtaining the corresponding standard deviation values for each given Weber fraction and its associated delay value (Figure 4.) A continuous set of paired Gaussian distributions were then obtained from the fitted power curve and can be visualised in Figure 5.


Guess Distribution at R = 0.85
Delay: 0.05, Username: 'OPMR' w = (0.29)
Guess Distribution at R = 0.85
Delay: 0.1, Username: 'ITAD' w = (0.31)
Guess Distribution at R = 0.85
Delay: 0.15, Username: 'ITOD' w = (0.16)
Guess Distribution at R = 0.85
Delay: 0.2, Username: 'EHAD' w = (0.20)
Guess Distribution at R = 0.85
Delay: 0.25, Username: 'ITAD' w = (0.18)
Guess Distribution at R = 0.85
Delay: 0.3, Username: 'EHAD' w = (0.18)
Guess Distribution at R = 0.85
Delay: 0.35, Username: 'ITOD' w = (0.24)
Guess Distribution at R = 0.85
Delay: 0.4, Username: 'IGTC' w = (0.21)
Guess Distribution at R = 0.85
Delay: 0.45, Username: 'IGTC' w = (0.25)
Guess Distribution at R = 0.85
Delay: 0.5, Username: 'OPMR' w = (0.24)
Guess Distribution at R = 0.85
Delay: 0.6, Username: 'OPMR' w = (0.08)
Guess Distribution at R = 0.85
Delay: 0.7, Username: 'IGTC' w = (0.16)
Guess Distribution at R = 0.85
Delay: 0.75, Username: 'EHAD' w = (0.20)
Guess Distribution at R = 0.85
Delay: 0.8, Username: 'EJHS' w = (0.15)
Guess Distribution at R = 0.85
Delay: 0.9, Username: 'ITOD' w = (0.14)
Guess Distribution at R = 0.85
Delay: 1.0, Username: 'HRGB' w = (0.10)
Left Dots
Right Dots
Probability Density
Number of Dots


Guess Distribution at R = 0.85
Delay: 1.1, Username: 'ITAD' w = (0.26)
Guess Distribution at R = 0.85
Delay: 1.2, Username: 'HRGB' w = (0.14)
Guess Distribution at R = 0.85
Delay: 1.25, Username: 'ITOD' w = (0.16)
Guess Distribution at R = 0.85
Delay: 1.3, Username: 'EHAD' w = (0.21)
Guess Distribution at R = 0.85
Delay: 1.5, Username: 'ITAD' w = (0.16)
Guess Distribution at R = 0.85
Delay: 1.6, Username: 'HRGB' w = (0.20)
Guess Distribution at R = 0.85
Delay: 1.7, Username: 'ITOD' w = (0.11)
Guess Distribution at R = 0.85
Delay: 1.75, Username: 'IGTC' w = (0.17)
Guess Distribution at R = 0.85
Delay: 1.8, Username: 'ITOD' w = (0.25)
Guess Distribution at R = 0.85
Delay: 1.9, Username: 'OPMR' w = (0.15)
Guess Distribution at R = 0.85
Delay: 2.0, Username: 'NJAD' w = (0.09)
Guess Distribution at R = 0.85
Delay: 2.1, Username: 'ITOD' w = (0.08)
Guess Distribution at R = 0.85
Delay: 2.25, Username: 'EHAD' w = (0.11)
Guess Distribution at R = 0.85
Delay: 2.3, Username: 'ITHS' w = (0.07)
Guess Distribution at R = 0.85
Delay: 2.5, Username: 'ITAD' w = (0.12)
Guess Distribution at R = 0.85
Delay: 2.75, Username: 'IGTC' w = (0.18)
Left Dots
Right Dots
Probability Density
Number of Dots

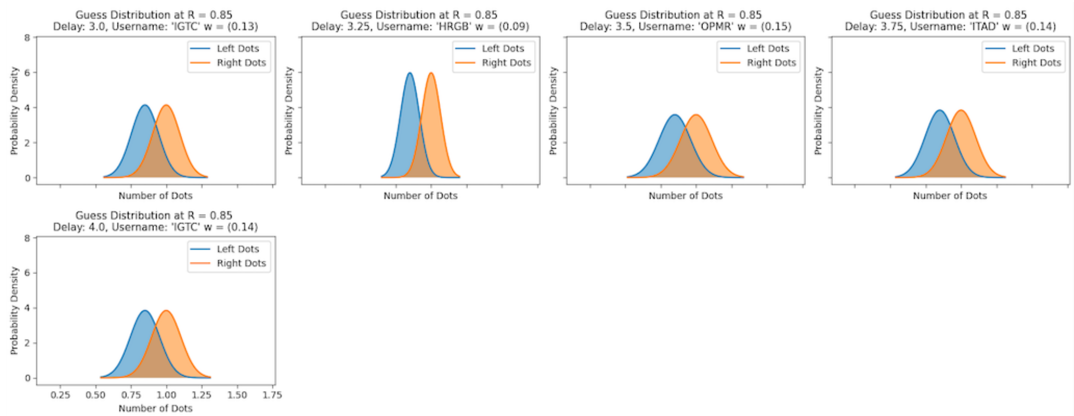


Figure 4: Paired Gaussian distributions at R = 0.85. Standard deviations for each respective user were first calculated by ensuring total area under the curve overlap equalled 0.5 at the dot ratio given by $1 - w$.

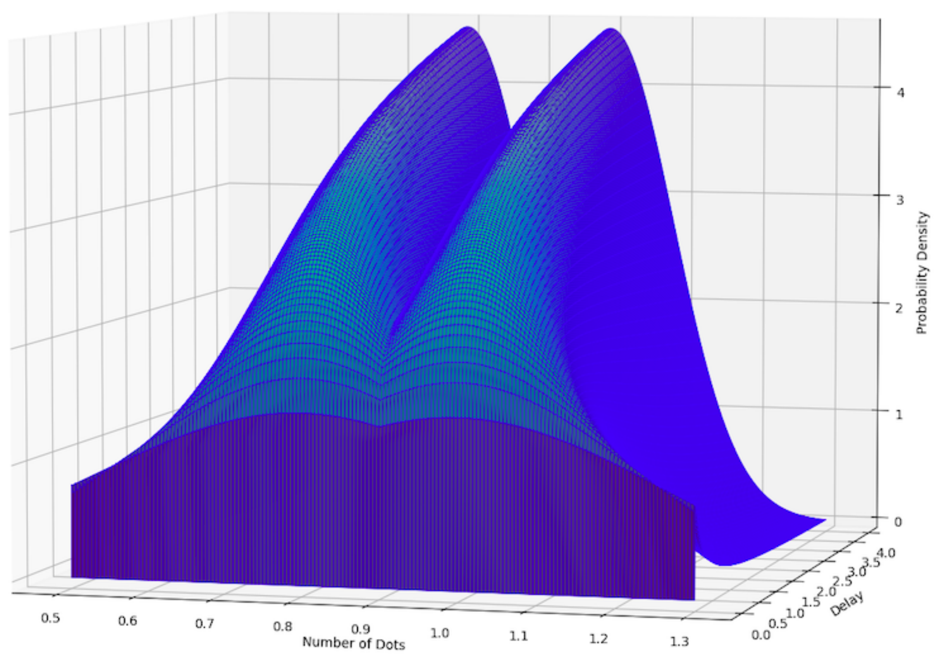


Figure 5: A continuous set of paired distributions, calculated by obtaining w values and corresponding standard deviations from the fitted power curve. Dot Ratio = 0.80.

Extending this visualisation further, we then plotted this function for a continuous set of dot ratios. Adjustment of the desired dot ratio, along with the incurred delay value, reveals an expected error rate, equalling half the area under the overlapping curves (Figure 6.)

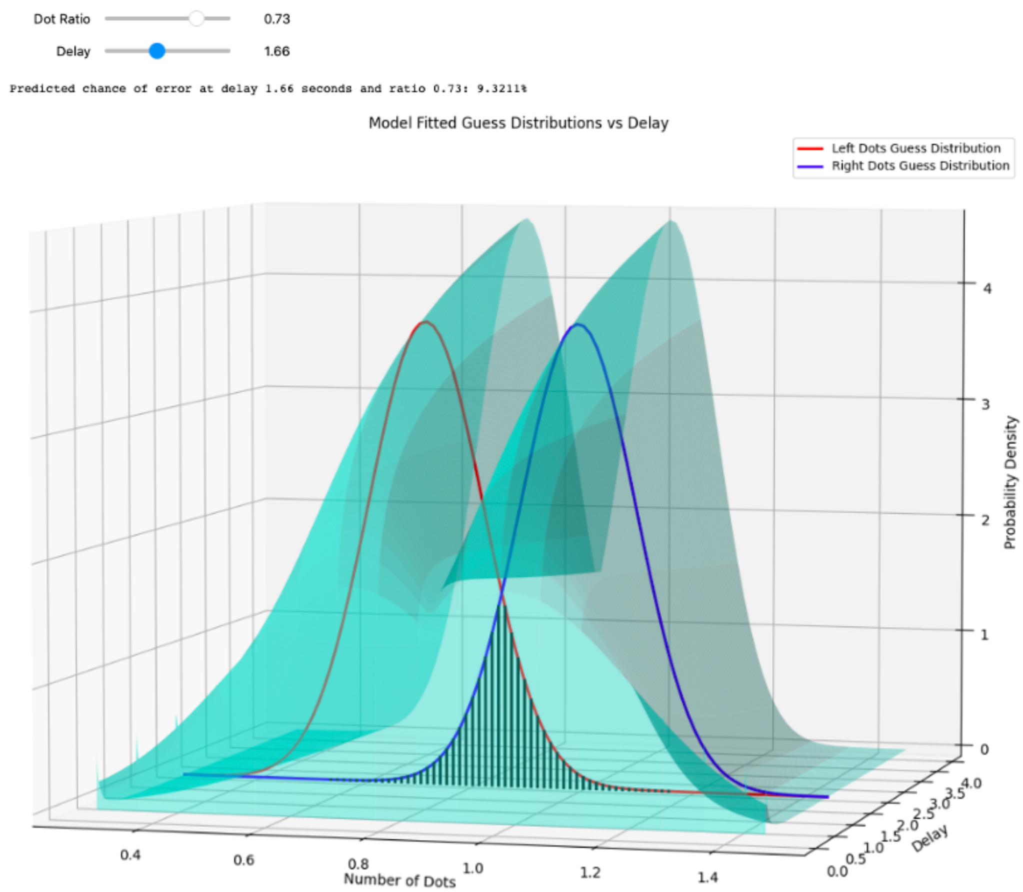


Figure 6: Visual tool for calculating the expected error as half the area under the overlapping curves, given a chosen dot ratio and delay period. Area under the overlapping curves is shown in black.

4 Discussion

These findings build upon the foundational work of Park et al. (2015) and others by providing a first insight into further factors which likely influence ANS ability (quantified by the Weber Fraction). Our analysis demonstrates the existence of clear scalar variability in ANS performance, as a result not only of innate ability of an individual, but also of an external time delay factor. The statistically significant relationship fitted by the OLS model, which explained 31.3% variance in Weber fractions resulting from changes in delay, underscores the impact of a temporal factor on ANS performance. The negative coefficient for the delay variable suggests that as the delay period increases, the success of intuitive numerical estimation improves, but up to a limit. This could be indicative of a switch from our embedded ANS ability, to our formal mathematical system of counting, wherein further increases in time are unlikely to yield gains.

As a result of this significant relationship, we were able to provide a general model for population estimated ANS performance across a continuous set of delay values and dot ratios. Given the previously discussed link between early ANS performance and learned mathematical ability, further investigation into the extent of adherence to the model by individuals with numerical cognition disorders, could not only help to consolidate the extent to this link, but it also provides potential for such a system to be used as a diagnostic tool.

That being said, there exists several limitations to our model. Primarily, although the sample size was sufficient to detect a significant relationship, it is not large enough to ensure the generalisability of our findings. Further research with a more diverse and larger sample could provide a more comprehensive understanding population wide ANS ability. Moreover, the necessity to manually proceed to the next round in our methodology, was designed to mitigate the effects of fatigue, but introduces an inconsistency variable, which may lead to further error. Future studies might explore automated progression with standardised rest periods to address this issue.

Overall, our investigation lays the framework for a higher dimensional model, which aims to account for a host of variable factors when estimating ANS performance. Discerning the precise influential effects of these numerous factors will not only provide new insights into innate quantitative ability but allow us to pinpoint the cognitive underpinnings that have propelled numerical reasoning throughout human and animal history. The implications of these findings in both educational and clinical settings will be insightful.

References

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Odic, D. & Starr, A. (2018), 'An introduction to the approximate number system', *Child Development Perspectives* **12**, 223–229.

Park, J. & Starns, J. J. (2015), 'The approximate number system acuity redefined: A diffusion model approach', *Frontiers in Psychology* **6**.